DSP Fall 2020

Quiz #1

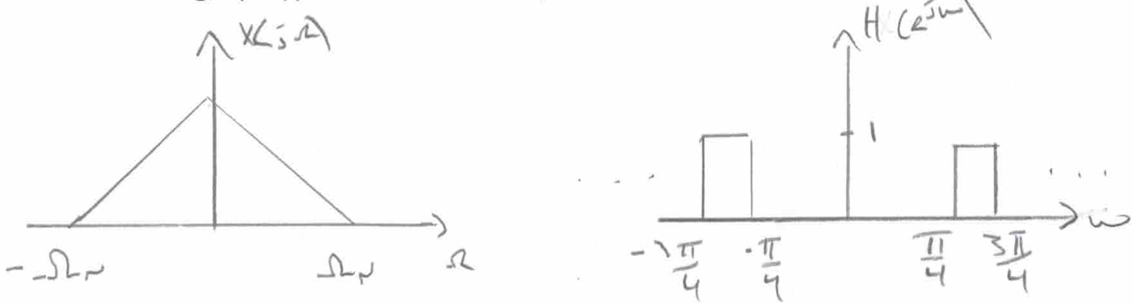
Name:

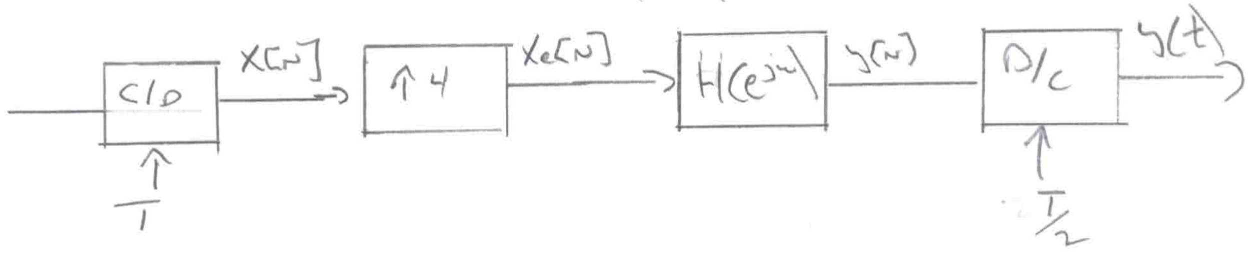
Consider the following system below, with input $X_c(j\Omega)$, and the frequency response of $H(e^{j\omega})$ are as shown. $X_c(t)$ is sampled at exactly Nyquist ($2\pi/T = 2\Omega_n$) and that the D/C converter is running at rate $\cancel{2T}$. $T/2$

a) (8 points) Sketch $X(e^{j\omega})$, $X_e(e^{j\omega})$, $Y(e^{j\omega})$, and $Y_c(j\Omega)$. Label all frequencies and amplitudes.

b) (1 point) Can you find an equivalent continuous time LTI system? If so, specify the system. If not, explain why not.

c) (1 point) Can you use a polyphase implementation to reduce the amount of computation used to perform the upsampling and filtering operations? If no, explain why not. If yes, sketch a block diagram showing an equivalent system implemented using a polyphase filter decomposition.

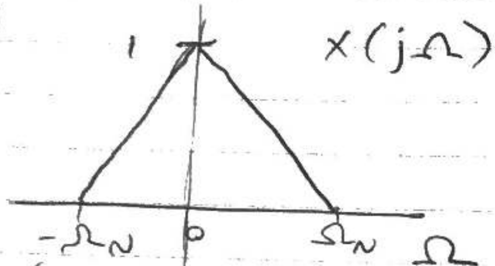




DSP Quiz 1

Jonathan Lam
9/23/2020

a)

