Ph213 – Section D Quiz 3

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9.9 ± 3.0

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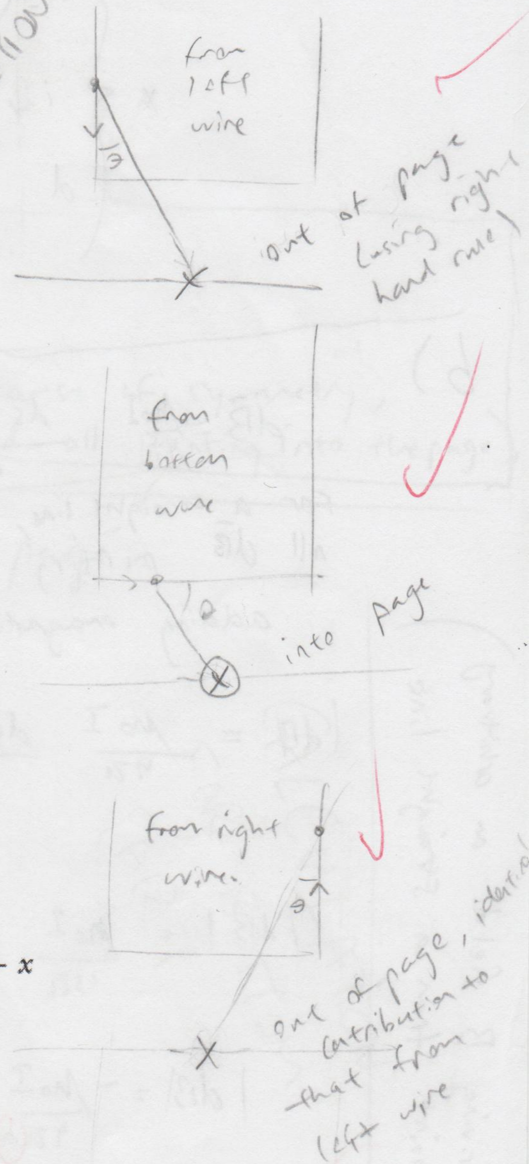
Using Biot-Savart, find the magnetic field (a **vector**) at the origin in terms of I, a, d.
(As the dotted lines suggest, the wires parallel to the y axis extend to y = +infinity.)

a) Setup the relevant integral(s). You must include the usual figure, showing one (or a few) "typical" contributions to the total field. [7 pts]

b) Solve. [7 pts]

Show ALL work. Words are often helpful.[1]

TYPICAL CONTRIBUTIONS



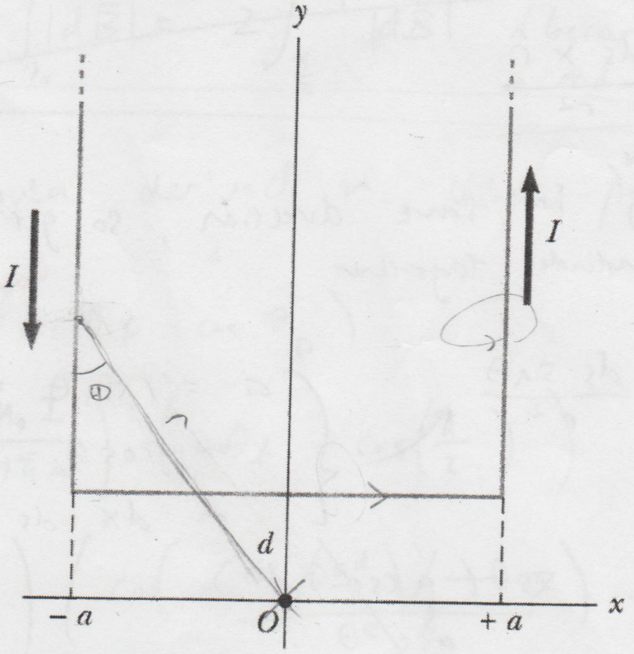


Figure P30.12

B field from left, right wires coming straight out of page, B field from bottom part going into page.
So we can do $|B_L| + |B_R| - |B_B| = 2|B_L| - |B_B|$

(because of symmetry b/t left and right wires)

[1] No 'trick' is required to solve this problem, and you will have to do integral(s) regardless, but you may find it somewhat useful to think for a bit before setting up the integrals.

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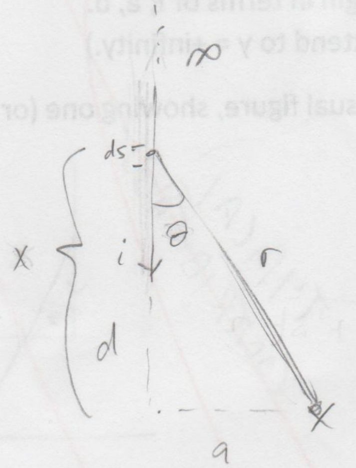
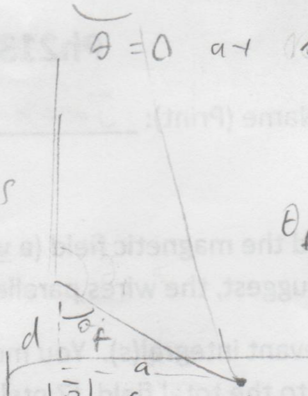
contribution

