On the correlation structure of continuous and discrete point rainfall

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[1] This paper proposes new analytical results based on the theory of stochastic processes that establish a theoretical dependence of point rainfall variance on the scale of observation under somewhat general assumptions. It is shown that power law scaling, commonly assumed by analyses and models of point rainfall, cannot hold over all scales of observation and that an "inner" regime exists for small sampling intervals where the variance is a quadratic function of interval length. A power law scaling regime is shown to exist for larger aggregation intervals, whose characteristics depend on the memory of the continuous rainfall process. The presence of a "transition" regime between the inner and the scaling regime is also shown and may explain the deviations from a power law scaling behavior in observed rainfall reported in the literature. Furthermore, the application of the theory to rainfall data from a representative variety of climate types shows that fractal and multifractal analysis techniques and models may indeed fail to capture observed rainfall properties over wide ranges of aggregation scales. Finally, correlation structures with "short-" and "long-"term memory are considered and different theoretical relationships expressing rainfall variance as a function of aggregation interval are derived and tested against rainfall observed at different locations with aggregation timescales ranging from 15 min to 2 days. The theoretical and observational analyses show that both finite and infinite memory rainfall processes may be observed in nature. *INDEX TERMS:* 1854 Hydrology: Precipitation (3354); 1869 Hydrology: Stochastic processes; *KEYWORDS:* precipitation, stochastic processes

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1. Introduction

[2] The statistical properties of point rainfall have been extensively studied, both theoretically and observationally, for the past 40 years by numerous researchers [*LeCam*, 1961; *Todorovic and Yevjevich*, 1969; *Zawadzki*, 1973; *Waymire and Gupta*, 1981]. Most of the early literature is focused on the study of rainfall aggregated at a single timescale of applicative interest: from the hourly [e.g., *Rodriguez-Iturbe et al.*, 1987, 1988] to the daily [e.g., *Smith*, 1987; *Todorovic and Woolisher*, 1974] to the climatic [e.g., *Pelletier and Turcotte*, 1997] timescales.

[3] More recent literature [e.g., *Lovejoy and Mandelbrot*, 1985; *Lovejoy and Schertzer*, 1985; *Burlando and Rosso*, 1996; *Onof et al.*, 1996; *Veneziano et al.*, 1996; *Rodriguez-Iturbe et al.*, 1998] has addressed the relationship between the statistical characters of rainfall measured at relatively long aggregation scales and the properties observed at shorter aggregation scales. This is clearly an interesting theoretical problem as it bears important conceptual and applicative consequences, like the possibility of obtaining information on rainfall at the hourly timescale, which is often necessary for hydrologic applications, from daily rainfall, for which long historical records and a wide measurement network exist.

[4] Current studies on the statistical links between rainfall properties at different sampling scales are usually conducted through fractal or multifractal frameworks [*Lovejoy and Mandelbrot*, 1985; *Veneziano et al.*, 1996; *Menabde et al.*, 1997], which postulate the absence of characteristic scales of variability, e.g., implying a power law scaling structure of rainfall statistical moments. This approach has proven to be fruitful and allowed the observation of scale-free behavior in a number of rainfall time series and over a significant range of scales [e.g., *Tessier et al.*, 1993; *Burlando and Rosso*, 1996; *Veneziano et al.*, 1996]. However, deviations from power law scaling in statistical moments have also been reported [e.g., *Olsson*, 1995; *De Lima and Grasman*, 1999; *Olsson and Burlando*, 2002] and it is important to determine whether they should be ascribed to non-pathological experimental fluctuations or, rather, to the intrinsic structure of rainfall processes. The presence or absence of characteristic scales in rainfall records, in fact, has important implications both from a conceptual and a practical point of view because it decisively impacts the requirements imposed on rainfall models.

[5] The field of point rainfall stochastic modeling has developed along two main approaches. The first approach uses clustered Poissonian processes to describe the arrival of storms and of rain cells within each storm [*LeCam*, 1961;

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Waymire and Gupta, 1981; *Rodriguez-Iturbe et al.*, 1987, 1988]. Stochastic models following this approach have been tested quite extensively [*Entekhabi et al.*, 1989; *Onof and Wheater*, 1993; *Bo et al.*, 1994] and are arguably the most robust modeling tool currently available [*Olsson and Burlando*, 2002]. Models based on clustered Poissonian arrivals inherently introduce characteristic timescales in the process, due to the exponential form assumed for interarrival times, and thus are not theoretically suitable to reproduce a power law scaling behavior. On the contrary, the other prevailing modeling approach moves from the hypothesis that the rainfall process is scale-free and that power laws describe the dependence of relevant statistical quantities on the scale of aggregation. Following this approach, generation schemes are defined that incorporate this scale-free behavior making use of mathematical constructs based, for example, on the formalism of random cascades [e.g., *Schertzer and Lovejoy*, 1987; *Menabde et al.*, 1997].

[6] According to the framework discussed above the most relevant questions in current point rainfall studies thus regard: (1) the existence of power law scaling in observed rainfall (in general, or at least in certain climate types) and its accurate identification; and (2) the ability of different classes of stochastic models to replicate power law/non-power law properties.