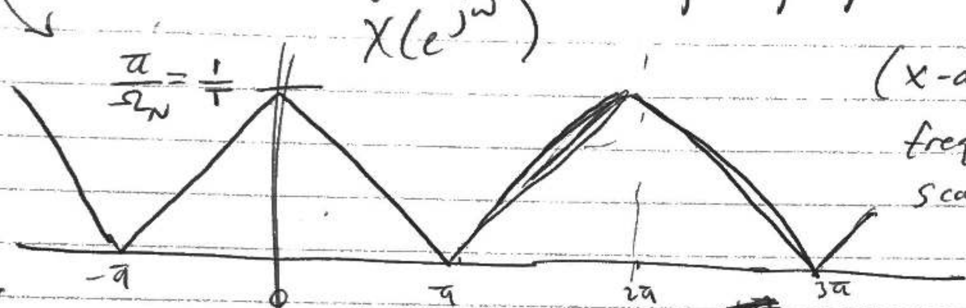


sampled @

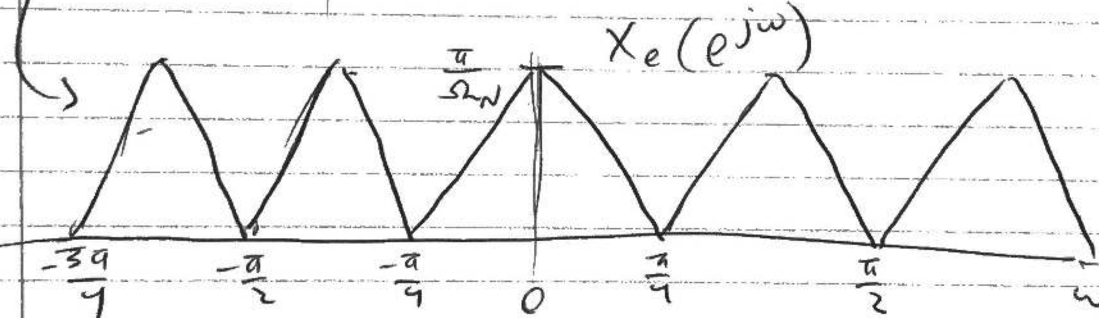
$$\Omega_s = \frac{2\pi}{T} = 2\Omega_N$$

$$\Rightarrow T = \frac{\pi}{\Omega_N}$$

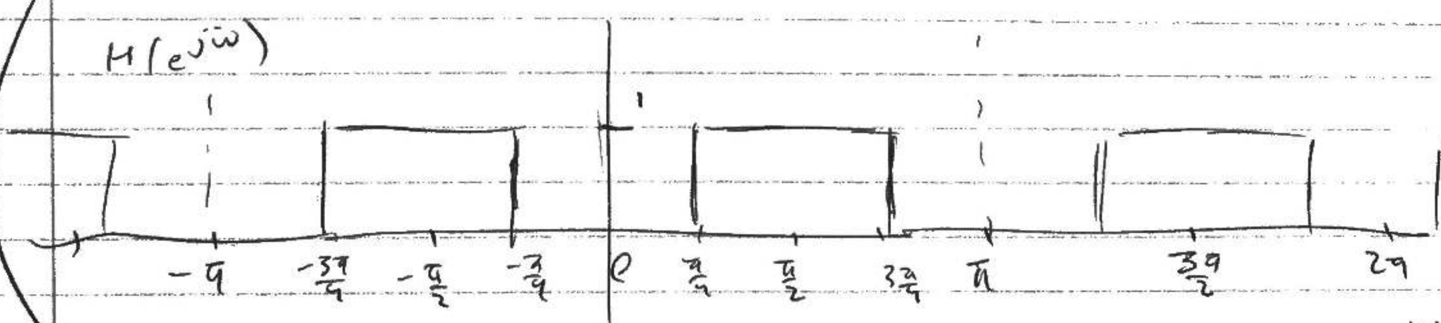
(don't need anti-aliasing filter, no aliasing is going to happen)

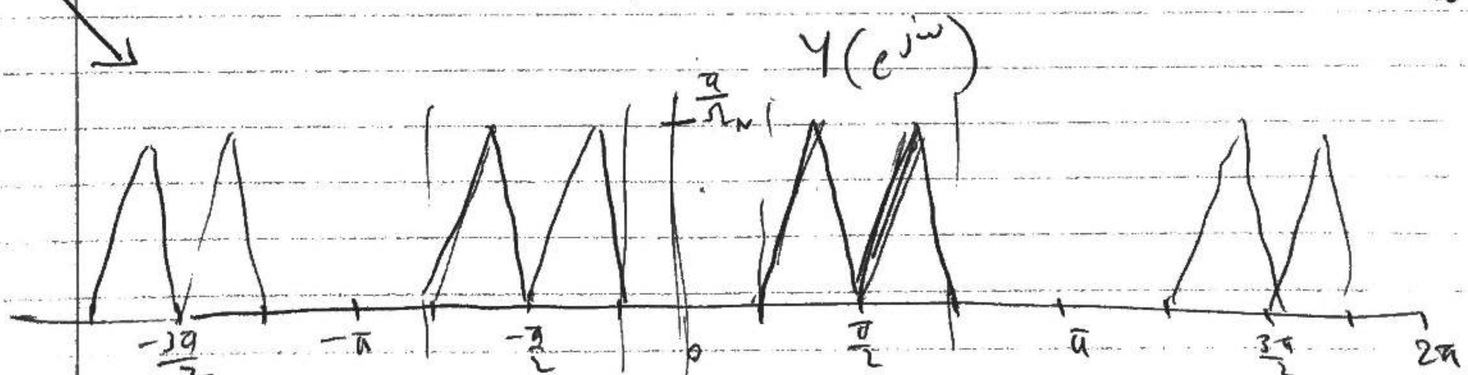


(x-axis scaled to digital frequency, magnitude scaled by $\frac{1}{T}$ (eq. 4.6))



