# Course Title

# Exam 1: Topic

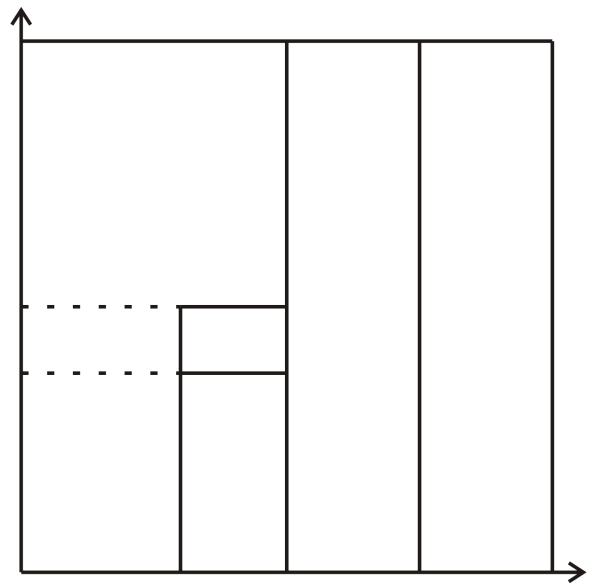
Class Number and Section
MM/DD/YYYY

**Name:** ______________________________
Please print your name clearly

**Read all of the following information before starting the exam:**

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit. When you do use your calculator (if a calculator is allowed), sketch all relevant graphs and explain all relevant mathematics.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 10 problems and is worth 100 points, partial credit will be given for incomplete answers. If you are not sure how to answer a problem give as much information as you can. It is your responsibility to make sure that you have all of the pages!
- You will have 1 hour 50 minutes to complete the exam.
- Good luck!

**1.** (*8 points*) Label each of the regions of the following flow regime map. Clearly label the x and y axis including any units and give the range of values for each of the flow regimes on the x and y axis.



**2.** (*8 points*) This problem involves setting up the truncation error analysis for the given fully implicit mass equation with constant positive velocity shown below.

$$\text{Difference Equation: } \frac{\rho_j^{n+1} - \rho_j^n}{\Delta t} + v\frac{\rho_j^{n+1} - \rho_{j-1}^{n+1}}{\Delta x_j} = 0$$

**a.** (*2 pts*) Plot an clearly state the variable that you are making the error evaluation about.

**b.** (*6 pts*) Write the Taylor series expansion for the two remaining variables about the point chosen above. (NOTE: you don't have to do anything beyond writing the Taylor series expansion.)

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(please do not remove this page from the test packet)

**Useful Equations**

$$f(x) = f(x_0) + (x - x_0)\left.\frac{df}{dx}\right|_{x=x_0} + (x-x_0)^2\,\frac{1}{2}\left.\frac{d^2 f}{dx^2}\right|_{x=x_0} + \cdots$$

$$f(x,y) = f(x_0,y_0) + \Delta x\left.\frac{\partial f}{\partial x}\right|_{x_0,y_0} + \Delta y\left.\frac{\partial f}{\partial y}\right|_{x_0,y_0} + \Delta x^2\frac{1}{2}\left.\frac{\partial^2 f}{\partial x^2}\right|_{x_0,y_0} + \Delta x\Delta y\left.\frac{\partial^2 f}{\partial x\partial y}\right|_{x_0,y_0} + \Delta y^2\frac{1}{2}\left.\frac{\partial^2 f}{\partial y^2}\right|_{x_0,y_0} + \cdots$$

$$g(x,y) = g(x_0,y_0) + \Delta x\left.\frac{\partial g}{\partial x}\right|_{x_0,y_0} + \Delta y\left.\frac{\partial g}{\partial y}\right|_{x_0,y_0} + \Delta x^2\frac{1}{2}\left.\frac{\partial^2 g}{\partial x^2}\right|_{x_0,y_0} + \Delta x\Delta y\left.\frac{\partial^2 g}{\partial x\partial y}\right|_{x_0,y_0} + \Delta y^2\frac{1}{2}\left.\frac{\partial^2 g}{\partial y^2}\right|_{x_0,y_0} + \cdots$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$$

$$\frac{\partial \rho e}{\partial t} + \nabla\cdot(\rho e\vec{v}) + P\nabla\cdot\vec{v} = q_{wl}$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}\nabla\cdot(\vec{v}) + \frac{1}{\rho}\nabla P + K\vec{v}^2 = 0$$

$$r = \frac{h_3}{h_2} = \frac{h_2}{h_1}$$

$$f_1 = f_{exact} + ah_1^p$$
$$f_2 = f_{exact} + ah_2^p$$
$$f_3 = f_{exact} + ah_3^p$$

$$\frac{f_3 - f_2}{f_2 - f_1} = \frac{h_2^p}{h_1^p} = r^p$$

$$p = \frac{\ln\left(\frac{f_3-f_2}{f_2-f_1}\right)}{\ln(r)}$$

$$f_{exact} = f_1 - \frac{f_2 - f_1}{r^p - 1}$$

$$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2 + \Delta P_l$$

$$\Delta P_l = \frac{1}{2}\rho v_1^2\left(1 - \frac{A_1}{A_2}\right)^2$$