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Assignment 1
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# Assignment 1

*Find the asymptotic equivalent of* $\frac{x}{\log x}$

1. Let
$$y = f(x) = \frac{x}{\log x}$$
Then
$$\log y = \log x - \log(\log x)$$
It is a fact (can show with L'Hospital's rule) that
$$\lim_{x\to\infty} \frac{\log(\log x)}{\log x} = 0$$
Therefore
$$\lim_{x\to\infty} \frac{\log y}{\log x} = \lim_{x\to\infty} \frac{\log x - \log(\log x)}{\log x} = 1$$
Hence
$$\begin{aligned} \log y &\sim \log x \\ y\log y &\sim y\log x \\ y\log y &\sim x \quad \text{by definition of } y \\ y\log y &\sim f^{-1}(y) \quad \text{by definition of } f \end{aligned}$$

Source: Hardy and Wright *An Introduction to the Theory of Numbers* (1975) pg. 10