Relating the scales of sediment burial to the characteristics of bedload transport

Kevin Pierce

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1 abstract

Bedload transport is intermittent, meaning it should really be described as a stochastic process. However, changes in river bed topography driven by sediment transport are usually interpreted by assuming bedload transport is a deterministic process. This work is a first stab at modeling sediment transport and bed elevations simultaneously. Applying a stochastic model within a control volume, we track the number of bed particles and the number of particles in motion at any instant. These two populations are subject to random transitions controlled by rates of entrainment and deposition. Since the number of particles in motion represents the bedload transport rate, and the number of particles at rest represents the bed elevation, our stochastic model describes the joint statistics of bedload transport and bed elevations. From the statistics of bed elevation, the time-scales of particle burial can be extracted. These burial time distributions have crucial implications for the interpretations of tracer experiments.

2 Introduction

The transport and storage of sediment within gravel bed rivers are primary controls on channel morphology, so these processes are important for a wide range of considerations, including river restoration [*Gaeuman et al.*, 2017], sediment budgeting [*Malmon et al.*, 2003], and contaminant transport [*Malmon et al.*, 2005]. In relation to these problems, predictions of the rate of downstream transport of sediment, and the timescales and depths of sediment burial within the riverbed are often needed, either to confine the rate of contaminant transport, or to interpret field studies based on the movement of tracer particles as they move downstream. Inquiries into transport and burial have a significant complication: under typical conditions in gravel bed rivers, both sediment transport rates and bed elevations evolve through time in an apparently random way. These random variations persist even under the most controlled laboratory conditions available, which suggests that transport rates and bed elevations should really be described statistically.

However, there is another factor which compounds this difficulty. It's conceptually clear that sediment transport and changes in bed elevation are not independent: erosion increases the number of particles in motion and decreases the bed elevation, while deposition decreases the number of particles in motion and increases the bed elevation. Accordingly, transport and bed elevations are random variables, but they are also correlated. In this paper, we fuse earlier work to develop a new understanding of this correlation. We ask: how do the timescales of sediment transport relate to the timescales of bed elevation change, and knowing this, how do the characteristics of sediment transport confine the timescales over which tracers or contaminants remain buried in a gravel bed river? To address these questions we develop a new theoretical approach.

Sediment within gravel bed rivers transports, under typical flow conditions, in an alternating sequence of rests and motions [*Einstein*, 1937]. When grains move, they do so by rolling, sliding, and bouncing along the bed, a state of motion referred to as bedload [*Einstein*, 1950; *Bagnold*, 1973; *Church*, 2006]. Owing at least to fluid turbulence and the variable configuration of grains on the bed surface [*Ferreira et al.*, 2015], the relative exposure of grains to the flow [*Egiazaroff*, 1965; *McEwan et al.*, 2004], and the stability of grains against forces and torques exerted by flow against their supporting contacts with other grains [*Coleman*, 1967; *Vowinckel et al.*, 2016], mobility varies from one stationary grain to the next [*Paintal*, 1971; *Celik*, 2011; *Vowinckel et al.*, 2016]. Accordingly, the mobilization of individual stationary grains is not predictable [*Einstein*, 1937], but on a population level, sampling over many turbulent fluctuations and granular configurations, the mobility of grains can be characterized statistically [*Einstein*, 1950; *Yano*

et al., 1969; *Yang and Sayre*, 1971; *Nakagawa and Tsujimoto*, 1976; *Dey et al.*, 2018]. Similarly, the trajectories of individual moving grains are not predictable, as they are subject to unknown turbulent fluctuations and random collisions with other stationary and moving grains [*Einstein*, 1937; *Wiberg and Smith*, 1985; *Bialik et al.*, 2015], but on a population level, motion characteristics – velocity, travel distance, travel time – lie on statistical distributions [*Yano et al.*, 1969; *Yang and Sayre*, 1971; *Nakagawa and Tsujimoto*, 1976; *Lajeunesse et al.*, 2010; *Fathel et al.*, 2015; *Heyman et al.*, 2016], motivating probabilistic descriptions of bedload transport rates [*Einstein*, 1950; *Paintal*, 1971; *Nakagawa and Tsujimoto*, 1976; *Sun and Donahue*, 2000; *Cheng*, 2004; *Ancey et al.*, 2008; *Ma et al.*, 2014].

