1 不规则网格的双线性插值

> 参考 pyresample 的实现方法。

对于不规则四边形的四个角点 P_1, P_2, P_3, P_4，目标点 $(\text{out}_x, \text{out}_y)$ 可以表示为：

- P_1: Upper Left； P_2: Upper right
- P_3: Lower Left； P_4: Lower right；**顺序要对，否则会产生错误结果。**

$$P(s,t) = (1-s)(1-t)P_1 + s(1-t)P_2 + (1-s)tP_3 + stP_4 \tag{1}$$

其中 s 和 t 是需要求解的参数，范围都在 [0,1] 区间内。

目标点 $(\text{out}_x, \text{out}_y)$ 满足：

$$\begin{aligned} \text{out}_x &= (1-s)(1-t)x_1 + s(1-t)x_2 + (1-s)tx_3 + stx_4 \\ &= x_1 - sx_1 - tx_1 + stx_1 + sx_2 - stx_2 + tx_3 - stx_3 + stx_4 \\ &= x_1 + s(x_2 - x_1) + t(x_3 - x_1) + st(x_1 - x_2 - x_3 + x_4) \end{aligned} \tag{2}$$

$$\begin{aligned} \text{out}_y &= (1-s)(1-t)y_1 + s(1-t)y_2 + (1-s)ty_3 + sty_4 \\ &= y_1 + s(y_2 - y_1) + t(y_3 - y_1) + st(y_1 - y_2 - y_3 + y_4) \end{aligned} \tag{3}$$

定义：

$$\begin{aligned} x_{21} = x_2 - x_1, y_{21} = y_2 - y_1 \\ x_{31} = x_3 - x_1, y_{31} = y_3 - y_1 \\ x_{42} = x_4 - x_2, y_{42} = y_4 - y_2 \end{aligned} \tag{4}$$

$$\text{out}_x - x_1 = sx_{21} + tx_{31} + st(x_{42} - x_{31})$$

$$s = \frac{\text{out}_x - x_1 - tx_{31}}{[x_{21} + t(x_{42} - x_{31})]} \tag{5}$$

同时，s也可以写成y的形式：

$$s = \frac{\text{out}_y - y_1 - ty_{31}}{[y_{21} + t(y_{42} - y_{31})]} \tag{6}$$

将式 5 中的s代入out_y：

$$\begin{aligned} \text{out}_y - y_1 = sy_{21} + ty_{31} + st(y_{42} - y_{31}) \\ \text{out}_y - y_1 = \left[\frac{\text{out}_x - x_1 - tx_{31}}{x_{21} + t(x_{42} - x_{31})}\right] y_{21} + ty_{31} + t\left[\frac{\text{out}_x - x_1 - tx_{31}}{x_{21} + t(x_{42} - x_{31})}\right](y_{42} - y_{31}) \end{aligned} \tag{7}$$

两边同乘以分母：

$$\begin{aligned} (\text{out}_y - y_1)[x_{21} + t(x_{42} - x_{31})] = \\ (\text{out}_x - x_1 - tx_{31})y_{21} + ty_{31}[x_{21} + t(x_{42} - x_{31})] + t(\text{out}_x - x_1 - tx_{31})(y_{42} - y_{31}) \end{aligned} \tag{8}$$

$$
\begin{aligned}
&(\text{out}_y - y_1)x_{21} + t(\text{out}_y - y_1)(x_{42} - x_{31}) = \\
&(\text{out}_x - x_1)y_{21} - tx_{31}y_{21} + ty_{31}x_{21} + t^2y_{31}(x_{42} - x_{31}) + t(\text{out}_x - x_1)(y_{42} - y_{31}) - t^2x_{31}(y_{42} - y_{31})
\end{aligned} \tag{9}
$$

将上式整理成$at^2 + bt + c = 0$的形式：

$$
\begin{aligned}
&[x_{31}(y_{42} - y_{31}) - y_{31}(x_{42} - x_{31})]t^2 + \\
&[(\text{out}_y - y_1)(x_{42} - x_{31}) + x_{31}\ y_{21} - y_{31}\ x_{21} - (\text{out}_x - x_1)(y_{42} - y_{31})]t + \\
&(\text{out}_y - y_1)x_{21} - (\text{out}_x - x_1)y_{21} = 0
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
a &= x_{31}(y_{42} - y_{31}) - y_{31}(x_{42} - x_{31}) = x_{31}y_{42} - y_{31}x_{42} \\
b &= (\text{out}_y - y_1)(x_{42} - x_{31}) + x_{31}\ y_{21} - y_{31}\ x_{21} - (\text{out}_x - x_1)(y_{42} - y_{31}) \\
&= \text{out}_y(x_{42} - x_{31}) - \text{out}_x(y_{42} - y_{31}) - x_{42}\ y_1 + x_{31}\ y_2 + x_1y_{42} - y_{31}\ x_2 \\
c &= (\text{out}_y - y_1)x_{21} - (\text{out}_x - x_1)y_{21} = \text{out}_yx_{21} - \text{out}_xy_{21} + x_1y_2 - y_1x_2
\end{aligned} \tag{11}
$$

之后，将 t 代入式 6 可得s。注意$t, s \in [0, 1]$

References

1. https://github.com/pytroll/pyresample/blob/main/pyresample/bilinear/_base.py
2. https://github.com/pytroll/pyresample/blob/main/pyresample/test/test_bilinear.py
3. http://www.ahinson.com/algorithms_general/Sections/InterpolationRegression/InterpolationIrregularBilinear.pdf