[7] General answers to the above issues are likely to be obtained through theory, rather than by observations alone, extensive as they may be. The present work contributes new analytical results, based on the theory of stochastic processes [*Cox and Isham*, 1980; *Vanmarke*, 1983], showing that deviations from power law scaling of rainfall variance must occur even in the absence of characteristic scales of rainfall fluctuation. A general, theoretical (non-power law) scaling structure for the coarse-grained variance (i.e., for the variance of data aggregated at progressively coarser timescales) is then introduced, which can account for rainfall features observed at different locations with aggregation scales ranging from 15 min to 2 days. A power law scaling regime is shown to be attained only asymptotically, and not necessarily at the aggregation scales of usual interest, with relevant conceptual and practical consequences.

2. Scaling Relationships

[8] Let $i(t)$ be the instantaneous rainfall intensity, regarded as a continuous stochastic process [*Cox and Isham*, 1980], which gives rise, by integration over intervals of duration T, to the observed realizations of aggregated rainfall depth h_T. If the process $i(t)$ is stationary, the variance $\sigma^2(t)$ of h_T may be related to the autocorrelation function $\rho_i(\tau)$ of $i(t)$ [*Vanmarke*, 1983]:

$$\sigma^2(T) = 2\,\sigma_i^2 \int_0^T (T-\tau)\rho_i(\tau)d\tau \quad (1)$$

where σ_i is the standard deviation of the continuous process $i(t)$. It is clear that equation (1) assumes the existence of the variance of the continuous process. This is often a reasonable assumption and the starting point for a theory that should be firstly based on the simplest possible hypotheses.

[9] A process may be said to have a finite memory if its autocorrelation function decays rapidly enough that $\rho_i(\tau)/\tau \to 0$ when $\tau \to \infty$. A characteristic integral scale, I, may then be defined [*Taylor*, 1935]:

$$I = \lim_{T\to\infty} \int_0^T \rho_i(\tau)d\tau \quad (2)$$

[10] By dividing equation (1) by T and by taking the limit for $T \to \infty$ it is seen that $\sigma^2(T)/T \to 2\sigma_i^2 I$ (notice that in this case $\tau/T \to 0$) and that $\sigma^2(T)$ is asymptotically proportional to T for finite memory processes.

[11] On the contrary, a process will here be said to have an infinite memory if its autocorrelation function asymptotically decreases slower than τ^{-1}, say $\rho_i(\tau) \xrightarrow{\tau\to\infty} \tau^{-\alpha}$ with $\alpha < 1$. In this case the integral scale I does not exist, and by dividing equation (1) by $T^{-\alpha}$ and by taking the limit for $T \to \infty$ it is seen that $\sigma^2(T)/T^{-\alpha} \to T^2$. Therefore $\sigma^2(T)$ is asymptotically proportional to T^β, with $\beta = 2 - \alpha$ $(1 < \beta < 2)$. It is worth noting that in the special case of $\rho(\tau) \propto \tau^{-1}$ the variance of the integral process behaves as $Tln(T)$.

[12] It is thus seen that the asymptotic behavior of $\sigma^2(T)$ reveals the correlation properties of the stochastic process generating the observed rainfall realizations. The above discussion shows that, in general, whenever the limit in (2) is finite, the process has a finite memory and $\sigma^2(T)$ asymptotically behaves as T. When such an integral is not finite (and its asymptotic behavior is not proportional to τ^{-1}) the process has an infinite memory and $\sigma^2(T)$ asymptotically behaves as T^β, with $1 < \beta < 2$.

[13] It is important to also establish the characteristics of $\sigma^2(T)$ near the origin to determine how the transition to the power law scaling regime takes place. When $T \approx 0$ one may approximate $\sigma^2(T)$ with a Taylor series arrested to the second term, where:

$$\frac{d\sigma^2(T)}{dT} = 2\sigma_i^2 \int_0^T \rho(\tau)d\tau$$
$$\frac{d^2\sigma^2(T)}{dT^2} = 2\sigma_i^2\rho(T) \quad (3)$$

Since $\sigma^2(T)$ and its first derivative are both zero for $T = 0$, the Taylor expansion for small T becomes:

$$\sigma^2(T) \cong \sigma_i^2\rho(0)\,T^2 = \sigma_i^2\,T^2 \quad (4)$$

Therefore $\sigma^2(T)$ behaves as T^2 near the origin.

[14] The overall shape of the function $\sigma^2(T)$ for the cases described above is shown in Figure 1. In this Figure the finite memory case was produced by assuming an exponential correlation function, while the infinite memory case is based on a power law tailed correlation. Both cases will be discussed in detail below.

[15] An inner regime near the origin may be identified in Figure 1, where $\sigma^2(T) \cong \sigma_i^2 T^2$ in all cases, as prescribed by equation (4).

[16] For large values of T a scaling regime is present where $\sigma^2(T) \propto T^\beta$, with β being larger or equal to one depending on the memory of the process.

