EE 300 Hw#1

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3)

$1(s) = 2(s) + 3(s) = S$; entire system functions → sss

$1(s) = 2(s) + 3(f) = S$; entire system functions → ssf

$1(s) = 2(f) + 3(s) = S$; entire system function → sfs

$1(f) = 2(s) + 3(s) = f$; system malfunction_fss

$1(f) = 2(f) + 3(s) = f$; system malfunction_ffs

$1(f) = 2(s) + 3(f) = f$; system malfunction-fsf

$1(f) = 2(f) + 3(f) = f$; system malfunction_fff

a) ssf, sfs, fss

b) ssf, sfs, fss, sss

c) sfs, ssf, sss

A' = opposite A
$\cup$ = list all outcomes
$\cap$ = list all outcomes that are same in variable.

d). c' = c not = fff, fsf, ffs, fss, sff

$A \cup C$ = A or C = ssf, sfs, fss, sss

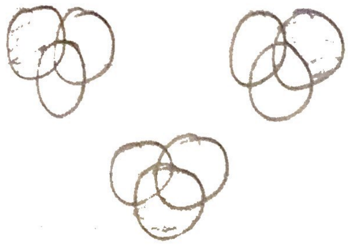
$A \cap C$ = A and C = ssf, sfs

$B \cup C$ = ssf, sfs, fss, sss

$B \cap C$ = ssf, sfs, sss

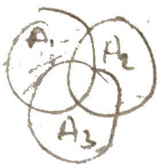
8)

a)



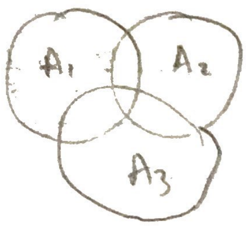
$(A_1 \cap A_2' \cap A_3') \cup (A_1' \cap A_2 \cap A_3') \cup (A_1' \cap A_2' \cap A_3)$

b)



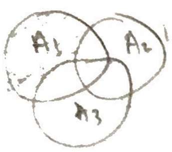
$(A_1 \cap A_2 \cap A_3)$

c)



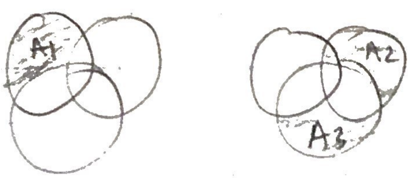
$(A_1 \cap A_2' \cap A_3')$

D)



$(A_1 \cap A_2' \cap A_3')$

e)



$(A_1 \cap A_2' \cap A_3') \cup (A_1' \cap A_2 \cap A_3)$

22)

a) $P(A \cup B) = PA + PB - P(A,B)$

$P(A,B) = PA + PB - P(A \cup B)$

↳ $0.4 + 0.5 - 0.7 = .2$, 20% probability

b) $0.4 - 0.2 = 0.2$, 20% probability

c) $0.1 + 0.5 - 0.2 = 0.4$, 40% probability

25)

A - GPS - 44%
B - sunroof - 55%
C - automatic transmission 70%

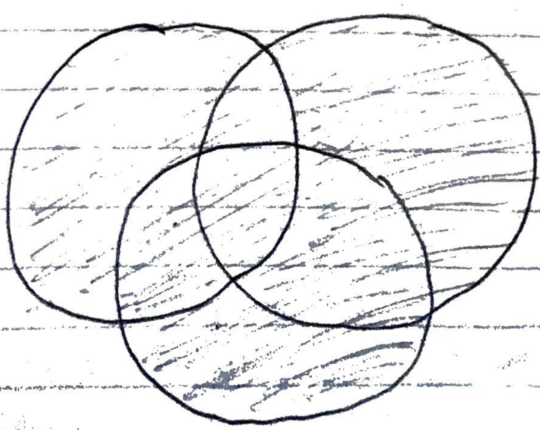
$A \cup B = 63\%$
$A \cup C = 77\%$
$B \cup C = 80\%$
$A \cup B \cup C = 85\%$

$P(A) = 0.44$, $P(B) = 0.55$, $P(C) = 0.7$

$P(A \cup B) = 0.63$, $P(A \cup C) = 0.77$, $P(B \cup C) = 0.80$

$P(A \cup B \cup C) = 0.85\%$

a) 85%, given



since $P(A \cup B \cup C) = 85\%$

we need to do Venn diagram first

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

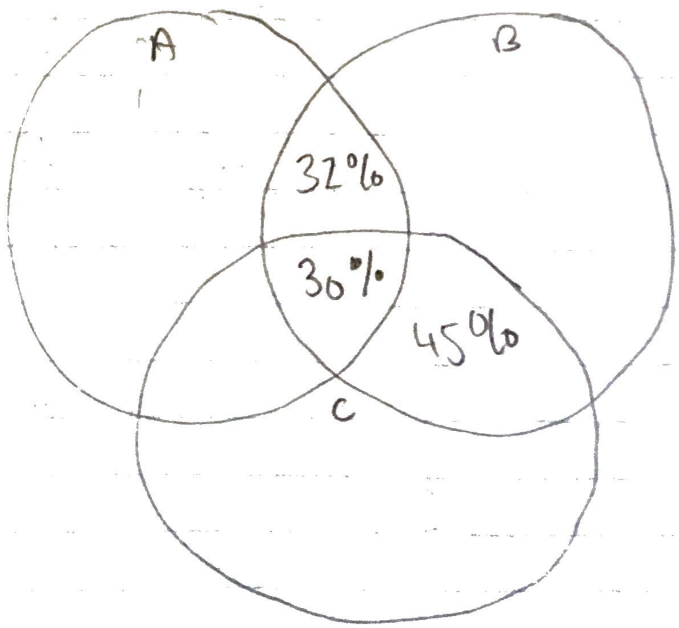
↳ $.40 + .55 - .63 = \underline{0.32} = 32\%$ A and B

