Intro to AI - Assignment 4

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1 Question 1

When giving an example of a simple inference procedure using model-checking in Ch.7.4.4, the textbook claims (the first paragraph in this section) that “P2,2 is true in two of the three models and false in one, so we cannot yet tell whether there is a pit in P2,2.” What are the two models and what is that one model? Show them.

Answer 1:

The knowledge base R1, R2, R3, R4 and R5 is true in 3 models, where P2,2 is true in 2 of those models and false in one. The following image from the book clearly shows the models where P2,2 is true and false. The red box indicates the models where P2,2 is true and the blue box indicates the model where P2,2 is false.

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	<u>*true*</u>
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Model representation:

Model 1 Where P$_{2,2}isfalse.$

	1	2	3	4
4				
3				
2				
1		B	P	

Model 2 where $\mathrm{P}_{2,2} istrue$:

	1	2	3	4
4				
3				
2		P		
1		B		

Model 3 where $\mathrm{P}_{2,2} istrue$:

	1	2	3	4
4				
3				
2		P		
1		B	P	

2 Question 2

In Ch.7.5.2, "Proof by resolution", it says "By the same process that led to R10 earlier, we can now derive the absence of pits in [2, 2] add [1, 3]....". Right after this sentence, the textbook shows R13 and R14, which are propositions that represent no pits in [2, 2] add [1, 3]. Your job in this question is to fill in the gaps between R12 and R13, R14 to show exactly how "by the same process that led to R10", R13, R14 are derived.

Answer 2:
Starting from R_{12} : $R_{12} : B_{1,2} \iff (P_{1,1} \lor P_{2,2} \lor P_{1,3})$
Apply biconditional elimination on $\mathrm{R}_{12} toget$

$$(B_{1,2} \implies (P_{1,1} \lor P_{2,2} \lor P_{1,3})) \land ((P_{1,1} \lor P_{2,2} \lor P_{1,3}) \implies B_{1,2})$$

Apply And-Elimination to obtain:

$$((P_{1,1} \lor P_{2,2} \lor P_{1,3}) \implies B_{1,2})$$

Logical equivalence for contrapositives gives

$$(\neg B_{1,2} \implies \neg(P_{1,1} \lor P_{2,2} \lor P_{1,3}))$$

Apply Modus Ponens

$$\neg(P_{1,1} \lor P_{2,2} \lor P_{1,3})$$

Apply De Morgan's rule

$$\neg P_{1,1} \land \neg P_{2,2} \land \neg P_{1,3}$$

Now with And-Elimination we can get

$$R_{13} : \neg P_{2,2}$$

And

$$R_{14} : \neg P_{1,3}$$

3 Question 3

Convert R6 (first appears in Ch.7.5.1) into CNF.

$$R_6 : (B_{1,1} \implies (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \implies B_{1,1})$$

Apply implication elimination

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land \underline{(\neg(P_{1,2} \lor P_{2,1})} \lor B_{1,1})$$

Apply de mogans for the underlined part

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land \underline{((\neg P_{1,2} \land \neg P_{2,1}) \lor B_{1,1})}$$

Apply distributive law for the underlined part to the CNF from

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$$

4 Question 4

Figure 7.14 in Ch.7.5.2 shows an example using resolution to prove ¬P1,2. In the same fashion, use resolution to prove ¬P2,1. Pay attention to the section "A resolution algorithm" in Ch.7.5.2 in which it describes how to carry out proof using resolution. Show your steps.

Answer 4:

$$If\ we\ consider\ our\ KB\ =\ (B_{1,1}\ \iff\ ({}_{P\,1,2} \lor P_{2,1})) \land \neg B_{1,1}\ as\ relevant.$$

$$We\ need\ to\ prove\ for\ \neg P_{2,1}$$

.

$$We\ end\ up\ with\ \alpha\ =\ \neg P_{2,1}.$$

The resolution procedure we should follow is (KB $\land\neg\alpha$).

We start by converting (KB $\land\neg\alpha) intoCNF$

Step 1:

$$\underline{(B_{1,1}\ \iff\ (P_{1,2} \lor P_{2,1}))} \land \neg B_{1,1} \land \neg(\neg P_{2,1})$$

$$Step\ 2:\ Apply\ biconditional\ elimination\ to\ the\ underlined\ part,\ and\ double\ negation\ elimination \neg(\neg P_{2,1})$$

$$\underline{(B_{1,1} \implies (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1})} \land \neg B_{1,1} \land P_{2,1}$$

Step 3: We know from the Question 3 the final CNF form of

$$(B_{1,1} \implies (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1})$$

is

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$$

Substituting we get

$$(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1}) \land \neg B_{1,1} \land P_{2,1}$$

We have 5 pairs.

$$a = (\neg B_{1,1} \lor P_{1,2} \lor P_{2,1})$$

$$b = (\neg P_{1,2} \lor B_{1,1})$$

$$c = (\neg P_{2,1} \lor B_{1,1})$$

$$d = \neg B_{1,1}$$

$$e = P_{2,1}$$

We get "a $\land b \land c \land d \land e$"

Combinations of "a" and "b"
1. P$_{1,2} \lor P_{2,1} \lor \neg P_{1,2}$
We know that P$_{1,2} \lor P_{2,1} \lor \neg P_{1,2} = P_{1,2} \lor \neg P_{1,2} \lor P_{2,1} = true \lor P_{2,1} = true, aclausewithcomplimentaryliteralsisnotresolvedfurther.$

2. $\neg B_{1,1} \lor P_{1,2} \lor B_{1,1}$
A clause with complimentary literal $\neg B_{1,1} and B_{1,1}$

Combinations of "a" and "c"
3. $\neg B_{1,1} \lor P_{1,2} \lor B_{1,1}$
A clause with complimentary literal $\neg B_{1,1} and B_{1,1}$
4. P$_{1,2} \lor P_{2,1} \lor \neg P_{2,1}$
A clause with complimentary literal $\neg P_{2,1} and P_{2,1}$

Combinations of "a" and "d"
5. P$_{1,2} \lor P_{2,1}$

Combinations of “a” and “e”
No useful resolution

Combinations of “b” and “c”
No useful resolution

Combinations of “b” and “d” 6. $\neg P_{1,2}$

Combinations of “b” and “e” No useful resolution

Combinations of “c” and “d”
7. $\neg P_{2,1}$

Combinations of “c” and “e”
8. $\mathrm{B}_{1,1}$

Combinations of “d” and “e”
No useful resolution

If further resolving the “e” and “7” will give us an empty clause. Hence KB entails $\alpha Combinationsof\backslash e” and\backslash 7” i.e.\backslash \neg P_{2,1}” and\backslash P_{2,1}$”