

1)

$$H(y) = - \sum_{i=1}^n P_i \log(P_i) =$$

$$- \left[\frac{6}{10} \log_2 \frac{6}{10} + \frac{4}{10} \log_2 \frac{4}{10} \right] = 0.97095$$

$$H(y) = 0.97095$$

2)

$$\text{Information Gain } IG = H_p - \left(\frac{C_1}{P} H_c^1 + \frac{C_2}{P} H_c^2 \right)$$

$$x_1 = \text{author}, x_2 = \text{long}, x_3 = \text{research}, x_4 = \text{grade}, x_5 = \text{lottery}$$

$$IG_{x_1} = 0.97095 - \left[\frac{6}{10} * -1 \left[\left(\frac{3}{6} \log_2 \frac{3}{6} + \frac{3}{6} \log_2 \frac{3}{6} \right) \right] + \frac{4}{10} * -1 \left[\left(\frac{3}{4} \log_2 \frac{3}{4} + \frac{1}{4} \log_2 \frac{1}{4} \right) \right] \right]$$

$$IG_{x_1} = 0.04644$$

$$IG_{x_2} = 0.97095 - \left[\frac{5}{10} * -1 \left[\left(\frac{5}{5} \log_2 \frac{5}{5} + \frac{0}{5} \log_2 \frac{0}{5} \right) \right] + \frac{5}{10} * -1 \left[\left(\frac{1}{5} \log_2 \frac{1}{5} + \frac{4}{5} \log_2 \frac{4}{5} \right) \right] \right]$$

$$IG_{x_2} = 0.60999$$

$$IG_{x_3} = 0.97095 - \left[\frac{7}{10} * -1 \left[\left(\frac{4}{7} \log_2 \frac{4}{7} + \frac{3}{7} \log_2 \frac{3}{7} \right) \right] + \frac{3}{10} * -1 \left[\left(\frac{2}{3} \log_2 \frac{2}{3} + \frac{1}{3} \log_2 \frac{1}{3} \right) \right] \right]$$

$$IG_{x_3} = 0.00580$$

$$IG_{x_4} = 0.97095 - \left[\frac{7}{10} * -1 \left[\left(\frac{5}{7} \log_2 \frac{5}{7} + \frac{2}{7} \log_2 \frac{2}{7} \right) \right] + \frac{3}{10} * -1 \left[\left(\frac{1}{3} \log_2 \frac{1}{3} + \frac{2}{3} \log_2 \frac{2}{3} \right) \right] \right]$$

$$IG_{x_4} = 0.09128$$

$$IG_{x_5} = 0.97095 - \left[\frac{3}{10} * -1 \left[\left(\frac{2}{3} \log_2 \frac{2}{3} + \frac{1}{3} \log_2 \frac{1}{3} \right) \right] + \frac{7}{10} * -1 \left[\left(\frac{4}{7} \log_2 \frac{4}{7} + \frac{3}{7} \log_2 \frac{3}{7} \right) \right] \right]$$

$$IG_{x_5} = 0.00580$$

You should split on x_2 for the root node because it has largest Information Gain.

3) TREE: If the statement is true, go to the right. If false, go to the left.

