THE COOPER UNION FOR THE ADVANCEMENT OF SCIENCE AND ART
ALBERT NERKEN SCHOOL OF ENGINEERING

Optimization by memoization of evaluation tasks in the Hazel structured programming environment

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This thesis was prepared under the direction of the Candidate's Thesis Advisor and has received approval. It was submitted to the Dean of the School of Engineering and the full Faculty, and was approved as partial fulfillment of the requirements for the degree of Master of Engineering.

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ABSTRACT

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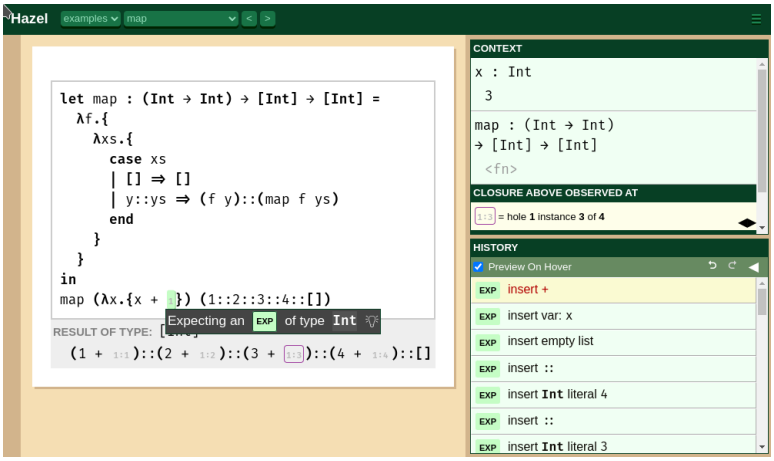

Figure 1: A screenshot of the Hazel live programming environment. Screenshot taken of the dev branch demo[2] on 02/06/2022.

1 Introduction

1.1 Problem statement

1.2 The contribution of this work

1.3 Structural overview

Section 2 provides a background on necessary topics in programming language (PL) theory and programming language implementations, in order to frame understanding for the Hazel live programming environment. Section 3 provides an overview of Hazel, in order to frame the work completed for this thesis project. Sections 4 to 6 describe the primary work completed for this project, as described in Section 1.2. Section 7 comprises an assessment of the work completed in terms of correctness and the theoretical performance. Section 8 is a discussion of future research directions that may be spawned off from this work. Section 9 concludes with a summary of findings and future work. The appendices contain additional information about the Hazel project not directly related to the primary contribution of this project, as well as selected source code snippets.

[2] `https://hazel.org/build/dev/`

2 Background

2.1 Functional programming

2.1.1 Recursion and mathematical induction

2.1.2 The λ-calculus

2.1.3 Purity and statefulness

2.1.4 Comparison to other programming paradigms

2.2 Implementations for programming languages

In order for a programming language to be practical, it must not only be defined as a set of syntax and semantics, but also have an *implementation* to run programs in the language. Hazel is implemented as an interpreted language, whose runtime is transpiled to Javascript so that it may be run as a client-side web application in the browser.

It is important to note that the definition of a language (its syntax and semantics) are largely orthogonal to its implementation. In other words, a programming language does not dictate whether it requires a compiler or interpreter implementation, and languages sometimes have multiple implementations.

2.2.1 Compiler vs. interpreter implementations

There are two general classes of programming language implementations: *interpreters* and *compilers*. Both types of implementations share the function of taking a program as input, and should be able to produce the same result (assuming an equal and determinstic machine state, equal inputs, correct implementations, and no exceptional behavior due to differences in resource usage).