Experiments indicate that, under steady laboratory conditions, bedload rates, which are a collective property of many individual motions, also fall on statistical distributions; and when mean rates rates are relatively low, the stochastic characteristics of bedload transport are emphasized, so that bedload rates express large deviations from mean values. This implies bedload distributions have relatively heavy tails [*Böhm et al.*, 2004; *Ancey et al.*, 2008; *Singh et al.*, 2009]. Earlier probabilistic models of the bedload flux predicted distributions which were Poissonian, with a thin tail [*Crickmore and Lean*, 1962; *Sayre and Hubbell*, 1965; *Yano et al.*, 1969], and because the observed magnitude of bedload fluctuations depends on sampling rate [*Ma et al.*, 2014], these thin-tailed distributions fit the classic experimental data, which were obtained at relatively low sampling rates, quite well. However, with increased technological capabilities supporting bedload experiments with higher sampling rates [*Böhm et al.*, 2004; *Ancey et al.*, 2008; *Singh et al.*, 2009], it became clear that instantaneous values of the bedload flux are often very large relative to mean values, and to describe this observation, some contemporary researchers have derived bedload flux distributions with heavier tails such as the negative binomial distribution [*Ancey et al.*, 2008; *Heyman et al.*, 2013; *Ma et al.*, 2014]. These ammended stochastic models rely on collective feedbacks in particle entrainment: when more particles are in motion, entrainment becomes more likely. These collective effects are not described in the classic literature [*Paintal*, 1971; *Shen*, 1980], and their origin, which may be associated to different processes, such as the migration of bedforms [*Dhont and Ancey*, 2018] or small granular avalanches from the bed surface [*Heyman et al.*, 2013] deserves further attention. Nevertheless, experiments are convincing that bedload fluctuations are wide when sampling intervals get small, and that collective feedbacks are one way to derive realistically wide fluctuations from statistical models. The model we develop here borrows this work.

Relative exposure is a key control on particle mobility. Stationary particles are preferentially mobilized or entrained from the bed in regions of relatively high elevation, or crests [*Sayre and Hubbell*, 1965; *Sayre and Conover*, 1967; *Yang and Sayre*, 1971; *Sawai*, 1987; *Wong et al.*, 2007], while moving particles tend to come to rest or deposit in regions of relatively low elevation, or troughs [*Sayre and Hubbell*, 1965; *Yang and Sayre*, 1971; *Charru et al.*, 2004; *Wong et al.*, 2007]. These relationships between mobility and relative bed elevation stem, in large part, from the contingence of applied fluid forces on the exposed area of grains [*Hofland*, 2005; *Dwivedi et al.*, 2012].These theoretically expected and experimentally observed trends on the differential entrainment and deposition characteristics of bed particles on crests or in troughs are a crucial component of our model, because they provide a reverting character that implies a stable distribution of bed elevations. We hypothesize these reverting attributes of entrainment and deposition describe experimentally observed bed elevation time series, which are random and almost always normally distributed, with typical deviations from mean values of one to five representative particle diameters [*Crickmore and Lean*, 1962; *Yang and Sayre*, 1971; *Wong et al.*, 2007; *Singh et al.*, 2009; *Voepel et al.*, 2013], and departures from normally distributed profiles when bed material is poorly consolidated or armored [*Singh et al.*, 2009].

Likewise, the entrainment and deposition of bedload particles at a location modifies the local bed elevation. *Nakagawa and Tsujimoto* [1980] used this correlation in a statistical model to describe the development of bedforms in time from a flat bed condition, although they did not consider the modification of entrainment and deposition rates with relative bed elevation, so their model eventually breaks down, as modelled bed elevations diverge to infinite values in absence of these negative feedbacks. Other statistical models have considered bed elevations at a location as a reverting random walk [*Voepel et al.*, 2013; *Martin et al.*, 2014], where the bed elevation changes up or down in random increments, and these random steps are bounded from above and below by some characteristic length, which other authors mentioned in relation to the active layer concept [*Wong et al.*, 2007; *Church*, 2017]. These random walk models have a different focus than *Nakagawa and Tsujimoto* [1980]: they are intended to describe the random changes in bed elevation caused by erosion and deposition at some location over an equilibrium bed, rather than the incipient development of bedforms on an adjusting bed. However, they are deficient because they do not relate the scales of bed elevation changes to the scales of sediment transport from which these changes emerge. That is, within these models, there is no link between the erosion and deposition rates (time scale) of individual sediment grains, the size of sediment grains (length scale) and the rate of bed elevation change (emergent time scale) and the upper and lower bounds of bed elevations at a location (emergent length scale). In this paper, we will combine earlier modelling approaches to study this link across scales, and make a first stab at understanding sediment storage by burial as an emergent property of sediment transport.

