Relation between Permutation and Combination

Mathematical Relation: $^nC_r = \frac{\lfloor n}{\lfloor n-r \lfloor r}$ | $^nP_r = \frac{\lfloor n}{\lfloor n-r}$

① So, $\boxed{^nP_r = {^nC_r} \times \lfloor r}$

② Total count of characters in power set: power set $= (1+1)^n = 2^n$

$2^n = {^nC_0} + {^nC_1} + {^nC_2} + {^nC_3} + \dots + {^nC_{n-3}} + {^nC_{n-2}} + {^nC_{n-1}} + {^nC_n}$

n-char n-char n-char n-char

n=3 (odd)

$^3C_0$ $^3C_3$

{}, {a}, {b}, {c}, {ab}, {ac}, {bc}, {abc}

$^3C_1$ $^3C_2$

In case where n is even, the mid set $^nC_{(n/2)}$ **will form n-char complement in itself**

$^4C_2$

n=4 (even) **{}, {a}, {b}, {c},{d}, {ab}, {ac}, {ad},{bc},{bd},{cd}, {abc}, {abd},{acd},{bcd},{abcd}**

Since there are 2^n sets, and 2 sets together gives 'n' char.

So, total number of chars in power set = ((2^n)/2)*n = 2^(n-1)*n

Permuation and combination in terms of arrangment :

Permuation: Arranging 'r' distinct items at 'n' positions. Eg. Arranging 2 distinct items(a,b) at 3 positions. $^3P_2 = 6$

Combination: Arranging 'r' identical items at 'n' positions. Eg. Arranging 2 (i,i) at 3 positions → $^3C_2 = 3$

permutation of 2 identical items at 3 positions nCr

$^3C_2 =$

i	i	_
i	_	i
_	i	i

permutation of 2 distinct items at 3 positions nPr

1	2	_	2	1	_
1	_	2	2	_	1
_	1	2	_	2	1

$\} \; ^3P_2 = 6$

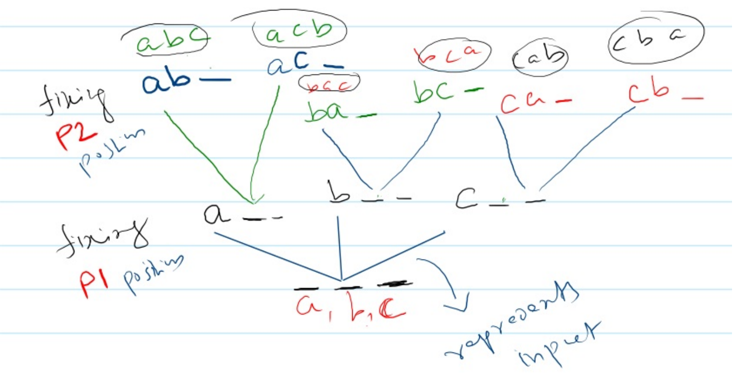
→ Since r =2, it means against each combination there will be r! copies(2!=2) of permuation.

$2^2 \times -$

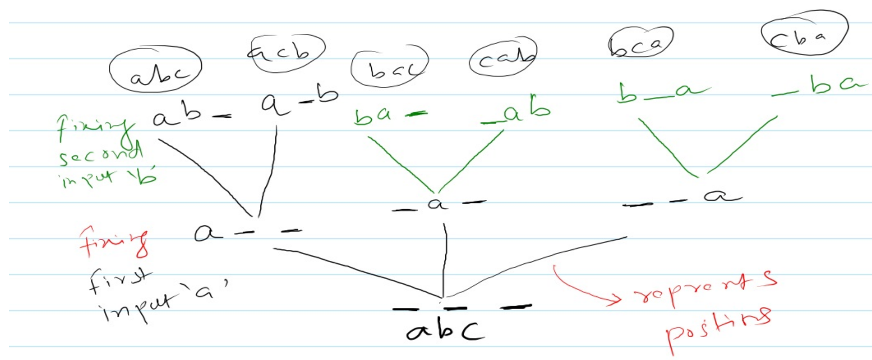
Approach for permuation tree formation:

1. By fixing the position and taking input elements as options
2. By fixing the input elemnt and taking positions as options