→ $P(A \cap C) \to 0.40 + 0.70 - 0.77 = \underline{0.33}$ $\underline{33\%}$ A and C

→ $P(B \cap C) \to 0.55 + 0.70 - 0.80 = \underline{0.45}$, $\underline{45\%}$ B and C

→ $P(A \cap B \cap C) \to P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(A \cap C) + P(B \cap C)$

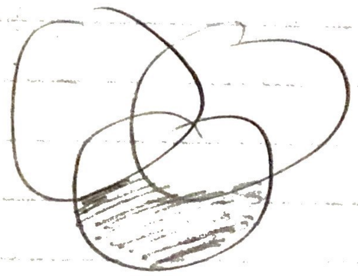
↳ $0.85 - 0.40 - 0.55 - 0.70 + 0.32 + 0.33 + 0.45$
$= \underline{0.30}$ or $\underline{30\%}$ of A and B and C



b) $P(A\cap B\cap C)' \longrightarrow P(1) - P(A) + P(B) + P(C)$

↳ $1-0.85 = \boxed{15\%}$

c) $P[A'\cap B'\cap C) = P(C) - P(A\cap C) - P(B\cap C)$
$+ P(A\cap B\cap C) \rightarrow 0.7 - 0.33 - 0.45 + 0.30 = \boxed{.22}$



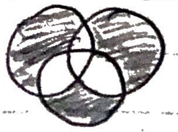
only transmission
$(A\cup B)'\cap C$

D) only 1 of each, already have C = $\underline{0.22}$ / 22%

for B = $P(A'\cap B\cap C') = P(B) - P(A\cap B) - P(B-C)$
$+ P(A\cap B\cap C) = 0.55 - 0.32 - 0.45 + 0.30 = \underline{0.08}$

for A = $P(A\cap B'\cap C') = P(A) - P(A\cap B) - P(A\cap C)$
$+ P(A\cap B\cap C) = 0.40 - 0.32 - 0.33 + 0.30$

↳ $\underline{0.05}$



Then add all of them: $0.22 + 0.05 + 0.08 = \boxed{0.35}$

34) 25 total failed keyboards

a) 6 electical defects

19 mechanical defects

$${}_nC_r = \frac{n!}{r!(n-r)!} \; ; n = 25, r = 5$$

$$\hookrightarrow {}_{25}C_5 = \frac{25!}{5! \cdot (25-5)!} = \frac{25!}{5! \cdot (20)} = 53{,}130$$

53,130 ways

b) $\binom{6}{2}$ = electrical → rest r mechanic = $\binom{19}{3}$

total

left over

$$\binom{6}{2}\binom{19}{3} = \frac{6!}{2!(6-2)} \cdot \frac{19!}{3!(19-3)}$$

$\hookrightarrow$ 14,535 ways

c) $P(E) = \frac{m}{n}$

$\binom{25}{5}$ 5 keyboards randomly selected from total of 25

$\binom{19}{4}\binom{6}{1} + \binom{19}{5}\binom{6}{0}$

$\frac{19!}{5!(19-5)} \cdot \frac{6!}{0!(6-0)}$

$\hookrightarrow 11,628$

$= \frac{19!}{4!(19-4)} \cdot \frac{6!}{1!(6-1)} = 23,256$

Add them up $= 23,256 + 11,628 = 34,884$

$P(A) = \frac{N(A)}{N} = \frac{34,884}{53,130} = \boxed{0.6566 \text{ or } 65.66\%}$

3a)

15 compact lightbulbs

5 - 13
33.33%

6 - 18W
40%

4 - 23W
26.66 %

$$\frac{15!}{3!(15-3)!} = 455 \text{ ways}$$

a) $$\frac{\binom{4}{2}\binom{11}{1}}{455} = \frac{\left(\frac{4!}{2!(4-2)!}\right)\left(\frac{11!}{1!(11-1)!}\right)}{455}$$

↳ 0.1450

b) $$\frac{\binom{5}{3}+\binom{6}{3}+\binom{4}{3}}{105}$$

↳ $$\left(\frac{5!}{3!(5-3)!}\right)+\left(\frac{6!}{3!(6-3)!}\right)+\left(\frac{4!}{3!(4-3)!}\right)$$

↳ 0.073

c) $$\frac{\binom{5}{1}\cdot\binom{6}{1}\binom{4}{1}}{105} = \frac{\left(\frac{5!}{1!(5-1)!}\right)\cdot\left(\frac{6!}{1!(6-1)!}\right)\left(\frac{4}{1!(4-1)!}\right)}{105}$$

↳ 0.2637

D) $$\frac{\binom{11}{4}}{\binom{15}{4}} = \left(\frac{\left(\frac{11!}{4!(11-4)!}\right)}{\left(\frac{15!}{4!(15-4)!}\right)}\right) = \boxed{0.241}$$

System

52) Pump #1 Pump #2

$P(A \cup B) = 7\%$; least 1 pump fails

$P(A \cap B) = 1\%$; both pumps fails

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.07 = q + q - 0.01$

$\to 0.07 = 2q - 0.01 \to -2q = -0.01 - 0.07$

$2q = 0.08$

$q = \frac{0.08}{2} \to q = 0.04$

Probability pump 1 will fail is

4 %