Stochastic birth-death model of heavy-tailed fluvial sediment resting times

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Key Points:

- We model fluvial bedload activity and local bed elevation as a two-species stochastic birth-death process.
- Resulting timescales of sediment storage by burial lie on heavy-tailed power-law distributions.
- Our model predicts that tracer particles will slow down indefinitely as their lifetime in the channel increases.

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Abstract

A consensus has formed that fluvial sediment expresses heavy-tailed statistical distributions of resting times due to the effect of sediment burial on its mobility, and this has key implications for the diffusion of sediment and evacuation of contaminants from river channels. Owing to observational difficulties, only a handful of experiments have resolved these heavy-tailed resting time distributions, and there have been few theoretical attempts to build understanding. These studies do not agree on the form or properties of resting time distributions. In this work, we develop a new stochastic theory which jointly describes bedload transport and bed elevation changes, and we derive resting time distributions for sediment undergoing burial in terms of the joint dynamics. Our theory corroborates a subset of existing experimental and theoretical work, expressing resting time distributions for sediment undergoing burial with heavy power-law tails, implying super-diffusion of bedload. Our key finding is that the largest bedload resting times are controlled not only by the erosion and deposition rates, as suggested by earlier studies, but also by the scaling of erosion and deposition rates with local changes in bed elevation.

1 Introduction

The classic studies of bedload transport in rivers try to link the bulk downstream flux of sediment to characteristics of the hydraulic forcing (e.g. Yalin, 1972; **?**). Although bulk fluxes are often described using deterministic models (Wilcock & Crowe, 2003, e.g.), the individual motions constituting them are more like a random walk, being an alternating sequence of motions and rests which are best characterized statistically (e.g. H. A. Einstein, 1937). The statistical properties of individual motions generate differences in the virtual velocities of grains, and these differences imply diffusion, or a spreading out of grains as they transport downstream (H. A. Einstein, 1937; Nakagawa & Tsujimoto, 1976; Yano, 1969). Diffusion is important in applications where the downstream transport of only a subset of grains is of interest, as in contaminant evacuation (Malmon, Reneau, Dunne, Katzman, & Drakos, 2005), river restoration (Gaeuman, Stewart, Schmandt, & Pryor, 2017), and bed texture adjustment (Hassan & Bradley, 2017), highlighting bedload diffusion as an important topic for study.

In the classic diffusion problems, such as the Brownian motion of pollen grains in water (e.g. A. Einstein, 1905), the variance of particle positions σ_x^2 scales with time t as $\sigma_x^2 \propto t$, a rate of spreading called Fickian or normal diffusion. Bedload diffusion is more nuanced. The nature of bedload diffusion is controlled by the probability distributions of step length and resting time. If either of these distributions has a heavy-tail, the diffusion is called anomalous instead of normal. A heavy-tailed step length or resting time distribution has an exceedance distribution $P(X > x) \sim x^{-\alpha}$ with tail parameter $\alpha < 2$. Anomalous diffusion is characterized by a non-linear scaling of the variance of particle position with time, as in $\sigma_x^2 \propto t^\gamma$ with $\gamma \neq 1$. In this expression, $\gamma < 1$ is called sub-diffusion and $\gamma > 1$ is called super-diffusion. In a strongly assymmetric random walk as in bedload transport, heavy-tailed step length distributions imply super-diffusion, while heavy-tailed resting time distributions imply either super or sub-diffusion depending on the value of the tail parameter α (Weeks & Swinney, 1998).

A consensus has formed that step lengths in gravel-bed rivers are thin-tailed (Bradley, 2017; Hassan, Voepel, Schumer, Parker, & Fraccarollo, 2013) while bedload diffusion can be anomalous (Bradley, Tucker, & Benson, 2010; Fan, Xie, & Nie, 2017; Martin, Jerolmack, & Schumer, 2012; Nikora, 2002; Phillips, Martin, & Jerolmack, 2013; Zhang, Meerschaert, & Packman, 2012), suggesting heavy-tailed resting times are the source of anomalous diffusion (Bradley, 2017; Martin et al., 2012). This perspective highlights the mechanism of heavy-tailed resting times as an important consideration. A dominant hypothesis is that heavy-tailed resting times stem from sediment burial (e.g. Bradley, 2017; Martin, Purohit, & Jerolmack, 2014; Voepel, Schumer, & Hassan, 2013). Conceptually, when grains rest on the bed surface, material transported from upstream can deposit over top of them, burying them away from the flow and preventing their entrainment until the overlying material is removed, increasing sediment resting times and imparting a heavy tail to the distribution.

Although a handful of field studies have resolved heavy-tailed resting times (Bradley, 2017; Olinde & Johnson, 2015; Pretzlav, 2016; Voepel et al., 2013), these do not resolve sediment burial and exhumation during floods, so they cannot discern whether burial is the mechanism generating heavy-tailed resting times. To our knowledge, only Voepel et al. (2013) and Martin et al. (2014) have provided direct support for the hypothesis that burial generates heavy-tailed resting times, although details of these works are inconsistent with one another. Martin et al. (2014) performed experiments in a narrow flume configured so grains formed a single layer in the cross-stream direction, providing direct resolution of sediment burial. Across a range of flow conditions, they obtained heavy-tailed power-law resting time distributions with a tail parameter $\alpha \approx 1$ which they directly attributed to sediment burial. Different flow conditions shifted the resting time above which the asymptotic power-law scaling applied, but did not obviously change the value of α. Using a bed activity timescale related to the rate of entrainment or deposition events, they managed to partially collapse their experimental distributions.

The theoretical component of the Martin et al. (2014) work is similar to that of Voepel et al. (2013). Both of these models treat local bed elevations as a bounded random walk, where the bed elevation steps randomly up or down with some probability, although they differ in that Voepel et al. (2013) introduces upper and lower bounds to the local bed elevation with hard reflecting barriers, while Martin et al. (2014) introduces them with a softer mean-reverting tendency. Both of these models are formally similar to classic stochastic theories of the resting times of sand grains on a duned bed (e.g. Nakagawa & Tsujimoto, 1980; Yang & Sayre, 1971). Within these models, the timescales of sediment burial are interpreted as the return time from above in the bed elevation time-series. From their assumptions, Voepel et al. (2013) conclude the resting time distribution should be a so-called tempered Pareto distribution, with an initial power-law decay which transitions to take on a thin-tailed exponential scaling (e.g.), and they note agreement between their predictions and the field data of Habersack (2001). In contrast, Martin et al. (2014) predicted a heavy-tailed power-law distribution of resting times which does not temper to an exponential distribution, and they successfully calibrated this model to their extensive set of flume experiments.

We believe the differences between these models deserve further theoretical clarification, and we note that although bedload transport is the underlying process driving changes in bed elevation, neither of these models directly incorporate it. In this paper, we leverage existing knowledge about bedload transport to study the resting times of coarse sediment under burial. We extend the stochastic bedload transport theory of Ancey, Davison, Böhm, Jodeau, and Frey (2008) to include bed elevation changes, and we use this model to compute the resting time distribution for sediment undergoing burial. From this theory, we

2 Theory

We define a control volume of downstream length Δx which contains some number n of moving particles within the water flow and some number m of stationary particles composing the bed at some instant. For simplicity we consider that all particles are approximately spherical with the same diameter $2a$, so that their mobility and packing characteristics are similar from one particle to the next. Within the control volume, we follow Ancey et al. (2008) and prescribe events which can occur from one instant to the next to modify the populations n and m, and we characterize these events using prob-