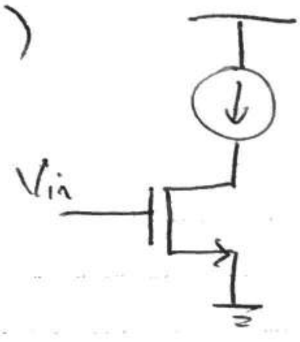
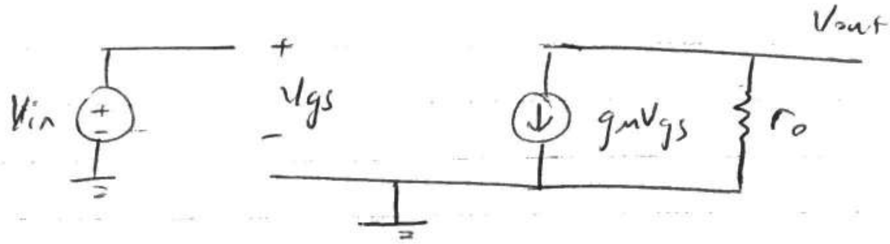
Common-source stage w/ current source load

a)





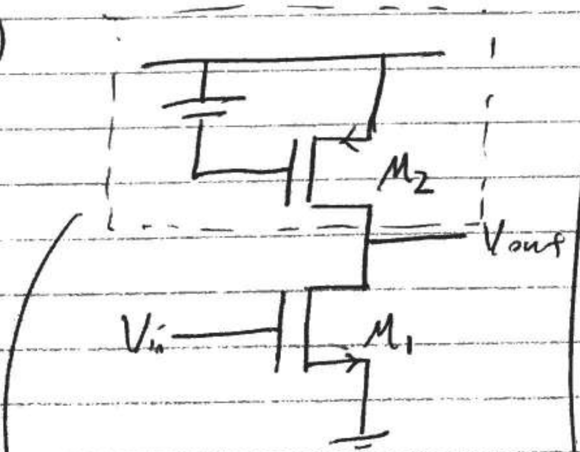
$V_{out} = -g_m V_{gs} r_o = -g_m V_{in} r_o$

$\Rightarrow \dfrac{V_{out}}{V_{in}} = -g_m r_o$

$R_{out} = r_o$ (since $V_{gs} = 0$, no current flows through the VCCS)

$$\boxed{\begin{aligned} A_v &= -g_m r_o \\ R_{out} &= r_o \\ R_{in} &= \infty \end{aligned}}$$

b.)



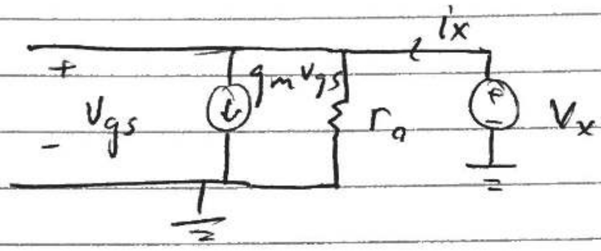
$$\boxed{\begin{aligned} A_v &= -g_{m1}(r_{out1} \parallel r_{out2}) \\ &= -g_{m1}(r_{o1} \parallel r_{o2}) \\ R_{out} &= r_{out1} \parallel r_{out2} = r_{o1} \parallel r_{o2} \\ R_{in} &= \infty \end{aligned}}$$

find impedance looking into the drain (R_{out2})

by the symmetry of NMOS and PMOS, impedance looking into the drain $R_{out2} = r_{o2}$

A diode-connected device.