[17] A transition regime exists between the inner and the scaling regimes, whose characteristics depend on the type and degree of correlation of the process. The position and extension of the transition regime are quite important. If the transition regime is, in fact, limited to very short timescales

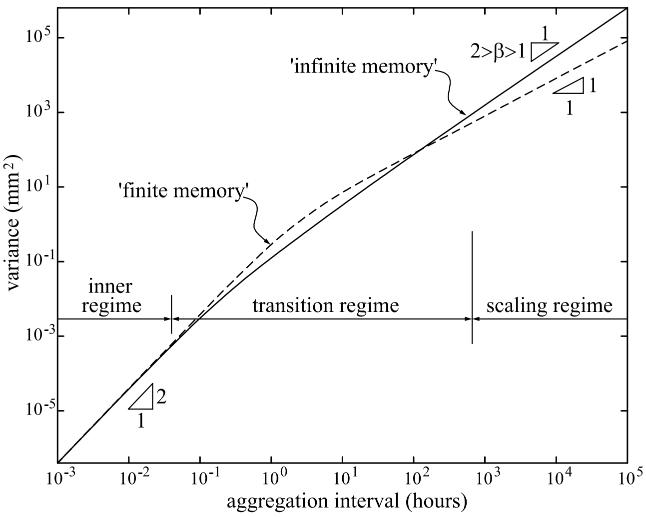

Figure 1. Variance of the aggregated rainfall process as a function of the aggregation interval. The "finite memory" case was obtained from a process with an exponential correlation function, while for the "infinite memory" case a power law correlation was assumed. In both cases the values assumed for the parameters (integral scale, exponents, etc.) are in the ranges observed in real data as obtained from the analyses below.

(i.e., smaller than, say, 1 hr), then the scaling regime can be safely assumed to be valid at all scales of usual observation. This hypothesis is implicitly assumed by simple- and multi scaling analyses [e.g., *Hubert et al.*, 1993; *Olsson*, 1995] and by fractal and multifractal models of rainfall [e.g., *Schertzer and Lovejoy*, 1987; *Menabde et al.*, 1997].

[18] On the contrary, if the transition regime is partially or completely in the range of scales of observation, then a power law dependance of variance on the aggregation interval is inadequate to describe real rainfall processes over all scales. In this case the usual procedure of fitting a power law to the coarse grained variance (such as in techniques involving statistical moments, used to determine the presence of multiscaling [see, e.g., *Tessier et al.*, 1993]) will necessarily yield an exponent larger than one to accommodate the transition toward the inner regime (where $\sigma^2(T) \cong \sigma_i T^2$) even when the process studied has a finite memory.

[19] The presence of the transition regime and the theory presented may also explain previous experimental findings according to which deviations from perfect power law scaling are often observed in point rainfall for the smallest sampling scales [e.g., *Olsson*, 1995; *De Lima and Grasman*, 1999; *Olsson and Burlando*, 2002]: such deviations are to be expected when the transition regime is located at the timescales of observation and are more evident for small values of T near the inner regime.

[20] Further, the theory discussed above suggests a method to determine the presence of long-term memory, which is based on fitting to the observations the function $\sigma^2(T)$ derived under different hypotheses (e.g., exponential or power law correlations) and then determining which form best interprets experimental data. This procedure will be applied to data from different locations in the following.

[21] Another important consideration arising from the above discussion regards the possibility of developing a downscaling procedure, i.e., a procedure which allows the determination of the variance of the process aggregated at a scale T_1 from observations aggregated at a scale $T_2 > T_1$. It is clear that attempting to use a single power law over the entire interval of temporal scales of interest will not lead to an accurate estimate of the variance at the smaller scale T_1 due to the presence of the transition and inner regimes. On the contrary, the use of a suitable correlation structure to derive an appropriate form of the function $\sigma^2(T)$ promises to yield the desired downscaling relationship.

3. Observational Data

[22] The theory discussed above and the analysis procedures derived from it were tested on a data set comprising four rainfall time series from locations with different climatic characteristics. The first measuring station is located in Marghera (Italy, Latitude: 12°14′30″N; Longitude: 45°27′11″Elevation: 0 m amsl), where a rain gauge recorded hourly rainfall in the period 1993–2000 (operated by Ente Zona Industriale di Porto Marghera - Italy). The second location considered is Ashover (UK, Latitude: 53.161°N; Longitude: 1.477°W; Elevation: 178 m amsl), where hourly rainfall was available for the period 1983–1988 (from the British Atmospheric Data Centre). The remaining two stations considered are located in the US and measured rainfall at the 15-minute aggregation. The stations are Matilija Dam (California, Latitude 34°29′00″N; Longitude: 119°18′00″W; Elevation: 320 m amsl, period: 1972–1992) and Lebanon Waterworks (Indiana, Latitude 40°04′00″N; Longitude: 86°28′00″W; Elevation: 290 m amsl, period: 1971–1992) (from the NCDC database). In all cases the ratio of recorded to observed (i.e., recorded plus missing) data was greater than 0.95.

[23] The climatic characteristics of the four stations may be seen in Figure 2. Ashover, Lebanon and Marghera are characterized by comparable values of average yearly rainfall (805 mm, 885 mm and 784 mm respectively), which are decisively larger than in the case of Matilija (622 mm). Marghera and Ashover present rather constant values of monthly average rainfall (Figure 2) with more pronounced values of the variance in the summer months, possibly in connection with the presence of short and intense convective storms. The two climates differ for the yearly average number of storms (189 for Marghera, 341 for Ashover), their average duration (2.1 hours and 3.0 hours respectively) and mean storm depth (4.2 mm and 2.4 mm respectively).

