

# **Enhanced Sensitivity Interferometry for Probing Post-Merger Gravitational Waves: Investigating a Novel L-Shaped Interferometer Configuration**

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Except where otherwise indicated, this thesis is my own original work.

Jessica Ross  
22 August 2024

to my xxx, yyy (yyy is the people you want to dedicated this thesis to.)



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# Acknowledgments

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Who do you want to thank?





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# Abstract

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Gravitational waves, first detected in 2015, have opened new avenues in astrophysics, cosmology, and fundamental physics. Among the various sources of gravitational waves, merging binary neutron stars offer unique insights into the physics of extreme environments and cosmological phenomena. While current ground-based detectors have observed over 90 compact binary merger events, post-merger signals remain poorly understood, particularly regarding the fate of the merging objects and the equation of state for neutron stars. This project aims to investigate a novel L-shaped interferometer configuration, which promises enhanced sensitivity to detect post-merger gravitational wave signals. Building upon previous research, this project will develop an optical model using Finesse in Python to characterise the interferometer's behaviour and assess its potential to extract information about the equation of state for binary neutron star mergers. By simulating the interferometer's performance and estimating event detection rates through Monte Carlo simulations, this study seeks to advance our understanding of post-merger signals and contribute to the development of future and current gravitational wave detectors. The proposed research program outlines a timeline for literature review, model development, testing, and analysis, aiming to yield significant insights into the physics of neutron stars and the broader field of gravitational wave astronomy.



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# Introduction

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## 1.1 Thesis Statement

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## 1.2 Thesis Outline

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# Gravitational Waves

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Gravitational wave, predicted by Einstein's General Theory of Relativity, are ripples in the fabric of spacetime generated by accelerating massive objects. These waves emerge when highly energetic processes, such as mergers of black holes or neutron stars, disturb spacetime's curvature. Gravitational waves propagate at the speed of light, carrying energy away from these systems and enabling a new window of exploration into the universe.

For decades, these waves remained theoretical, as the necessary technology to detect such minuscule distortions in spacetime was far beyond our reach. However, this changed with the advent of highly sensitive laser interferometers. In 2015, the Laser Interferometer Gravitational-Wave Observatory (LIGO) detected the first direct signal of gravitational waves from a binary black hole merger (GW150914) [Abbott, 2016b], the first gravitational wave signal can be shown in 2.1 **note: trouble inputting figure**, showing the inspiral, merger and ringdown phases. Since 2015, over 90 compact binary merger events have been observed by current generation detectors, including Virgo and KAGRA [Zhang et al., 2023]. This groundbreaking detection opened a new window into the universe, allowing astrophysicists to observe cosmic events that were previously invisible.

The basic principle behind these detectors is relatively simple: gravitational waves stretch and compress spacetime, altering the distances between objects. Laser interferometers, such as LIGO and Virgo, detect these changes by measuring the interference patterns of light travelling between mirrors spaced kilometres apart. When a gravitational wave passes through the detector, it causes one arm to lengthen and the other to shorten, creating a detectable phase shift in the laser beams.

## 2.1 Theoretical Foundation of Gravitational Waves

Einstein's field equations of General Relativity describes how mass and energy cause spacetime to curve, influencing the motion of objects. When massive bodies accelerate, especially in non-spherical configurations like binary star systems, they produce time-dependent changes in the curvature of spacetime that propagate outward as

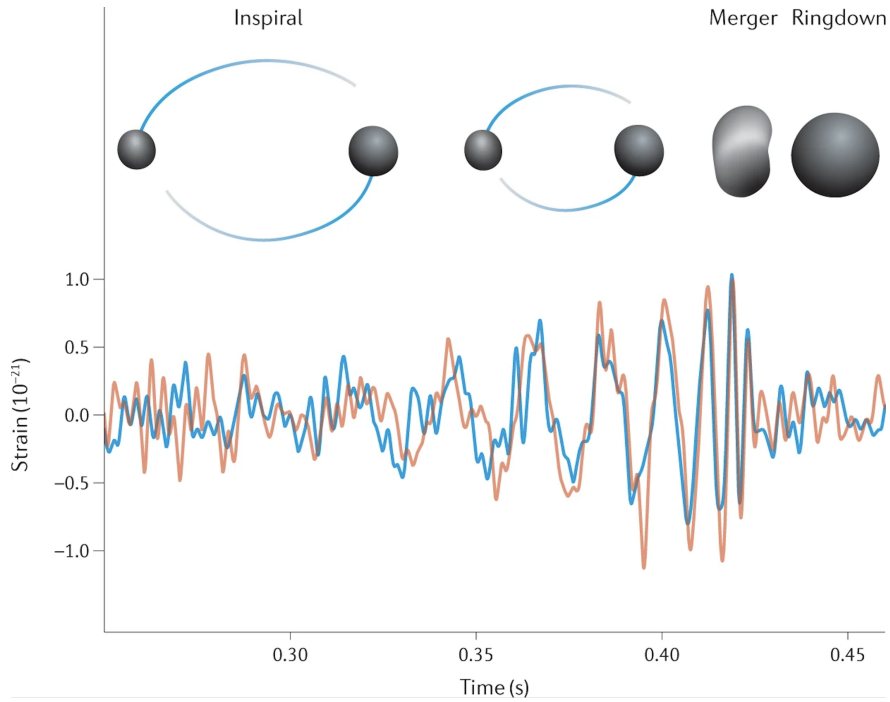


Figure 2.1: The detected gravitational-wave strain amplitude as a function of time for GW150914 the first signal detected nearly simultaneously by the LIGO Hanford and Livingston observatories on September 14, 2015. Credit: Bailes [2021]

gravitational waves. These waves carry energy, reducing the mass and orbital energy of the objects generating them. The amplitude of the waves decreases with distance from the source, but they are powerful enough to affect spacetime across vast distances [Abbott, 2016b].

