Regularized Anderson Acceleration for Off-Policy Deep Reinforcement Learning

Wenjie Shi, Shiji Song, Hui Wu, Ya-Chu Hsu, Cheng Wu, Gao Huang
Tsinghua University, Beijing, China

MOTIVATION

- Sample inefficiency remains a major limitation of RL for problems with continuous and high dimensional state spaces.
- Sample inefficiency makes learning in real physical systems impractical and severely prohibits the applicability of RL approaches in challenging scenarios.

Existing methods

- To learn the model of system dynamics.
- Applying off-policy training scheme to reuse past experience.

Our method

- RL is closely linked to fixed-point iteration: the optimal policy can be found by solving a fixed-point problem of Bellman operator.
- Anderson acceleration is a method capable of speeding up the computation of fixed point iterations.

METHOD

Anderson Acceleration for Policy Iteration

For a policy iteration, suppose that estimates have been computed up to iteration k, and that in addition to the current estimate Q^{π_k}, the $m-1$ previous estimates $Q^{\pi_{k-1}},\ldots, Q^{\pi_{k-m+1}}$ are also known. Then, a linear combination of estimates Q^{π_i} with coefficients α_i reads

$$Q_\alpha^k = \sum_{i=1}^{m} \alpha_i Q^{\pi_{k-m+i}} \text{ with } \sum_{i=1}^{m} \alpha_i = 1.$$

We define combined Bellman operator $\mathcal{T}_c$ as follows

$$\mathcal{T}_c Q_\alpha^k = \sum_{i=1}^{m} \alpha_i \mathcal{T} Q^{\pi_{k-m+i}}.$$

Then, one searches a vector α^k that minimizes the objective function defined as the combined Bellman residuals among the entire state-action space $\mathcal{S}\times\mathcal{T}$,

$$\alpha^k = \underset{\alpha\in\mathbb{R}^m}{\operatorname{argmin}}\, J(\alpha) = \underset{\alpha\in\mathbb{R}^m}{\operatorname{argmin}} \left\| \sum_{i=1}^{m} \alpha_i (\mathcal{T} Q^{\pi_{k-m+i}} - Q^{\pi_{k-m+i}}) \right\|, \quad \text{s.t.} \sum_{i=1}^{m} \alpha_i = 1.$$

Regularized variant with function approximation

Challenges

- Sweeping entire state-action space is intractable.
- Function approximation errors are unavoidable.
- The solution may suffer from ill-conditioning when the squared Bellman residuals matrix is rank-deficient.

Adding a regularization term to the objective function,

$$\widetilde{\alpha}^k = \underset{\alpha\in\mathbb{R}^m}{\operatorname{argmin}} \left\| \sum_{i=1}^{m} \alpha_i (\mathcal{T} Q^{\pi_{k-m+i}} - Q^{\pi_{k-m+i}} + e_{k-m+i}) \right\| + \lambda \|\alpha\|^2, \quad \text{s.t.} \sum_{i=1}^{m} \alpha_i = 1,$$

where e_{k-m+i} represents the perturbation induced by function approximation errors, λ controls the scale of regularization. The solution can be obtained analytically

$$\widetilde{\alpha}^k = \frac{(\widetilde{\Delta}_k^T \widetilde{\Delta}_k + \lambda I)^{-1}\mathbf{1}}{\mathbf{1}^T(\widetilde{\Delta}_k^T \widetilde{\Delta}_k + \lambda I)^{-1}\mathbf{1}}, \qquad \widetilde{\Delta}_k = [\widetilde{\delta}_{k-m+1},\ldots,\widetilde{\delta}_k] \in \mathbb{R}^{N_A\times m}, \quad \widetilde{\delta}_i = \mathcal{T}Q^{\pi_i} - Q^{\pi_i} + e_i \in \mathbb{R}^{N_A}$$

Proposition 1: *Consider two identical PIs $\mathcal{I}_1$ and $\mathcal{I}_2$ with function approximation. $\mathcal{I}_2$ is implemented with RAA and takes into account approximation errors, whereas $\mathcal{I}_1$ is only implemented with vanilla Anderson acceleration. Let α^k and $\widetilde{\alpha}^k$ be the coefficient vectors of $\mathcal{I}_1$ and $\mathcal{I}_2$ respectively. Then, we have the following bounds*

$$\|\widetilde{\alpha}^k\| \le \sqrt{\frac{\lambda + \|\widetilde{\Delta}_k\|^2}{m\lambda}}, \quad \|\widetilde{\alpha}^k - \alpha^k\| \le \frac{\|\widetilde{\Delta}_k^T\widetilde{\Delta}_k - \Delta_k^T\Delta_k\| + \lambda}{\lambda}\|\alpha^k\|.$$