or



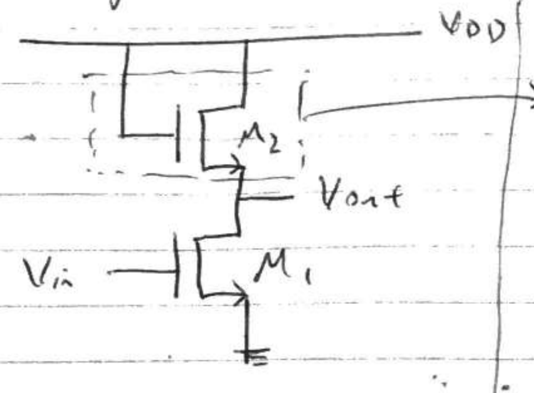
$$i_x = g_m V_x + \frac{V_x}{r_o}$$

$$i_x = V_x\left(g_m + \frac{1}{r_o}\right)$$

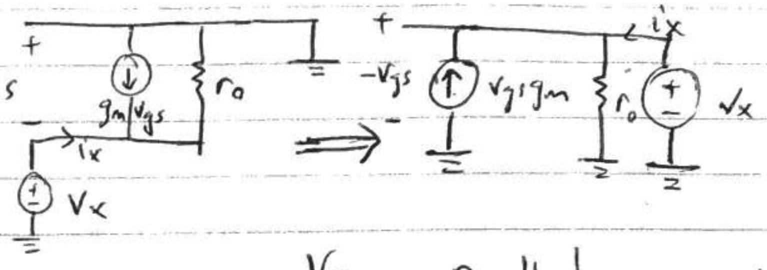
$$R_{out} = \frac{V_x}{i_x} = \frac{1}{g_m + \frac{1}{r_o}} \cdot \frac{\frac{r_o}{g_m}}{\frac{r_o}{g_m}} = \frac{r_o \cdot \frac{1}{g_m}}{r_o + \frac{1}{g_m}} = \left(r_o \parallel \frac{1}{g_m}\right)$$

$$\boxed{R_{out} = r_o \parallel \frac{1}{g_m}}$$

CS Stage w/ Diode-connected load.



→ impedance looking into the source:



$\frac{V_x}{i_x} = r_o \parallel \frac{1}{g_m}$ again

b) assume $\lambda \neq 0$:

$R_{out} = R_{out,1} \parallel$ (Impedance looking into source of M_2)

$= r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}}$

$A_v = -g_{m1} R_{out} = -g_{m1} \left(r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}} \right)$

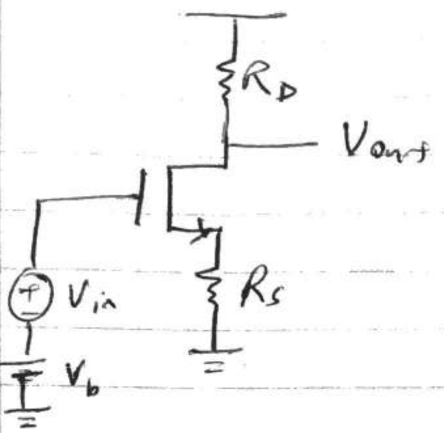
$R_{in} = \infty$.

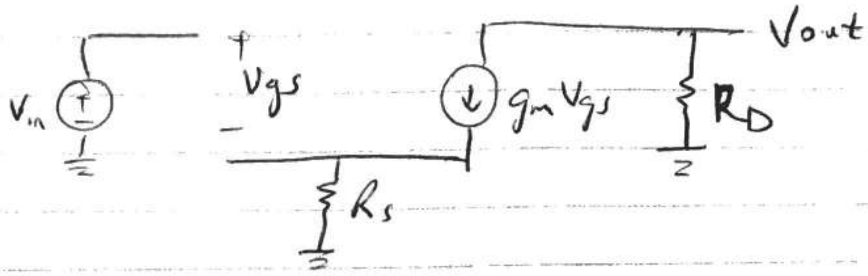
for $\lambda = 0$. case, assume $r_{o1} = r_{o2} = \infty$.

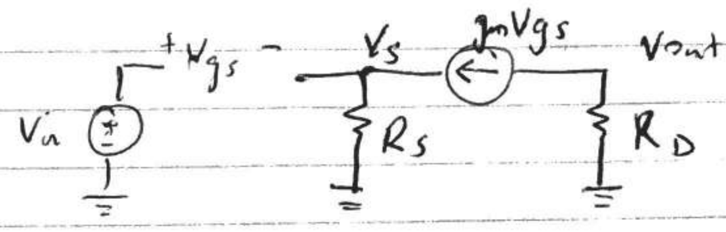
a) $\lambda = 0$	b) $\lambda \neq 0$
$A_v = -\frac{g_{m1}}{g_{m2}}$	$A_v = -g_{m1} \left(r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}} \right)$
$R_{in} = \infty$	$R_{in} = \infty$
$R_{out} = \frac{1}{g_{m2}}$	$R_{out} = r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}}$

Common-Source w/ Degeneration.

