stochastic sediment transport modeling
annotated bibliography

Kevin Pierce (kevinkayaks@gmail.com) // University of British Columbia

---

This annotated bibliography is in preparation of a publication of a high resolution sediment transport dataset alongside a new sediment transport theory based upon Markov random process theory. As such the bibliography contains papers from three categories: (1) similar sediment transport experiments; (2) applied mathematics and computer vision papers relating to experiment analysis; and (3) papers on stochastic mathematics or sediment transport theory.

---

annotations:

[1] S. Abbott and S. Francis, "Saltation and suspension trajectories of solid grains in a water stream," *Royal Transations of the Royal Society of London A.*, vol. 294, no. 1321, pp. 225–254, 1977.

> This is the followup paper to Francis (1973) which is probably the earliest photography based experiment resolving sediment transport characteristics in a water stream. The authors take many thousands of photos to obtain positions, velocities, and accelerations of bed and wash load particles within a flume. They discriminate position and velocity components into modes of motion (as in rolling, sliding, suspended). Wide variation is noted, but the dataset is too small to access distributions of velocity or acceleration. This paper is significant, but much like Shields (1936) its significance has become entirely historical as its database contribution is completely overshadowed by modern datasets (see Heyman et al. (2016)).

[2] C. Ancey, P. Bohorquez, and E. Bardou, "Sediment Transport in Mountain Rivers," *ERCOFTAC Bulletin.*, vol. 100, pp. 37–52, 2014.

> "Gilbert (1914), which is credited with the first comprehensive experimental investigation into bed load transport in inclined flumes [5]." "The introduction of a finite-size volume in Eq. (3) leads to additional problems: how can we distinguish fluctuations that are intrinsic to the phenomenon and those that are induced by the average process?" "Planform structures (e.g., bars and meanders) as well as bed forms (e.g, rip- ples, dunes, steps and pools) have attracted considerable attention from geomorphologists and hydraulicians. " "The mainstream view is that bed structures arise from a loss of stability of the bed due to the coupling between the turbulent water stream, sediment trans- port, and bed topography [15]. The main difficulty is that depth-averaged equations such as the Saint-Venant- Exner equations (see 3.1)

are linearly stable for Froude numbers as large as 2. The calculation of bed form initiation and propagation then requires a more elaborate framework." The idea is to count the number of moving particles in a control volume or in an array of adjacent volumes of length x. In this paper they review the Markov model for number fluctuations, including their two methods of solution. First the master equation is written expressing transport/entrainment/deposition among and in an array of cells. Then they review two methods of solution. The first is to do a Kramer's Moyal expansion to obtain a Fokker Planck Equation. Unfortunately all terms are important: it's a strongly interacting situation. The second is to do a Poisson transformation. In this way they obtain the moments of the distribution. In summary it's somehow revealed to be a negative binomial distribution for the particle number. As collective entrianment is turned off it becomes a Poisson distribution– the same as the result of Einstein 1937. They further develop a Lagevin equation approach to find that the velocity distribution should be truncated Gaussian. They do not even nearly resolve an exponential distribution as was noted by Furbish. They note that number fluctuations drive sediment transport fluctuations, a point which is corroborated by Furbish and Radice. They convolve velocity and number distributions to obtain a probability distribution of the sediment transport rate (numerical computation). They note only a few other authors have done this. Einstein did not actually do this– Hamamori (1962) and Turowski (2010). They find poor agreement with their experiment on 6mm glass beads. They propose either (1) Velocities and Numbers are correlated, or (2) They did not consider multiple modes of motion, and rolling and saltating regimes move at much different paces. There is some other stuff I did not read. I did not really finish the paper as it got dry and long-winded.

[3] C. Ancey, P. Bohorquez, and J. Heyman, "Stochastic interpretation of the advection-diffusion," *Journal of Geophysical Research : Earth Surface*, vol. 120, pp. 325–345, 2015.

In this paper they use the Kramers-Moyal expansion to solve their Markov model over an array of cells with c = n/L the continuum concentration as L goes to 0. "This article is concerned with the microscopic foundation of the advection-diffusion equation with a source term. This is the best Ancey paper and deserves a careful read. They do the multidimensional Poisson transform here to solve their Markov equation on an array of cells."

[4] C. Ancey, A. C. Davison, T. Böhm, M. Jodeau, and P. Frey, "Entrainment and motion of coarse particles in a shallow water stream down a steep slope," *Journal of Fluid Mechanics*, vol. 595, no. 2008, pp. 83–114, 2008.

need to write the annote 4 dis

[5] C. Ancey and J. Heyman, "A microstructural approach to bed load transport: mean behaviour and fluctuations of particle transport rates," *Journal of Fluid Mechanics*, vol. 744, no. 2014, pp. 129–168, 2014.

