Relation between signed & unsigned Integer.

Let's say we have 3 bits to represent the counting.

C	B	A

C B A		
000	{}	0
001	A	1
010	B	2
100	C	4
011	AB	3
110	BC	6
101	AC	5
111	ABC	7

$\Rightarrow 2^3 = 8$

So, total allowed representation is

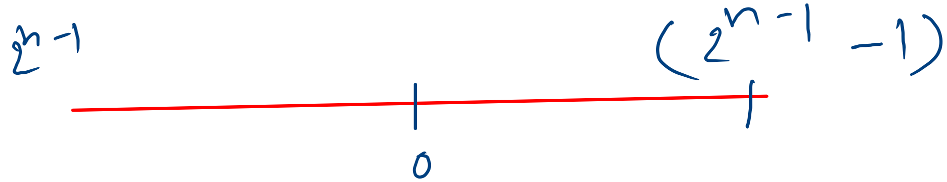
$\underline{2^n}$ where n is given bits.

Total unsigned numbers = $\underline{2^n}$

Total signed numbers = $2 \times 2^{n-1} = (2^{n-1} + 2^{n-1})$

-ve numbers   -ve numbers

Range of +ve & -ve numbers



Example: given bits = 3.

total unsigned numbers = $2^3 = 8$

range = 0 to 7.