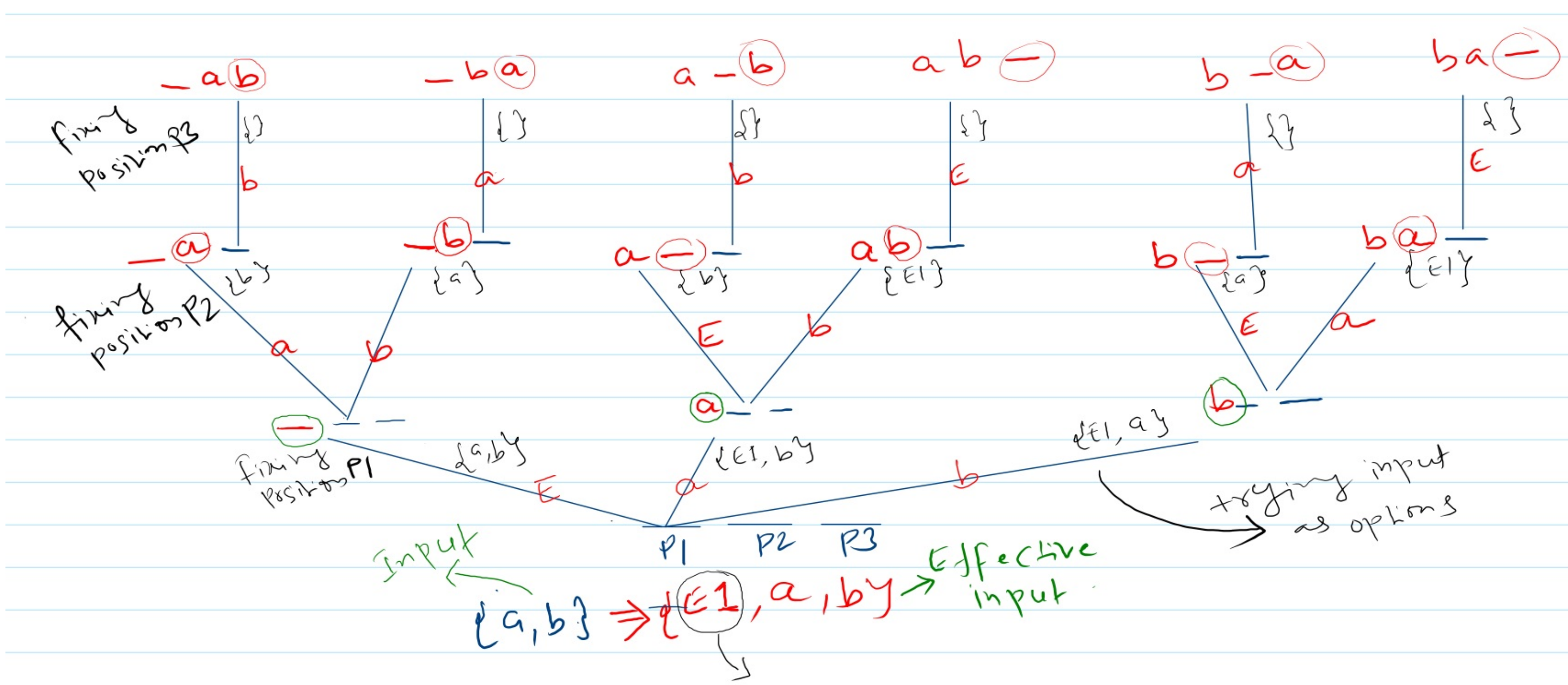
1A. By fixing the position and taking input elements as options where input_count == position_count



2. By fixing the input elemnt and taking positions as options where input_count <= position_count



1B. By fixing the position and taking input elements as options where input_count < position_count



Empty_count = 3 -2 = 1

Note: since input_count is smaller than postion_count so, input options to try at level will get exhausted before reaching to leaf level. This is why 'empty' need to be treated as special input.