In this development, and in accord with previous investigations, we interpret burial as the return time from above in the bed elevation time-series [*Voepel et al.*, 2013; *Martin et al.*, 2014; *Bradley*, 2017]. For example, if the bed elevation is y at the instant a deposition occurs, shifting it to some larger value, the return time from above is the time which elaspses from this instant to the next instant the bed elevation is again y, with the intervening interval occupied by some sequence of a number of particle entrainment and deposition events, each occurring at random instants in the interval, but necessarily, to bring the elevation back to y, occurring in equal proportions. Since the temporal pattern of entrainment and deposition events is random, the return time from above, or the burial time, is also random, and it its scales emerge from the statistics of sediment transport. The burial time distribution is of special interest for tracer studies and contaminant transport, as the phenomenon of anomalous super-diffusion of tracer particles can be described in terms of it [*Schumer et al.*, 2009; *Martin et al.*, 2012; *Zhang et al.*, 2012]. However, the form of the burial time distribution is contentious. Only a handful of theoretical and experimental studies have been performed, and their findings do not show clear agreement. For example, one set of experimental and theoretical investigators have measured or considered a tempered distribution for the burial time, meaning it is a power law distribution at smaller times up until some crossover point, when it becomes exponential [*Zhang et al.*, 2012; *Voepel et al.*, 2013]. Other investigators measured power law distributions without any crossover to exponential behavior *Martin et al.* [2012, 2014]; *Olinde and Johnson* [2015]; *Bradley* [2017]. *Martin et al.* [2012, 2014] are based upon the same flume experiment, while *Olinde and Johnson* [2015] was a field experiment with RFID tracers which could only measure up to 10hrs and may have failed to resolve tails.

- first discuss the burial time experimentally: results have been either tempered distribution or power law - tempered distributions are a power law with an exponential tail - field studies are Voepel 2013 who analyzed Habersack 2001, Olinde 2015, Bradley 2016 - Voepel 2013 saw, from RFID data of Habersack, tempered distribution with exponential crossover typically around 40-50 minutes - Olinde 2015 used RFID tags and saw power law distributions with tail parameters during different flow events ranging from 0.24 to 0.72. Their resolution was limited to 10hrs. - Bradley 2017 used RFID pit tags over a 9 year tracer study to get the best experimental resolution yet. He gets 0.67 as the tail parameter. Very heavy. Smaller means heavier. - laboratory studies are martin2012 and martin2014 - martin 2012 gets tail parameter 0.85 power law and develops the interpretation that anomalous diffusion is due to heavy-tailed waiting times which are induced by burial. - martin 2014 gets tail parameters from a quasi-2d channel with bidisperse spheres for sediment of about 1 using image analysis techniques

theoretical studies: -yang and sayre 1971 develop exponential return time tail on cdf according to Martin 2014 - nakagawa and tsujimoto 1980 develop exponential tail on cdf according to Martin 2014 - Zhang 2012 assumed tempered distribution for the sake of computing anomalous diffusion -voepel 2013 gets exponential tail on cdf with a reverting random walk model which has hard reflecting boundaries at maximum and minimum elevations -martin 2014 gets power law tail of approx 1 or slightly steeper on cdf with a reverting random walk model which has soft reflecting boundaries at maximum and minimum elevations. they say their distributions are not truncated because of their soft reflecting quality. They highlight their tails truncate when they inadvertantly run the simulation for not enough time.

- my conclusion? tails from my model should be power law at least in accord with the martin 2014 experiments. CDF like 1/T - For the interpretation of field results the statement of Bradley 2017 about two return time problems ticking simultaneously is cutting

For example, *Voepel et al.* [2013] analyzed tracer data obtained in the field by *Habersack* [2001], and concluded that the experimental burial time distribution was tempered, meaning it is a power law distribution at shorter timescales, and it transitions at some crossover point to an exponential distribution at longer timescales, and *Voepel et al.* [2013] modelled the tempered distribution using a random walk model of bed elevations with hard reflecting boundaries at the maximum and minimum bed elevations. In contrast, *Martin et al.* [2012] tracked tracers within flume experiments to determine a distribution that scaled as a power law across all burial times, with no tempered crossover, which *Martin et al.* [2014] modelled with a random walk model of bed elevations with soft reverting boundaries. Similarly, *Bradley* [2017] analyzed 9 years of data from a bedload tracer experiment to conclude the burial time distribution scaled as a power law, with no tempered crossover. T

There are a few theoretical and experimental suggestions that the burial time distribution is tempered, meaning it has an initial power law scaling, which transitions at some crossover point, possibly controlled by the upper and lower bounds of the bed elevation, into a thin tailed exponential distribution [*Zhang et al.*, 2012; *Voepel et al.*, 2013; *Olinde and Johnson*, 2015; *Bradley*, 2017; *Fan et al.*, 2017], however, the experiments of *Martin et al.* [2012] seem to show a power law tail instead, and *Martin et al.* [2014] reproduced this observation with a random-walk model of bed elevations similar to the *Voepel et al.* [2013] model, but with a soft reverting character at the limits of maximum and minimum bed elevation, rather than a hard barrier.

e will investigate this burial time distribution from a different angle, and show that, in our theoretical model, it also appears tempered, and is appropriately described by the two-parameter Pareto distribution which *Bradley* [2017] fit to field data, and we relate the crossover time between power-law and exponential regimes to the entrainment and deposition rates which statistically characterize bedload transport.