A compiler is a programming language implementation that converts the program to some low-level representation that is natively executable on the hardware architecture (e.g., x86-64 assembly for most modern personal computers, or the virtualized JVM architecture) before evaluation. This process typically comprises *lexing* (breaking down into atomic tokens) the program text, *parsing*

the lexed tokens into a suitable *intermediate representation* (IR) such as LLVM, performing optimization passes on the intermediate representation, and then generating the target bytecode (such as x86-64 assembly). The bytecode outputted from the compilation process is used for evaluation. Compiled implementations tend to produce better runtime efficiency, since the compilation steps are performed separate of the evaluation, and because there is little to no runtime overhead.

An interpreter is a programming language implementation that does not compile down to native bytecode, and thus requires an interpreter or *runtime*, which performs the evaluation. Interpreters still require lexing and parsing, and may have any number of optimization stages, but do not generate bytecode for the native machine, instead evaluating the program directly.

In certain contexts (especially in the ML spheres), the term *elaboration* is used to the process of transforming the *external language* (a well-formed, textual program) into the *internal language* (IR). The interior language may include additional information not present in the external language, such as types generated by type inference (e.g., in SML/NJ) or bidirectional typing (e.g., in Hazel).

The distinction between compiled and interpreted languages is not a very clear line: some implementations feature just-in-time (JIT) compilation that allow "on-the-fly" compilation (e.g., the JVM or CLR), and some implementations may perform the lexing and parsing separately to generate a non-native bytecode representation to be later evaluated by a runtime. A general characterization of compiled vs. interpreted languages is the amount of runtime overhead required by the implementation.

Hazel is a purely interpreted language implementation, as optimizations for speed are not among its main concerns. However, performance is clearly one of the main concerns of this thesis project, but the gains will be algorithmic and use the nature of Hazel's structural editing and hole calculus to benefit performance, rather than changing the fundamental implementation. There is, however, a separate endeavor to write a compiled interpretation of Hazel[3], which is outside the scope of this project.

[3] `https://github.com/hazelgrove/hazel/tree/hazelc`

2.2.2 The substitution and environment models of evaluation

Evaluation in Hazel was performed using originally using a *substitution model of evaluation*, which is a theoretically simpler model. In this model, variables that are bound by some construct are substituted into the construct's body. For example, the variable(s) bound using a `let`-expression pattern are substituted in the `let`-expression's body, and the variable(s) bound during a function application are substituted into the function's body, and then the body is evaluated.

In this formulation, variables are "given meaning" via substitution; once evaluation reaches an expression, all variables in scope (in the typing context) will have been replaced by their value by some containing binding expression. In other words, variables are never evaluated directly; they are substituted by their values when bound, and their values are evaluated. The substitution model is useful for teaching purposes because it is simple and close to its mathematical definition: a variable can be thought of as an equivalent stand-in for its value.

However, for the purpose of computational efficiency, a model in which values are lazily expanded ("looked-up") only when needed is more efficient. This is called the *environment model of evaluation*, and generally is more efficient because the runtime does not need to perform an extra substitution pass over subexpressions; untraversed (unevaluated) branches do not require substituting; and the runtime does not need to carry an expression-level IR of the language. The last point is due to the fact that the substitution model manipulates expressions, while evaluation does not; this means that the latter is more amenable for compilation, and is how compiled languages tend to be implemented: each frame of the theoretical stack frame is a de facto environment frame. While switching from the substitution to environment model is not an improvement in asymptotic efficency, these effects are useful especially for high-performance and compiled languages.

Note that the substitution model does not imply a lazy (i.e., normal-order, call-by-name, call-by-need) evaluation as in languages such as Haskell or Miranda, in which bound variables are (by default) not evaluated until their value is required. Laziness is conceptually tied to substitution, but the substitution model does not require laziness. Like most programming languages, Hazel only has strict (i.e., applicative-order, call-by-value) evaluation: the expressions bound to variables are

evaluated at the time of binding.

The implementation of evaluation with environments differs from that of evaluation with substitution primarily in that: an evaluation environment is required to look up bound variables as evaluation reaches them; binding constructs extend the evaluation environment rather than performing substitution; and λ abstractions are bound with their evaluation environment at runtime to form (lexical) closures.