Ancey obtains a macroscopic exner equation from a birth death equation. This exner equation is a linear advection diffusion equation with a source term. This paper introduces linked birth-death processes between cells and takes the limit as the width of cells goes to zero to obtain advection diffusion formulation of sediment transport.

Number fluctuations in the ith cell are considered the result of four factors: (1) transport into the cell from its neighboring upstream cell; (2) individual or collective entrainment within the cell; (3) deposition within the cell; and (4) migration out of the cell to the neighboring cell downstream.

[6] C. Ancey, "Stochastic modeling in sediment dynamics: Exner equation for planar bed incipient bed load transport conditions," *Journal of Geophysical Research: Earth Surface*, vol. 115, no. F2, pp. n/a–n/a, 2010.

need to write the annote 4 dis

[7] C. Ancey, T. Böhm, M. Jodeau, and P. Frey, "Statistical description of sediment transport experiments," *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics*, vol. 74, no. 1, pp. 1–14, 2006.

need to write the annote 4 dis

[8] Anderson P.W., "More is Different," pp. 393–396, 1972.

need to write the annote 4 dis

[9] C. Arteta, V. Lempitsky, and A. Zisserman, "Interactive Object Counting," *ECCV*, vol. 8691, 2014.

Authors pursue object counting through ridge regression. The target is a field of 2d Kronecker deltas at the point annotation of each object. The output is a 2d density map. There is no requirement of object segmentation: point annotations are sufficient. Integrals of the density map give object counts. This is a conventional algorithm which does not use neural learning.

[10] F. Ballio, D. Pokrajac, A. Radice, and S. Hosseini Sadabadi, "Lagrangian and Eulerian Description of Bed Load Transport," *Journal of Geophysical Research: Earth Surface*, pp. 384–408, 2018.

need to write the annote 4 dis

[11] F. Ballio, V. Nikora, and S. E. Coleman, "On the definition of solid discharge in hydro-environment research and applications," *Journal of Hydraulic Research*, vol. 52, no. 2, pp. 173–184, 2014.

need to write the annote 4 dis

[12] T. Böhm, C. Ancey, P. Frey, J. L. Reboud, and C. Ducottet, "Fluctuations of the solid discharge of gravity-driven particle flows in a turbulent stream," *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics*, vol. 69, no. 6 1, pp. 1–13, 2004.

need to write the annote 4 dis

[13] J. Campagnol, A. Radice, R. Nokes, V. Bulankina, A. Lescova, and F. Ballio, "Lagrangian analysis of bed-load sediment motion: Database contribution," *Journal of Hydraulic Research*, vol. 51, no. 5, pp. 589–596, 2013.

need to write the annote 4 dis

[14] E. Cand‘es and J. Romberg, “Practical Signal Recovery from Random Projections,” *IS&T/SPIE's 17th Annual Symposium on Electronic Imaging*, vol. 2291, no. 626, p. 18, 2005.

need to write the annote 4 dis

[15] F. Charru, H. Mouilleron, and O. Eiff, “Erosion and deposition of particles on a bed sheared by a viscous flow,” *Journal of Fluid Mechanics*, vol. 519, no. 2004, pp. 55–80, 2004.

need to write the annote 4 dis

[16] J. Crocker and D. Grier, “Methods of Digital Video Microscopy for Colloidal Studies,” *Journal of Colloid and Interface Science*, vol. 179, no. 1, pp. 298–310, 1996.

need to write the annote 4 dis

[17] T. G. Drake, R. L. Shreve, W. E. Dietrich, P. J. Whiting, and L. B. Leopold, “Bedload transport of fine gravel observed by motion-picture photography,” *Journal of Fluid Mechanics*, vol. 192, no. 1988, p. 193, 1988.

need to write the annote 4 dis

[18] H. A. Einstein, “Bedload transport as a probability problem,” Ph.D. dissertation, Wasserbau Eidg. Tech. , Hoschsch, Zurich, 1937.

Einstein seeded tracer particles into an artificial flume and observed their movement character- istics. He interpreted particle transport as a switching process between two phases: the first is a resting phase, and the second is a movement phase. The parameter relevant to the resting phase is the time spent there, the resting time. The parameter relevant to the movement phase is the distance covered within it, the particle hop length. Time spent in motion was considered irrelevant, since the time spent in motion is very short relative to resting time. Within this formulation, the travel distance of a tracer particle in a time interval emerges from considering the phase to phase switching, in which one accounts for the total of all hop distances and the total of all resting times through the switching which occurred in the time interval.