EMPTY_COUNT = POSITION_COUNT - INPUT_COUNT

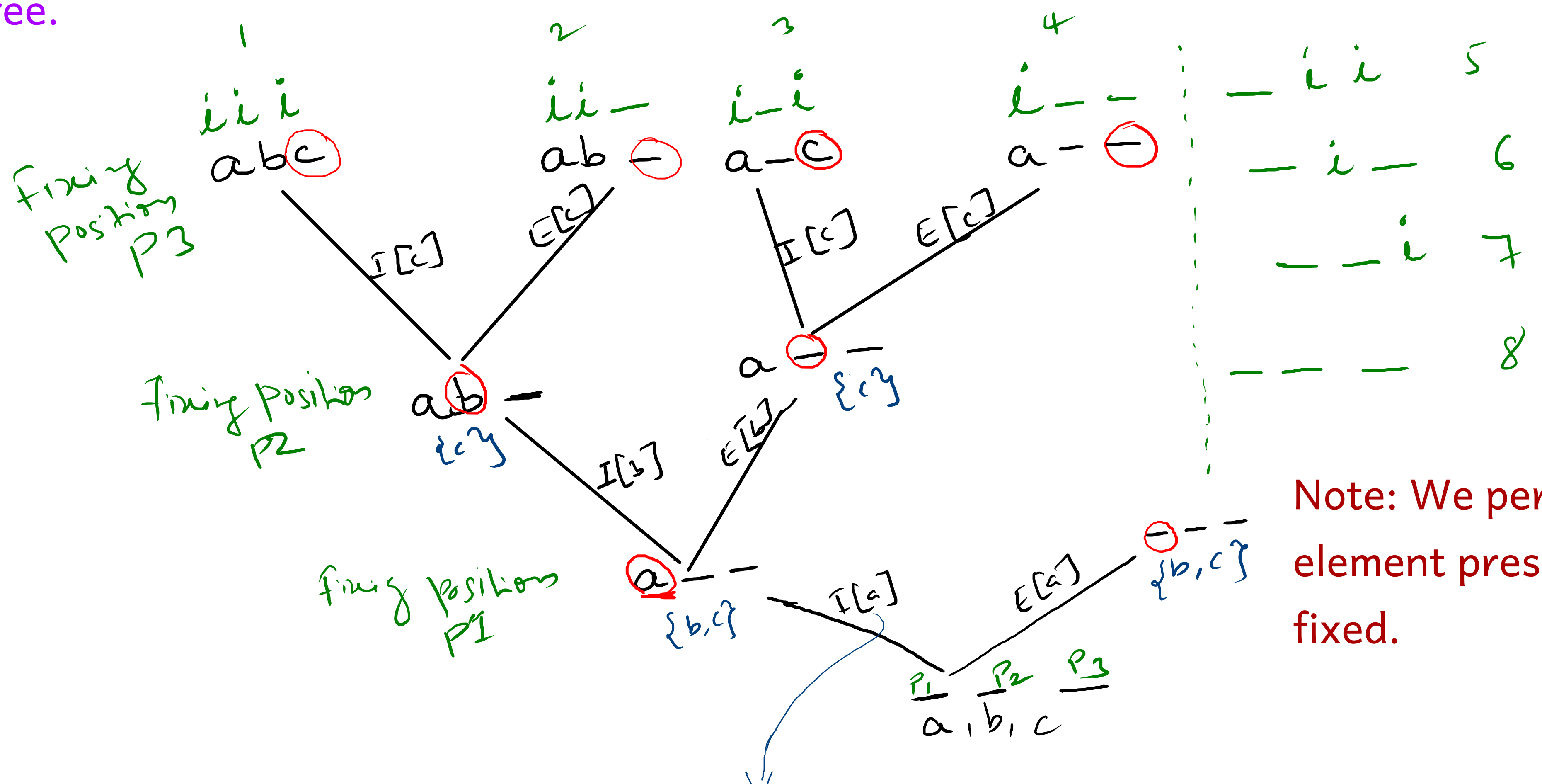
Approaches for Combination tree formation:

1. Pascal_Identity based Include_Exclude_Tree tree by fixing position
2. Pascal_Identity_Expansion based Include_Tree by fixing position

1. Pascal_Identity based Include_Exclude_Tree tree by fixing position where

input_count <= postion_count

Note: Position is fixed at each level, and include(i) & exclude(i) are taken as options i.e. branches of tree.