2.3 Classifications of type systems

2.3.1 Primer on programming language semantics and reasoning

2.3.2 Static vs. dynamic typing

2.3.3 Strong vs. weak typing

2.3.4 Inferred vs. manifold typing

2.3.5 Gradual typing

2.3.6 Other classifications

2.4 Approaches to programming interfaces

2.4.1 Structure editors

2.4.2 Graphical editors

2.4.3 Intentional, generative, and meta-programming

2.4.4 Applications to programming education

2.4.5 Drawbacks of non-textual editors

3 An overview of the Hazel programming environment

Hazel is the experimental language that implements the Hazelnut bidirectionally-typed edit static semantics with holes and the Hazelnut Live dynamic semantics, and it is also the name of the reference implementation. It is intended to serve as a proof-of-concept of the semantics with holes that attempt to mitigate the gap problem; however, the implementation is becoming increasingly practical with additional research effort. The reference implementation is an interpreter written in OCaml and transpiled to Javascript using the `js_of_ocaml` (JSOO) library [2] so that it may be run client-side in the browser. A screenshot of the reference implementation[4] is shown in Figure 1. The source code may be found on GitHub[5].

Hazel's syntax and semantics resembles languages in the ML (Meta Language) family of languages such as OCaml, although Hazel does not support polymorphism at this time. Hazel can be characterized as a purely functional, statically-typed, bidirectionally-typed, strict-order evaluation, structured editor language. Hazel semantically differs most significantly from other ML languages in the last respect due to its theoretic foundations in solving the gap problem.

[4] `https://hazel.org/build/dev/`
[5] `https://github.com/hazelgrove/hazel`

3.1 Hazelnut static semantics

3.1.1 Expression and type holes

3.1.2 Bidirectional typing

3.1.3 Example of bidirectional type derivation

3.2 Hazelnut Live dynamic semantics

3.2.1 Example of elaboration

3.2.2 Example of evaluation

3.2.3 Example of hole instance numbering

3.3 Hazel programming environment

3.3.1 Explanation of interface

3.3.2 Implications of Hazel

4 Implementing the environment model of evaluation

4.1 Hazel-specific implementation

In the case of Hazel (which does not prioritize speed of evaluation in its implementation, and is not a compiled language), evaluation with (reified) environments offers an additional (performance) benefit over the substitution model: the ability to easily identify (and thus memoize) operations over environments. This is useful for the optimizations described later in this paper.

The implementation of evaluation in Hazel differs from a typical interpreter implementation of evaluation with environments in three regards: we need to account for hole environments; environments are uniquely identified by an identifier for memoization (in turn for optimization); and any closures in the evaluation result should be converted back into plain λ abstractions, for reasons that will be discussed later TODOREF.

4.2 Formalization

Omar et al. [1] describes evaluation with the substitution model using a little-step semantics with an evaluation context $\mathcal{E}$, reproduced in TODOREF.

Figure 2 is an analogous small-step description of the substitution model, also using the little-step semantics.

The Hazel implementation follows a big-step evaluation model, so a big-step formalization is also displayed in Figure 3.

4.3 The evaluation boundary and post-processing

4.4 A strict evaluation boundary

4.5 Post-processing memoization

4.6 Purity of implementation

$\boxed{E \vdash d \to d'}$ Internal expression d takes an instruction transition to d' given environment E

$$\frac{}{E, x \leftarrow d \vdash x \to d}\ \text{EvalS-Var}$$

$$\frac{}{E \vdash (\lambda x : \tau.d) \to ([E]\lambda x : \tau.d')}\ \text{EvalS-Lam} \qquad \frac{E \vdash d \to d'}{E \vdash \text{fix}\, f.d \to \text{fix}\, f.d'}\ \text{EvalS-Fix}$$