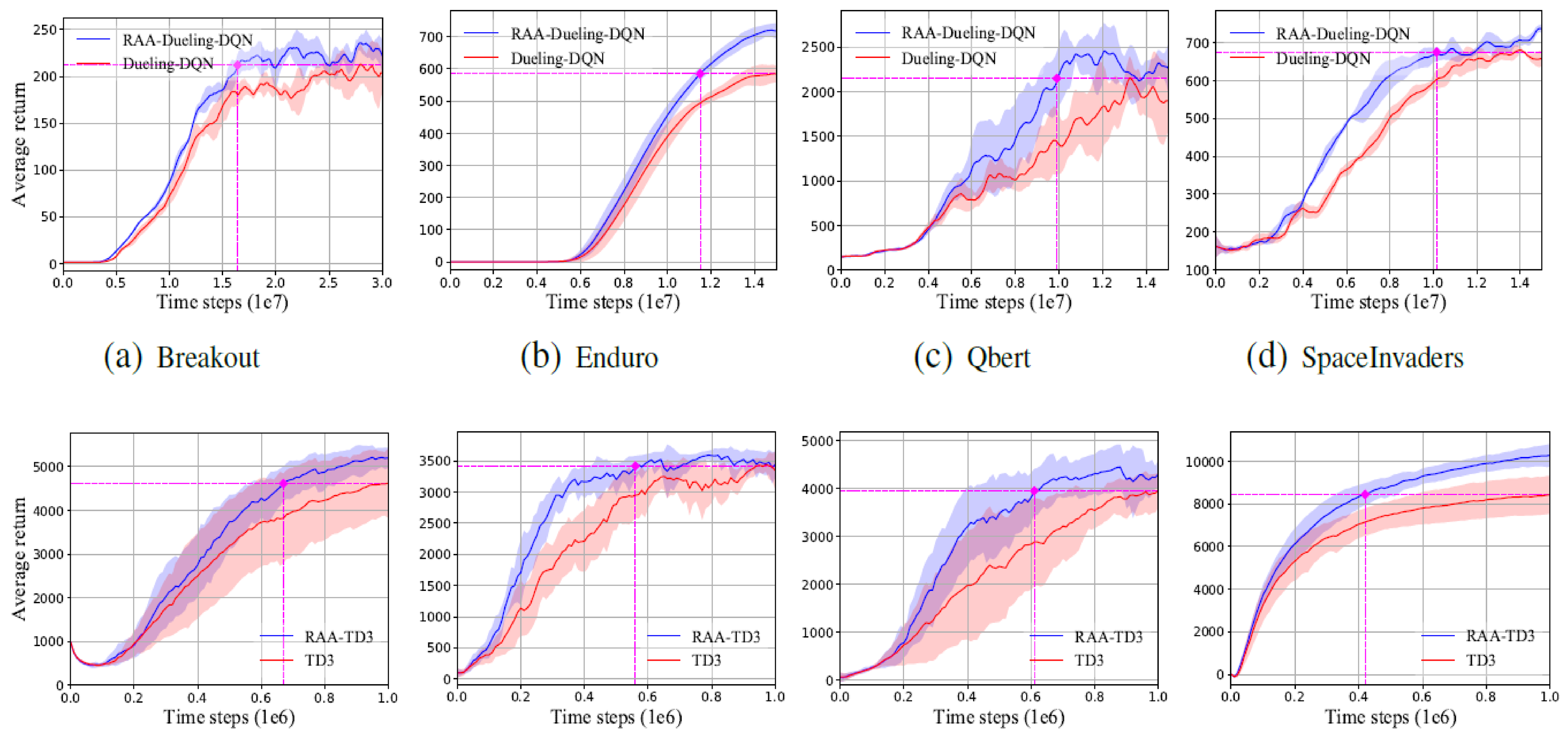
Regularized Anderson acceleration for actor-critic

Under the paradigm of off-policy deep RL (actor-critic), RAA variant of policy iteration degrades into the Bellman equation

RAA-Dueling-DQN: $Q_\theta(s_t,a_t) = \mathbb{E}_{s_{t+1},r_t}\left[r_t + \gamma \sum_{i=1}^{m} \widetilde{\alpha}_i \max_{a_{t+1}} Q_{\theta^i}(s_{t+1},a_{t+1})\right],$

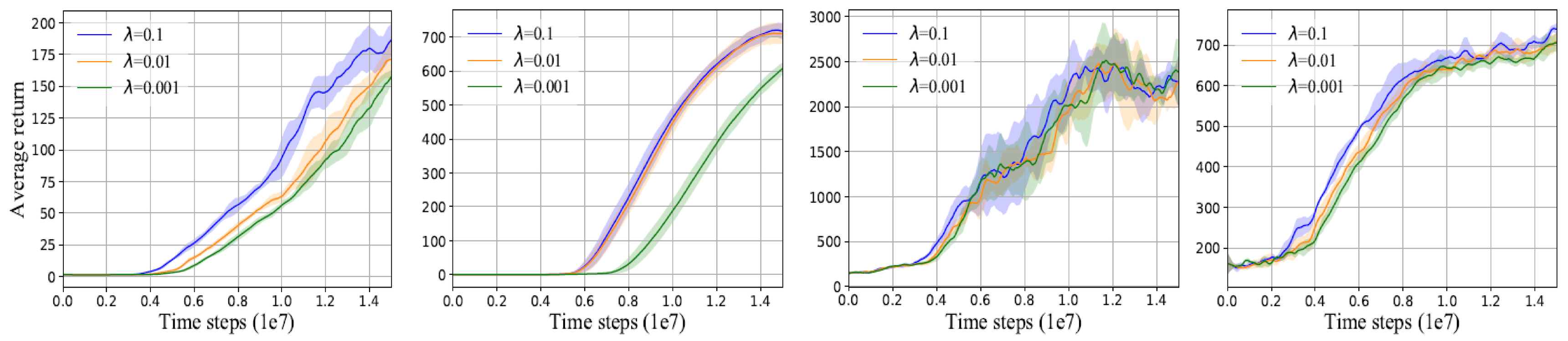
RAA-TD3: $Q_\theta(s_t,a_t) = \mathbb{E}_{s_{t+1},r_t}\left[r_t + \gamma \sum_{i=1}^{m} \widetilde{\alpha}_i \widehat{Q}_{\theta^i}(s_{t+1},\pi_{\phi'}(s_{t+1})+\epsilon)\right], \ \widehat{Q}_{\theta^i}(s_t,a_t) = \min_{j=1,2} Q_{\theta^i_j}(s_t,a_t).$

