POGIL Polymers

Guided-Inquiry Activities
for Polymer Chemistry and Polymer Physics

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The current source for these materials is accessible on Github:
https://github.com/jlaaser/pogil-polymers

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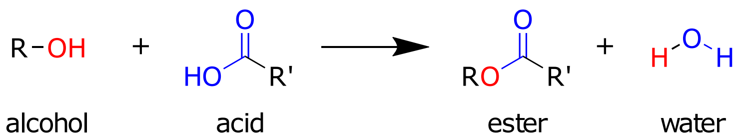
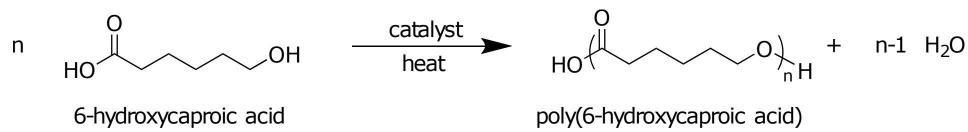
Activity 1: Chemistries of Step-Growth Polymerizations

Model 1: Synthesis of a Polyester

Esterification reactions are a common type of reaction used to produce polymers by step-growth polymerization. In a typical esterification reaction, an alcohol and a carboxylic acid react to form an ester bond:



One example of a polymerization reaction using this chemistry is the synthesis of poly(6-hydroxycaproic acid) from 6-hydroxycaproic acid monomers:



Critical Thinking Questions:

1. Consider the 6-hydroxycaproic acid monomer shown in Model 1:

   a) As drawn, what type of functional group is on the *left* side of the monomer?

   b) As drawn, what type of functional group is on the *right* side of the monomer?

Information:

When a monomer used in a step-growth polymerization has different reactive functional groups on each end, it is called an "AB-type" monomer.

When a monomer used in a step-growth polymerization has the same reactive functional group on each end, it is called an "AA-type" or "BB-type" monomer.

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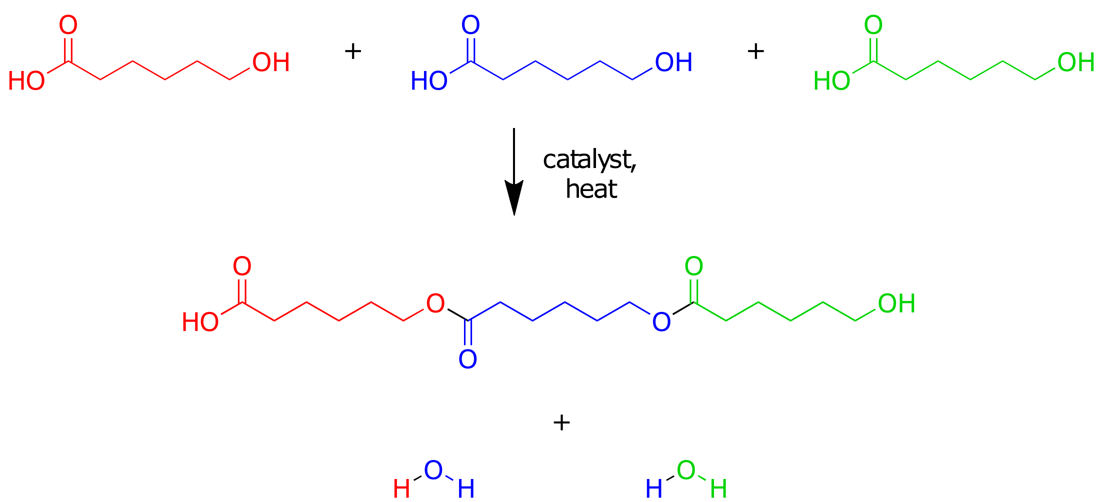
Critical Thinking Questions:

2. Would you classify the poly(6-hydroxycaproic acid) monomer used in this synthesis as an AA-type monomer or an AB-type monomer? Briefly explain your answer in 1-2 complete sentences.

3. The synthesis of a short 6-hydroxycaproic acid oligomer from three monomers is shown explicitly, below:

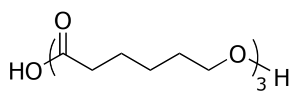


The molecules are color-coded so that you can see which atoms in the oligomer came from which monomer.

a) What type of bonds connect the different monomers in the polymer backbone?

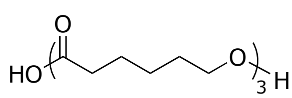
b) Explain, in one or two complete sentences, why you think we classify this polymer as a “polyester”:

c) When we write the structure of this oligomer as

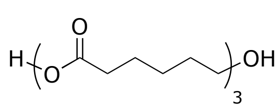


how many monomers make up each repeat unit in the polymer chain?

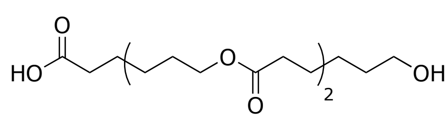
d) Explain, in one or two complete sentences, why we generally abbreviate the product of this reaction as



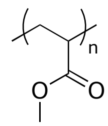
rather than as



or



4. Consider the following polymer:



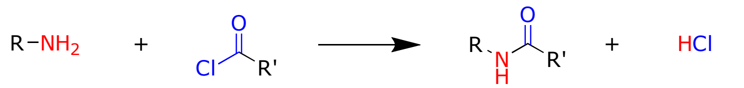
a) Does this polymer have ester bonds in the polymer backbone?

b) Would you be able to produce this polymer by esterification reactions of small molecules? Why or why not?

c) Based on your answers to the previous two questions, would you classify this polymer as a polyester? Why or why not?