The most significant sources of gravitational waves is compact binary coalescence's (CBC's), involving mergers of objects such as black holes and neutron stars. These mergers create a distinct gravitational wave signature with three phases: inspiral, merger, and ringdown. During the inspiral phase, as the two objects orbit each other and spiral inward, the frequency and amplitude of the gravitational waves gradually increase in what is referred to as a “chirp”. The final moments of the merger produce a sharp peak in the signal, and the ringdown phase marks the settling of the merged object into a stable state. These phases offer insights into the masses, spins, and other properties of the objects [Buonanno and Damour, 1999; Maggiore et al., 2020].

## 2.2 Gravitational Wave Detection and Observations

The detection of gravitational waves relies on highly sensitive instruments such as the Laser Interferometer Gravitational-Wave Observatory (LIGO) and Virgo. These



detectors use laser interferometry to measure minute distortions in spacetime caused by passing gravitational waves. The first direct detection of gravitational waves was made in 2015 by LIGO from a binary black hole merger, GW150914 [Abbott, 2016b]. This groundbreaking discovery confirmed Einstein’s theory and opened a new field of observational astronomy.

Since then, numerous other detections have followed, including both black hole and neutron star mergers. These observations have provided critical tests of General Relativity in the strong-field region and revealed a population of black holes with masses previously unknown through electromagnetic methods [Abbott, 2017b]. Gravitational wave astronomy has also allowed scientists to investigate the nature of neutron stars, providing insights into the behaviour of matter at nuclear densities.

## 2.3 Gravitational Waves as Probes of Fundamental Physics

As the detection of gravitational waves allows for unprecedented tests of General Relativity and alternate theories of gravity in the strong-field region, by comparing observed waveforms with theoretical predictions, researchers can place constraints on deviations from Einstein’s theory. This has significant implications for fundamental physics, including the possibility of enhancing current theories or placing limits on phenomena like the existence of extra dimensions or modifications to gravity at high energies [Yunes and Siemens, 2013].

Gravitational waves also provide a direct probe of regions of the universe that are otherwise hidden from view, such as the interiors of supernovae or the immediate vicinity of black holes. Unlike electromagnetic waves, gravitational waves are not absorbed or scattered by matter, allowing them to carry information directly from these regions to Earth [Rosswog et al., 2018].

## 2.4 Implications for Cosmology

Gravitational waves offer an innovative way to measure cosmic distances and the expansion of the universe. The amplitude of a gravitational wave signal depends on the distance to the source, allowing researchers to infer the distance directly. When combined with redshift measurements from electromagnetic observations, this method provides a way to determine the Hubble constant, making gravitational waves a form of “standard siren” analogous to the role of Type Ia supernovae as standard candles in cosmology [Abbott, 2017d,a].

The binary neutron star merger GW170817 was a major event towards cosmology, as it allowed for an independent measurement of the Hubble constant. This measurement is particularly significant given the current tension between the estimates of the Hubble constant obtained from the cosmic microwave background and those based

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on local distance measurements using supernovae. Gravitational wave-based measurements offer a new perspective on this discrepancy and could help resolve the ongoing debate [Abbott, 2017b; Hotokezaka et al., 2019].

## 2.5 The Stochastic Gravitational Wave Background

One of the most captivating possibilities for future gravitational wave observations is the detection of the stochastic gravitational wave background [Abbott, 2016a]. This background is composed of the superposition of many unresolved sources of gravitational waves, such as the mergers of distant black holes and neutron stars, or even gravitational waves generated during the early universe. Detecting this background would provide a new way to study the history of the universe, including the processes that shaped its evolution and the formation of large-scale structures like galaxies and galaxy clusters [Pan et al., 2020; Maggiore et al., 2020].

The stochastic gravitational wave background may also carry information about the early universe, potentially providing evidence for phenomena such as cosmic inflation or phase transitions in the early universe. Detecting these primordial gravitational waves would offer a unique glimpse into the universe’s infancy, complementing observations of the cosmic microwave background and large-scale structures [Maggiore et al., 2020].

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# Neutron Stars

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## 3.1 Neutron Star Mergers and the Equation of State

Neutron stars are among the densest objects in the universe, formed when massive stars undergo supernovae explosions and their cores collapse. They typically have masses between 1.4 and 2 solar masses, but their radii are only about 10 km [Hua, 2022], creating densities up to  $10^{15}$  g/cm<sup>3</sup> [Chatziioannou, 2020; Weber, 2005]. Neutron star mergers provide a unique opportunity to study the properties of ultra-dense matter. Unlike black holes, neutron stars are composed of nuclear matter, and their collisions offer insights into the equation of state of such matter. The equation of state connects the relationship between pressure, density, and the temperature inside neutron stars. The gravitational waves emitted during the inspiral phase of a neutron star merger are influenced by the tidal deformability of the stars, which in turn depends on the equation of state. By comparing observed waveforms with theoretical models, scientists can place constraints on the equation of state and gain a deeper understanding of nuclear physics [Fattoyev et al., 2018].