[19] R. Ettema and C. F. Mutel, “Hans Albert Einstein: Innovation and Compromise in Formulating Sediment Transport by Rivers,” *Journal of Hydraulic Engineering*, vol. 130, no. 6, pp. 477–487, 2004.

need to write the annote 4 dis

[20] S. Fathel, D. Furbish, and M. Schmeeckle, “Parsing anomalous versus normal diffusive behavior of bedload sediment particles,” *Earth Surface Processes and Landforms*, vol. 41, no. 12, pp. 1797–1803, 2016.

need to write the annote 4 dis

[21] S. L. Fathel, D. J. Furbish, and M. W. Schmeeckle, “Experimental evidence of statistical ensemble behavior in bed load sediment transport,” *Journal of Geophysical Research F: Earth Surface*, vol. 120, no. 11, pp. 2298–2317, 2015.

need to write the annote 4 dis

[22] D. J. Furbish, A. E. Ball, and M. W. Schmeeckle, "A probabilistic description of the bed load sediment flux: 4. Fickian diffusion at low transport rates," *Journal of Geophysical Research: Earth Surface*, vol. 117, no. 3, pp. 1–13, 2012.

need to write the annote 4 dis

[23] D. J. Furbish, S. L. Fathel, and M. W. Schmeeckle, "Particle Motions and Bed Load Theory : The Entrainment Forms of the Flux and the Exner Equation," in *Gravel-Bed Rivers: Processes and Disasters*, 1st ed., D. Tsutsumi and J. B. Laronne, Eds. John Wiley & Sons Ltd., 2017, ch. 4, pp. 97–120.

need to write the annote 4 dis

[24] D. J. Furbish, S. L. Fathel, M. W. Schmeeckle, D. J. Jerolmack, and R. Schumer, "The elements and richness of particle diffusion during sediment transport at small timescales," *Earth Surface Processes and Landforms*, vol. 42, no. 1, pp. 214–237, 2017.

need to write the annote 4 dis

[25] D. J. Furbish, P. K. Haff, J. C. Roseberry, and M. W. Schmeeckle, "A probabilistic description of the bed load sediment flux: 1. Theory," *Journal of Geophysical Research: Earth Surface*, vol. 117, no. 3, 2012.

need to write the annote 4 dis

[26] D. J. Furbish, J. C. Roseberry, and M. W. Schmeeckle, "A probabilistic description of the bed load sediment flux: 3. the particle velocity distribution and the diffusive flux," *Journal of Geophysical Research: Earth Surface*, vol. 117, no. 3, 2012.

need to write the annote 4 dis

[27] D. J. Furbish and M. W. Schmeeckle, "A probabilistic derivation of the exponential-like distribution of bed load particle velocities," *Water Resources Research*, vol. 49, no. 3, pp. 1537–1551, 2013.

need to write the annote 4 dis

[28] D. J. Furbish, M. W. Schmeeckle, R. Schumer, and S. L. Fathel, "Probability distributions of bed load particle velocities, accelerations, hop distances, and travel times informed by Jaynes's principle of maximum entropy," *Journal of Geophysical Research: Earth Surface*, vol. 121, no. 7, pp. 1373–1390, 2016.

need to write the annote 4 dis

[29] M. A. Hassan and D. N. Bradley, "Geomorphic controls on tracer particle dispersion in gravel bed rivers," *Gravel-Bed Rivers 8*, vol. 30, no. September, pp. 3–9, 2015.

need it

[30] M. A. Hassan, M. Church, and A. P. Schick, "Distance of movement of coarse particles in gravel bed streams," *Water Resources Research*, vol. 27, no. 4, pp. 503–511, 1991.

need to write the annote 4 dis

[31] J. Heyman, P. Bohórquez, and C. Ancey, "Exploring the physics of sediment transport in non-uniform super-critical flows through a large dataset of particle trajectories," *J. Geophys. Res:Earth Surf.*, vol. submitted, 2015.

need to write the annote 4 dis

[32] J. Heyman, P. Bohorquez, and C. Ancey, "Entrainment, motion, and deposition of coarse particles transported by water over a sloping mobile bed," *Journal of Geophysical Research: Earth Surface*, vol. 121, no. 10, pp. 1931–1952, 2016.