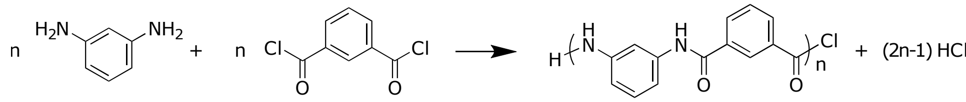
Model 2: Synthesis of a Polyamide

Amidation reactions are another type of reaction used to produce polymers by step-growth polymerizations. For example, acid chlorides can be reacted with primary amines to form an amide bond:



amine | acid chloride | amide | hydrochloric acid

Commercially, this reaction is used to produce Nomex, a heat-resistant polymer used in oven mitts and firefighters' protective clothing, among other applications. A reaction scheme for the synthesis of Nomex is shown below:



monomer 1 | monomer 2 | Nomex

Critical Thinking Questions:

5. Why is this polymer classified as a polyamide?

6. What functional groups does monomer 1 have? Would you classify this monomer as an AA-type monomer or an AB-type monomer?

7. What functional groups does monomer 2 have? Would you classify this monomer as an AA-type monomer or an AB-type monomer?

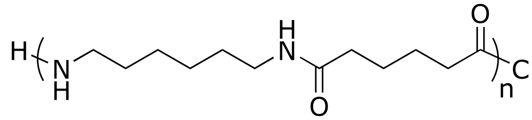
8. Explain, in one or two complete sentences, why we might describe this reaction as an "AA+BB"-type polymerization:

9. How many monomers make up each repeat unit?

10. A very similar reaction can be used to make Kevlar, the high-strength polymer used in bulletproof vests and cut-resistant gloves. Given that Kevlar is produced from the following two monomers,

predict the structure of the Kevlar polymer:

11. A similar chemistry can also be used to prepare nylon-6,6, a polymer used in many consumer goods. The structure of nylon-6,6 is shown below:



What two monomers would you need to combine to make this polymer?

Information:

A polymerization reaction is called a *condensation* polymerization if the reaction produces a small-molecule byproduct that is not part of the polymer chain.

Critical Thinking Questions:

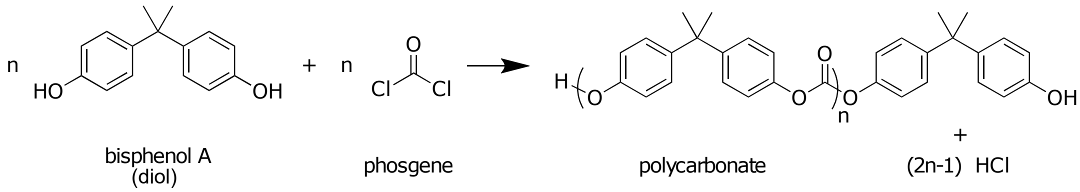
12. Is the esterification reaction in Model 1 a condensation polymerization? If so, what is the small-molecule byproduct that is produced?

13. Is the amidation reaction in Model 2 a condensation polymerization? If so, what is the small-molecule byproduct that is produced?

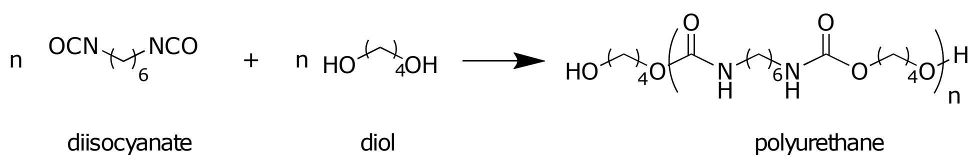
Model 3: Other Chemistries used for Step-Growth Polymerization

Shown below are synthetic schemes for a variety of other commercially-important polymers produced by step-growth polymerization:

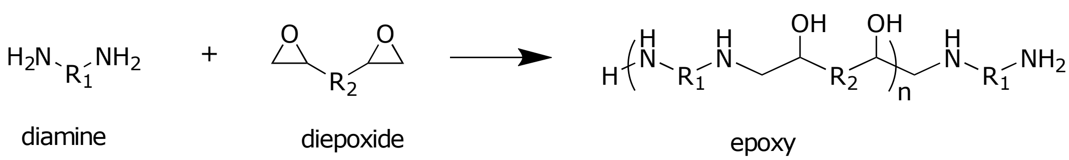
a) polycarbonate (high transparency and impact resistance; used in DVDs, glasses, etc.)



b) polyurethanes (foams; thermoplastic elastomers, e.g. spandex)



c) epoxies (adhesives; coatings)



Critical Thinking Questions:

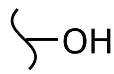
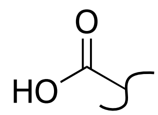
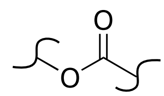
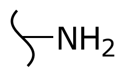
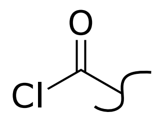
14. Classify each of the reactions in the above Model as either an “AB-type” or “AA+BB-type” polymerization:

a)

b)

c)

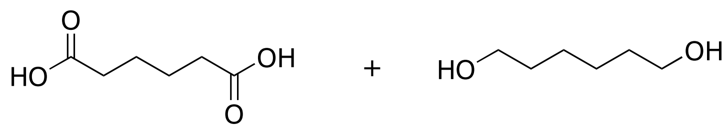
15. Complete the following table for the polymerizations depicted in Models 1-3:

Polymer	“A” reactive group	“B” reactive group	“ab” bond formed	Small Molecule Byproduct
Polyester				
Polyamide				HCl
Polycarbonate				
Polyurethane				
Epoxy				

16. Which of the above polymerization reactions would you classify as condensation polymerizations? Briefly explain your answer in 1-2 complete sentences.

