

# Lsn 8

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## Admin

## Power

Recall, in MA206 we tested Hypothesis such as:

$$H_0 : \mu = \mu_0 \text{ vs } H_a : \mu \neq \mu_0$$

Assuming that  $H_0$  is true, we created a standardized statistic:

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

Which looks like:

Using  $\alpha = 0.05$  our *Rejection Region* can be generated, which gives the values of  $t$  that we would conclude  $\mu \neq \mu_0$  for. Note that if  $\mu = \mu_a$  we *should* reject  $H_a$ , however there is a chance that we will not:

This region gives the probability of committing a Type-II error for  $\mu = \mu_a$ . Statistical power is  $1 - P(\text{Type-II error})$  or the probability that the researchers find evidence for the alternative hypothesis when the alternative hypothesis is true.

Note that in order to actually find Power or Type-II error we need to specify a value of  $\mu_a$ . Going back to our picture we see:

So the value chosen for  $\mu_a$  impacts Power, another thing that impacts Power is  $n$ .

For a two-sample or one-sample t-test, finding power in R is straight forward with the command `power.t.test`. Here we have to input  $n$ ,  $\Delta = |\mu_0 - \mu_a|$  which can be thought of as how big of an effect do we want to observe,  $s$ , and/or power. Note that one of these must be blank and is solved for. For instance, if we want to observe an effect of 1 and we assume  $s = 1$ , and we have  $n = 10$  observations for group and we are testing  $H_0 : \mu_1 = \mu_2$  vs  $H_a : \mu_1 \neq \mu_2$  we can run

```
power.t.test(n=10,delta=1,sd=1,power = NULL,type="two.sample",
             alternative="two.sided")
```

```
##
##      Two-sample t test power calculation
##
##              n = 10
##            delta = 1
##              sd = 1
##      sig.level = 0.05
##            power = 0.5619846
##      alternative = two.sided
##
## NOTE: n is number in *each* group
```

And we see that the power associated with this test is 0.56, or the probability of committing a Type-II error in this instance is 0.44. If we want to determine how big of a sample we would need to have 80 % power we would run:

```
power.t.test(n=NULL,delta=1,sd=1,power=0.8,type="two.sample",
             alternative="two.sided")
```

```
##
##      Two-sample t test power calculation
##
##              n = 16.71477
##            delta = 1
##              sd = 1
##      sig.level = 0.05
##            power = 0.8
##      alternative = two.sided
##
## NOTE: n is number in *each* group
```

To see that we would need 17 people in each group.

The same thing can be done with ANOVA tests using `library(pwr)` which must be installed.

The function `pwr.anova.test` can then be used in a similar way. The only tricky thing is that it relies on an effect size not given by  $R^2$  but rather given by  $f = \sqrt{R^2/(1 - R^2)}$ . This value is called Cohen's  $f^2$ , which is different than our F statistic. For two samples there's a similar value called Cohen's  $d$  statistic which is

found via  $\frac{\bar{x}_1 - \bar{x}_2}{s}$ . Using Cohen's  $f$  we can find power via:

```
library(pwr)
R2=.1
f=sqrt(R2/(1-R2))
pwr.anova.test(f=f,k=5,n=10)
```

```
##
##      Balanced one-way analysis of variance power calculation
##
##          k = 5
##          n = 10
##          f = 0.3333333
##      sig.level = 0.05
##          power = 0.3968709
##
## NOTE: n is number in each group
```

Which matches Table 1.6.4. We can then change:

```
R2=.3
f=sqrt(R2/(1-R2))
pwr.anova.test(f=f,k=5,n=10)
```

```
##
##      Balanced one-way analysis of variance power calculation
##
##          k = 5
##          n = 10
##          f = 0.6546537
##      sig.level = 0.05
##          power = 0.9574163
##
## NOTE: n is number in each group
```

Or change People within group

```
R2=.1
f=sqrt(R2/(1-R2))
pwr.anova.test(f=f,k=5,n=20)
```

```
##
##      Balanced one-way analysis of variance power calculation
##
##          k = 5
##          n = 20
##          f = 0.3333333
##      sig.level = 0.05
##          power = 0.7431771
##
## NOTE: n is number in each group
```