To do this, we pursue a fusion of the statistical model of bedform formation by *Nakagawa and Tsujimoto* [1980] with the *Einstein* [1950]-inspired stochastic bedload transport theory of *Ancey et al.* [2008] and the empirical relationships between relative exposure and local entrainment and deposition rates obtained by *Sawai* [1987] and *Wong et al.* [2007].This fusion hinges on the concept of a control volume containing some volume of bed with particles in motion above it. All resting and moving particles are idealized as identical spheres. Within this volume, we track the coordination and dynamics of two populations: first is the number of particles at rest, which changes randomly in accord with entrainment and deposition [*Nakagawa and Tsujimoto*, 1980; *Turowski*, 2009]; and second is the number of particles in motion, which changes in accord with migration into the control volume from upstream, entrainment within the control volume, deposition within the control volume, and migration out of the control volume to downstream [*Ancey et al.*, 2008].

Using the velocity at which particles travel, we can relate the bedload rate to the number of particles in motion [*Einstein*, 1950; *Charru et al.*, 2004; *Ancey et al.*, 2008; *Furbish et al.*, 2012], and using the porosity of the bed which characterizes the number of particles per unit volume of bed, we can relate the mean bed elevation within the control volume to the number of particles at rest within it. From this conceptual basis, with a two-population stochastic model, we can obtain the joint probability distribution of the number of moving particles, which is a proxy for the bedload rate [*Ancey et al.*, 2008], and the number of stationary particles, which is a proxy for the bed elevation [*Nakagawa and Tsujimoto*, 1980]. In appropriate limits, this model reduces to both of the stochastic models it's built upon. With this conceptual picture of two populations, we can study sediment burial in terms of sediment transport, and add a new perspective to the debate around the form of distributions of sediment burial time which underlie questions of tracer diffusion and contaminant transport in gravel bed rivers.

3   Theoretical Development

Within a control volume, we consider the joint stochastic dynamics of two populations which may change at any instant as a result of birth, death, and migration type processes. These populations are the number of moving particles within the control volume, which we denote as n, and the number of stationary particles within the control volume, which we denote as m. The conceptual idea is depicted in figure 1. Of course, both of these populations are positive integers including zero. Following earlier writers, we assume that bedload transport characteristics are not contingent on their past history, so we can relate the rates of population change at any instant to the populations at that instant – the Markov hypothesis [*Cox and Miller*, 1965; *Ancey et al.*, 2008]. This type of two-population Markov birth-death model is commonly applied to provide statistical understanding of ecological populations [*Cox and Miller*, 1965; *Pielou*, 1977; *Swift*, 2002] and chemical reactions [*Gardiner*, 1983; *van Kampen*, 1992; *Gillespie*, 2007]. In river science, a similar two-population approach has been used to understand alluvial cover and erosion within bedrock streams [*Turowski*, 2009].

To apply a Markov birth-death-migration model to the joint dynamics of bed elevations and bedload transport rates, we just need to outline the transitions which can modify the populations of resting and moving particles, and then specify their probabilities of occurence in a small interval of time as functions of these populations. Using these rates, we can consider the flow of probability through time to form a Master equation [*Cox and Miller*, 1965; *Pielou*, 1977; *Gardiner*, 1983; *Gillespie*, 1992; *van Kampen*, 1992] governing the joint probability distribution $P(n, m; t)$ that there are n moving particles and m stationary particles within the control volume at time t. In this model, we incorporate four distinct transitions which can shift the populations at any instant: these are migration into the control volume from upstream, entrainment within the control volume, deposition within
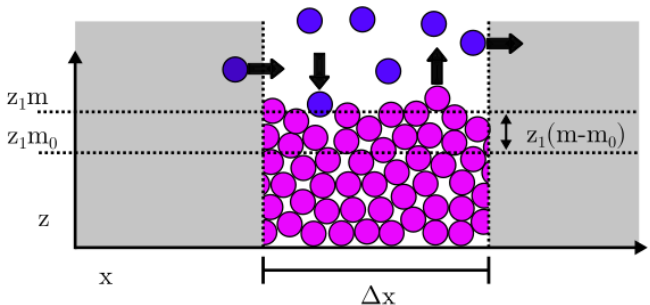