Exercises:

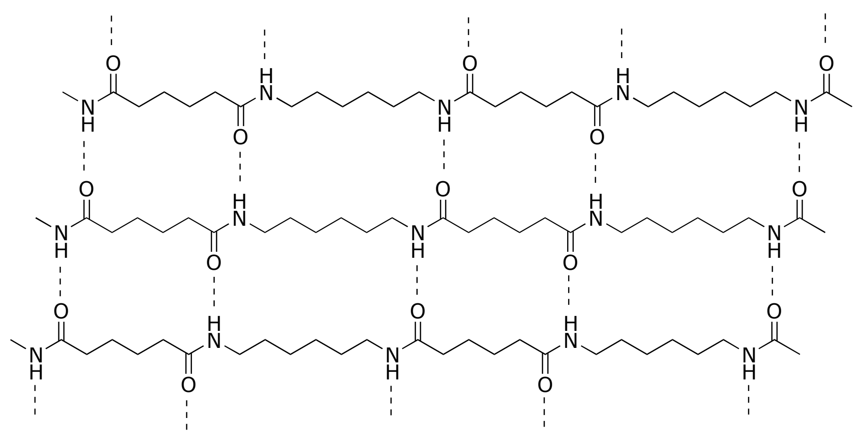
1. Although the polymers formed by AB-type and AA+BB-type step-growth polymerizations are similar, there are some subtle but important differences. Consider the synthesis of a polyester from the following two monomers:



   a) Draw the structure of the polymer that would be formed from this pair of monomers.

   b) Compare this structure to the polymer produced from the AB-type monomer in Model 1 (hint: you may find it useful to explicitly draw out a few repeat units). Are they the same, or different? If they are different, briefly describe what is different about the two structures.

2. One of the reasons that polyamides have such useful properties is that the amide groups can form hydrogen bonds between chains, as shown below:



   These inter-chain hydrogen bonds significantly improve the mechanical properties (e.g. stiffness, resilience, etc.) of the material.

   Draw an analogous structure for the AA+BB-type polyester that you drew in Exercise 1. Can this polymer form inter-chain hydrogen bonds? Briefly explain your answer, and discuss how you expect the physical properties of the polyester to compare to those of the polyamide.

3. The epoxidation reaction shown in Model 3 formed a linear polymer with secondary amines. However, secondary amines can also attack epoxides. Draw out the polymer structure that you would expect to generate if this occurs. How would you describe this polymer architecture?

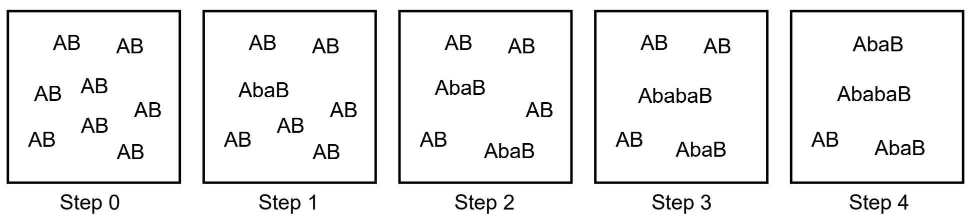
Activity 2: Degree of Polymerization in Step-Growth Polymerizations

Model 1: Polymerization of "AB"-Type Monomers

The simplest type of step-growth polymerization is one in which each monomer has one "A"-type reactive group and one "B"-type reactive group. These types of monomers are referred to as "AB"-type monomers.

In each step of the polymerization, an "A" group on one molecule reacts with a "B" group on another molecule to form an "ab" bond, as shown below:

For example, for a simple reaction mixture containing 8 "AB"-type monomers, the evolution of the reaction mixture might look something like this:



In this diagram, each string of letters represents one molecule; each molecule may be either an unreacted monomer ("AB") or a growing polymer chain (e.g. "AbaB", "AbabaB", etc.).

Critical Thinking Questions:

1. Fill out the following table for the reaction mixture shown in Model 1:

Step	Number of unreacted "A" groups	Number of molecules
0		
1		
2		
3		
4		

2. Explain, in a complete sentence, how the number of molecules in the mixture is related to the number of unreacted "A" groups.

Information:

At any given time, the number-average degree of polymerization, N_n, is the total *initial* number of monomers divided by the total number of molecules *currently* present. That is,

$$N_n = \frac{\text{initial number of monomers}}{\text{current number of molecules}}$$

Critical Thinking Questions:

3. In Model 1, there were 8 monomers in the initial reaction mixture. Using this information, calculate the number-average degree of polymerization for each step shown in Model 1.

Step	**0**	**1**	**2**	**3**	**4**
$\mathbf{N_n}$					

4. Suppose that you had initially started with 100 monomers. Then, suppose that at some time later, you had only 8 unreacted "A" groups left.

   a) How many molecules would there be in the reaction mixture at this point?

   b) What would the number-average degree of polymerization be at this point?