assume $\lambda = 0$







$$V_{gs} = V_{in} - V_S$$

$$g_m(V_{in} - V_s) = \frac{V_S}{R_s} = -\frac{V_{out}}{R_D}$$

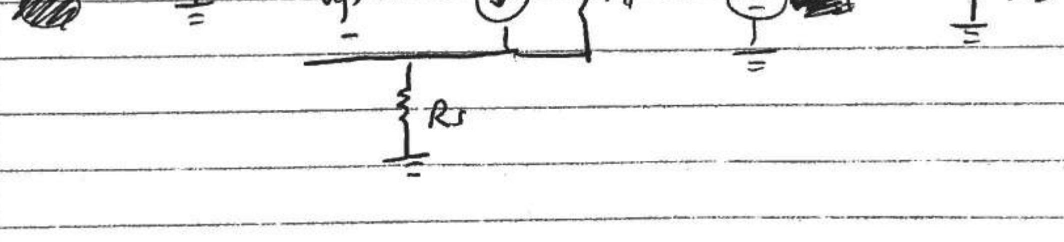
$$V_s = \frac{R_S}{R_D} V_{out}$$

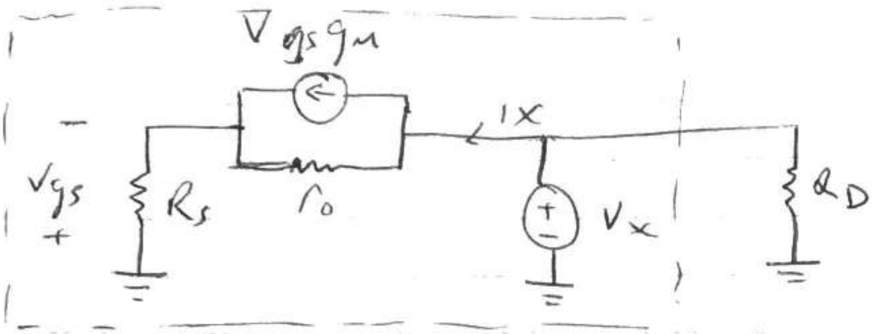
$$g_m\left(V_{in} + \frac{R_S}{R_D} V_{out}\right) = -\frac{V_{out}}{R_D}$$

$$V_{in}(g_m R_D) = -V_{out}(1 + g_m R_S)$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_m R_D}{1 + g_m R_S} = \boxed{-\frac{R_D}{\frac{1}{g_m} + R_S} = A_v \quad (\text{assuming } \lambda = 0)}$$

R_{out} (assuming $\lambda \neq 0$).





To make calcs easier ignore R_D for now

$$i_x = \frac{-V_{gs}}{R_s} = g_m V_{gs} + \frac{V_x + V_{gs}}{r_o}$$

$$\Rightarrow V_{gs} = -i_x R_s$$

$$i_x = g_m(-i_x R_s) + \frac{V_x}{r_o} + \frac{-i_x R_s}{r_o}$$

$$i_x\left(1 + g_m R_s + \frac{R_s}{r_o}\right) = V_x\left(\frac{1}{r_o}\right)$$

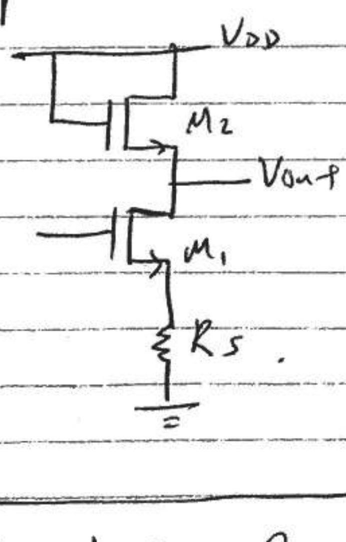
$$R_{out} = \frac{V_x}{i_x} = r_o\left(1 + g_m R_s + \frac{R_s}{r_o}\right) = r_o(1 + g_m R_s) + R_s$$

$$\cancel{R_{out} = r_o(1 + g_m R_s) + R_s}$$

Note that this is in parallel with R_D

So:
$$\boxed{\begin{aligned} R_{in} &= \infty \\ R_{out} &= (r_o(1 + g_m R_s) + R_s) \parallel R_D \end{aligned}}$$

Example.