Figure 1: The conceptual picture of two populations of particles within a control volume. The relative bed elevation z is a function of the packing fraction, particle diameter, and control volume size. The bed is depicted in an aggraded state where $m > m_0$. In this state, entrainment is emphasized and deposition is de-emphasized. The four allowed transitions– migration in, entrainment, deposition, and migration out– are depicted with arrows. The system is con-

the control volume, and migration out of the control volume to downstream. We characterize these transitions by rates, or probabilities per unit time, and by the changes they enact in the populations of moving and stationary particles within the control volume. The entrainment and deposition rates are expected to depend on relative bed elevation of the control volume [*Yang and Sayre*, 1971; *Sawai*, 1987; *Wong et al.*, 2007], which we relate to the number of particles at rest within the volume, and hold relative to some baseline number of particles at rest, m_0:

$$z(m) = \frac{\pi a^2}{\phi \Delta x}(m - m_0) = z_1(m - m_0), \tag{1}$$

where Δx is the downstream length of the control volume, a is the radius of the idealized spherical bed particles we consider, and ϕ is the packing fraction of these particles, expected to be a constant value [*Cundall and Strack*, 1979]. The factor $z_1 = \pi a^2/(\phi \Delta x)$ is an important fundamental scale of the problem. This length scale characterizes the change in elevation (in an average sense across the control volume) associated with the addition or removal of a single particle.

Following *Ancey et al.* [2008], *Turowski* [2009], *Ancey and Heyman* [2014] and others, the changes in the populations these transitions enact and their probabilities of occurence in a very small time interval dt can be written as

migration in	$(n, m) \to (n + 1, m)$	νdt	(2)
entrainment	$(n, m) \to (n + 1, m - 1)$	$[\lambda(m) + \mu(m)n]dt$	(3)
deposition	$(n, m) \to (n - 1, m + 1)$	$\sigma(m)ndt$	(4)
migration out	$(n, m) \to (n - 1, m)$	$\gamma ndt,$	(5)

except, in a generalization of *Ancey et al.* [2008] to incorporate the effects of relative exposure on particle mobility, we parameterize the entrainment and deposition rates by the bed elevation [*Sawai*, 1987; *Wong et al.*, 2007]. According to *Sawai* [1987], the rate of entrainment scales linearly with relative elevation, and according to *Wong et al.* [2007], it scales exponentially. Since the linear scaling is simpler and results from the *Wong et al.* [2007] relationship as the limiting case of small relative bed elevations, we hold the entrainment rate linearly proportional to $z(m)$:

$$\lambda(m) + \mu(m)n = (\lambda_0 + \mu_0 n)\left(1 + \frac{z_1}{(2l)^2}z(m)\right), \qquad \text{entrainment rate at } (n, m). \tag{6}$$

Conceptually, the ratio z_1/l controls the sensitivity of the entrainment rate to bed elevation changes. The additional constant factors are incorporated without loss of generality to simplify subsequent results and interpretations. Physically, z_1 is the bed elevation change induced by a single particle, and, as we will see, l is related to the standard deviation of bed elevation. To our knowledge, there is no data about the dependence of bedload deposition rates on relative bed elevation. However, *Wong et al.* [2007] hypothesized that the scaling of deposition with bed elevation was opposite of the scaling of entrainment, and we follow this:

$$\sigma(m)n = \sigma_0\left(1 - \frac{z_1}{(2l)^2}z(m)\right)n, \qquad \text{deposition rate at } (n, m). \tag{7}$$

$$\tag{8}$$

These equations incorporate the reasonable and expected trends that entrainment happens preferentially from crests, while deposition happens preferentially from troughs.

Using these transitions and their rates, we can write the master equation for the probability $P(n, m; t)$ that there are n moving particles and m stationary particles at time t in the standard way [*Cox and Miller*, 1965; *Ancey et al.*, 2008]:

$$\frac{\partial P}{\partial t}(n, m; t) = \nu P(n - 1, m; t) + \{\lambda(m + 1) + [n - 1]\mu(m + 1)\}P(n - 1, m + 1; t)$$
$$+ [n + 1]\sigma(m - 1)P(n + 1, m - 1; t) + [n + 1]\gamma P(n + 1, m; t)$$
$$- \{\nu + \lambda(m) + n\mu(m) + n\sigma(m) + n\gamma\}P(n, m; t). \tag{9}$$

The master equation 9 reduces to the master equations of *Ancey et al.* [2008] and *Nakagawa and Tsujimoto* [1980] under appropriate limits. When $z_1/l \to 0$, meaning bed elevation changes have negligible effects on particle mobility, we obtain the *Ancey et al.* [2008] results for bedload transport. When $z_1/l \to \infty$, meaning bed elevation changes dominate particle mobility, we obtain the *Nakagawa and Tsujimoto* [1980] results for bed elevations. In the intermediate region, bed elevation changes are a negative feedback on bedload transport.

