Likelihoods for Random Scale Detection Function

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1 Likelihood formulations for the random scale detection function

The function *fitadmb* in *RandomScale* has an argument likelihood that can have values "g","f1","f2", or "fixed". Also the function *fitdata* has an argument *wrong* which when set to FALSE is equivalent to "g" likelihood and when set to TRUE is equivalent to "f1". The "f1" likelihood will produce incorrect results unless the data are untruncated. To avoid confusion, the "f1" likelihood was dropped from the paper but it is shown here because it remains in the code. The default for fitadmb is "f2" which we have found converges better than the "g" likelihood with our code. The "fixed" likelihood is the half-normal without a random component.

1.1 g likelihood

Using the random scale detection function, the marginalized likelihood for the sample of n observed distances can be derived directly from equations 2.39 and 2.40 in Borchers and Burnham (2004). In comparison with the covariate approach using fixed effects from above, we think of the random effect as a covariate with unknown values and integrate over the random effect. This is accomplished by including an integral over the unknown random effect in both the numerator and denominator:

$$L_g(\beta, \sigma_\epsilon) = \prod_{i=1}^{n} \frac{\int_{-\infty}^{\infty} g(x_i|\epsilon)\, N(\epsilon, 0, \sigma_\epsilon) d\epsilon}{\int_{-\infty}^{\infty} \int_0^w g(u|\epsilon) du\, N(\epsilon, 0, \sigma_\epsilon) d\epsilon} \tag{1}$$

where the x_i refer to the distances to the detected objects with $i = 1, 2, ..., n$. We denote L_g with subscript g indicating that here we use a properly defined detection function $g(x|\epsilon)$ with $g(0) = 1$ (for comparison see L_f below). Also, in this formulation the scale intercept is denoted with β_g. The numerator of eq 1 is the marginal probability that the ith object was seen at x_i:

$$\int_{-\infty}^{\infty} g(x_i|\epsilon) N(\epsilon, 0, \sigma_\epsilon) \frac{1}{w} d\epsilon \quad , \tag{2}$$

while the denominator of eq 1 is the marginal probability that the object was seen within truncation width w:

$$\int_{-\infty}^{\infty} \int_0^w g(u|\epsilon) du \, N(\epsilon, 0, \sigma_\epsilon) \frac{1}{w} d\epsilon \quad . \tag{3}$$

We note that in contrast to point transects, the availability function for line transects $h(x) = 1/w$ from eqs 2 and 3 cancel in eq 1.

1.2 f2 likelihood

An alternative likelihood is:

$$L_f(\beta, \sigma_\epsilon) = \prod_{i=1}^{n} \frac{\int_{-\infty}^{\infty} f(x_i|\epsilon) \, N(\epsilon, 0, \sigma_\epsilon) d\epsilon}{\int_{-\infty}^{\infty} \int_0^w f(u|\epsilon) du \, N(\epsilon, 0, \sigma_\epsilon) d\epsilon} \tag{4}$$

where

$$f(x|\epsilon) = 2 \left(\sqrt{2\pi} \gamma(\epsilon) \right)^{-1} \exp\left(-x^2/(2\gamma(\epsilon)^2) \right), \qquad x \geq 0$$

is the pdf in absence of truncation ($w = \infty$).

1.3 f1 likelihood

One might be tempted to use the following likelihood:

$$L_{f1}(\beta, \sigma_\epsilon) = \prod_{i=1}^{n} \int_{-\infty}^{\infty} \frac{g(x_i|\epsilon)}{\int_0^w g(u|\epsilon) du} N(\epsilon, 0, \sigma_\epsilon) d\epsilon \quad . \tag{5}$$

Compared to eq 1, the normal density is omitted from the denominator and the random effect is integrated out simultaneously over the numerator and denominator. But this is not correct and will produce biased estimates. If $w = \infty$, then the relationship $\beta_{f1} = \beta_g + \sigma_\epsilon^2$ holds where β_{f1} is derived from eq 5. However, it does not hold if the data are truncated as is generally the case for wildlife studies.

References

Borchers, D. and Burnham, K. (2004). *Advanced Distance Sampling.*, chapter General formulation for distance sampling. Oxford University Press, Oxford.