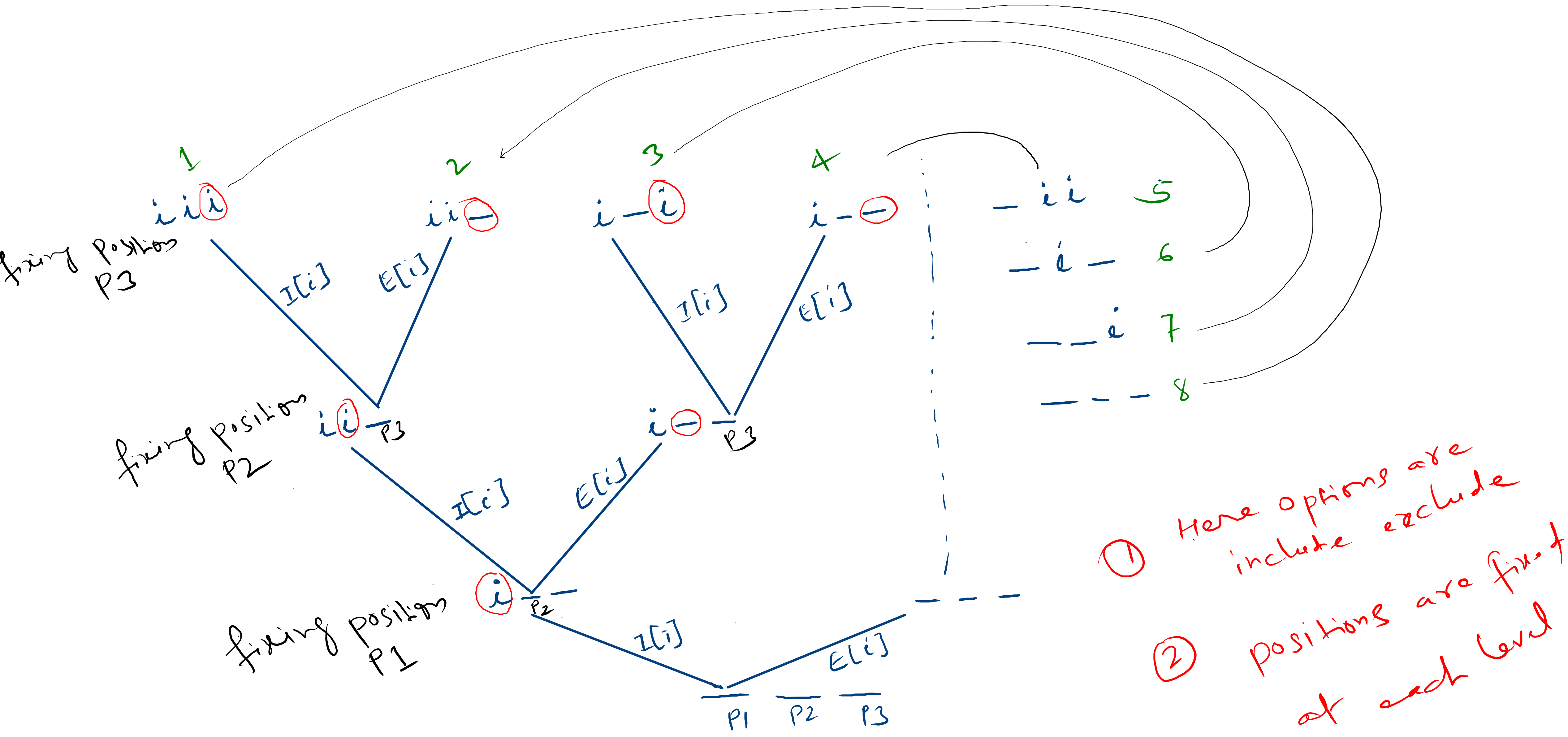
Note: We perform include exclude on element present at the postion to be fixed.

performing opertation include and exclude on input element persent at postion 'p1' i.e. position to be fixed.