This master equation for the populations (n, m) equivalently describes the probability of a given bedload rate q_s and bed elevation $z(m)$, because the bedload rate, in units of number per unit time, can be written in terms of n as a sum of velocities of each moving particle, $q_s = \sum_{i=1}^{n} u_i$, and the spatially averaged bed elevation across the control volume is related to m via equation 1. In general, the particle velocities u_i are identically distributed random variables which lie on a thin-tailed distribution [*Lajeunesse et al.*, 2010; *Furbish et al.*, 2012; *Fathel et al.*, 2015], although many authors have treated them as constants [*Wiberg and Smith*, 1985; *Charru et al.*, 2004; *Ancey et al.*, 2008]. A conflicting factor is that particles in different modes of motion (rolling, sliding, bouncing) may require separate velocity distributions under certain conditions, in order to properly characterize the transport [*Frey*, 2014]. For simplicity, we neglect these issues.

The structure of the master equation 9, as an infinite system of non-linear partial differential equations, probably restricts any analytical solution for $P(n, m; t)$. Furthermore, the statistical moments of n and m are difficult to obtain, although generating function approaches are regularly applied to similar problems [*Swift*, 2002]. In this case, the moment equations derived from 9 do not close, meaning they are also an infinite hirearchy, so the problem is not much simpler than solving the original master equation. Accordingly, we have to take a numerical approach to proceed. Luckily, numerical simulation schemes for these types of population dynamics have been a topic of intense research, as there is a requirement to solve similar equations in chemical physics. From these algorithms, weighing simplicity against efficiency, we choose the classic and straightforward stochastic simulation algorithm (SSA) of Gillespie [*Gillespie*, 1977, 2007], which simulates exact realizations (time series) of the joint population dynamics $(n(t), m(t))$, consistent with the master equation 9. We collect many such realizations from repeated simulation, and we use this approximate ensemble to study the statistics of n and m, i.e., of q_s and z. In the next section, we present simulation results regarding the statistics of bedload transport, bed elevation changes, and their emergent property, sediment burial.

4 Simulation Results

This results are tentative, and I need to do more simulations. However, this is a start.

4.1 bedload transport and bed elevation

4.2 sediment burial

5 Discussion

6 Conclusion

References

Ancey, C., and J. Heyman, A microstructural approach to bed load transport: mean behaviour and fluctuations of particle transport rates, *Journal of Fluid Mechanics*, *744*, 129–168, 2014.

Ancey, C., A. C. Davison, T. Böhm, M. Jodeau, and P. Frey, Entrainment and motion of coarse particles in a shallow water stream down a steep slope, *Journal of Fluid Mechanics*, *595*, 83–114, 2008.

Bagnold, R. A., The nature of saltation and of 'bed-load' transport in water, *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, *332*, 473–504, 1973.

Bialik, R. J., V. I. Nikora, M. Karpiński, and P. M. Rowiński, Diffusion of bedload particles in open-channel flows: distribution of travel times and second-order statistics of particle trajectories, *Environmental Fluid Mechanics*, *15*, 1281–1292, 2015.

Böhm, T., C. Ancey, P. Frey, J. L. Reboud, and C. Ducottet, Fluctuations of the solid discharge of gravity-driven particle flows in a turbulent stream, *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics*, *69*, 1–13, 2004.

Bradley, D. N., Direct Observation of Heavy-Tailed Storage Times of Bed Load Tracer Particles Causing Anomalous Superdiffusion, *Geophysical Research Letters*, *44*, 12,227–12,235, 2017.

Celik, A. O., Experimental investigation of the role of turbulence fluctuations on incipient motion of sediment, Phd thesis, Virginia Polytechnic Institute and State University, 2011.

Charru, F., H. Mouilleron, and O. Eiff, Erosion and deposition of particles on a bed sheared by a viscous flow, *Journal of Fluid Mechanics*, *519*, 55–80, 2004.

Cheng, N. S., Analysis of bedload transport in laminar flows, *Advances in Water Resources*, *27*, 937–942, 2004.

Church, M., Bed Material Transport and the Morphology of Alluvial River Channels, *Annual Review of Earth and Planetary Sciences*, *34*, 325–354, 2006.

Church, M., What is the "active layer"?, *Water Resources Research*, *53*, 5–10, 2017.

Coleman, N. L., A theoretical and experimental study of drag and lift forces acting on a sphere resting on a hypothetical streambed, in *12th IAHR Congress*, Littleton, CO, 1967.

Cox, D. R., and H. Miller, *The Theory of Stochastic Processes*, Chapman and Hall, London, 1965.

Crickmore, M., and G. Lean, The measurement of sand transport by means of radioactive tracers, *Proc. Roy. Soc. A.*, *266*, 402–421, 1962.

Cundall, P. A., and O. D. L. Strack, A discrete numerical model for granular assemblies, *Géotechnique*, *29*, 47–65, 1979.

Dey, S., S. Z. Ali, S. Dey, and S. Z. Ali, Review Article : Advances in modeling of bed particle entrainment sheared by turbulent flow Review Article : Advances in modeling of bed particle entrainment sheared by turbulent flow, *Physics of Fluids*, *30*, 2018.