There is a very nice outline here of the way different approaches to stochastic sediment transport theories have arrived at the same advection diffusion formulation (as in Parker or Ancey et al 2014). This is used to introduce an overarching problem termed 'lack of closure' to stochastic models. Closure is the relationship between the parameters required by advection diffusion forumations of sediment transport rate to measurable channel parameters. In this paper Heyman presents an experiment where particle motion was tracked from the side. Median background subtraction was employed to get particle velocity and acceleration distributions. There are a few unique contributions such as average particle velocity as a function of particle elevation. Two methods are described for calculating particle diffusivity. Analysis of particle properties seems to follow something similar to Ballio et al 2018, involving Gaussian smoothing kernels.

[33] J. Heyman, H. B. Ma, F. Mettra, and C. Ancey, "Spatial correlations in bed load transport: Evidence, importance, and modeling," *Journal of Geophysical Research: Earth Surface*, vol. 119, no. 8, pp. 1751–1767, 2014. [Online]. Available: http://doi.wiley.com/10.1002/2013JF003003

need to write the annote 4 dis

[34] ——, "Spatial correlations in bed load transport : Evidence , importance , and modeling," *Journal of Geophysical Research : Earth Surface*, pp. 1751–1767, 2014.

need to write the annote 4 dis

[35] J. Heyman, F. Mettra, H. B. Ma, and C. Ancey, "Statistics of bedload transport over steep slopes: Separation of time scales and collective motion," *Geophysical Research Letters*, vol. 40, no. 1, pp. 128–133, 2013.

need to write the annote 4 dis

[36] T. B. Hoey, "Temporal variations in bedload transport in rates and sediment storage," *Progress in Physical Geography*, vol. 16, no. 3, pp. 319–338, 1992.

need to write the annote 4 dis

[37] A. A. Kalinske, "Movement of sediment as bed load in rivers," *Eos, Transactions American Geophysical Union*, vol. 28, no. 4, pp. 615–620, 1947.

need to write the annote 4 dis

[38] H. Kuhn, “The Hungarian Method for the Assignment Problem,” *Naval Research Logistics Quarterly*, vol. 2, no. 1, pp. 83–97, 1955.

This is the classic paper introducing the Hungarian method for the assignment problem. The assignment problem is the cheapest way to assign workers to jobs provided each worker has a specific cost to compete a specific job.

[39] E. Lajeunesse, L. Malverti, and F. Charru, “Bed load transport in turbulent flow at the grain scale: Experiments and modeling,” *Journal of Geophysical Research: Earth Surface*, vol. 115, no. 4, 2010.

need to write the annote 4 dis

[40] E. Lajeunesse, O. Devauchelle, and F. James, “Advection and dispersion of bedload tracers,” no. November, pp. 1–19, 2017.

This is a nice physics-like paper. They take the Charru 2004 model for advection of bedload tracers and solve it both numerically and exactly in asympototic regimes. They note super-super diffusion (to use G Parker’s term) at short timescales and Brownian diffusion (I think) at long timescales. They analyze scaling behavior of mean tracer position, variance of tracer position, and skew of tracer position, and find two scaling regimes in each. These analyses all applied for steady transport conditions. They extended their analysis to unsteady flow conditions as one finds in nature by incorporating an intermittency factor which rescales time using the fraction of above-threshold flows. They also eliminate time from the scaling equations on variance and skew, expressing variance as a function of travel distance. ”According to this model, it should be possible to estimate the particle flight length and the average bed load transport rate from the evo- lution of the variance and the skewness of a plume of tracers in a river.”

[41] H. Ma, J. Heyman, X. Fu, F. Mettra, C. Ancey, and G. Parker, “Bed load transport over a broad range of timescales: Determination of three regimes of fluctuations,” *Journal of Geophysical Research: Earth Surface*, vol. 119, no. 12, 2014.

need to write the annote 4 dis

[42] P. A. Nelson, D. Bellugi, and W. E. Dietrich, “Delineation of river bed-surface patches by clustering high-resolution spatial grain size data,” *Geomorphology*, vol. 205, pp. 102–119, 2014.

need to write the annote 4 dis

[43] M. Pietikäinen, M. Turk, L. Wang, G. Zhao, and L. Cheng, “Machine learning in motion analysis: New advances,” *Image and Vision Computing*, vol. 31, no. 6-7, pp. 419–420, 2013.

need to write the annote 4 dis

[44] A. Radice, S. Malavasi, and F. Bailio, “Solid transport measurements through image processing,” *Experiments in Fluids*, vol. 41, no. 5, pp. 721–734, 2006.

need to write the annote 4 dis

[45] A. Radice, F. Ballio, and V. Nikora, “On statistical properties of bed load sediment concentration,” *Water Resources Research*, vol. 45, no. 6, pp. 1–8, 2009.