1. Pascal_Identity based Include_Exclude_Tree tree by fixing position where

input_count <= postion_count

power set by placing 'i' on n given position



2. Pascal_Identity_Expansion based Include_Tree by fixing position where

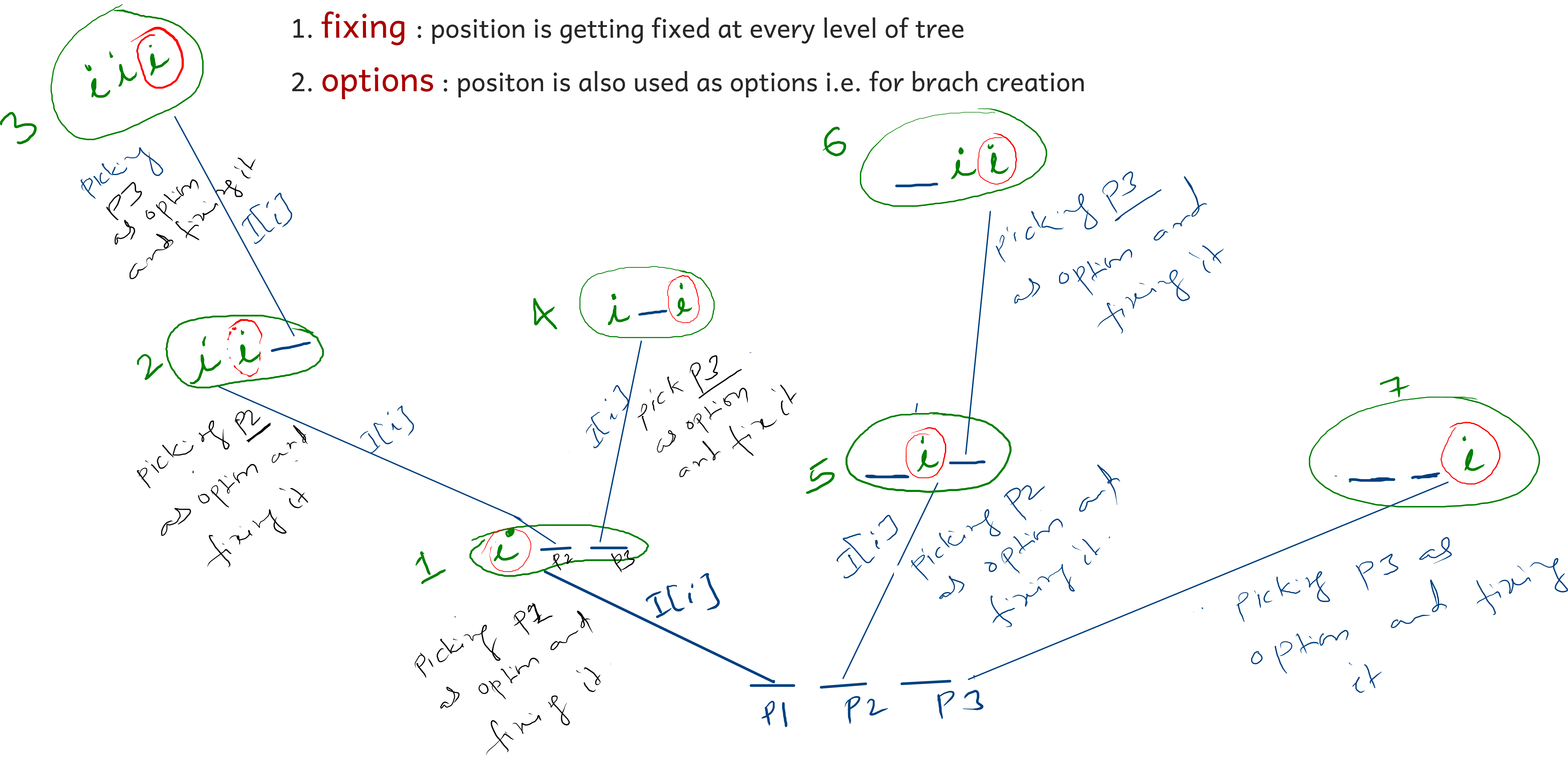
input_count <= position_count

Power set by placing 'i' at 'n' given positions

In pascal identity Expansion strategy position is used for both :