$$R_D = r_{o_2} \parallel \frac{1}{g_{m_2}}$$

$$R_{out} = (r_{o_1}(1 + g_{m_1} R_s) + R_s) \parallel \left(r_{o_2} \parallel \frac{1}{g_{m_2}}\right)$$

assume $\lambda = 0 \Rightarrow r_{o_1} = r_{o_2} = \infty$

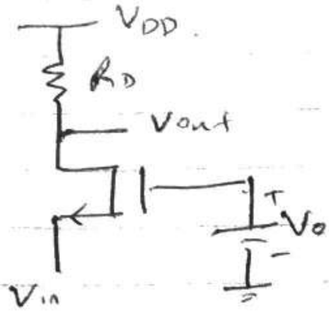
$$R_D = \frac{1}{g_{m_2}}$$

$$R_{out} = \infty \parallel \infty \parallel \frac{1}{g_{m_2}} = \frac{1}{g_{m_2}}$$

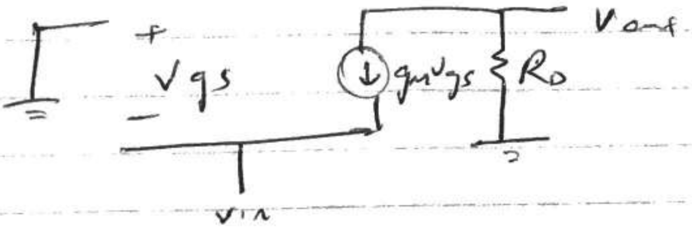
$$A_V = -\frac{R_D}{\frac{1}{g_{m_1}} + R_s} = \frac{-\frac{1}{g_{m_2}}}{\frac{1}{g_{m_1}} + R_s}$$

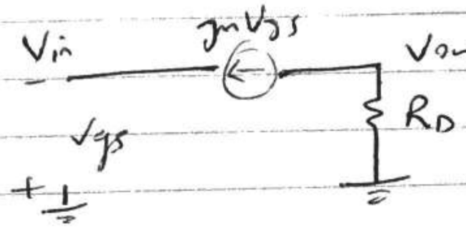
$$\boxed{\text{assuming } \lambda = 0,\ R_{out} = \frac{1}{g_{m_2}},\ A_V = -\frac{\frac{1}{g_{m_2}}}{\frac{1}{g_{m_1}} + R_s}}$$

Common-Gate Topology



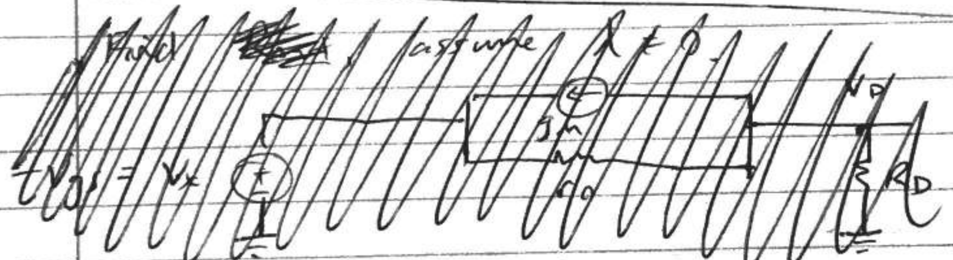
Find A_v, assume $\lambda = 0$.





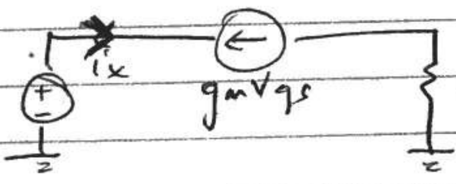
$$V_{out} = -g_m V_{gs} R_D = g_m V_{in} R_D$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = g_m R_D = A_v.$$



Find R_{in}, assume $\lambda = 0$.

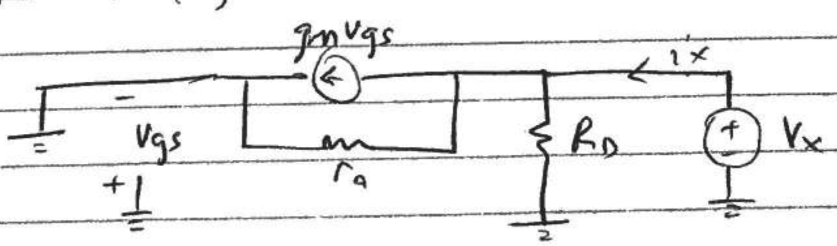
$-V_{gs} = V_x$



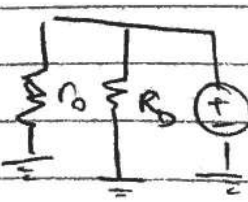
$$i_x = -g_m V_{gs} = g_m V_x$$

$$\Rightarrow \frac{V_x}{i_x} = \frac{1}{g_m} = R_{in}.$$

Find R_{out}, assume $\lambda \neq 0$.



