4. Interpretation of Coefficients

(1) Simple logistic regression $logit(p(x)) = \beta_0 + \beta_1 x$
(2) Intercept β_0 determines $p(0)$, or the probability when $x = 0$. For continuous x this is typically uninteresting - unless explanatory variables are centered.
(3) Increase value of x by 1. Then, independently of x

$$logit(p(x+1)) - logit(p(x)) = \beta_1$$

Or, equivalently

$$OR(x+1, x) = exp(\beta_1)$$

In other words, regression coefficients can be interpreted via odds ratios.
(4) Important: units of measurement matter!

(5) Example. Explanatory variable is an indicator, $x = 1$ for an exposed, $x = 0$ for a non-exposed individual. Then, $\beta_0 = logit(p(0))$ for the non-exposed, and $exp(\beta_1)$ is the odds ratio between the exposed and non-exposed. Then, if $p(1), p(0)$ are small, we have $OR \approx RR$

(6) Consider two explanatory varibles

$$logit(p(x,t)) = \beta_0 + \beta_1 x + \beta_2 t$$

Or, the effects are *additive in the logit scale*

(7) Then, $OR[(x+1,t),(x,t)] = exp(\beta_1)$ for *all x and t*

(8) Moreover, $OR[(x+1,t+1),(x,t)] = exp(\beta_1 + \beta_2) = OR[(x+1,t),(x,t)]OR[(x,t+1),(x,t)]$. So, the effects are *multiplicative* in the OR scale!

(9) Example. Suppose $x = 1$ for the exposed and $x = 0$ for the non-exposed. Then, t has the same effect for both groups, and the odds ratio does not depend on t. (True?)
(10) If the effects are not additive, there could be *interactions*:

$$logit(p(x,t)) = \beta_0 + \beta_1 x + \beta_2 t + \beta_3 xt$$

(11) Then, e.g., $OR[(x+1,t),(x,t)] = exp(\beta_1 + \beta_3 t)$, so the effect of x depends on t
(12) Example. Suppose $x = 1$ for the exposed and $x = 0$ for the non-exposed. Then, β_3 gives the difference in response among the two groups to a unit increase in t. (13) A word of caution: *All* the above chacterizations assume that *all* relevant explanatory variables are included in the model and there are no *functional dependencies* among them.

(14) ANOVA parametrizations: groups $k = 1, \ldots, K$ with models

$$logit(p(k,x)) = \alpha_k^1 + \beta x, \quad k = 1, \ldots, K$$

(15) Alternatively

$$logit(p(k,x)) = \mu + \alpha_k^2 + \beta x, \quad k = 1, \ldots, K,$$

where $\alpha_1 = 0$.
(16) Third possibility

$$logit(p(k,x)) = \mu + \alpha_k^3 + \beta x, \quad k = 1, \ldots, K$$

where $\sum_{k=1}^{K} \alpha_k = 0$. The parametrizations are *equivalent*.
(17) Possible *multicollinearity* changes the interpretation:

$$logit(p(x)) = \beta_0 + \beta_1 x + \beta_2 x^2$$

Centering can help interpretation.

(18) Representing *nonlinear* effects in the *logit*-scale. Suppose $a \leq x \leq b$. Define a division of the interval:
$a = a_0 < a_1 < a_2 < \dots a_n = b$, so that
$[a, b] = [a_0, a_1) \cup [a_1, a_2) \cup \dots \cup [a_{n-1}, a_n]$.

For each of the n subintervals define a coefficient γ_i such that the effect of x is γ_i if $x \in [a_{i-1}, a_i), i = 1, \dots n$. Then, for such x we have

$$logit(p(x)) = \beta_0 + \gamma_i.$$