Mutual recursion

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I want to understand mutual recursion for myself, so here it is. Mutual recursion using a pair and fixpoint form.

```
let (a, b) = (fun x -> b x, fun x -> a x)
```

$$
\begin{aligned}
(a, b) &= \operatorname{fix} a.\, \operatorname{fix} b.(\lambda x.b\ x, \lambda x.a\ x) && (1)\\
&= \operatorname{fix} b.(\lambda x.b\ x, \lambda x.(\operatorname{fix} a.\, \operatorname{fix} b.(\lambda x.b\ x, \lambda x.a\ x))\ x) && (2)\\
&= (\lambda x.(\operatorname{fix} b.(\lambda x.b\ x, \lambda x.(\operatorname{fix} a.\, \operatorname{fix} b.(\lambda x.b\ x, \lambda x.a\ x))\ x))\ x, \lambda x.(\operatorname{fix} a.\, \operatorname{fix} b.(\lambda x.b\ x, \lambda x.a\ x))\ x) && (3)
\end{aligned}
$$

or

$$
\begin{aligned}
a &= \lambda x.(\operatorname{fix} b.(\lambda x.b\ x, \lambda x.(\operatorname{fix} a.\, \operatorname{fix} b.(\lambda x.b\ x, \lambda x.a\ x))\ x))\ x && (4)\\
b &= \lambda x.(\operatorname{fix} a.\, \operatorname{fix} b.(\lambda x.b\ x, \lambda x.a\ x))\ x && (5)
\end{aligned}
$$