1. **fixing** : position is getting fixed at every level of tree

2. **options** : positon is also used as options i.e. for brach creation



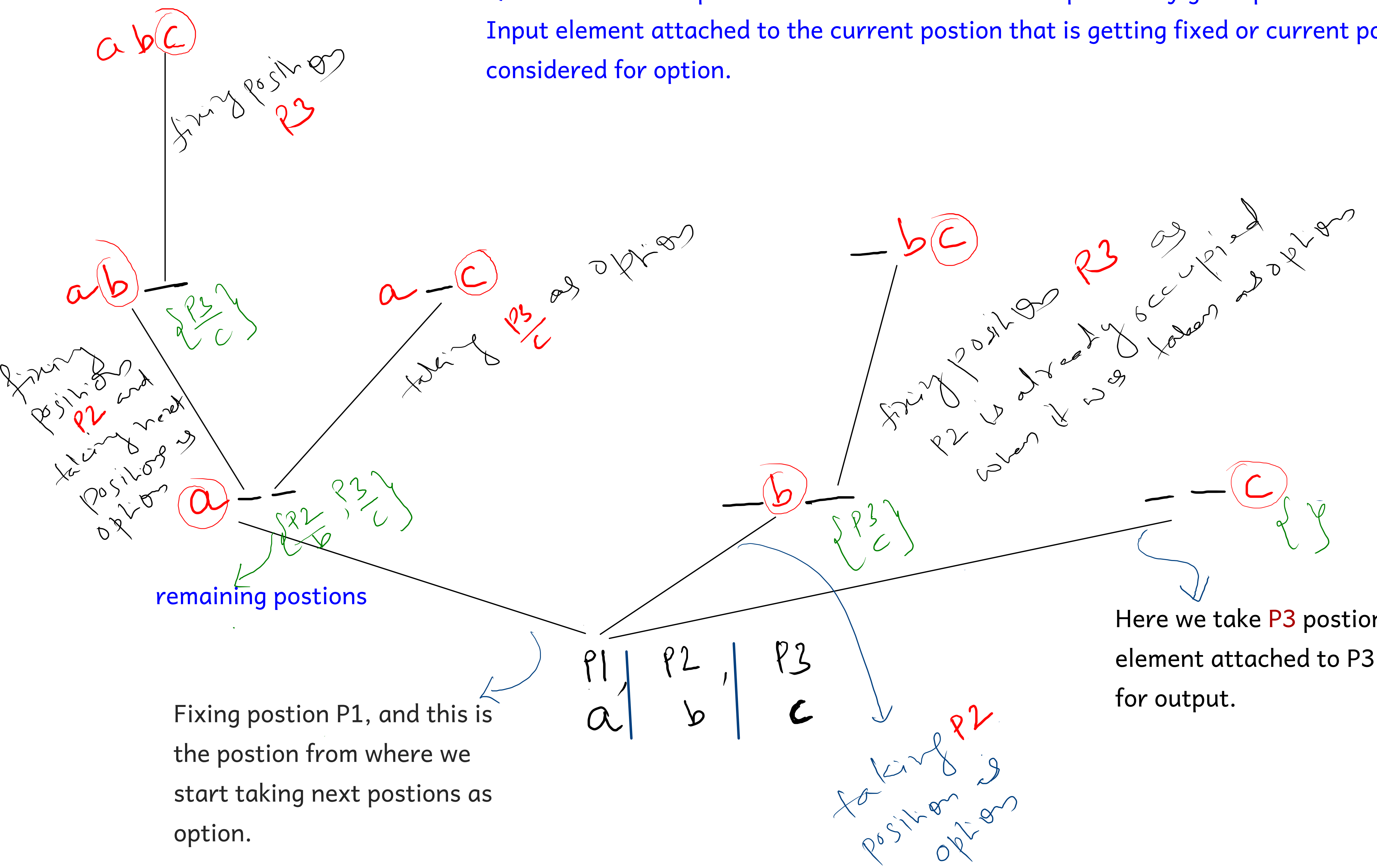
2. Pascal_Identity_Expansion based Include_Tree by fixing position where input_count

Power set by placing 'abc' at '3' given positions

Note : Input elements are hard-attached with respective postions.

Question: Which input element is considered for output at any given postion ?

