## Epic Questions based on the role playing game Dungeons and Dragons. And discrete probability.

First, some notation, there are lots of dice in Dungeons and Dragons, the notation XdY means roll a Y sided die X times and sum the result. For example 3d6 means roll a 6 sided die three times, and sum the result. 2d10 means roll a 10 sided die twice and sum. Assume all dice are fair.

**Epic Question 1. Dice rolls and character creation.** (20 points) In Dungeons and Dragons, an ability score of 18 indicates the highest innate ability a human can have. To generate an ability score, roll 3d6. A score of 3 is the lowest possible value a human can have, and a score of 18 is the highest. A value of 9 is considered average. There are 6 ability scores (strength, dexterity, intelligence, wisdom, constitution and charisma). The fun method for creating a character is to generate 3 values per ability score, and keep the highest one. Answer the following questions.

- a) What is the probability that any one roll of 3 dice, you generate an ability score of 18?
- b) What is the probability, using the fun method of generating 3 scores and keeping the highest, that you generate an ability score of 18?
- c) What is the probability using the fun method, you generate a character that is the most perfect form of human possible (i.e. a character not unlike Prof Fred Fontaine), that has 18's in all ability scores?
- d) What is the probability using the fun method, you generate the most average, uninteresting character possible (i.e. a character remarkably similar to Prof. Keene), that has 9's in all ability scores? Hint: there are 25 ways to roll a 9 with 3 dice.

**Epic Question 2. Wizards, Trolls and Warriors.** (30 points) Despite being completely average, there is a mighty and powerful wizard named Keene, who is suddenly ambushed by a menacing group of trolls. The reason for the trolls hatred of Keene is unclear, but they are often heard screeching the word "stochastics" in a bloody rage. Every troll has 1d4 hit points, which are the amount of damage they can take before dying. The wizard Keene has unfortunately been out drinking the night before, and can only remember 1 spell, FIREBALL, which does 2d2 damage to everything within range. For example if a troll has 3 HP, and the FIREBALL does 3 points of damage, the troll is dead.

- a) What is the average number of hit points that each troll has? What is the average amount of damage the FIREBALL spell does? Find a bound on the probability that the FIREBALL does greater than 3 points of damage.
- b) Write the probability mass functions for the amount of damage the FIREBALL does and the amount of hit points trolls have.
- **c)** Suppose there are 6 trolls, and they are all within range of the FIREBALL. What is the probability that Keene slays all the trolls with this spell?
- **d)** Unfortunately, the FIREBALL kills all but 1 of the trolls. Given that 5 trolls perished, what is the expected amount of hit points that the remaining troll has?
- e) The lone survivor, Shitvam the Troll, flees back to his master, the evil and powerful warrior Shedjam. Shedjam brandishes two weapons, the Sword of Tuition and the Hammer of Tenure Denial, which do 2d6 and 1d4 points of damage respectively. Shedjam furiously attacks, and needs to roll an 11 or greater on a 20 sided die in order to hit the wizard Keene with his Sword. If Shedham hits with his Sword, he may also attempt an attack with the Hammer, again needing a 11 or greater to hit. What is the expected amount of damage Shedjam would do to the Wizard Keene per attack?
- f) (extra credit) Powerless against the mighty Shedjam, as the hungover Keene has cast his only spell, he summons the mighty tenured warrior Roby Tumblesnatch to defend him. Roby has fashioned a shield made of recycled cardboard that is remarkably effective, rendering Shedjam's weapons ineffective. While Roby and Shedjam battle to a stalemate, Keene recalls that knowledge is power, and thus reasons total knowledge is total power. Unfortunately, Keene is terrible at math, and is unable to compute the joint pdf he desires. What is the probability, given this exam question, that Keene receives tenure?