Dhont, B., and C. Ancey, Are Bedload Transport Pulses in Gravel Bed Rivers Created by Bar Migration or Sediment Waves?, *Geophysical Research Letters*, *45*, 5501–5508, 2018.

Dwivedi, A., B. Melville, A. J. Raudkivi, A. Y. Shamseldin, and Y.-M. Chiew, Role of Turbulence and Particle Exposure on Entrainment of Large Spherical Particles in Flows with Low Relative Submergence, *Journal of Hydraulic Engineering*, *138*, 1022–1030, 2012.

Egiazaroff, I., Calculation of non-uniform sediment concentrations, *Journal of Hydraulic Engineering*, *91*, 225–248, 1965.

Einstein, H. A., Bed-load transport as a probability problem, Ph.D. thesis, ETH Zurich, 1937.

Einstein, H. A., The Bed-Load Function for Sediment Transportation in Open Channel Flows, *Tech. Rep. 1026*, United States Department of Agriculture, Washington, DC, 1950.

Fan, N., Y. Xie, and R. Nie, Bed load transport for a mixture of particle sizes: Downstream sorting rather than anomalous diffusion, *Journal of Hydrology*, *553*, 26–34, 2017.

Fathel, S. L., D. J. Furbish, and M. W. Schmeeckle, Experimental evidence of statistical ensemble behavior in bed load sediment transport, *Journal of Geophysical Research F: Earth Surface*, *120*, 2298–2317, 2015.

Ferreira, R. M. L., M. A. Hassan, and C. Ferrer-Boix, *Rivers – Physical, Fluvial and Environmental Processes*, Springer International, Switzerland, 2015.

Frey, P., Particle velocity and concentration profiles in bedload experiments on a steep slope, *Earth Surface Processes and Landforms*, *39*, 646–655, 2014.

Furbish, D. J., A. E. Ball, and M. W. Schmeeckle, A probabilistic description of the bed load sediment flux: 4. Fickian diffusion at low transport rates, *Journal of Geophysical Research: Earth Surface*, *117*, 1–13, 2012.

Gaeuman, D., R. Stewart, B. Schmandt, and C. Pryor, Geomorphic response to gravel augmentation and high-flow dam release in the Trinity River, California, *Earth Surface Processes and Landforms*, *42*, 2523–2540, 2017.

Gardiner, C., *Handbook of stochastic methods for physics, chemistry, and the natural sciences*, Springer-Verlag, 1983.

Gillespie, D. T., Exact stochastic simulation of coupled chemical reactions, *Journal of Physical Chemistry*, *81*, 2340–2361, 1977.

Gillespie, D. T., *Markov Processes: An Introduction For Physical Sciences*, Academic Press, Inc., 1992.

Gillespie, D. T., Stochastic Simulation of Chemical Kinetics, *Annual Review of Physical Chemistry*, *58*, 35–55, 2007.

Habersack, H. M., Radio-tracking gravel particles in a large braided river in New Zealand: A field test of the stochastic theory of bed load transport proposed by Einstein, *Hydrological Processes*, *15*, 377–391, 2001.

Heyman, J., F. Mettra, H. B. Ma, and C. Ancey, Statistics of bedload transport over steep slopes: Separation of time scales and collective motion, *Geophysical Research Letters*, *40*, 128–133, 2013.

Heyman, J., P. Bohorquez, and C. Ancey, Entrainment, motion, and deposition of coarse particles transported by water over a sloping mobile bed, *Journal of Geophysical Research: Earth Surface*, *121*, 1931–1952, 2016.

Hofland, B., Rock & Roll, Doctoral dissertation, Delft University of Technology, 2005.

Lajeunesse, E., L. Malverti, and F. Charru, Bed load transport in turbulent flow at the grain scale: Experiments and modeling, *Journal of Geophysical Research: Earth Surface*, *115*, 2010.

Ma, H., J. Heyman, X. Fu, F. Mettra, C. Ancey, and G. Parker, Bed load transport over a broad range of timescales: Determination of three regimes of fluctuations, *Journal of Geophysical Research*, *119*, 1–21, 2014.

Malmon, D. V., T. Dunne, S. L. Reneau, S. The, and N. September, Stochastic Theory of Particle Trajectories through Alluvial Valley Floors, *The Journal of Geology*, *111*, 525–542, 2003.

Malmon, D. V., S. L. Reneau, T. Dunne, D. Katzman, and P. G. Drakos, Influence of sediment storage on downstream delivery of contaminated sediment, *Water Resources Research*, *41*, 1–17, 2005.

Martin, R. L., D. J. Jerolmack, and R. Schumer, The physical basis for anomalous diffusion in bed load transport, *Journal of Geophysical Research: Earth Surface*, *117*, 1–18, 2012.