For example, the detection of the neutron star merger GW170817 provided crucial data on the equation of state. The observed tidal deformability constrained several possible models, ruling out overly soft or stiff equations of state and suggesting an intermediate compressibility for neutron star matter. This has profound implications for nuclear physics, as it provides constraints on the possible compositions of neutron star interiors, including exotic states of matter like quark-gluon plasma [Abbott, 2017b; Chatziioannou, 2020].

## 3.2 Post-Merger Dynamics and High-Frequency Gravitational Waves

The post-merger phase of neutron star collisions provides insights into the behaviour of matter under extreme conditions. If the merger results in a hyper massive neutron star gravitational waves are emitted in the 2-4 kHz range [Ackley, 2020]. Observing these high-frequency signals is key to understanding the properties of ultra-dense matter and the equation of state at higher densities and temperatures. However,

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current detectors like LIGO and Virgo have limited sensitivity at these frequencies, preventing direct observation of post-merger signals.

### 3.3 Multi-Messenger Astronomy and Neutron Star Mergers

Neutron star mergers also play a critical role in multi-messenger astronomy, where gravitational wave observations are combined with electromagnetic counterparts to provide a more comprehensive view of astrophysical events [Abbott, 2017d]. For example, the detection of GW170817 was accompanied by a short gamma-ray burst (GRB) and a kilonova explosion [Abbott, 2017b]. This multi-messenger observation confirmed the connection between neutron star mergers and the production of heavy elements via r-process nucleosynthesis, such as gold and platinum [Abbott, 2018, 2017d; Thielemann et al., 2017].

### 3.4 Gravitational Waves and High-Energy Astrophysics

In high-energy astrophysics, gravitational waves provide a new means of studying phenomena that are otherwise difficult to observe. For example, the collapse of massive stars into black holes, or supernova explosions, are expected to produce gravitational waves, although no direct detection of such waves from a supernova has yet been made. Theoretical models predict that core-collapse supernovae could emit gravitational waves during the collapse and subsequent explosion of the star, particularly if the explosion is asymmetric or involves rapid rotation [Kuroda et al., 2016].

The formation of black holes in the aftermath of supernovae or neutron star mergers offers opportunities to study the behaviour of matter in extreme gravitational fields. Gravitational waves emitted during these processes provide a direct probe of the collapse dynamics, potentially revealing new insights into the mechanisms that drive supernova explosions and the formation of black holes [Rosswog et al., 2018; Ackley, 2020].

Neutron stars serve as natural laboratories for testing fundamental physics. Their extreme magnetic fields, which can reach  $10^{15}$  Gauss in magnetars, influence the behaviour of matter at the quantum level, impacting the equation of state and other astrophysical phenomena [Kaspi and Beloborodov, 2017]. Neutron stars can also be observed through their thermal emissions. Young neutron stars, like those found in supernova remnants, emit X-rays due to the cooling of their surfaces [Lattimer, 2010]. Studying these X-rays provides information about the star's surface composition and magnetic field strength. Additionally, neutron stars offer unique environments for studying the strong nuclear force and the behaviour of matter at densities exceeding that of atomic nuclei [Weber, 2005].

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Neutron stars allow researchers to explore regimes of quantum chromodynamics (QCD) that are inaccessible in terrestrial experiments. These conditions may permit the existence of exotic phases, such as colour superconductivity, or strong quark matter, challenging our current understanding of nuclear matter [Weber, 2005; Yunes et al., 2016].



# Detector Importance and Topologies

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## 4.1 Binary Black Hole Mergers and Implications for Astrophysics

Binary black hole mergers are among the most frequently detected sources of gravitational waves. These systems contain two black holes orbiting each other, losing energy through gravitational radiation until they merge. The frequency and amplitude of the emitted waves depends primarily on the masses of the black holes, with more massive black holes producing lower frequency signals but higher amplitudes. Observations of black hole mergers have revealed a population of stellar mass black holes with masses far exceeding previous expectations based on electromagnetic observations alone [Abbott, 2016b].

In addition to mass, the spin of black holes plays a crucial role in shaping the gravitational wave signal. If the spins of the black holes are misaligned with the orbital angular momentum, the system can exhibit precession, which introduces complexity to the waveform. By studying these effects, researchers gain insights into the formation history of the binary, such as whether the black holes formed through isolated binary evolution or dynamical encounters in dense environments like globular cluster [Rodriguez et al., 2016].

## 4.2 Gravitational Waves and Black Hole Thermodynamics

The study of black hole mergers through gravitational waves has implications for black hole thermodynamics, a field that explores the connections between black hole properties and the laws of thermodynamics. In particular, the areas of a black hole's event horizon, which increases during a merger, is closely related to the black hole's entropy, as described by Bekenstein-Hawking formula [Bekenstein, 1973]. The dynamics of black hole mergers offer a unique opportunity to study these thermodynamic relationships in action, providing new insights into the nature of entropy and

energy in gravitational systems [Buonanno and Damour, 1999; Sathyaprakash and Schutz, 2009].

### 4.3 Future Prospects for Gravitational Wave Astronomy

The future of gravitational wave astronomy is particularly exciting as new detectors come online and existing ones are upgraded. Next generation observatories like the Einstein Telescope and Cosmic Explorer are expected to increase sensitivity by an order of magnitude, allowing for the detection of weaker and more distant sources. This will enable scientists to observe previously undetectable events, such as the mergers of intermediate-mass black holes or the stochastic gravitational wave background from the early universe [Maggiore et al., 2020].