(x-axis compressed by factor of 4, no scaling of magnitude (eq. 4.85))

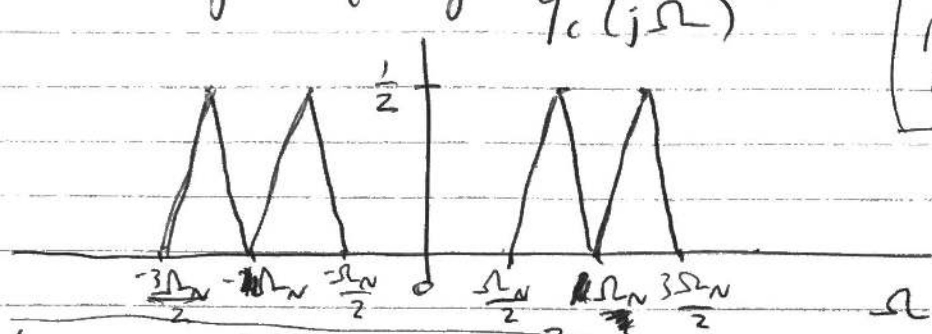




DSP Quiz 1

JONATHAN LAM

a, cont'd.) D/C converter involves rescaling x-axis to analog frequency, rescaling magnitude by T, and passing through an anti-imaging LPF



D/C converter @ sampling rate $\frac{2}{T}$

$\omega = \Omega T$

here $T = \frac{T_0}{2} = \frac{\pi}{2\Omega_N}$ where T_0 was the C/D sampling period

$$\Omega = \omega\left(\frac{2\Omega_N}{\pi}\right)$$

b) There is no LTI equivalent to this system, because of sampling at different frequencies (i.e., the C/D and D/C operate at different rates) and is thus not TI.

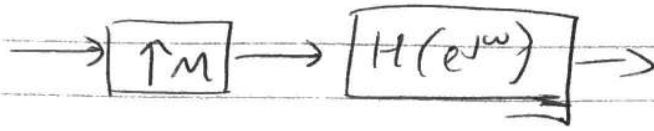
Another way to tell that it is not LTI is that the support of $Y(j\Omega)$ includes frequencies that were not present in the support of $X(j\Omega)$, which is impossible if the system is linear.

DSP QUIZ 1

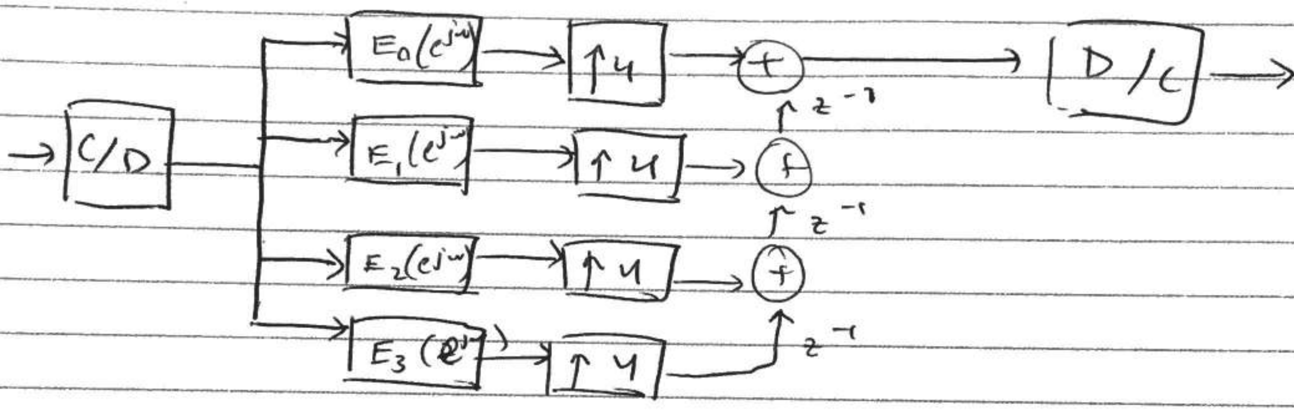
JONATHAN LAM

c) a

Yes, a polyphase implementation can be useful here. In particular we have the pattern:



which means that the system H is operating at the higher (upsampled) frequency, and can be benefitted by ~~writing this as a polyphase~~ operating at the original frequency, i.e.,:



where $\{E_i\}$, $0 \le i < 4$ are the polyphase components of $H(e^{j\omega})$.

I apologize for my handwriting