5. More generally, suppose you started with v_A^0 monomers, and at some time later, you had only v_A unreacted "A" groups left. What would the number average-degree of polymerization be at this point, in terms of v_A^0 and v_A?

Information:

Usually, we find it more useful to work in terms of the *fraction* of all "A" groups that have reacted, rather than the total *number* of "A" groups that have reacted.

In step-growth polymerizations, we refer to the fraction of "A" groups that have *reacted* as the "extent of reaction", p. When the fraction of "A" groups that *have* reacted is p, the fraction of "A" groups that have *not* reacted is $1 - p$.

Critical Thinking Questions:

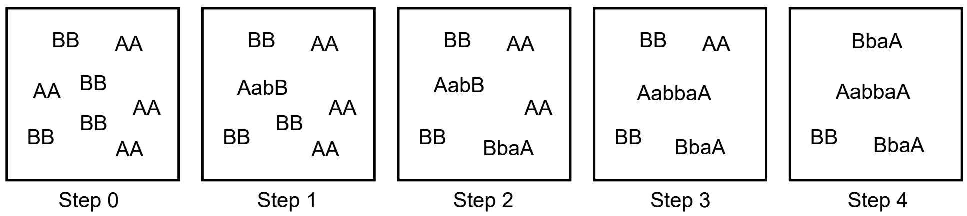
6. If we start with v_A^0 "A" groups, what is v_A (that is, how many "A" groups are still unreacted) when the extent of reaction is equal to p?

7. Using your answers to critical thinking questions 5 and 6, derive an expression for N_n in terms of p.

Model 2: Polymerization of "AA" and "BB"-Type Monomers

Now, let's consider a slightly more complicated reaction, with two different types of monomers that each have *either* two "A" reactive groups *or* two "B" type reactive groups. We call monomers with two "A" groups "AA"-type monomers, and we call monomers with two "B" groups "BB"-type monomers.

Suppose we start with 8 monomers, 4 of which are "AA"-type monomers and 4 of which are "BB"-type monomers. In this case, the evolution of the reaction mixture might look something like this:



Critical Thinking Questions:

8. For the reaction mixture shown in Model 2, fill out the following table:

Step	**Number of unreacted "A" groups**	**Number of molecules**	$\mathbf{N_n}$
0			
1			
2			
3			
4			

9. Compare your answers in question 8 with those from question 1. What similarities and/or differences do you notice?

10. Consider the following statement:

    "In polymerizations of AA- and BB-type monomers, we should be able to use the same expressions to calculate N_n as we did for polymerizations of AB-type monomers."

    In two or three complete sentences, briefly critique or defend this statement, making sure to explain your reasoning.

Information:

> A reaction is *stoichiometrically balanced* if the initial reaction mixture contains exactly the same number of "A" and "B" reactive groups.

Critical Thinking Questions:

11. Are the reactions in Models 1 and 2 stoichiometrically balanced? Briefly explain your answer in one or two complete sentences.

12. Predict what the reaction mixtures in Models 1 and 2 might look like if you let them react until no more reactions could take place:

Model 1　　Model 2

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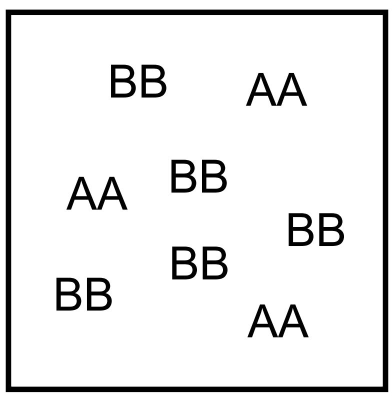
13. Calculate the number-average degree of polymerization for both of the “final” states you drew in response to the previous question:

Model 3: A Stoichiometrically-Imbalanced Reaction Mixture

Practically speaking, it is often very difficult to ensure that a reaction mixture is perfectly stoichiometrically-balanced, and there is often a small excess of one type of monomer or the other.

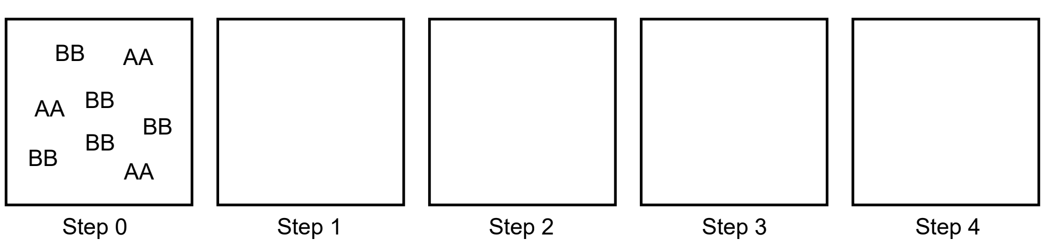
In this model, consider a reaction mixture that starts with 3 AA-type monomers and 5 BB-type monomers:



Step 0

Critical Thinking Questions:

14. Fill in the blank spaces in the figure below with reasonable predictions for what the reaction mixture might look like in each successive step.