$$\frac{E \vdash d_2\ \mathsf{final} \qquad E', x \leftarrow d_2 \vdash d_1 \to d_1'}{E \vdash ([E']\lambda x : \tau.d_1)(d_2) \to ([E']\lambda x : \tau.d_1')(d_2)}\ \text{EvalS-Ap}_1$$

$$\frac{E \vdash d_2\ \mathsf{final} \qquad E', x \leftarrow d_2 \vdash d_1\ \mathsf{final}}{E \vdash ([E']\lambda x : \tau.d_1)(d_2) \to d_1}\ \text{EvalS-Ap}_2$$

$$\frac{E \vdash d_2\ \mathsf{final} \qquad E', f \leftarrow ([E']\lambda x : \tau.d_1), x \leftarrow d_2 \vdash d_1 \to d_1'}{E \vdash \text{fix}\,.([E']\lambda x : \tau.d_1)(d_2) \to \text{fix}\,.([E']\lambda x : \tau.d_1')(d_2)}\ \text{EvalS-Ap}_3$$

$$\frac{E \vdash d_2\ \mathsf{final} \qquad E', f \leftarrow ([E']\lambda x : \tau.d_1), x \leftarrow d_2 \vdash d_1\ \mathsf{final}}{E \vdash \text{fix}\,.([E']\lambda x : \tau.d_1)(d_2) \to d_1}\ \text{EvalS-Ap}_4$$

$$\frac{E \vdash d_2\ \mathsf{final} \qquad E, x \leftarrow d_2 \vdash d_1 \to d_1'}{E \vdash \texttt{let}\ \ x = d_2\ \ \texttt{in}\ \ d_1 \to \texttt{let}\ \ x = d_2\ \ \texttt{in}\ \ d_1'}\ \text{EvalS-Let}_1$$

$$\frac{E \vdash d_2\ \mathsf{final} \qquad E, x \leftarrow d_2 \vdash d_1\ \mathsf{final}}{E \vdash \texttt{let}\ \ x = d_2\ \ \texttt{in}\ \ d_1 \to d_1}\ \text{EvalS-Let}_2$$

$$\frac{}{E \vdash (\!|)\!|_{\varnothing} \to (\!|)\!|_{E}}\ \text{EvalS-EHole} \qquad \frac{E \vdash d\ \mathsf{final}}{E \vdash (\!|d|\!)_{\varnothing} \to (\!|d|\!)_{E}}\ \text{EvalS-NEHole}$$

$$\frac{}{E \vdash \underline{n_1} + \underline{n_2} \to \underline{n_1 + n_2}}\ \text{EvalS-Op}$$

Figure 2: Small-step semantics for the environment model of evaluation

$\boxed{E \vdash d \Downarrow d'}$ Internal expression d evaluates to d' given environment E

$$\frac{E \vdash d \ \mathsf{final}}{E \vdash d \Downarrow d} \text{ EvalB-Final} \qquad \frac{}{E, x \leftarrow d \vdash x \Downarrow d} \text{ EvalB-Var}$$

$$\frac{E \vdash d \Downarrow d'}{E \vdash \mathrm{fix}\, f.d \Downarrow \mathrm{fix}\, f.d'} \text{ EvalB-Fix} \qquad \frac{}{E \vdash (\lambda x : \tau.d) \Downarrow [E](\lambda x : \tau.d')} \text{ EvalB-Lam}$$

$$\frac{E \vdash d_1 \Downarrow d_1' \qquad d_1' \neq ([E']\lambda x : \tau.d) \qquad d_1' \neq \mathrm{fix}\, f.d \qquad E \vdash d_2 \Downarrow d_2'}{E \vdash d_1(d_2) \Downarrow d_1'(d_2')} \text{ EvalB-Ap}_1$$

$$\frac{E \vdash d_1 \Downarrow ([E']\lambda x : \tau.d_1') \qquad E \vdash d_2 \Downarrow d_2' \qquad E', x \leftarrow d_2' \vdash d_1' \Downarrow d}{E \vdash d_1(d_2) \Downarrow d} \text{ EvalB-Ap}_2$$