a) From left and right wires





$\theta_f = \tan^{-1}\left(\frac{d}{a}\right)$

$d\bar{B}$ from Biot-Savart and next

$|\bar{B}|$ from left wire $= \int_0^{\theta_f} |d\bar{B}|$

Out of the page

Equal to $\bar{B}$ contribution from right b/c of symmetry and b/c both are in the same direction

b)

$$d\bar{B} = \frac{\mu_0 I}{4\pi} \frac{d\bar{s} \times \hat{r}}{r^2}$$

For a straight line, all $d\bar{B}$ pointing in same direction, so get magnitude by adding magnitudes together

(deriving B field at arbitrary point from a straight line)

$$|d\bar{B}| = \frac{\mu_0 I}{4\pi} \frac{ds \sin\theta}{r^2}$$

$a = r\sin\theta \Rightarrow r = a\csc\theta$

$x = r\cos\theta = a\csc\theta\cos\theta = a\cot\theta$

$\Rightarrow dx = ds = -a\csc^2\theta \, d\theta$

$$|d\bar{B}| = \frac{\mu_0 I}{4\pi} \frac{\sin\theta(-\cancel{a\csc^2\theta}\, d\theta)}{\cancel{a^2\csc^2\theta}}$$

$$|d\bar{B}| = -\frac{\mu_0 I}{4\pi a} \sin\theta \, d\theta$$

only for left and right sections.

(-.5)

$$\text{so } |\bar{B}| = \frac{\mu_0 I}{4\pi a}\int_{\theta_0}^{\theta_f} -\sin\theta \, d\theta = \frac{\mu_0 I}{4\pi a}\left(\cos\theta_f - \cos\theta_0\right)$$

θ_f should be $\tan^{-1}\left(\frac{a}{d}\right)$, otherwise you are effectively multiplying by a $\ominus$ sign.

for left and right, $\theta_f = \tan^{-1}\left(\frac{d}{a}\right)$, $\theta_0 = 0$

~~integrate from~~ (-.5)

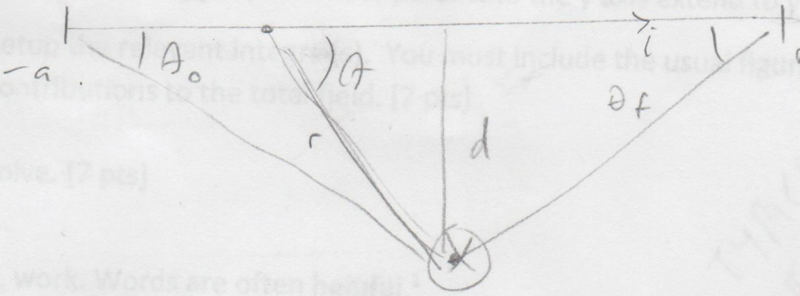
$$\text{so } \frac{\mu_0 I}{4\pi a}\left(\cos\left(\tan^{-1}\left(\frac{d}{a}\right)\right) - \cos(0)\right) = \frac{\mu_0 I (-1)}{4\pi a}\left(\frac{d}{\sqrt{a^2+d^2}} - 1\right)$$

from left and right wires each

)

From bottom part

$$\theta_0 = \tan^{-1}\left(\frac{-d}{a}\right) \qquad \theta_f = \tan^{-1}\left(\frac{d}{a}\right)$$



overall contribution from bottom:

$$|\bar{B}| = \int_{\theta_0}^{\theta_f} |d\bar{B}| = 2\int_{\frac{\pi}{2}}^{\theta_f} |d\bar{B}|$$ (because of symmetry, and all pointing into the page)

b) using formula derived on other page:

$$|B| = \frac{\mu_0 I}{4\pi a}\left(\cos\theta_f - \cos\theta_0\right)$$

d now

$$= 2\left(\frac{\mu_0 I}{4\pi a}\right)\left(\cos\theta_f - \cos\left(\frac{\pi}{2}\right)\right)$$

$$= 2\left(\frac{\mu_0 I}{4\pi a}\right)\left(\cos\left(\tan^{-1}\left(\frac{-d}{a}\right)\right) - 0\right)$$

$$= \frac{\mu_0 I}{2\pi a}\left(\frac{a}{\sqrt{a^2+d^2}}\right)$$

$a \to d$

CALCULATION FOR BOTTOM PART

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adding all contributions together:

wires on left and right

wire on bottom

$$|B_{origin}| = 2\left(\frac{\mu_0 I}{4\pi a}\left(\frac{d}{\sqrt{a^2+d^2}} - 1\right)\right) - \frac{\mu_0 I}{2\pi a}\left(\frac{a}{\sqrt{a^2+d^2}}\right)$$

$$= \boxed{\frac{\mu_0 I}{2\pi a}\left(\frac{d-a}{\sqrt{a^2+d^2}} - 1\right)}$$ (pointing out of the page)