15. Which type of reactive group is the "limiting reagent" in this reaction? Briefly explain your reasoning.

16. Predict what the reaction mixture in Model 3 might look like if you let it react until no more reactions could take place:

Model 3

17. Calculate the number-average degree of polymerization for the "final" state you drew in response to the previous question:

18. Is the final degree of polymerization for this stoichiometrically-unbalanced reaction smaller than, equal to, or larger than the final degree of polymerization you calculated for the stoichiometrically-balanced reactions in Models 1 and 2?

19. Which type of reactive group is on the ends of all of the chains you drew in question 16?

20. Briefly critique or defend the following statement:

    "When drawing the structure of a polymer produced by a step-growth polymerization, we should always make sure that we draw end groups consistent with whichever reactive species was present in excess."

Information:

In stoichiometrically-imbalanced step-growth polymerizations with an excess of B groups, we define a parameter r that reflects the ratio of A groups to B groups. If the initial number of A groups is v_A^0 and the initial number of B groups is v_b^0, then

$$r = \frac{v_A^0}{v_B^0}$$

For a reaction mixture with stoichiometric imbalance r at extent of reaction p, the number-average degree of polymerization is given by

$$N_n = \frac{1+r}{1+r-2pr}$$

Critical Thinking Questions:

21. Using this expression, fill in the following table with the expected number-average degree of polymerization for different combinations of r and p values:

	$p = 0.9$	$p = 0.99$	$p = 0.999$
$r = 0.9$			
$r = 0.99$			
$r = 0.999$			

22. On the basis of your answers to the previous question, briefly critique or defend the following statement: *"Achieving high molecular weights in step-growth polymerizations requires both very precise measurement of the reagents, and reaction conditions which strongly favor the bond-forming reaction."*

Exercises:

1. Often, when calculating extents of reaction and degrees of polymerization, our usual rules regarding significant figures don't work well. To see why, do the following:

   a) Rearranging our equation for N_n for a stoichiometrically-balanced reaction and solving for p, we find that $p = \frac{N_n - 1}{N_n} = 1 - \frac{1}{N_n}$.
   Using this equation, calculate the extent of reaction necessary to reach each of the following degrees of polymerization. Write out as many digits as your calculator gives you.

N_n	p
10	
100	
300	
320	
1000	
10000	

b) How many significant figures are given in each of the N_n values in the preceding problem? What value(s) would you get for p if you round your answer following the usual sig fig rules?

c) In the context of your results, does the "usual" rule for sig figs make sense when calculating extents of reactions? Explain your reasoning in 1-2 complete sentences.

d) Propose an alternate sig fig rule that would make more sense for calculating extents of reaction.

2. In the stoichiometrically-balanced reactions in models 1 and 2, the number of molecules was equal to the number of unreacted 'A' groups in each step. Is the same thing true for the stoichiometrically-imbalanced reaction in Model 3? Why or why not?

3. In this activity, we only calculated the number-average *degree of polymerization* of the polymers produced in step-growth polymerizations. However, usually, we want to be able to calculate the *molecular weight* of the polymers as well.

   a) In Model 1, we considered a reaction of AB-type monomers. If each monomer had mass m_{AB}, how would you calculate the number-average molecular weight, M_n, of the polymer produced when the extent of reaction is equal to p?

   b) In Model 2, we considered a stoichiometrically-balanced reaction of AA- and BB-type monomers. If the AA-type monomers each had mass m_{AA} and the BB-type monomers each had mass m_{BB}, how would you calculate the number-average molecular weight of the polymer produced when the extent of reaction is equal to p?
   Note: this question is a little tricky - remember that N_n counts monomers, *but in this reaction, not all of the monomers have the same molecular weight. How might you be able to correct for this?*

4. In Model 3, we considered a stoichiometrically-imbalanced reaction of AA- and BB-type monomers. However, another important limit occurs when we have equal numbers of AA- and BB-type monomers, but add in an extra monofunctional reagent "Bx" that can only react on one side.

   In this exercise, suppose that we have v_A^0 'A' groups from AA-type monomers, v_B^0 'B' groups from BB-type monomers, and $v_{B'}^0$ 'B' groups from Bx-type molecules.

   a) Consider the following statements from two students:
      - **Student 1:** *"The total number of 'B' groups is just $v_B^0 + v_{B'}^0$, so we can account for the presence of monofunctional Bx molecules by replacing $r = \frac{v_A^0}{v_B^0}$ with $r' = \frac{v_A^0}{v_B^0 + v_{B'}^0}$."*
      - **Student 2:** *"One Bx molecule has the same effect on the degree of polymerization as one BB-type molecule. Since Bx-type molecules have the same effect with half as many 'B' groups, that means that a 'B' group from a Bx molecule is twice as effective at stopping chain growth as a 'B' group from a BB molecule, so we should replace r with $r' = \frac{v_A^0}{v_B^0 + 2v_{B'}^0}$."*

      Which student do you agree with, and why?

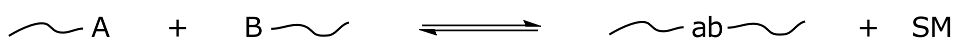
   b) Monofunctional reagents are a common impurity in supplies of difunctional monomers. Briefly explain why this means it is necessary to rigorously purify the starting materials used in step-growth polymerizations.

Activity 3: Equilibrium in Condensation Polymerizations

Model 1: Equilibria in Condensation Reactions

So far, we have written all reactions as unidirectional, proceeding only from *reactants* to *products*. However, in reality, the reactions used to produce polymers by step-growth polymerization are typically reversible, and proceed under equilibrium conditions.