$V_g = V_s = 0 \Rightarrow V_{gs} = 0 \Rightarrow g_m V_{gs} = 0 \Rightarrow$



$\Rightarrow R_{out} = R_D \parallel r_o.$

if $\lambda = 0$, $r_o = \infty$, $R_{out} = R_D$.

Common Gate Topology

$\lambda = 0$

$A_v = g_m R_D$

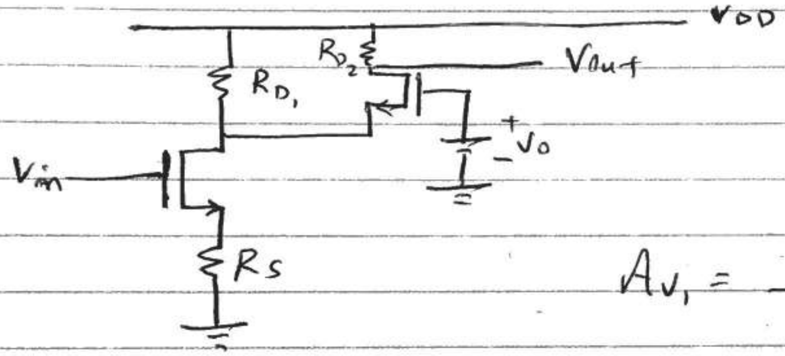
$R_{out} = R_D$

$R_{in} = \frac{1}{g_m}$

$\lambda \neq 0$

$R_{out} = r_o \| R_D$

Cascade Example



$\lambda = 0$

$A_v = A_{v_1} A_{v_2}$

$$A_{v_1} = -\frac{R_D \| R_{in_2}}{\frac{1}{g_{m_1}} + R_s}$$

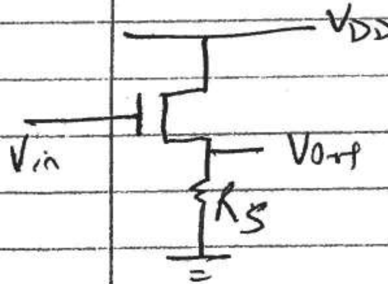
$A_{v_2} = g_{m_2} R_{D_2}$

$R_{in_2} = \frac{1}{g_{m_2}}.$

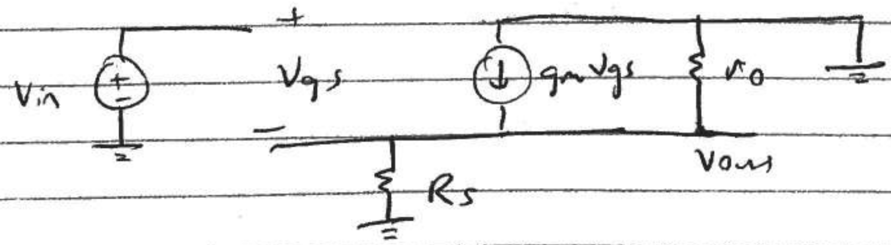
$$A_v = -\left(\frac{R_D \| \frac{1}{g_{m_2}}}{\frac{1}{g_{m_1}} + R_s}\right)\left(g_{m_2} R_{D_2}\right).$$

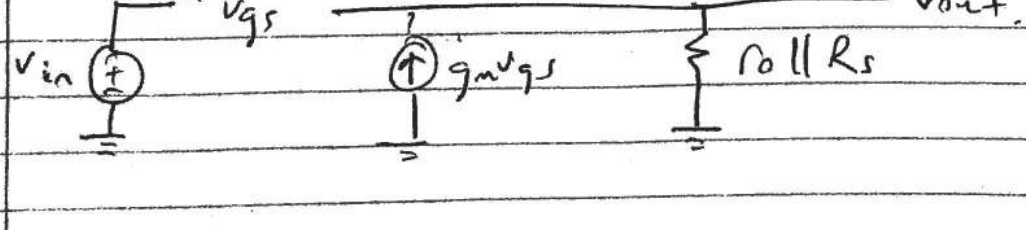
Source Follower.

$\lambda \neq 0.$



Finding A_v.