### 4.4 The Role of Gravitational Waves in Astrophysical Transients

Gravitational wave astronomy has complemented the study of astrophysical transients, which are short lived events that produce intense bursts of energy across multiple wavelengths. These transients, such as gamma-ray bursts (GRBs) and kilonovae, often accompany the mergers of compact objects like neutron stars or black holes. Gravitational wave detectors enable researchers to pinpoint the locations of these mergers, facilitating coordinated observations across the electromagnetic spectrum.

The multi-messenger approach allows astronomers to piece together a more complete picture of these events, including the processes leading to the emission of gravitational waves and the accompanying electromagnetic signals. For example, short gamma-ray bursts are believed to be produced during the final stages of neutron star mergers, when the resulting black hole or hyper massive neutron star drives jets of materials at relativistic speeds. The gravitational wave data provide information about the masses and spins of the merging neutron stars, while the gamma-ray and optical signals offer insight into the energetic processes that occur during and after the merger [Rosswog et al., 2018; Thielemann et al., 2017].

Kilonovae, the optical and infrared counterparts to neutron star mergers, are driven by the radioactive decay of heavy elements produced during the merger. The gravitational wave observations allow for precise measurements of the merger dynamics, while the electromagnetic observations provide information about the synthesis of heavy elements through r-process nucleosynthesis [Abbott, 2017d; Thielemann et al., 2017]. This has important implications for our understanding of the origin of these elements in the universe and the role of neutron star mergers in cosmic chemical evolution.



## 4.5 Current and Future Topologies

The detection of these events is made possible by advanced interferometric detectors such as LIGO, Virgo, and KAGRA, which use variations of the Michelson interferometer topology to measure the minuscule distortions in spacetime caused by gravitational waves. These detectors are remarkable in their ability to resolve displacements on the scale of  $10^{-18}$  metres, orders of magnitude smaller than the diameter of a proton [Abbott, 2016b]. The unprecedented precision of these instruments is a testament to the remarkable engineering and technological innovations that underpin gravitational wave detection.

The Michelson interferometer, the foundation design of these detectors, splits a single laser beam and sends the two beams down two perpendicular arms (Figure 4.1). These beams are reflected back by suspended mirrors and recombined at a photodetector, creating interference patterns that are highly sensitive to changes in arm length. The passage of a gravitational wave disturbs the relative lengths of the arms, altering the interference pattern and allowing the detection of the wave. Enhancing the sensitivity of the interferometer involves several strategies, such as incorporating Fabry-Perot cavities within the arms to increase the effective path length by reflecting the laser beams multiple times before recombination [Cahillane and Mansell, 2022; Adhikari, 2014].

In LIGO, these Fabry-Perot cavities are paired with power recycling and signal recycling mirrors that further enhance the interferometer's sensitivity by increasing the laser power circulating in the system and broadening the detector's frequency response [Abbott, 2020a]. Virgo, operating with a 3 km interferometer, similarly utilises these techniques but also incorporates monolithic suspensions, which minimise thermal noise by using silica fibres to suspend the mirrors. This design innovation improves Virgo's performance, especially in the low-frequency regime [Acernese, 2015]. Meanwhile, KAGRA introduces a distinct innovation with its cryogenically cooled mirror, which are aimed at reducing thermal noise at low frequencies, a critical advancement that enhances the sensitivity of the detector to gravitational waves from events such as black hole mergers and neutron star collisions [Aso et al., 2013].

While current detectors like LIGO, Virgo, and KAGRA have achieved remarkable success, they are still limited in their frequency range, particularly in detecting high-frequency gravitational waves. Future generations of detectors aim to address these limitations, the proposed Cosmic Explorer and Einstein Telescope are poised to revolutionise gravitational wave detection. The Cosmic Explorer, with its proposed 40 km arms, aims to increase sensitivity by a factor of ten compared to LIGO, allowing for the detection of gravitational waves from much earlier in the universe's history [Evans, 2021]. The Einstein Telescope, designed with a triangular configuration and underground location, seeks to achieve continuous detection of gravitational waves across a wide range of frequencies, from a few hertz to several kilohertz. This will