$$\frac{\begin{array}{c} E \vdash d_1 \Downarrow (\mathrm{fix}\, f.([E']\lambda x : \tau.d_1')) = d_1'' \\ E \vdash d_2 \Downarrow d_2' \qquad E', f \leftarrow d_1'', x \leftarrow d_2' \vdash d_1' \Downarrow d \end{array}}{E \vdash d_1(d_2) \Downarrow d} \text{ EvalB-Ap}_3$$

$$\frac{E \vdash d_2 \Downarrow d_2' \qquad E, x \leftarrow d_2' \vdash d_1 \Downarrow d}{E \vdash \texttt{let}\ x = d_2\ \texttt{in}\ d_1 \Downarrow d} \text{ EvalB-Let}$$

$$\frac{E \vdash d_1 \Downarrow d_1' \qquad E \vdash d_2 \Downarrow d_2' \qquad (d_1 \neq \underline{n_1} \vee d_2 \neq \underline{n_2})}{E \vdash d_1 + d_2 \Downarrow d_1' + d_2'} \text{ EvalB-Op}_1$$

$$\frac{}{E \vdash (\!|)_\varnothing \Downarrow (\!|)_E} \text{ EvalB-EHole} \qquad \frac{E \vdash d \Downarrow d'}{E \vdash (\!|d|\!)_\varnothing \Downarrow (\!|d'|\!)_E} \text{ EvalB-NEHole}$$

$$\frac{E \vdash d_1 \Downarrow \underline{n_1} \qquad E \vdash d_2 \Downarrow \underline{n_2}}{E \vdash d_1 + d_2 \Downarrow \underline{n_1 + n_2}} \text{ EvalB-Op}_2$$

Figure 3: Big-step semantics for the environment model of evaluation

```
let a = ⦇⦈1 in
let b = λ x . { a + x + ⦇⦈2 } in
let c = ⦇⦈3 in
⦇⦈4 + b 1 + f ⦇⦈5
```

Figure 4: A seemingly innocuous Hazel program

```
let a = ⦇⦈1 in
let b = ⦇⦈2 in
let c = ⦇⦈3 in
let d = ⦇⦈4 in
let e = ⦇⦈5 in
let f = ⦇⦈6 in
let g = ⦇⦈7 in
...
let x = ⦇⦈n in
⦇⦈n+1
```

Figure 5: A Hazel program that generates an exponential (2^N) number of total hole instances

5 Memoizing hole instance numbering using environments

5.1 Issues with the current implementation

Consider the program shown in Figure 4.

A performance issue appears with the existing evaluator with the program shown in Figure 5.

5.2 Hole instances and closures

5.3 Algorithmic concerns and a two-stage approach

5.4 Memoization and unification with closure post-processing

5.5 Differences in the hole instance numbering

6 Implementation of fill-and-resume

6.1 CMTT interpretation of fill-and-resume

6.2 Memoization of recent actions

6.3 UI changes for notebook-like editing

7 Evaluation of methods

8 Future directions

8.1 FAR for all edits

8.2 Stateless and efficient notebook environment

9 Conclusions and recommendations

REFERENCES

[1] Cyrus Omar, Ian Voysey, Ravi Chugh, and Matthew A. Hammer. Live functional programming with typed holes. *PACMPL*, 3(POPL), 2019.

[2] Jérôme Vouillon and Vincent Balat. From bytecode to javascript: the js_of_ocaml compiler. *Software: Practice and Experience*, 44(8):951–972, 2014.

APPENDICES

A Formalization of evaluation with environments

B Additional contributions to Hazel

B.1 Additional performance improvements

B.2 Documentation and learning efforts

C Related concurrent research directions in Hazel

C.1 Improved hole renumbering

D Selected code samples