# Mutual recursion

Jonathan Lam

2022/02/18

I want to understand mutual recursion for myself, so here it is. Mutual recursion using a pair and fixpoint form. The part I was missing was having the projection out of the tuple. The fixpoint form is described in the following sections of Harper's PFPL: Chapter 10 (Plotkin's PCF) and section 11.3 (Primitive and Mutual Recursion).

```
let rec even x = if x = 0 then true  else odd  (x - 1)
and     odd  x = if x = 0 then false else even (x - 1)
```

This translates to the tuple of mutually-dependent functions:

$$e' \equiv \lambda x.\, \text{if } x = 0.\{\mathrm{T} \mid \text{self}\ .1\ (x-1)\} \tag{1}$$
$$o' \equiv \lambda x.\, \text{if } x = 0.\{\mathrm{F} \mid \text{self}\ .0\ (x-1)\} \tag{2}$$
$$\text{fix}_{eo} = \text{fix}\,\text{self}\,.(e', o') \tag{3}$$
$$(e, o) \leftarrow \text{fix}_{eo} \tag{4}$$
$$\Downarrow (\lambda x.\, \text{if } x = 0.\{\mathrm{T} \mid \text{fix}_{eo}\ .1\ (x-1)\}, \lambda x.\, \text{if } x = 0.\{\mathrm{F} \mid \text{fix}_{eo}\ .0\ (x-1)\}) \tag{5}$$
$$e \leftarrow \lambda x.\, \text{if } x = 0.\{\mathrm{T} \mid \text{fix}_{eo}\ .1\ (x-1)\} \tag{6}$$
$$o \leftarrow \lambda x.\, \text{if } x = 0.\{\mathrm{F} \mid \text{fix}_{eo}\ .0\ (x-1)\} \tag{7}$$

Now, let us consider the evaluation of `even 2`:

$$e\ 2 \tag{8}$$
$$(\lambda x.\, \text{if } x = 0.\{\mathrm{T} \mid \text{fix}_{eo}\ .1\ (x-1)\})\ 2 \tag{9}$$
$$\text{if } 2 = 0.\{\mathrm{T} \mid \text{fix}_{eo}\ .1\ (2-1)\} \tag{10}$$
$$\text{fix}_{eo}\ .1\ 1 \tag{11}$$
$$(\lambda x.\, \text{if } x = 0.\{\mathrm{T} \mid \text{fix}_{eo}\ .1\ (x-1)\}, \lambda x.\, \text{if } x = 0.\{\mathrm{F} \mid \text{fix}_{eo}\ .0\ (x-1)\}).1\ 1 \tag{12}$$
$$(\lambda x.\, \text{if } x = 0.\{\mathrm{F} \mid \text{fix}_{eo}\ .0\ (x-1)\})\ 1 \tag{13}$$
$$\text{if } 1 = 0.\{\mathrm{F} \mid \text{fix}_{eo}\ .0\ (1-1)\} \tag{14}$$
$$\text{fix}_{eo}\ .0\ 0 \tag{15}$$
$$(\lambda x.\, \text{if } x = 0.\{\mathrm{T} \mid \text{fix}_{eo}\ .1\ (x-1)\}, \lambda x.\, \text{if } x = 0.\{\mathrm{F} \mid \text{fix}_{eo}\ .0\ (x-1)\}).0\ 0 \tag{16}$$
$$(\lambda x.\, \text{if } x = 0.\{\mathrm{T} \mid \text{fix}_{eo}\ .1\ (x-1)\})\ 0 \tag{17}$$
$$\text{if } 0 = 0.\{\mathrm{T} \mid \text{fix}_{eo}\ .1\ (0-1)\} \tag{18}$$
$$\mathrm{T} \tag{19}$$