James K. Pringle
550.621 Probability
Dr. Jim Fill
Assignment 3
March 26, 2013

Assignment 3

Problem 6.3.7 in Chung

6.3.7 Show: If $F_n \xrightarrow{v} F$ and $G_n \xrightarrow{v} G$, then $F_n * G_n \xrightarrow{v} F * G$.

Proof. Let F_n, G_n, F, and G be d.f.'s with ch.f.'s f_n, g_n, f, and g, respectively. By the properties of convolutions, $F_n * G_n$ and $F * G$ are d.f.'s, so let their respective p.m.'s be μ_n and μ. Let $F_n \xrightarrow{v} F$ and $G_n \xrightarrow{v} G$. Thus by theorem 6.3.1, $f_n \to f$ and $g_n \to g$ uniformly on every finite interval. In particular, that convergence holds on $[t-\epsilon, t+\epsilon]$ for all $t \in \mathbb{R}$ and for all finite $\epsilon > 0$. This implies

$$f_n \to f \quad \text{and} \quad g_n \to g \tag{1}$$

pointwise on $\mathbb{R}$. By theorems 6.1.4 and 3.3.4, it follows that $F_n * G_n$ has ch.f. $f_n g_n$ and $F * G$ has ch.f. fg. By properties of a limit and (1), $f_n g_n \to fg$ pointwise in $\mathbb{R}$. By the properties of characteristic functions, fg is continuous at 0. Hence by theorem 6.3.2, $\mu_n \xrightarrow{v} \mu$, and it follows that $F_n * G_n \xrightarrow{v} F * G$. $\square$