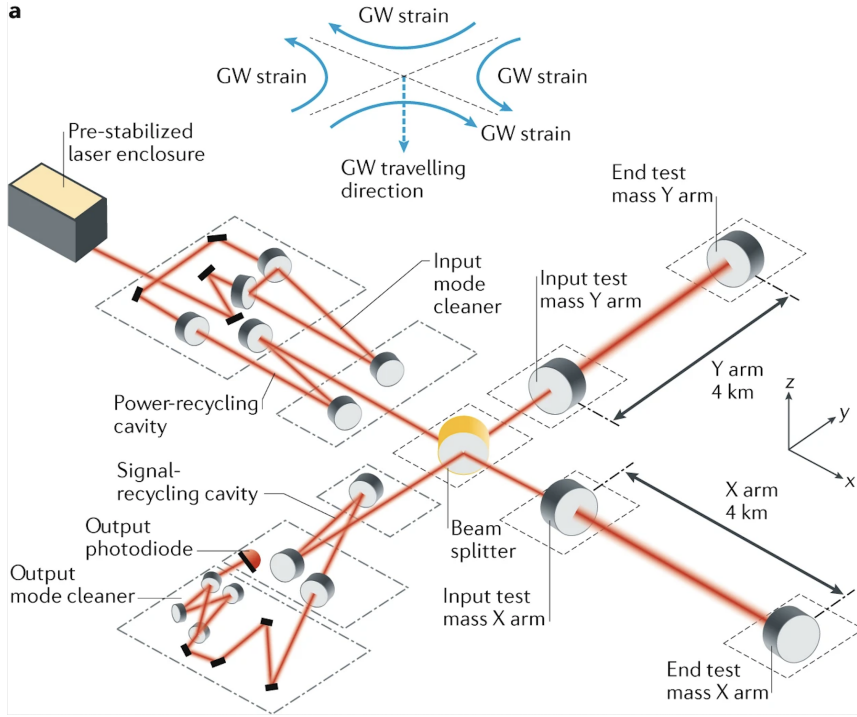


Figure 4.1: A schematic of a working laser interferometer gravitational wave detector. A simplified view of the Advanced LIGO interferometer, showing the optical configuration including the laser, input mode cleaner, power-recycling and signal recycling cavities, the 4-km-long arm cavities and the output mode cleaner. The gravitational wave (GW) signal is recorded on the output photodiode after the output mode cleaner. Credit: Bailes [2021]

be particularly advantageous for detecting the post-merger phase of neutron star collisions and binary black hole mergers [Maggiore et al., 2020]. The relative sizes compared to the current 4-km detector LIGO is shown in 4.2

One of the most promising advancements in the field of gravitational wave detection lies in high-frequency detectors, such as the Neutron Star Extreme Matter Observatory (NEMO). Current detectors, with their sensitivity limited to frequencies below 2 kHz, struggle to detect the high-frequency gravitational waves produced during the post-merger phase of neutron star collisions. NEMO, specifically designed to operate in the 2-4 kHz range [Ackley, 2020], targets this frequency regime where critical information about the neutron star equation of state can be obtained [Ackley, 2020]. This high-frequency regime holds the key to understanding the behaviour of matter at nuclear densities, potentially revealing the presence of exotic states of matter such as hyperons or deconfined quarks [Abbott, 2017c, 2018, 2019, 2020b; Chatziioannou, 2020; Pan et al., 2020; Cahillane and Mansell, 2022].

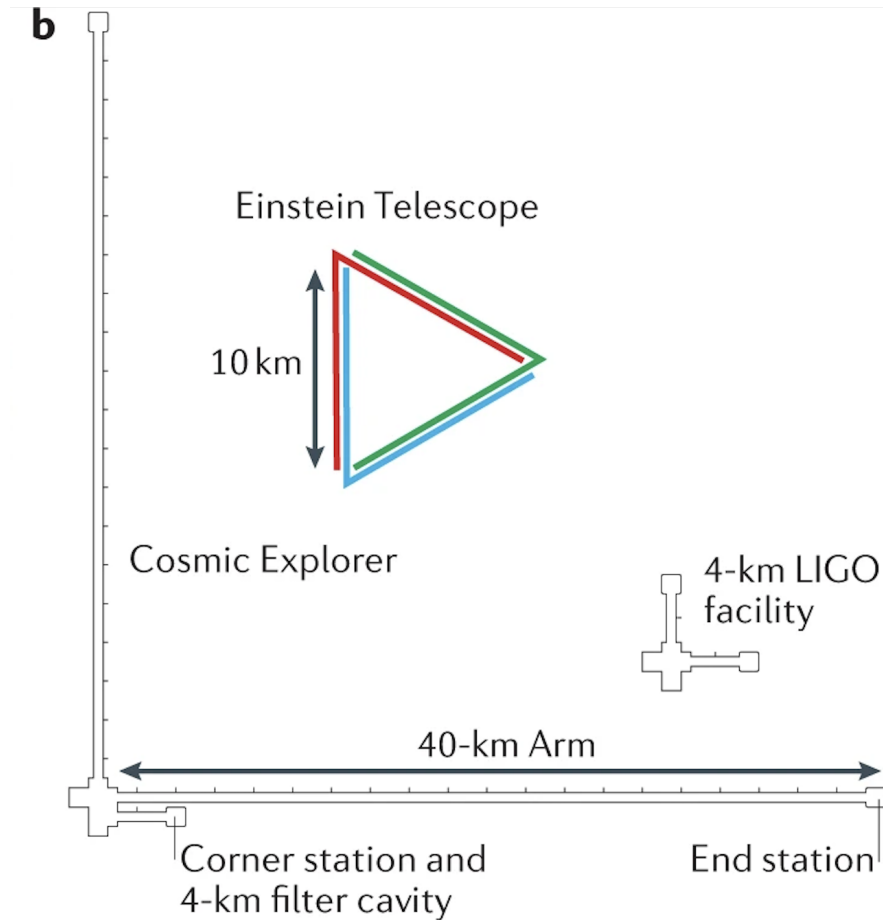


Figure 4.2: A comparison of the Einstein Telescope and Cosmic Explorer detector configurations. For comparison, the current Advanced LIGO detector is shown. Credit Bailes [2021]

## 4.6 Problem Statement

My project will investigate the operation of a so-called L-shape interferometer. It is based on a 50 km long linear cavity, which is folded to  $90^\circ$  at the mid-point between the two cavity mirrors. Additional central mirrors will construct a signal input and output path, where a power recycling and signal recycling mirror can be placed. This configuration has the potential to significantly enhance the sensitivity beyond that of the Cosmic Explorer or the Einstein Telescope, as well as bringing insight into current astronomical telescopes. However, as this is a new optical configuration, an understanding of its operation and limits are required to fully characterise its potential.