Signed numbers $\Rightarrow 2 \times 2^2 = (2^2 + 2^2)$

+ve   -ve

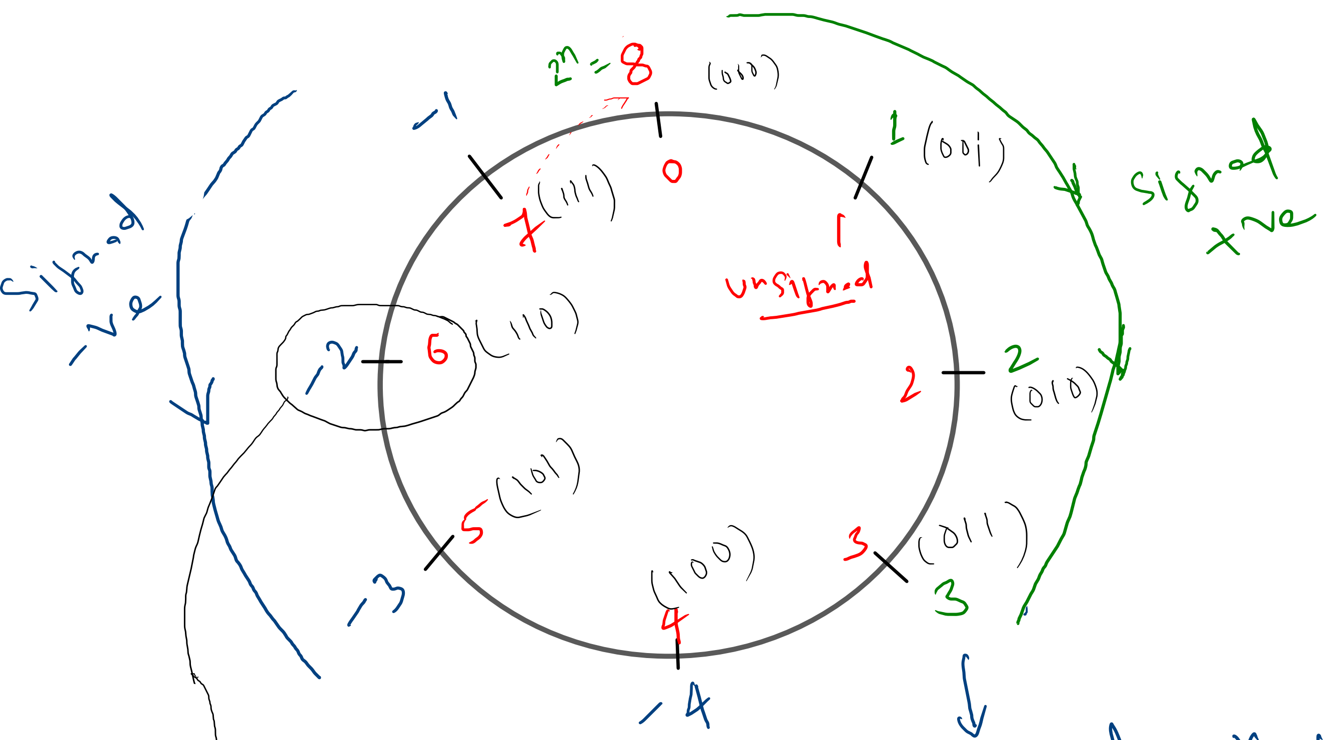
+ve number range = 0 to $2^2 - 1$

$\Rightarrow$ 0 to 3

-ve number range = -2^2 to -1

= -4 to -1

Signed & Unsigned relations



Signed max + 1 = Unsigned Min

Signed & Unsigned relations

$$\boxed{2^n + \underset{\text{negative}}{\text{Signed}} = \text{Unsigned}}$$

$$2^3 + (-1) = 7$$

Same bit representations

1	1	1

NEGATIVE SIGNED & UNSIGNED RELATION WITH ITS 2'S COMPLEMENT

unsigned = 2^n + signed negative

eg = 7 = 2^3 −1 → signed negative

unsigned

same bit representation

1	1	1

sign bit

Signed value

→ Since its the signed representation so first we have to get its 2's complement

$111 \xrightarrow{\text{1's complement}} 000 \xrightarrow{\text{add } 1} 001$

$0\times2^2 + 0\times2^1 + 1\times2^0 = 1$

→ Since sign bit is 1 so value is −1

Unsigned value

$1\times2^2 + 1\times2^1 + 1\times2^0 = 7$

Q. How to calculate bit representation of −ve number.

→ Get the 2's complement of its absolute value.

eg. −1, absolute value of −1 is 1, i.e. 001 and its 2's complement is 111

POSITIVE SIGNED & UNSIGNED RELATION WITH ITS 2'S COMPLEMENT

→ Unsigned & +ve signed have same bit representation.

e.g. Let's see the value of 3 in '3' bit representation.

0	1	1
2^2	2^1	2^0

Signed value is 3

Since it is a signed number so we need to first calculate 2's complement to see the relation

$$011 \xrightarrow{\text{1's complement}} 100 \xrightarrow{\text{add } 1} 101$$

$$101 \Rightarrow 1\times2^2 + 0\times2^1 + 1\times2^0 = 5$$

$$\boxed{\text{Positive signed value} = 2^N - (\text{2's complement})}$$

Both Unsigned value & +ve signed value are same.

$$0\times2^2 + 1\times2^1 + 1\times2^0 = 3$$

or $\underbrace{\text{2's complement}}_{\text{Unsigned}} = 2^N - (\text{+ve signed})$

$$3 = 2^3 - 5$$

3 → +ve signed

5 → 2's complement or Unsigned

Unsigned $= 2^N + (\text{signed } -\text{ve})$

2's complement $= 2^N - (+\text{ve signed})$

eg 1

$7 = 2^3 + (-1)$

$7 = 2^3 - (1)$

eg 2

$5 = 2^3 + (-3)$ [unsigned $= 2^N + (-\text{ve signed})$]

||

$5 = 2^3 - (3)$ [2's complement $= 2^N - (+\text{ve signed})$]

$$\begin{cases} \text{Unsigned} = 2^N + (-\text{ve signed}) \\ \underline{\text{Unsigned}} = 2^N - (+\text{ve signed}) \end{cases}$$

2's complement.

So, Unsigned value is 2's complement.

eg:

$$\underline{5} = 2^3 + \underline{(-3)}$$

bit representation of 2's complement of (-3) and unsigned 5 are same

2's complement is 1's complement + 1

Since $(2^N - \text{Number})$ gives 2's complement

$$\boxed{\text{So, } (2^N - 1 - \text{Number}) \text{ gives } \underline{1}\text{'s complement.}}$$

Q1 What is the 2's complement of $x = 3$ in 3 bit system?

or

What is the bit representation of $x = -3$ in 3 bit system?

Sol[n] (1): Find the 2's complement of absolute value of x i.e. 3.

Sol[n] 2: Unsigned bit representation of $(2^{n} - x)$ i.e $(2^{3} - 3) = 5$

Q2 What is the 1's complement of $x = 3$ in 3 bit system?

Sol[n] = $(2^{n} - 1 - x)$ i.e $2^{3} - 1 - 3 = 4$.