[24] The two stations from the U.S. exhibit a somewhat stronger yearly cycle both in the monthly average and the monthly variance. Matilija is characterized by few storms (on the average 57 every year) concentrated during winter which are quite intense (indeed the most intense among the sites considered, the average storm depth being 10.9 mm) and relatively long-lived (average storm duration: 2.4 hours). The data from Lebanon reveal an evident yearly cycle, though less pronounced than in the case of Matilija, with larger amounts of rainfall being delivered during the summer by storms which are on the average the shortest among the sites considered (mean yearly number of storms: 155; average storm duration: 1.5 hours; mean storm depth: 5.7 mm).

[25] The locations considered have been selected on the basis of their climatic characteristics. The wide variety of

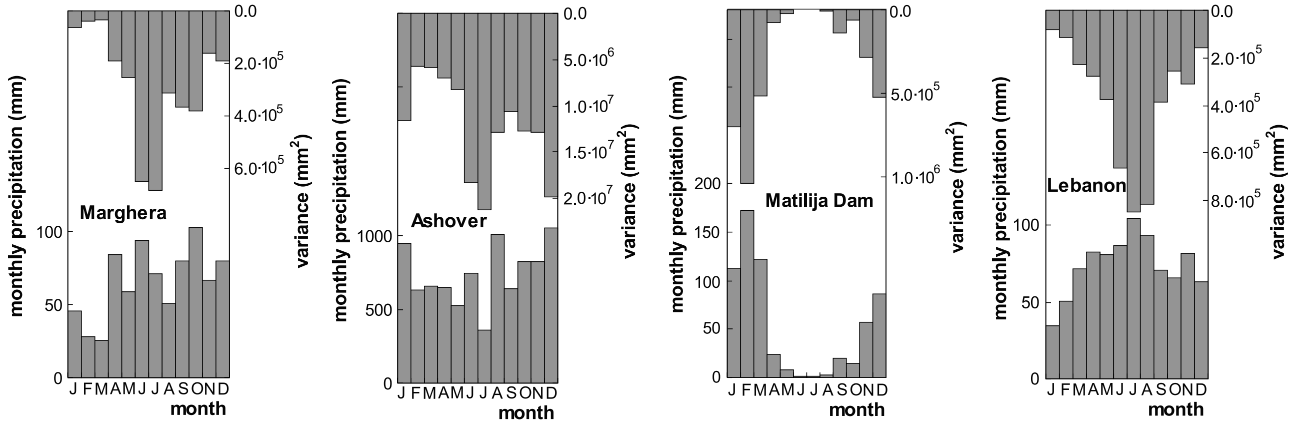

Figure 2. Monthly average and variance of hourly rainfall for the data from the four stations of observation considered.

climate types represented by the stations selected allows the testing of the theory introduced above on a representative sample of correlation properties.

4. Observed and Predicted Scaling Properties

[26] In this section the theory introduced above will be applied to the analysis of the experimental data from the four locations selected. Two (contrasting) cases will be considered. The finite memory case will be represented through an exponentially correlated process with $\rho_i(\tau) = \exp(-\tau/I)$ (whose integral over all positive real numbers is equal to I). Clearly, other choices of processes with a finite memory are possible, but an exponential correlation is a very common modeling assumption, e.g., typical of clustered Poissonian processes, whose exponentially correlated cell inter-arrival time and duration induce an exponential autocorrelation function [e.g., *Cox and Isham*, 1980; *Rodriguez-Iturbe et al.*, 1987, 1988]. From equation (1) one obtains in this case:

$$\sigma^2(T) = 2\,\sigma_i^2\, I\left[I\left(e^{-T/I} - 1\right) + T\right] \tag{5}$$

[27] The infinite memory case is represented by introducing an autocorrelation function which asymptotically decreases as $\tau^{-\alpha}$, with $\alpha < 1$. Since a power law diverges for $\tau \to 0$, an exponential correlation nucleus is included in the definition:

$$\rho_i(\tau) = \begin{cases} e^{-\frac{\alpha\tau}{\epsilon}} & \tau \le \epsilon \\ \left(\dfrac{\epsilon}{e\,\tau}\right)^{\alpha} & \tau \ge \epsilon \end{cases} \tag{6}$$

This autocorrelation function is continuous with continuous first derivative everywhere (and in particular in $\tau = \epsilon$) and has the interesting property of yielding an exponential autocorrelation when $\epsilon \to \infty$. Further, the definition (6) offers the possibility of describing a process with a rapidly decaying correlation for small lags, given by the exponential nucleus, and a long-term memory for large lags, assured by its power law tail. It is seen that the temporal scale I is not defined for correlation (6) since the integral in (2) is not finite. By use of (1) one finds:

$$\sigma^2(T) = 2\,\sigma_i^2\,\frac{\epsilon}{\alpha}\left[\frac{\epsilon}{\alpha}\left(e^{-\frac{\alpha T}{\epsilon}} - 1\right) + T\right] \qquad T \le \epsilon$$

$$\begin{aligned}\sigma^2(T) = 2\,\sigma_i^2\Bigg[&\frac{\epsilon^{\alpha}e^{-\alpha}}{(1-\alpha)(2-\alpha)}T^{2-\alpha} + \frac{\epsilon}{\alpha}\left(1 - \frac{e^{-\alpha}}{1-\alpha}\right)T \\ &+ \frac{\epsilon^2}{\alpha^2}\left(e^{-\alpha} - 1\right) + 2\,\frac{\epsilon^2 e^{-\alpha}}{\alpha(2-\alpha)}\Bigg] \qquad T \ge \epsilon.\end{aligned} \tag{7}$$

which behaves proportionally to T^{β} with $\beta = 2 - \alpha$ for large values of T as required by the theory.