This issue is particularly important for condensation reactions, which produce a small-molecule byproduct. Consider the reaction of an "A" functional group with a "B" functional group to produce an "ab" bond:



Here, we have included the small molecule byproduct, "SM" on the right side of the reaction. We have also written the reaction arrow as a double arrow ($\rightleftharpoons$) to indicate that the reaction is reversible.

Critical Thinking Questions:

1. Write an expression for the equilibrium constant for this reaction, K_{eq}, in terms of the concentrations of A groups ([A]), B groups ([B]), ab bonds ([ab]), and released small molecules ([SM]):

2. Suppose we start with v_A^0 A groups. If the reaction is stoichiometrically balanced, that means we also start with v_A^0 B groups.

   Using this information, complete the following ICE table:

	A	**B**	**ab**	**SM**
Initial	v_A^0	v_A^0	0	0
Change	$-x$			
Equilibrium		$v_A^0 - x$		

3. Plug your values from the equilibrium line of this table into your expression from question 1 to find an expression for K_{eq} in terms of v_A^0 and x.

   Note: your expression from question 1 is written in terms of concentrations, while the values in the table in question 2 are written in terms of numbers of molecules. In this problem, however, it turns out that everything cancels such that you can just plug the values from question 2 into the expression from question 1 directly.

4. When the extent of reaction is equal to p, the number of A groups that have reacted is pv_A^0, so $x = pv_A^0$. Using this information, rewrite K_{eq} only in terms of p.

   Hint: to make the next question easier, leave the denominator in the form $(1-p)^2$ *instead of multiplying it out.*

5. Solve for p in terms of K_{eq}.

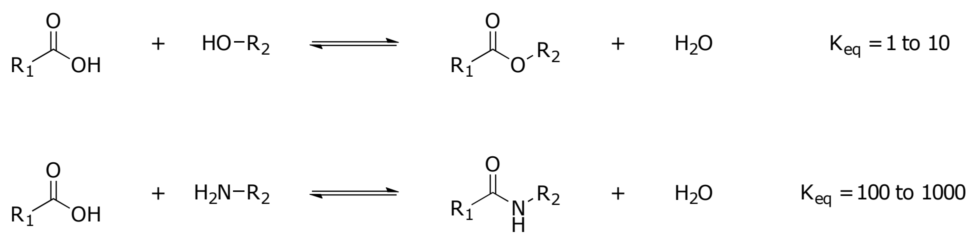
   Hint: start by taking the square root of both sides of the equation.

6. Finally, recalling that the number-average degree of polymerization is related to p by $N_n = \frac{1}{1-p}$, find an expression for N_n in terms of K_{eq}.

Model 2: Equilibrium in Condensation Polymerizations

Polyesters and polyamides are two important classes of polymers formed by condensation polymerization.

Shown below are the bond-forming reactions and range of equilibrium constants for typical esterification and amidation reactions:



Critical Thinking Questions:

7. Calculate the range of N_n values you would expect for a polyester synthesized under equilibrium conditions.

8. Calculate the range of N_n values you would expect for a polyamide synthesized under equilibrium conditions.

Information:

Commercial applications of polyesters and polyamides typically require degrees of polymerization of 100 or more.

Critical Thinking Questions:

9. Can commercial polyesters and polyamides be produced under equilibrium conditions? Why or why not?

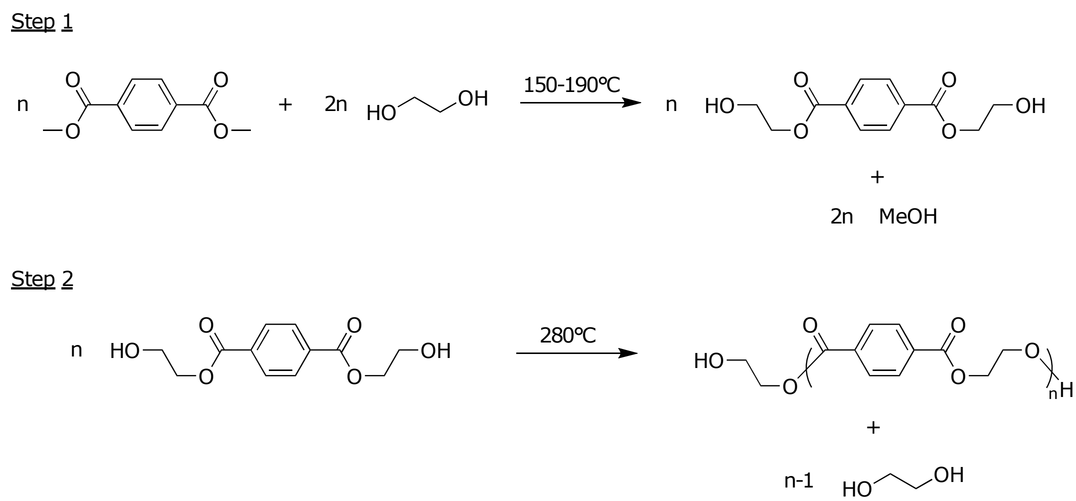
10. Propose at least one way that you could increase the degree of polymerization for polymers produced using the reactions shown in Model 2. Explain your proposed solution in 2-3 complete sentences.

    Hint: you will probably want to think about this question in the context of Le Chatelier's principle.

