Example Problem

Suppose we want to find the maximum value of $\mathbf{z = 4x_1 + 6x_2}$ where $x_1, x_2 \geq 0$, subject to the following constraints:

$$-x_1 + x_2 \leq 11$$
$$-x_1 - x_2 \geq -27$$
$$2x_1 + 5x_2 \leq 90$$

Source for the question: [Link]

Solution to Example Problem

Step 1: To transform the given linear form into Standard Form

$$-x_1 + x_2 \leq 11$$
$$-x_1 - x_2 \geq -27 \rightarrow x_1 + x_2 \leq 27$$
$$2x_1 + 5x_2 \leq 90$$
$$z = 4x_1 + 6x_2 \rightarrow z - 4x_1 - 6x_2 = 0$$

Step 2: Determine the Slack Variable

$$-x_1 + x_2 + s_1 = 11$$
$$x_1 + x_2 + s_2 = 27$$
$$2x_1 + 5x_2 + s_3 = 90$$

Step 3: Setting up the Tableau

	z	x_1	x_2	s_1	s_2	s_3	b
	1	-4	-6	0	0	0	0
s_1	0	-1	1	1	0	0	11
s_2	0	1	1	0	1	0	27
s_3	0	2	5	0	0	1	90

Step 4: Check Optimality
The values of x1 and x2 in the highlighted row are negative, hence they are not optimized.

Step 5: Identify the pivot variable
As -6 is the smallest number in the highlighted row, pivot = -6

	z	x_1	x_2	s_1	s_2	s_3	b	
	1	-4	-6	0	0	0	0	
s_1	0	-1	1	1	0	0	11	11/1 = 11
s_2	0	1	1	0	1	0	27	27/1 = 27
s_3	0	2	5	0	0	1	90	90/5 = 18

Step 6: Create the new Tableau

1. The indicator row variable is replaced with the pivot column variable.

	z	x_1	x_2	s_1	s_2	s_3	b
x_2							
s_2							
s_3							

2. Divide the indicator row by pivot

	z	x_1	x_2	s_1	s_2	s_3	b
x_2	0	-1	1	1	0	0	11
s_2							
s_3							

3. Filling up the remaining table

New tableau value = (Negative value in old tableau pivot column) x (value in new tableau pivot row) + (Old tableau value)

Example,

New tableau value for highlighted x_1 = (6) x (-1) + (-4) = -10

	z	x_1	x_2	s_1	s_2	s_3	b
		-10					
x_2	0	-1	1	1	0	0	11
s_2							
s_3							

Similarly calculating for all the other values,

	z	x_1	x_2	s_1	s_2	s_3	b
	1	-10	0	6	0	0	66
x_2	0	-1	1	1	0	0	11
s_2	0	2	0	-1	1	0	16
s_3	0	7	0	-5	0	1	35

Step 7: Check Optimality

The values of x1 in the highlighted row is negative, hence they are not optimized.

Step 8: Identify the pivot variable

As -10 is the smallest number in the highlighted row, pivot = -10

	z	x_1	x_2	s_1	s_2	s_3	b	
	1	-10	0	6	0	0	66	
x_2	0	-1	1	1	0	0	11	11/-1 = -11
s_2	0	2	0	-1	1	0	16	16/2 = 8
s_3	0	7	0	-5	0	1	35	35/7 = 5

Step 9: Create the new Tableau

	z	x_1	x_2	s_1	s_2	s_3	b
	1	0	0	-8/7	0	10/7	116
x_2	0	0	1	2/7	0	1/7	16
s_2	0	0	0	3/7	1	-2/7	6
x_1	0	1	0	-5/7	0	1/7	5

Step 10: Check Optimality
The values of s1 in the highlighted row is negative, hence they are not optimized.

Step 11: Identify the pivot variable
As -8/7 is the smallest number in the highlighted row, pivot = -8/7

	z	x_1	x_2	s_1	s_2	s_3	b	
	1	0	0	-8/7	0	10/7	116	
x_2	0	0	1	2/7	0	1/7	16	56
s_2	0	0	0	3/7	1	-2/7	6	14
x_1	0	1	0	-5/7	0	1/7	5	-7

Step 12: Create the new Tableau

	z	x_1	x_2	s_1	s_2	s_3	b
	1	0	0	0	8/3	2/3	132
x_2	0	0	1	0	-2/3	1/3	12
s_1	0	0	0	1	7/3	-2/3	14
x_1	0	1	0	0	5/3	-1/3	15

Step 13: Check Optimality
The values in the highlighted row are positive, hence we have found the optimal solution.

Step 14: Identify Optimal Values
From table, $x_1 = 15$, $x_2 = 12$, $s_1 = 14$, $s_2 = 0$, $s_3 = 0$, $z = 132$
$z = 4x_1 + 6x_2 = 4(15) + 6(12) = 60 + 72 = 132$