Input element attached to the current postion that is getting fixed or current postion considered for option.



remaining postions

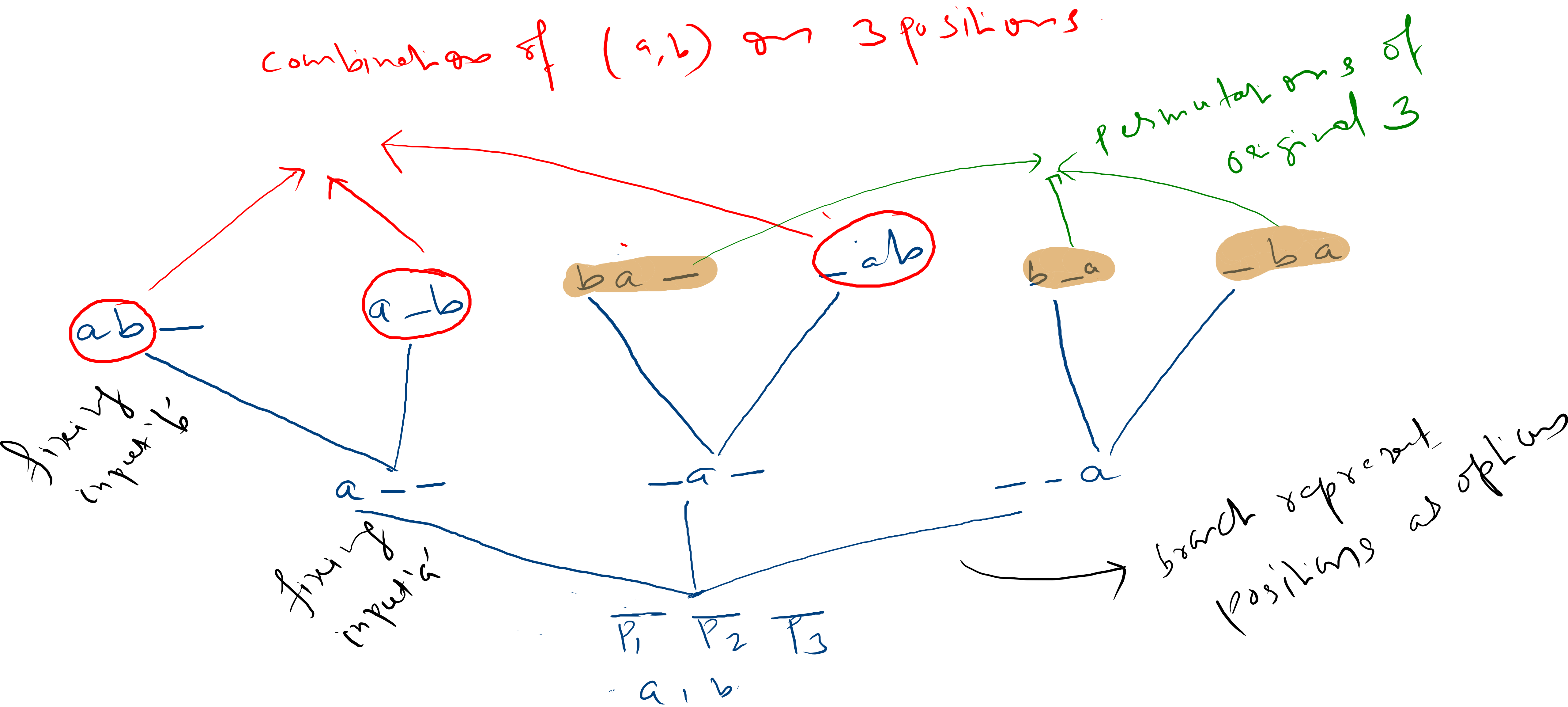
Fixing postion P1, and this is the postion from where we start taking next postions as option.

Here we take P3 postion as option, so the element attached to P3 i.e. 'C' is considered for output.

Relation between Permutation and Combination

Question : Print combination using prermuation strategy of fixing input and taking position as options

Note: If we allow only to place the input in lexicographic order, then we will get combinations from permutation strategy.



Rule of thumb for picking what to be fixed and what to be taken as options in recursion tree

Observation:

1. For case where options to try at level get exhausted before leaf level:
 - We have to cover lots of corner cases with lots of if and buts.
2. For case where options to try at level get exhausted at leaf level or remain unexhausted:
 - Solution remain simple and straight-forward.

Rule of Thumb to pick the approach to tackle the recursive problem:

The parmeter whose count is smaller than the other need to be picked as fixing at levels.

Example: input :{a,b}; positions:{_,_,_,_} ;

Here, inputs count are 2 and positions count are 4 and since input_count is smaller than the position_count so we will pick 'input' as to fix at levels.