[28] In order for the analyses to be coherent with the stationarity requirement of the theory, the rainfall records from each month of the year were considered to be generated by different stochastic processes. Each of the twelve stochastic processes thus introduced may then be considered stationary with an acceptable degree of approximation. Twelve functions $\sigma^2(T)$ are therefore obtained for each station by aggregating the rainfall time series over non-overlapping intervals of increasing duration T. The variance for every month in every year of observation is then computed and an "ensemble estimate" for the variance of each month is obtained by averaging the values for the given month from all years. This procedure yields the results in Figure 3, which shows rainfall variance as a function of the aggregation interval for some sample months and for the different observation sites. Notice that the curves in Figure 3 have been arbitrarily shifted in the vertical direction in order to improve its readability. Fitted analytical forms of $\sigma^2(T)$ obtained by assuming $\sigma^2(T) = \sigma_0^2 T^{\gamma}$ and by use of equations (5) and (7) are plotted for comparison. The theoretical slope of the inner regime (2:1) and the characteristic slope of a finite memory process (1:1) have also been indicated for reference. The forms (5) and (7) were fitted by minimizing (through a numerical procedure) the sum of the squared differences between the logarithms of observed ($\sigma^2_{obs}(T_i)$) and predicted ($\sigma^2_{pr}(T_i)$) values of the variance, i.e., by minimizing the objective function: $RMSE = \Sigma_i[log(\sigma^2_{obs}(T_i)) - log(\sigma^2_{pr}(T_i))]^2 = \Sigma_i[log(\sigma^2_{obs}(T_i)/\sigma^2_{pr}(T_i))]^2$. The same objective function was minimized to fit the power law form of $\sigma(T)$ by applying the classical analytical procedure