I will see to develop an optical model to characterise its behaviour, using Finesse in python, and try to evolve that with numerical time domain signals to understand its detector potential to extract the equation of state for binary neutron star mergers.



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# Background and Related Work

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## 5.1 Optical Cavity Understanding

Optical cavities are critical components of modern interferometric gravitational wave detectors, playing a crucial role in enhancing sensitivity by amplifying the light circulating within the interferometer. This amplification is essential due to the incredibly faint signals generated by passing gravitational waves, which perturb spacetime on the order of  $10^{-21}$  metres ref. To increase this sensitivity, detectors like LIGO and Virgo use Fabry-Perot cavities in each arm to increase the interaction length between light and the gravitational wave. This arrangement maximises the probability of detecting weak signals, such as those originating from binary neutron star mergers [Ackley, 2020; Abbott, 2016b].

### 5.1.1 Fabry-Perot Cavity

The Fabry-Perot cavity is a key element of optical interferometers used in gravitational wave detectors. It consists of two highly reflective mirrors placed at a distance, creating a resonant cavity for light. When light enters the cavity, it is reflected multiple times between the mirrors which amplifies the circulating power. Resonance occurs when the round-trip phase accumulation of the light is a multiple of  $2\pi$ , i.e. when the following resonance condition is met:

$$m\lambda = 2L, \quad (5.1)$$

where  $m$  is an integer,  $\lambda$  is the wavelength of light, and  $L$  is the length of the cavity. Achieving resonance allows for maximal transmission of light through the cavity, increasing the interferometer's sensitivity to gravitational waves.

In these cavities, the electric field transmitted through the cavity can be expressed as:

$$E_{out} = E_{in} \frac{T_1 T_2 e^{i\phi}}{1 + R_1 R_2 e^{i\phi}}, \quad (5.2)$$

where  $T_1$  and  $T_2$  are the transmission coefficients of the mirrors,  $R_1$  and  $R_2$  are the reflection coefficients, and  $\phi$  is the round-trip phase shift. This equation highlights how

the interaction between light and the cavity's mirror governs the overall behaviour of the optical cavity.

The transmission coefficient  $t_c$  and the reflection coefficient  $r_c$  for a Fabry-Perot cavity are given by:

$$t_c = \frac{t_1 t_2 e^{-ikL}}{1 - r_1 r_2 e^{-i2kL}} r_c = -r_1 + \frac{t_1^2 r_2}{1 - r_1 r_2 e^{-i2kL}}, \quad (5.3)$$

where  $t_1$  and  $t_2$  are the transmission coefficients of the mirrors, and  $r_1$  and  $r_2$  are the reflection coefficients,  $k$  represents the wave number, which is related to the wavelength  $\lambda$  by  $k = \frac{2\pi}{\lambda}$ .

These coefficients dictate how much light is transmitted or reflected by the cavity, and the design of these parameters is essential to maximising the sensitivity of the interferometer.

### 5.1.2 Importance of Amplified Light in Interferometers

Amplified light in interferometers is crucial for detecting the extremely faint gravitational wave signals generated by events such as binary neutron star mergers. Optical cavities enhance the interaction time of light with gravitational waves by increasing the effective path length that the light travels. This amplification increases the sensitivity of the interferometer to these minuscule spacetime distortions. In a Fabry-Perot cavity, circulating power is amplified by a factor of  $F$ , which is determined by the reflectivity of the mirrors:

$$F = \frac{1}{1 - R}, \quad (5.4)$$

where  $R$  is the mirror reflectivity. The circulating power in the cavity directly impacts the detector's sensitivity, particularly at high frequencies, such as those in the 2-4 kHz range where binary neutron star mergers emit gravitational waves. High-reflectivity mirrors can result in a substantial power buildup, magnifying the signal from a passing gravitational wave. For example, in the LIGO detector, the circulating power in each arm can reach hundreds of kilowatts, greatly enhancing sensitivity to even the tiniest spacetime distortion.

### 5.1.3 Michelson Interferometer

The Michelson interferometer forms the foundation of gravitational waves detectors. It consists of two perpendicular arms, each containing a Fabry-Perot cavity. A beam-splitter divides incoming light into two beams that travel down the arms, reflecting off mirrors at the ends, and recombine at the beamsplitter. Changes in the interference pattern of the recombined beams signal the passage of a gravitational wave. The change in arm length  $\Delta L$  caused by a gravitational wave is proportional to the strain  $h$ , given by:

$$\Delta L = hL, \quad (5.5)$$

where  $L$  is the arm length. Longer arms increase sensitivity, as the displacement  $\Delta L$  becomes more significant for a given gravitational wave strain. LIGO's 4km arms, for instance, enable it to detect gravitational waves from sources billions of light years away [Saulson, 1990; Ackley, 2020]. Thus a proposed interferometer with 25 km arms will be able to reach even more distant objects.