Martin, R. L., P. K. Purohit, and D. J. Jerolmack, Sedimentary bed evolution as a mean-reverting random walk: Implications for tracer statistics, *Geophysical Research Letters*, *41*, 6152–6159, 2014.

McEwan, I., M. Sørensen, J. Heald, S. Tait, G. Cunningham, D. Goring, and B. Willetts, Probabilistic Modeling of Bed-Load Composition, *Journal of Hydraulic Engineering*, *130*, 129–139, 2004.

Nakagawa, H., and T. Tsujimoto, On Probabilistic Characteristics of Motion of Individual Sediment Particles on Stream Beds, in *Hydraulic Problems Solved by Stochastic Methods: Second International IAHR Symposium on Stochastic Hydraulics*, Lund, Sweden, 1976.

Nakagawa, H., and T. Tsujimoto, Sand bed instability due to bed load motion, *Journal of the Hydraulics Division-ASCE*, 1980.

Olinde, L., and J. P. Johnson, Using RFID and accelerometer-embedded tracers to measure probabilities of bed load transport, step lengths, and rest times in a mountain stream, *Water Resources Research*, *51*, 7572–7589, 2015.

Paintal, A. S., A Stochastic Model Of Bed Load Transport, *Journal of Hydraulic Research*, *9*, 527–554, 1971.

Pielou, E., *Mathematical Ecology*, 1 ed., John Wiley & Sons Ltd., New York, NY, 1977.

Sawai, K., Dispersion of bed load particles, *Bull. Disas. Prev. Res. Inst., Kyoto Univ.*, *37*, 1987.

Sayre, W. W., and W. J. Conover, General two-dimensional stochastic model for the transport and dispersion of bed-material sediment particles, in *Proceedings of the 12th Congress IAHR*, pp. 88–95, Colorado State University, Fort Collins, Co, USA, 1967.

Sayre, W. W., and D. W. Hubbell, Transport and Dispersion of Labeled Bed Material North Loup River , Nebraska, *Transport of Radionuclides by Streams*, 1965.

Schumer, R., M. M. Meerschaert, and B. Baeumer, Fractional advection-dispersion equations for modeling transport at the Earth surface, *Journal of Geophysical Research: Earth Surface*, *114*, 1–15, 2009.

Shen, H., Stochastic sediment bed load models, in *Application of Stochastic Processes in Sediment Transport*, edited by H. Shen and C. Fatt, Water Resources Publications, Littleton, CO, 1980.

Singh, A., K. Fienberg, D. J. Jerolmack, J. Marr, and E. Foufoula-Georgiou, Experimental evidence for statistical scaling and intermittency in sediment transport rates, *Journal of Geophysical Research: Earth Surface*, *114*, 1–16, 2009.

Sun, Z., and J. Donahue, Statistically derived bedload formula for any fraction of nonuniform sediment, *Journal of Hydraulic Engineering*, *126*, 105–111, 2000.

Swift, R. J., A Stochastic Predator-Prey Model, *Bulletin of the Irish Mathematical Society*, *48*, 57–63, 2002.

Turowski, J. M., Stochastic modeling of the cover effect and bedrock erosion, *Water Resources Research*, *45*, 1–13, 2009.

van Kampen, N. G., *Stochastic Processes in Physics and Chemistry*, Elsevier Science Publishers B.B., Amsterdam, 1992.

Voepel, H., R. Schumer, and M. A. Hassan, Sediment residence time distributions: Theory and application from bed elevation measurements, *Journal of Geophysical Research: Earth Surface*, *118*, 2557–2567, 2013.

Vowinckel, B., R. Jain, T. Kempe, J. Frohlich, J. Fr??hlich, and J. Frohlich, Entrainment of single particles in a turbulent open-channel flow: A numerical study, *Journal of Hydraulic Research*, *54*, 158–171, 2016.

Wiberg, P. L., and J. D. Smith, A theoretical model for saltating grains in water, *Journal of Geophysical Research*, *90*, 7341, 1985.

Wong, M., G. Parker, P. DeVries, T. M. Brown, and S. J. Burges, Experiments on dispersion of tracer stones under lower-regime plane-bed equilibrium bed load transport, *Water Resources Research*, *43*, 1–23, 2007.

Yang, C. T., and W. W. Sayre, Stochastic model for sand dispersion, *Journal of the Hydraulics Division, ASCE*, *97*, 1971.

Yano, K., Y. Tsuchiya, and M. Michiue, Studies on the Sand Transport in Streams with Tracers, *Bull. Disas. Prev. Res. Inst. Kyoto Univ.*, *18*, 1969.

Zhang, Y., M. M. Meerschaert, and A. I. Packman, Linking fluvial bed sediment transport across scales, *Geophysical Research Letters*, *39*, 1–6, 2012.