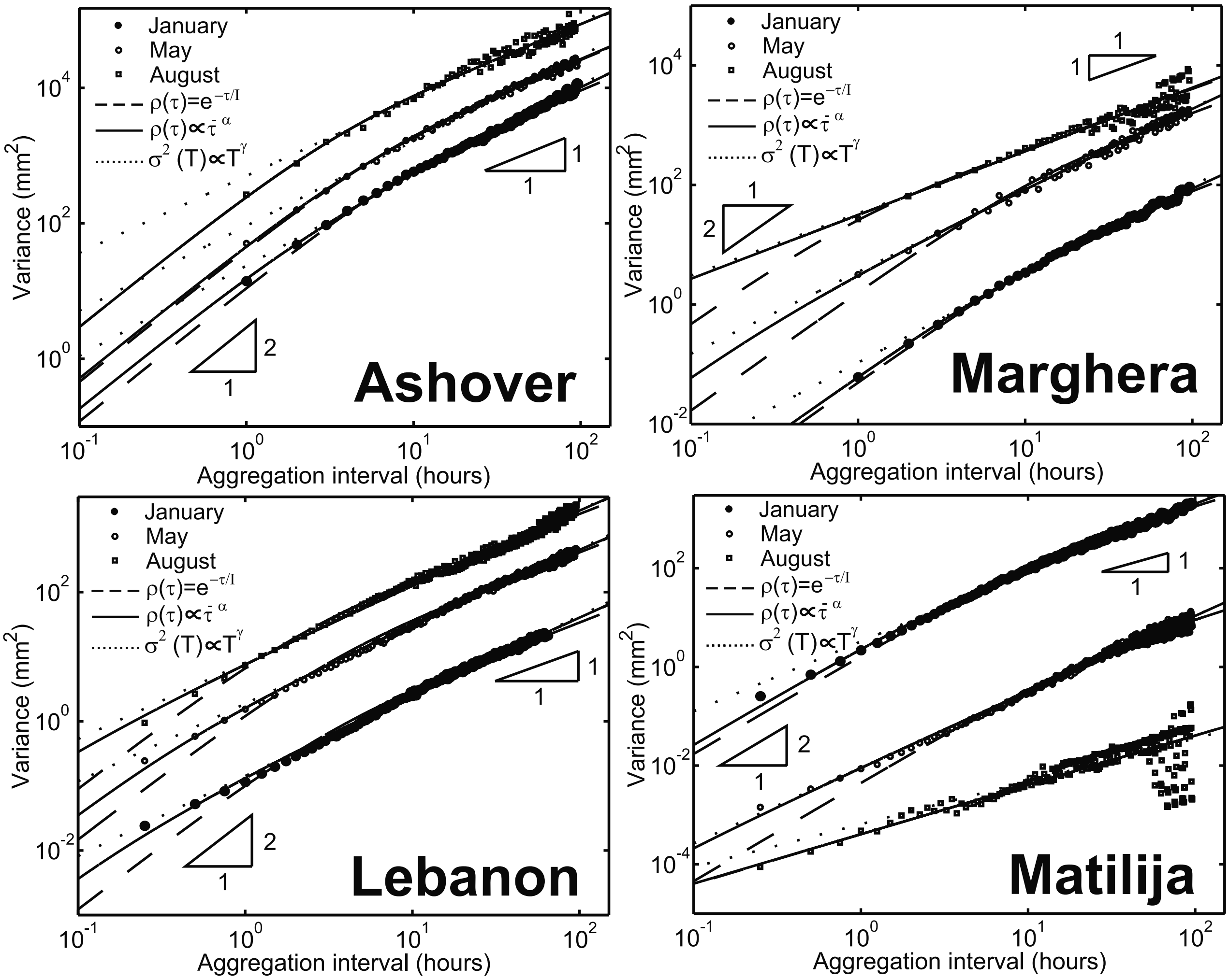

Figure 3. Rainfall variance $\sigma^2(T)$ as a function of the aggregation interval T for some sample months and for the four measurement stations considered. The experimental values are fitted assuming a power law variance and the analytical forms obtained from exponential and power law autocorrelation functions.

to the logarithms of T and observational $\sigma(T)$. The months of January, May and August were arbitrarily chosen to sample the seasonal dependence of the form of $\sigma^2(T)$ but results from remaining months are all qualitatively similar to those presented. The values of the parameters obtained for the months in Figure 3 are shown in Table 1, which also lists the values of the root mean square error (RMSE).

[29] It is seen that the assumption of a power law variance function yields RMSE values which are usually larger and at most equal to those obtained from both the forms (5) and (7). Further, a power law variance function often severely overestimates the variance for small aggregation scales (e.g., Ashover all months, Marghera January, Lebanon August and Matilija January and August), most likely because the observed $\sigma^2(T)$ must tend to the quadratic form of the inner regime. It is interesting to note that the fit of a single power law to all the aggregation scales considered yields exponents γ decisively larger than unity (except in the case of Matilija in August, Table 1) seemingly indicating the presence of strong persistence in the signal. This is not supported by considerations based on the fitted (5) and (7). It is true that (see Table 1) the data from all stations present the symptoms of persistence for the month of January: the exponent α obtained by fitting (7) is smaller than unity (and the RMSE for the infinite-memory assumption is smaller than both the power law variance and the finite-memory case (5)). Nevertheless, data from Ashover, Marghera and Matilija for the month of August, do not show the presence of infinite memory: the exponent α is much larger than unity (and thus $\beta = 2 - \alpha$ is smaller than one) and the RMSE obtained for the forms (5), (7) are the same, showing that the correlation (6) falls back to an exponential form in the range of aggregation scales considered.

[30] These findings are consistent with physical interpretations. It is quite common in winter to observe a sequence of long and correlated events, e.g., due to storm systems at the synoptic scale, inducing long-term memory in the rainfall signal. On the contrary, in many locations summer events tend to be due to isolated convective events which preserve little memory of preceding storms.

[31] Observed characteristics exhibit an intermediate and mixed behavior during spring. The variance from Marghera and Matilija shows the signs of persistence ($\alpha < 1$), while Ashover and Lebanon seem to be characterized by less correlated rainfall temporal patterns. The theory introduced

Table 1. Parameter Values Obtained From Fitting a Power Law Variance, $\sigma^2(T) = \sigma_0{}^2 T^\gamma$, and the Analytical Forms (5) and (7) to Experimental Variance Values for Three Sample Periods of the Year and the Four Stations of Observation Considered

	$\sigma^2(T) = \sigma_0{}^2 T^\gamma$			$\rho(\tau) = e^{-\tau/I}$			$\rho(\tau) \propto \tau^{-\alpha}$			
	σ_0^2, mm^2	γ	RMSE $\times 10^{-2}$	σ_i^2, mm^2	I, h	RMSE $\times 10^{-2}$	σ_i^2, mm^2	ϵ, h	α	RMSE $\times 10^{-2}$
					January					
Ashover	35.5	1.3	4.3	17.9	4.0	4.9	27.90	1.5	0.98	3.2
Marghera	0.20	1.5	4.3	0.10	8.4	4.7	0.01	3.8	0.90	3.6
Lebanon	0.18	1.2	3.6	0.02	1.4	5.4	0.59	0.13	0.93	3.2
Matilija	1.97	1.4	5.5	1.04	5.5	5.5	1.60	1.7	0.88	4.0
					May					
Ashover	27.3	1.2	5.6	13.6	2.9	3.4	15.40	3.93	1.73	3.3
Marghera	0.51	1.4	5.5	0.25	5.0	7.7	0.96	0.210	0.71	5.4
Lebanon	0.85	1.2	4.2	0.69	1.4	5.1	1.81	0.269	1.06	3.6
Matilija	0.04	1.5	8.9	0.02	11.1	10	0.14	0.0351	0.48	8.9
					August					
Ashover	43.9	1.1	9.1	26.40	1.5	7.7	26.30	44.	30.	7.7
Marghera	0.71	1.1	13	1.08	0.40	13	1.08	9.6	24.	13
Lebanon	1.68	1.2	6.2	1.93	0.88	7.5	14.90	0.035	0.94	6.2
Matilija	0.01	0.9	39	435	$1 \cdot 10^{-5}$	39	55.70	0.0060	7.7	39

here may thus be used to detect the presence of persistence and shows that conclusions drawn by assuming a single power law variance may be misleading.