EXPERIMENTS



(a) Breakout (b) Enduro (c) Qbert (d) SpaceInvaders
(e) Ant-v2 (f) Hopper-v2 (g) Walker2d-v2 (h) HalfCheetah-v2

Figure 1: Learning Curves of Dueling-DQN, TD3 and their RAA variants on discrete and continuous control tasks.



(a) Breakout (b) Enduro (c) Qbert (d) SpaceInvaders

Figure 2: Sensitivity of RAA-Dueling-DQN to the scaling of regularization on discrete control tasks.

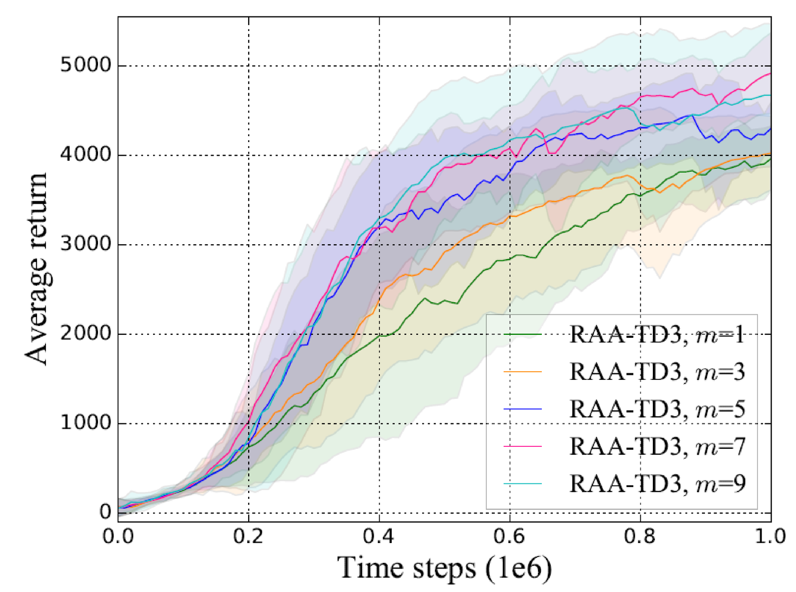


Figure 3: Learning Curves of RAA-TD3 on Walker2d-v2 with different m.

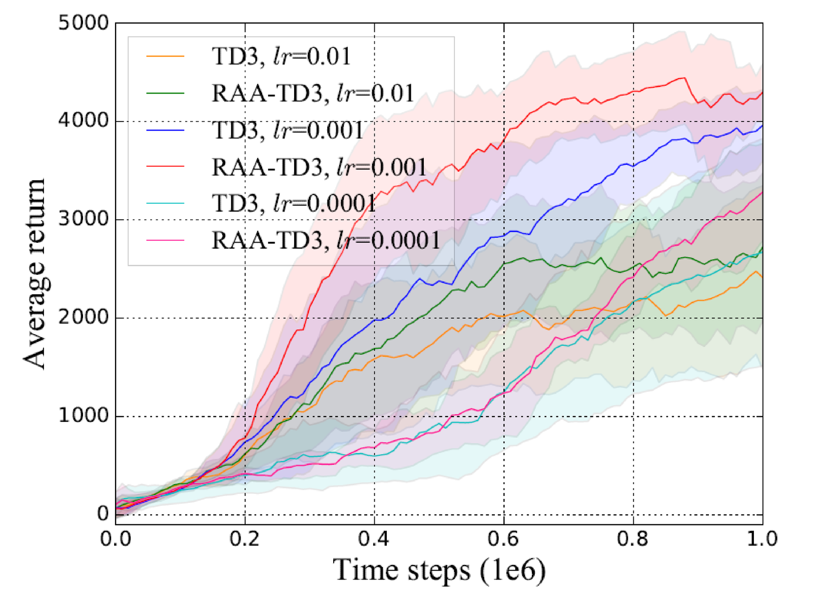


Figure 4: Performance comparison on Walker2d-v2 with different learning rates.

Thirty-third Conference on Neural Information Processing Systems (NeurIPS 2019)