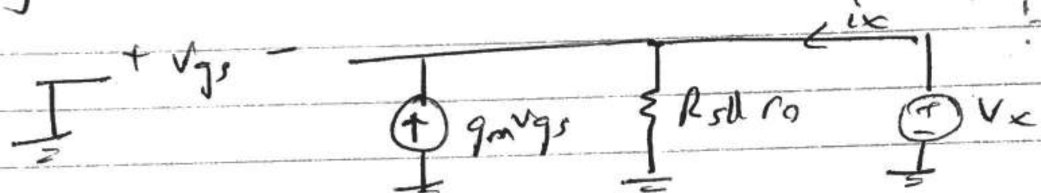
$$V_{out} = g_m V_{gs} (R_s \parallel r_o)$$
$$= g_m (V_{in} - V_{out}) (R_s \parallel R_o)$$

$$V_{out} (1 + g_m (R_s \parallel r_o)) = V_{in} (g_m (R_s \parallel r_o))$$

$$A_v = \frac{V_{out}}{V_{in}} = \frac{g_m (R_s \parallel r_o)}{1 + g_m (R_s \parallel r_o)} = \frac{R_s \parallel r_o}{\frac{1}{g_m} + (R_s \parallel r_o)}$$

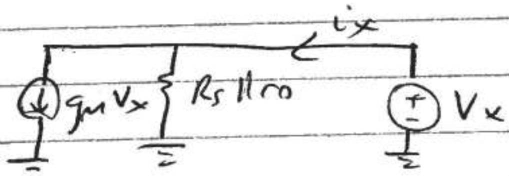
Finding R_{in}, $\lambda \neq 0$. $R_{in} = \infty$.

Finding R_{out}, $\lambda \neq 0$.



$V_{gs} = -V_x$

$\Downarrow$



impedance of current source in parallel w/ resistance

$$R_{out} = \frac{1}{g_m} \parallel (R_s \parallel r_o)$$

Source Follower

a) $\lambda = 0$

$$A_v = \frac{R_s}{\frac{1}{g_m} + R_s}$$

$R_{in} = \infty$

$$R_{out} = \frac{1}{g_m} \parallel R_s$$

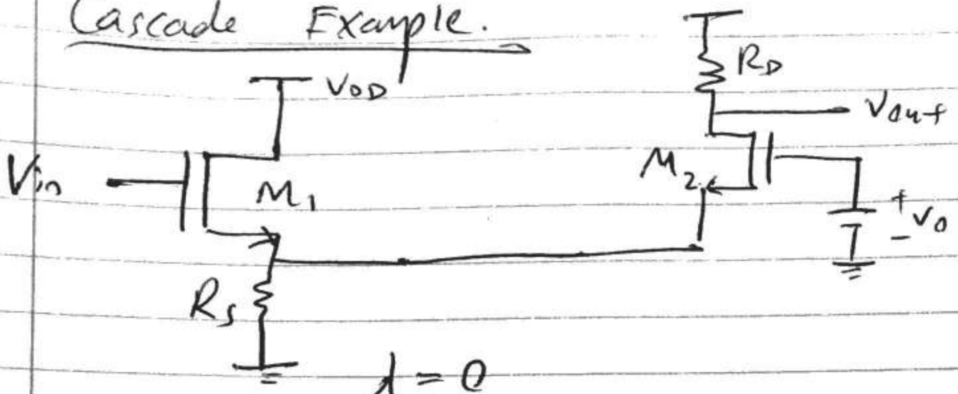
b) $\lambda \neq 0$.

$$A_v = \frac{R_s \parallel r_o}{\frac{1}{g_m} + R_s \parallel r_o}$$

$R_{in} = \infty$

$$R_{out} = \frac{1}{g_m} \parallel R_s \parallel r_o$$

Cascade Example.



$\lambda = 0$

$$A_V = A_{V_1} A_{V_2}$$

$$A_{V_1} = \frac{R_S \parallel R_{in2}}{\frac{1}{g_{m_1}} + R_S \parallel R_{in2}}.$$

$$R_{in\,2} = \frac{1}{g_{m_2}}$$

$$A_{V_2} = g_{m_2} R_D$$

$$\boxed{A_V = \left(\frac{R_S \parallel \frac{1}{g_{m_2}}}{\frac{1}{g_{m_1}} + R_S \parallel \frac{1}{g_{m_2}}}\right)\left(g_{m_2} R_D\right).}$$