[32] Comparisons in Figure 3 and Table 1 also show that exponential-correlation best fits often tend, in the cases considered, to underestimate observed variance at short scales of aggregation whereas power law-correlation best fits tend to better interpret the observed shape of $\sigma^2(T)$. This is certainly to be expected since the power law correlation is defined through three parameters (σ_i, α, ϵ) while the exponential correlation only depends on two parameters (σ_i, I). Nevertheless the power law-correlation fits were found to show little sensitivity on the value of ϵ (results not shown here for brevity) and the RMSE to be robustly smaller than in the exponential-correlation case as may be seen in Table 1.

[33] The results in Figure 3 and Table 1 suggest that the power law-tailed autocorrelation structure (6) better describes the observed $\sigma^2(T)$ curves than does a finite-memory, exponential, one or a power law form for $\sigma^2(T)$. The variance $\sigma^2(T)$ for the last two cases is, in fact, unable to reproduce the experimental values at all scales of aggregation and for the variety of climates considered.

[34] The form (6) is the most flexible among those considered as it can suitably represent both finite and infinite-memory processes and has the advantage of correctly incorporating the inner and transition regimes, which cannot be accounted for by a simple power law variance.

[35] Figure 3 indicates that the experimental variance as a function of the aggregation interval clearly exhibits a convex-up shape with a decreasing slope in all cases analyzed (the same behavior is also found for the months not included in Figure 3). This suggests that the transition regime is located within the range of aggregation intervals of observation, implying that, in general, a single power law cannot adequately describe the observed coarse-grained variance. The great variety of climates explored suggests that this circumstance is quite common in nature and that the validity of the usual power law scaling assumption at the basis of fractal and multifractal models and analyses of rainfall is confined to limited ranges of aggregation scales. This limitation not only regards breaks in scaling regimes at large scales, but more importantly relates to a scaling break which is located at small scales, suggesting that models based on scaling assumptions of statistical moments are not suitable for downscaling procedures.

[36] In further support of the above considerations Figure 4 plots $\sigma^2(T)/T$ versus T. The theory presented requires this function to tend to $2\sigma_i^2 T$ for small T. In Figure 4 the theoretical tangent for $T \rightarrow 0$ is plotted as a reference (also see the insets of Figure 4), assuming the values of σ_i obtained through the fitting procedures applied above. The results in Figure 4 are quite representative of the analyses performed on all the months and stations considered. Much of the observational data well conforms to the theoretical behavior for the inner regime (e.g., Marghera in January, Ashover in August and Lebanon in May). In other cases, the data are compatible with an inner regime located at timescales smaller than those of observation. In other cases still (e.g., Matilija in January, which also exhibits the largest fluctuations in the observed variance) the experimental $\sigma^2(T)/T$ functions exhibit slopes decisively smaller than theoretical predictions. This fact might be linked to a diverging value of the variance $\sigma_i{}^2$ of the continuous process, but the issue awaits further clarifications for which rainfall observations with higher temporal resolution are needed.

[37] The theory presented also requires the function $\sigma^2(T)/T$ to asymptotically tend to a power law with exponent $1 - \alpha$, in the case of infinite memory process, and to a horizontal line in the case of a finite memory process. All the data considered tend to the theoretical asymptotic behavior, which was computed using the parameters described above and is indicated in Figure 4. The fact that the asymptotic power law (or horizontal line) does not describe the observed behavior for small T is a further confirmation of the existence of the inner and transition regimes and of the inadequacy of simple variance scaling assumptions.

[38] Finally, the slope changes in Figure 4 indicate that the border between the transition regime and the scaling