This is a long duration video observation of bedload transport. A nice quote from the introduction: many empirical and phenomenological formulations for predicting solid discharge in open channels have been proposed. However, the estimates of the solid transport rate using these conventional equations proved to be highly uncertain [e.g., Gomez and Church, 1989; Martin, 2003], leading to increased attention to the small-scale details of sediment transport, which could help in advancing the existing relationships.” They also note several observations that bedload velocity is only weakly dependent on shear stress conditions at the bed, meaning activity or concentration fluctuations drive sediment transport fluctuations. This is in agreement with Ancey 2008. They measure water flow in 1D with ADV. They find an inertial range with -5/3 exponent. They find scaling behavior in sediment concentration fluctuations: exponent -5/3 was obtained characteristic of the fluid and concentration profiles. The experiment observed dyed tracer grains from above at low transport stages with no sediment recirculation.

[46] J. C. Roseberry, M. W. Schmeeckle, and D. J. Furbish, “A probabilistic description of the bed load sediment flux: 2. Particle activity and motions,” *Journal of Geophysical Research: Earth Surface*, vol. 117, no. 3, 2012.

need to write the annote 4 dis

[47] M. Saletti, P. Molnar, A. E. Zimmermann, M. A. Hassan, and M. Church, “Temporal variability and memory in sediment transport in an experimental step-pool channel,” *Water Resour. Res.*, vol. 51, pp. 1649–1670, 2015.

need to write the annote 4 dis

[48] W. Sayre and D. Hubbell, “Transport and Dispersion of Labeled Bed Material North Loup River , Nebraska,” *Transport of Radionuclides by Streams*, 1965.

need to write the annote 4 dis

[49] M. W. Schmeeckle, J. M. Nelson, and R. L. Shreve, “Forces on stationary particles in near-bed turbulent flows,” *Journal of Geophysical Research: Earth Surface*, vol. 112, no. 2, pp. 1–21, 2007.

need to write the annote 4 dis

[50] J. Schmidhuber, “Deep Learning in neural networks: An overview,” *Neural Networks*, vol. 61, pp. 85–117, 2015.

need to write the annote 4 dis

[51] G. Seizilles, E. Lajeunesse, O. Devauchelle, and M. Bak, “Cross-stream diffusion in bedload transport,” *Physics of Fluids*, vol. 26, no. 1, 2014.

need to write the annote 4 dis

[52] A. Singh, K. Fienberg, D. J. Jerolmack, J. Marr, and E. Foufoula-Georgiou, "Experimental evidence for statistical scaling and intermittency in sediment transport rates," *Journal of Geophysical Research: Earth Surface*, vol. 114, no. 1, pp. 1–16, 2009.

This is one of two papers I know of which measure a time series of bed elevations within a location in a flume experiment.

[53] B. M. Sumer, L. H. C. Chua, N.-S. Cheng, and J. Fredsøe, "Influence of Turbulence on Bed Load Sediment Transport," *Journal of Hydraulic Engineering*, vol. 129, no. 8, pp. 585–596, 2003.

need to write the annote 4 dis

[54] J. G. Venditti, P. A. Nelson, R. W. Bradley, D. Haught, and A. B. Gitto, "Bedforms, Structures, Patches, and Sediment Supply in Gravel-Bed Rivers," *Gravel-Bed Rivers*, pp. 439–466, 2017.

need to write the annote 4 dis

[55] H. Voepel, R. Schumer, and M. A. Hassan, "Sediment residence time distributions: Theory and application from bed elevation measurements," *Journal of Geophysical Research: Earth Surface*, vol. 118, no. 4, pp. 2557–2567, 2013.

need to write the annote 4 dis

[56] Y. Xue, N. Ray, J. Hugh, and G. Bigras, "A novel framework to integrate convolutional neural network with compressed sensing for cell detection," *IEEE Transations on Image Processing*, pp. 2319–2323, 2017.

The authors use a convolutional neural network which takes in an image containing objects and gives out a vector. This vector is a compressed representation of the pixel level positions of objects within the image. The vector is formed by multiplying the sparse array encoding pixel level positions by a Gaussian random matrix. Compressed sensing theory provides a way to recover the encoded signal with high accuracy using L1 optimization.