#### 5.1.4 Dual-Recycled Fabry-Perot Michelson Interferometers

In advanced detectors, the dual recycled Fabry-Perot Michelson interferometer configuration incorporates both power and signal recycling mirrors in addition to the Fabry-Perot cavities in the arms. Power recycling increases the amount of laser power circulating within the interferometer, boosting sensitivity, while signal recycling enhances sensitivity to specific frequency bands, such as the 2-4 kHz range critical for detecting binary neutron star mergers [Abbott, 2016b]. However, in order to enhance the signals at the 2-4 kHz regime, there must be a trade-off with the sensitivity to the lower frequencies where detectors like LIGO focus on. The frequency response of the interferometer can be optimised by adjusting the signal recycling mirror, thereby tuning the detector to frequencies where gravitational wave signals from specific astrophysical events are strongest.

This configuration is crucial for upcoming detectors that aim to explore the 2-4 kHz frequency range, where signals from binary neutron star mergers are prominent. Enhancing sensitivity in this range will enable more detailed observations of these events, probing valuable data for constraining neutron star equations of state and probing the behaviour of matter under extreme densities [Ackley, 2020].

#### 5.1.5 Fundamental Noise Contributions

Several fundamental noise sources limit the sensitivity of gravitational wave detectors, including quantum noise, thermal noise, and seismic noise. The contributions of all the noise losses are shown in 5.1.

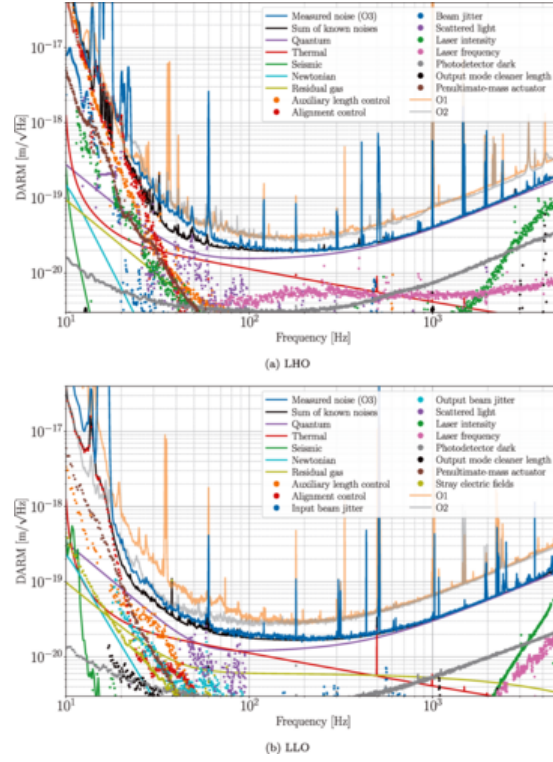


Figure 5.1: Full noise budget of (a) LIGO Hanford Observatory (LHO) and (b) LIGO Livingston Observatory (LLO). Calculated noise terms are given as solid lines, while measured contributions are given as dots. Credit: Buikema [2020]

### 5.1.5.1 Quantum Noise

Quantum noise is an intrinsic limitation arising from the Heisenberg uncertainty principle. It affects both the amplitude (shot noise) and phase (radiation pressure noise) of the light in the interferometer. Shot noise dominates at high frequencies, while radiation pressure noise dominates at low frequencies. The noise in the position of the test masses (mirrors) due to quantum effects can be expressed as:

$$\Delta x = \frac{\hbar}{2m\omega}, \quad (5.6)$$

where  $\hbar$  is the reduced Planck constant,  $m$  is the mass of the test mass, and  $\omega$  is the angular frequency of the light [Saulson, 1990].

### 5.1.5.2 Thermal Noise

Thermal noise arises from the random motion of atoms in the mirrors and suspension systems, driven by thermal energy. It limits sensitivity in the mid-frequency range of the detector and is governed by the fluctuation-dissipation theorem, which relates



thermal noise to the material properties of the detector:

$$S_x(f) = \frac{4k_B T}{\omega Q}, \quad (5.7)$$

where  $S_x(f)$  is the power spectral density,  $k_B$  is the Boltzmann constant,  $T$  is the temperature,  $\omega$  is the frequency, and  $Q$  is the quality of the material [Ackley, 2020].

### 5.1.5.3 Seismic Noise

Seismic noise is caused by ground vibrations and affects the low-frequency performance of the interferometer. This noise is mitigated through isolation systems, but it remains a limiting factor at frequencies below 10 Hz.

### 5.1.6 Amplitude Spectral Density of a Detector

The sensitivity of a gravitational wave detector is characterised by its amplitude spectral density (ASD), which describes the noise level as a function of frequency. The ASD, typically measured in units of  $\text{Hz}^{-1/2}$ , qualifies the detector's sensitivity at different frequencies. Detectors like LIGO have an optimal sensitivity around 100 Hz but exhibit reduced sensitivity at higher frequencies due to quantum and thermal noise. Enhancing sensitivity at 2-4 kHz range, where binary neutron star mergers as expected to produce gravitational wave signals, is a key goal for this project and next-generation detectors.

Continual improvements in the sensitivity of interferometers will unlock new potential for gravitational wave astronomy. Enhancing sensitivity in the 2-4 Hz frequency range will be particularly beneficial for detecting and studying binary neutron star mergers. The proposed upgrade to a 25 km arm-length interferometer, with a dual-recycled Fabry-Perot Michelson configuration, will significantly improve sensitivity, enabling the detection of weaker and more distant signals. This will allow astronomers to probe the extreme environments, shedding light on phenomena such as black holes, neutron stars, and the very nature of spacetime itself.