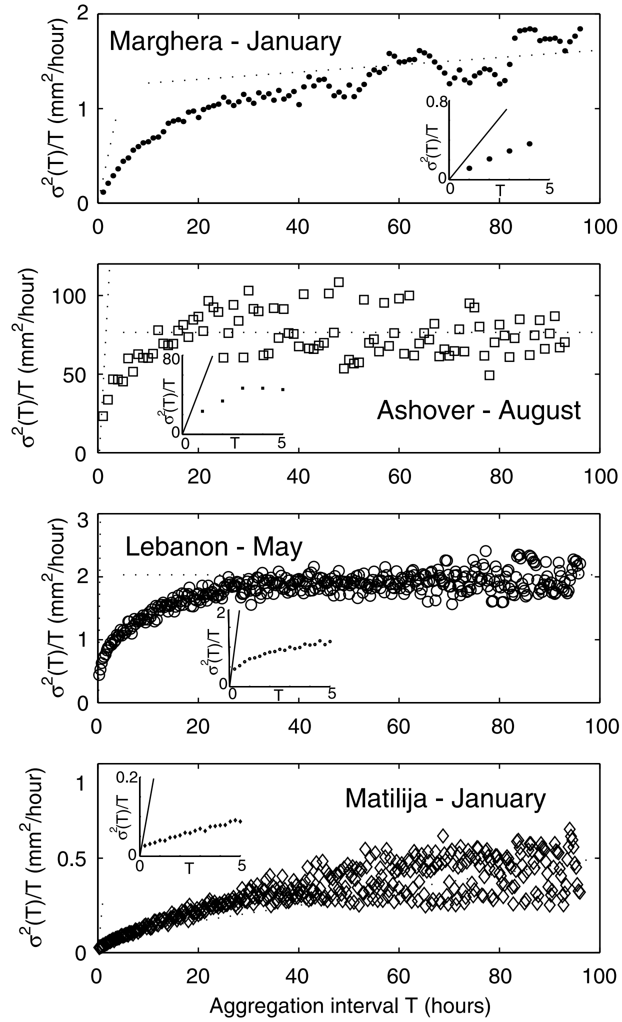

Figure 4. Plots of $\sigma^2(T)/T$ versus aggregation interval T for some sample months and for the four measurement stations considered. A linear relationship representative of the inner regime (whose slope is obtained from parameter values obtained from fitting observed variances) and a power law representative of the scaling regime at large aggregation times (with the appropriate exponent obtained from fitting) are shown as references. The insets show enlargements of data behavior near the origin.

regime is located approximately at T = 60 *hours* for Marghera in January, at T = 20 *hours* for Ashover in August, at T = 35 *hours* for Lebanon in May and around $T = 60 \div 80$ *hours* for Matilija in January. The exact identification of the beginning of the inner regime is uncertain for the observations used and will require the use of rainfall data with greater temporal resolution. Nevertheless, its presence is quite visible in the sharp increase, near the origin, of the slope of the experimental curves of Figure 4, and importantly affects the scaling properties of the variance.

5. Conclusions

[39] The paper introduced new theoretical relationships linking point rainfall variance to the aggregation scale. It was shown, in particular, that the functional dependence of rainfall variance with the sampling scale exhibits three distinct regimes: (1) an inner regime, where $\sigma^2(T)$ is quadratic, independently from the correlation characteristics of the underlying continuous rainfall process; (2) a scaling regime, where $\sigma^2(T)$ is a power law, whose exponent depends on the memory of the precipitation process. Such exponent is larger than unity for infinite memory processes and is equal to one for processes with finite memory; (3) a transition regime between the inner and the scaling regimes.

[40] Using observed time series from a wide range of climates and covering a wide interval of aggregation scales, it was shown that the assumption of a power law form of $\sigma^2(T)$ for all values of T is unable to capture actual rainfall properties. In particular, fitting a single power law to the observed coarse-grained variance yields exponents larger than unity even when the process is a finite-memory one, because the transition regime may be located within the aggregation scales of the observations.

[41] The existence of the transition regime may also provide an explanation for the capability of rainfall models based on clustered Poisson processes (e.g., rectangular pulses models) of reproducing rainfall variance at different scales even in the presence of power law scaling with exponents larger than unity [e.g., *Rodriguez-Iturbe et al.*, 1987, 1988; *Burlando and Rosso*, 1996; *Onof et al.*, 1996]. Such stochastic models, in fact, by definition only possess short-term memory (for which it was seen that asymptotically $\sigma^2(T) \propto T$) but may be tuned to reproduce the observed variance over a limited range of aggregation scales even when $\sigma^2(T) \propto T^\beta$ (with $\beta > 1$). This can be explained by considering that the tuning procedure artificially "stretches" the $\sigma^2(T)$ function of the model to position its transition regime, which can locally approximate a power law with a larger-than-one exponent to cover the scales of calibration. In this manner the linear scaling regime is forcefully shifted to the right of the range of calibration scales. Therefore rectangular pulses models should be expected to underestimate the actual variance for aggregation scales larger than those of calibration (due to the fact that their variance increases linearly with T and thus slower than T^β) and to overestimate it for scales smaller than those of calibration (due to a slower transition to the inner, T^2-regime, for decreasing T). It may thus be concluded that models based on poisson processes are not suitable to perform rainfall downscaling: they should not be tuned at one scale of aggregation and then used to generate synthetic rainfall at smaller scales.

[42] Theory and observations indicate that a proper interpretation of rainfall variance as a function of the aggregation interval requires the use of certain theoretical forms for $\sigma^2(T)$, here developed under different hypotheses. The use of these forms can allow the detection of infinite or finite memory processes and the development of appropriate downscaling procedures. This can for example be accomplished by determining the parameters in equation (7) through fitting to variance data at the available timescale (e.g., daily and higher) and by then using it to estimate the value of the variance at smaller timescales (e.g., hourly).

[43] Finally, it should be noticed that any measurement procedure consists of integrating a signal over a finite time or space interval (or volume) and that the properties of the

resulting observations (provided that the underlying continuous process has a finite variance) must thus conform to the theoretical framework proposed above.

[44] Therefore the theory and results presented herein are not related solely to rainfall observations and models, but seemingly apply to a broad class of natural processes.

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