Model 3: Synthesis of Polyethylene Terephthalate

One commercially-important polyester produced by condensation polymerization is polyethylene terephthalate, or PET. PET is produced in quantities of more than 9 billion pounds per year, and is used in drink bottles, plastic films, and many other products.

PET is produced in a two-step transesterification process, as shown below:



Critical Thinking Questions:

11. What small molecule is released in the transesterification reaction shown in Step 1?

12. What small molecule is released in the transesterification reaction shown in Step 2?

Information:

The boiling point of methanol is 64.7°C.

The boiling point of ethylene glycol ($HOCH_2CH_2OH$) is 196°C.

Critical Thinking Questions:

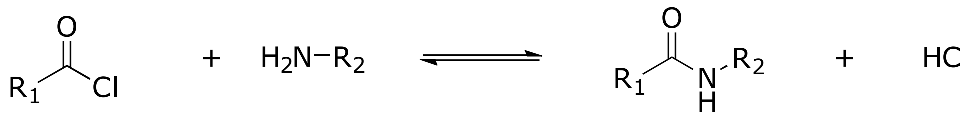
13. Why is the reaction in Step 1 carried out between 150 and 190°C? Briefly explain your answer in 1-2 complete sentences.

14. Why is the reaction in Step 2 carried out at 280°C? Briefly explain your answer in 1-2 complete sentences.

15. If we attempted to run the reaction in Step 1 at 280°C, would we form the desired polymer? Why or why not?

Exercises:

1. Both of the reactions shown in Model 2 produced water (H_2O) as the small-molecule byproduct. However, if we had started with acid chloride reagents, we might instead produce HCl as the small-molecule byproduct:



   Explain why, in this case, adding a stoichiometric amount of triethylamine should help promote formation of high molecular weight products.

2. Explain, in complete sentences, why the reactions shown in Model 3 are both referred to as "trans-esterification" reactions.

   Hint: what types of bonds are broken in each reaction, and what types of bonds are formed? How does this differ from the other esterification reactions you have studied in this class?

3. Why does the first reaction in Model 3 (i.e. "Step 1") produce only a small oligomer rather than a large polymer chain? Explain your answer in terms of the reaction stoichiometry.

4. The reactions in Model 3 produce PET by transesterification of AA and BB-type monomers.

   Propose alternate monomers that could be used to produce the same polymer by...

   a) ... esterification of AA and BB-type monomers:

   b) ... transesterification of an AB-type monomer:

   c) Both the AA+BB-type trans-esterification shown in Model 3 and the AA+BB-type esterification you drew in part (a) of this problem are used commercially to produce PET. However, PET is never (or only rarely) produced from AB-type monomers. Why do you think this is true?

5. Although our focus in this exercise was on how to increase the degree of polymerization by removing the small-molecule byproduct, it is sometimes advantageous to push the equilibrium in the other direction.

   If you were to take a sample of PET and heat it in the presence of a large excess of water, what products would you expect to result? Propose at least one application in which this process might be useful.

Activity 4: Molecular Weight Distributions in Step-Growth Polymerizations

Model 1: Probabilities of Forming Different Chain Lengths

Suppose we perform a step-growth polymerization of AB-type monomers and stop the polymerization at extent of reaction p (i.e. we stop the polymerization when the fraction of A groups that have reacted is equal to p).

Suppose we then select a single molecule from this reaction mixture. This molecule will have an unreacted A group on one end, and an unreacted B group on the other.

Consider the following argument:

1. The unreacted 'B' group was originally part of an 'AB' monomer. Of these AB monomers ($i = 1$),
   - The fraction in which the A group monomer did *not* react, and the molecule remained an AB monomer, is $1 - p$.
   - The fraction in which the A group *did* react, and the selected molecule is at least an AbaB dimer, is p.
2. Of the molecules that reacted to form AbaB dimers ($i = 2$),
   - The fraction *of dimers* in which the A group did not react, and the molecule remained an AbaB dimer, is $1 - p$. The total fraction *of molecules* that are AbaB dimers is thus

     $$\begin{gathered}(\text{fraction of molecules that form dimers}) \cdot (\text{fraction of dimers that don't react further}) \\ = p(1-p)\end{gathered}$$

   - The fraction *of dimers* in which the A group did react, and the selected molecule is at least an AbabaB trimer, is p. The total fraction *of molecules* that form at least an AbabaB trimer is thus

     $$\begin{gathered}(\text{fraction of molecules that form dimers}) \cdot (\text{fraction of dimers that react further}) \\ = p \cdot p = p^2\end{gathered}$$

3. Of the molecules that reacted to form AbabaB trimers ($i = 3$),
   - The fraction *of trimers* in which the A group did not react, and the molecule remained an AbabaB trimer, is $1-p$. The total fraction *of molecules* that are AbabaB trimers is thus

     $$\begin{gathered}(\text{fraction of molecules that form trimers}) \cdot (\text{fraction of trimers that don't react further}) \\ = p^2(1-p)\end{gathered}$$

   - The fraction *of trimers* in which the A group did react, and the selected molecule is at least an AbababaB tetramer, is p. The total fraction *of molecules* that form at least an AbababaB tetramer ($i = 4$) is thus

     $$\begin{gathered}(\text{fraction of molecules that form trimers}) \cdot (\text{fraction of trimers that react further}) \\ = p^2 \cdot p = p^3\end{gathered}$$

Critical Thinking Questions:

1. Following this reasoning,

   a) How would you calculate the fraction of *molecules* that *remain* as AbababaB tetramers ($i = 4$)? Write your answer in both words and symbols.

   b) How would you calculate the fraction of molecules that react further to form at least an AbababababaB pentamer ($i = 5$)? Write your answer in both words and symbols.