## 5.2 Motivation

### 5.3 Related work

The recent study by Zhang et al. [2023] represents a significant advancement in gravitational wave detection, particularly aimed at improving sensitivity in the high-frequency range of 2-4 kHz, which is crucial for observing postmerger signals from neutron stars. This study investigates how a new interferometric configuration could surpass the quantum loss limit of the traditional Fabry-Perot Michelson interferometer used in gravitational wave observatories like LIGO and Virgo.

The core innovation presented in Zhang et al. [2023] lies in the design of an advanced interferometer optimised for postmerger neutron star observations. They propose modifications to the traditional Fabry-Perot Michelson interferometer, incorporating concepts such as speedometer configurations and squeezed vacuum states. These changes aim to reduce quantum noise and enhance detector sensitivity beyond the standard quantum limit, particularly in the high-frequency bands where postmerger neutron stars emit most of their gravitational radiation.

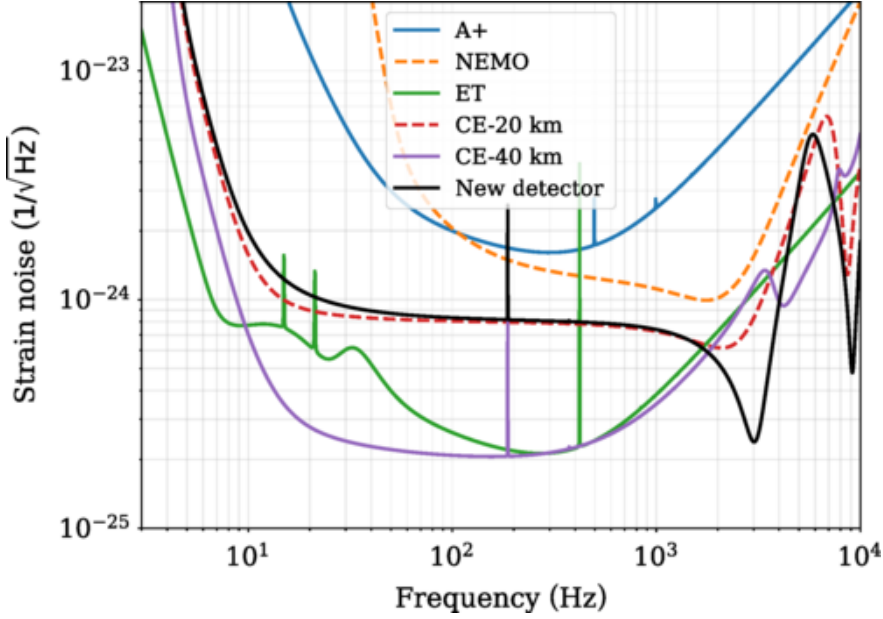


Figure 5.2: **Figure 4** from Zhang et al. [2023] Strain sensitivity of the 25-km new detector in comparison to A+, NEMO, ET, the “postmerger” tuned CE-20-km and the CE-40-km detectors. The sensitivity of ET corresponds to the combined three 10-km triangular configurations. The new detector gives superior sensitivity in the frequency range of 2–4 and 8–10 kHz.

Figure 4 (5.2) of Zhang et al. [2023], presents the strain sensitivity of their proposed gravitational wave detector for post merger neutron stars, comparing it against existing detectors such as Advanced LIGO. The figure illustrates how the novel detector design significantly enhances sensitivity in the crucial 2-4 kHz frequency range, which is highly relevant for observing signals from binary neutron star mergers. The sensitivity improvement is primarily due to the detector’s ability to circumvent the quantum loss limit inherent to Fabry-Perot Michelson interferometers, allowing it to achieve strain sensitivities beyond current limits. However this comes to a trade off with lower frequencies which current detectors are sensitive to.

In my thesis, I aim to replicate this enhancement in strain sensitivity, focusing on optimising the optical cavity configuration within the interferometer. The dual-

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recycled Fabry-Perot Michelson interferometer configuration I am working on is designed to amplify sensitivity in the 2-4 kHz range by leveraging longer arm lengths (up to 25 km) and exploring ways to minimise quantum noise.

The strain sensitivity curve in Figure 4 (4.2) shows a marked reduction in noise at high frequencies, attributed to the new cavity and interferometer design proposed by Zhang et al. [2023]. Achieving a similar reduction in noise within my own setup would require fine-tuning the cavity parameters, especially the mirror reflectivity and resonance conditions. Additionally, incorporating advanced techniques like quantum noise suppression, as highlighted by Zhang et al. [2023]., could further enhance the sensitivity of my design. By replicating the results shown in Figure 4 (4.2), my goal is to push the sensitivity of next generation detectors to new levels, enabling the observation of more detailed and distant astrophysical phenomena, particularly postmerger neutron star events.

The analysis of Figure 3 (4.2) underlines the importance of integrating cutting-edge cavity configurations and noise mitigation strategies in gravitational wave detectors. For my thesis, aligning with these innovations is critical for achieving the sensitivity improvements necessary to detect high-frequency gravitational wave signals with greater precision.

## 5.4 Summary

Summary what you discussed in this chapter, and mention the story in next chapter. Readers should roughly understand what your thesis takes about by only reading words at the beginning and the end (Summary) of each chapter.



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