2. Using the information in the model, and your answers to the previous question, fill in the following table:

i	**Fraction of molecules that contain exactly i monomers**
1	
2	
3	
4	
5	

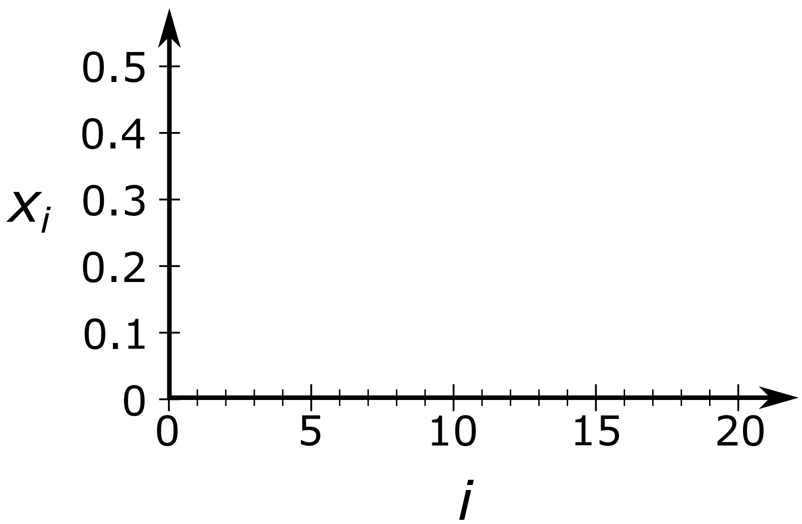
3. What pattern do you notice in these values? Briefly describe your observations in 1-2 complete sentences.

4. Complete the following statement:
   "The fraction of molecules, x_i, that are composed of exactly i monomers is __________."

5. Using this expression, calculate the fraction of molecules that have exactly length i for both $p = 0.5$ and $p = 0.9$ at the following values of i:

i	x_i when $p = 0.5$	x_i when $p = 0.9$
1		
2		
3		
5		
10		
15		
20		

6. Plot your results on the following axes. Make sure to use a different symbol for points corresponding to $p = 0.5$ than for the points corresponding to $p = 0.9$.



7. How are the plots for $p = 0.5$ and $p = 0.9$ similar, and how are they different? Briefly describe your observations in 2-3 complete sentences.

8. What is the *most probable* chain length for each value of p?

9. Can the fraction of chains with length $i + 1$ ever be *greater* than the fraction of chains with length i? Justify your answer in 1-2 complete sentences.

Model 2: M_n and M_w for Step-Growth Polymerizations

To calculate M_n and M_w, we need to know n_i, or the total number of chains with i monomers.

If we started with v_A^0 monomers, then when the extent of reaction is equal to p, there will be $(1-p)v_A^0$ unreacted A groups left. Recalling that the number of unreacted A groups is equal to the number of molecules in the reaction mixture, this lets us write

$$\begin{aligned} n_i &= \text{(fraction of molecules that have length } i\text{) x (number of molecules in reaction mixture)} \\ &= \left(p^{i-1}(1-p)\right)\left((1-p)v_A^0\right) \\ &= p^{i-1}(1-p)^2 v_A^0 \end{aligned}$$

If we plug this expression into our equation for M_n, we get

$$M_n = \frac{\sum_i n_i M_i}{\sum_i n_i} = M_0 \frac{\sum_i p^{i-1}(1-p)^2 i}{\sum_i p^{i-1}(1-p)^2}$$

where M_0 is the molecular weight of the monomer ($M_i = M_0 i$).

Evaluating these sums is a bit tedious, but if we do so, we obtain

$$M_n = \frac{M_0}{1-p} \qquad \text{or} \qquad N_n = \frac{M_n}{M_0} = \frac{1}{1-p}$$

which is exactly what we expected (whew - our math worked!).

Similarly, if we plug this expression into our equation for M_w and work through the sums, we get

$$M_w = \frac{\sum_i n_i M_i^2}{\sum_i n_i M_i} = M_0 \frac{1+p}{1-p} \qquad \text{or} \qquad N_w = \frac{M_w}{M_0} = \frac{1+p}{1-p}$$

Critical Thinking Questions:

10. Calculate the dispersity for a step-growth reaction with extent of reaction p.

11. What is the value of the dispersity when $p = 0$? Briefly comment on whether or not this answer makes sense.

12. What is the value of the dispersity when $p = 1$?

13. Can the dispersity of a polymer produced by step-growth polymerization ever be greater than 2? Briefly defend your answer in 1-2 complete sentences.

Exercises:

1. Suppose you synthesized a polymer by step-growth polymerization and found that it had a dispersity of 1.86.

    a) What must the extent of reaction have been in this polymerization?

    b) What would you expect the number-average degree of polymerization of this polymer to be?

Chapter 4

Free-Radical Polymerization

Chapter 5

Controlled Polymerizations

Chapter 6

Copolymers

Part III

Polymer Physics

Chapter 7

Conformations of Polymer Chains

Chapter 8

Mechanical Properties of Polymers

Chapter 9

Phase Behavior of Polymers & Their Solutions

Chapter 10

Thermal Properties of Polymers