

# ECE300 – Project 1

Jonathan Lam

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Simulating analog modulation schemes in MATLAB

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# 1 Overview

Analog modulation schemes are at the core of all modern communication schemes. In this project, four common modulation schemes – conventional amplitude modulation, single side-band amplitude modulation, phase modulation, and frequency modulation – are implemented in MATLAB. In this report, we first review some of the modulation theory. We demonstrate that these signals faithfully recreate the original message after modulation and demodulation, and that the modulation is correct. Then, after adding additive white gaussian noise (AWGN) random processes, simulations are performed to analyze the effect of modulation scheme parameters and noise variance on the signal-to-noise ratio (SNR) on the different schemes. The relationship between SNR and noise power, and the relationships between SNR and the various parameters of the modulation schemes, generally agreed with the theory. The theoretical SNR calculations are compared with the experimentally-computed SNR values. SNR values comparable to theoretical values are obtained for all but FM. In addition to the requirements of this project, some additional checks were performed to verify the behavior of the modulated signals and also generally agreed with the theory.

## 1.1 Procedure

The assignment called for the following:

1. Read in an audio signal (and its sampling rate).
2. Write modulator/demodulator implementations for each modulation scheme. Make sure that the behavior seems correct. Verify Carson's rule.
3. Generate three zero-mean, different-variance AWGN noise processes. Add these to the modulated outputs of each system, demodulate them, and calculate the SNR of the demodulated signals experimentally. Compare these with the theoretical results.
4. Choose a noise process for each of conventional AM, FM, and PM which makes the demodulated output qualitatively noisy (but recognizable). Vary the modulation indices of each of these schemes by a factor of two in both directions, and calculate the SNR of the demodulated signals experimentally. Compare these with the theoretical results.

Reading in the audio file is described in 8. The implementations of the modulator and demodulator functions for each scheme are described in 2. Plots of the results appear in 3 and are discussed in 4.

## 1.2 Execution environments and resource usage

In order to emulate reasonable frequencies for the modulation schemes, I originally had the carrier and sampling frequencies set fairly high. The carrier frequency was set around 600kHz for amplitude modulation and around 1MHz for angle modulation, and the sampling rates were set to ten times those frequencies. Even though this AM carrier is on the lower end of the AM spectrum, and the FM carriers are far lower than real carriers, the lower end of which is at 88MHz [1], this was quite heavy on my computers. This caused many vectors with dozens of millions of double floating-point samples, which took dozens of seconds to run and took over 30GB of RAM when running on my Intel S5520UR server.

Another problem I encountered with these high frequencies is that there was often significant numerical error, especially with my FM signal. Interestingly, this happened very clearly when advancing the sampling rate to over 5MHz, or the carrier frequency to over 500kHz.

As a result of both the resource consumption issues and numerical errors associated with the multi-megahertz frequency ranges, I lowered the carrier and sampling frequencies for the angle modulation. While this becomes even less unrealistic for angle modulation, it did help alleviate both of these problems. A plot of the FM demodulated signal is Figure 16, and a plot of the FM demodulated signal with the lower carrier and sampling frequency is Figure 5.

## 2 Review of modulation schemes

Conventional AM, SSB AM, FM, and PM schemes are covered in this section. We also covered DSB AM and vestigial AM in class, but these are not covered here.

### 2.1 Conventional (CONV) AM modulation

#### 2.1.1 Overview

Conventional AM (henceforth denoted CONV) is amplitude modulation with an offset, such that the modulation envelope is always positive and thus can be cheaply extracted with an envelope detector. Given a message signal  $m(t)$ , where  $|m(t)| \leq 1$ , modulation index  $0 \leq a \leq 1$  (greater values of  $a$  are possible but cause overmodulation, or clipping), carrier frequency  $f_c$ , and carrier amplitude  $A_c$ , the modulation involves scaling and shifting  $m$ , and then mixing this with a carrier tone. The modulated signal  $u(t)$  is:

$$u_{conv}(t) = A_c(1 + am(t)) \cos(2\pi f_c t) \quad (1)$$

Demodulation can be performed in two ways:

1. By mixing with the carrier signal (i.e., an I-Q demodulator with only an I component), which can be obtained by using a narrowband filter similar to in DSB-SC AM demodulation.
2. By rectifying and low-pass-filtering the signal (i.e., performing an envelope detection). Either full-wave or half-wave rectification can be used. This method is ubiquitous because it is cheap to implement in hardware.

Additionally, the demodulator has to:

- DC block (e.g., with a HPF) to remove the offset
- Divide the output by  $aA_c$  to get back to the original output amplitude (power)
- Scale the output up in order to compensate for energy lost during the LPF/HPF

#### 2.1.2 Analysis

Let  $P_u$  be the total transmitted (bandpass) (modulated signal) power,  $P_c$  is the power in the transmitted signal sent in the carrier,  $P_y$  is the power in the transmitted signal for the message signal, and  $P_m$  is the power of the original normalized message signal.

$$P_u = P_c + P_m \quad (2)$$

$$P_c = \frac{A_c^2}{2} \quad (3)$$

$$P_y = \frac{a^2 A_c^2}{2} P_m \quad (4)$$

Since  $P_m < 1$  (since  $|m(t)| < 1$ ) and  $|a| < 1$ , at least half of the power is always sent in the carrier.

The SNR can be estimated by assuming an I-Q demodulation scheme. This doesn't account for rectification noise and is thus not perfect, but it is good enough. Let  $r(t)$  be the noiseless received

message signal after mixing with the carrier tone (and ignoring the carrier power, which can be removed with a DC block). Let  $y(t)$  denote the message part of the signal, and  $n(t)$  denote the additive noise incurred during transmission.

$$r(t) = \frac{1}{2}A_c(1 + am(t))\cos(2\pi f_c t) + n_I(t)\cos(2\pi f_c t) + n_Q(t)\sin(2\pi f_c t) \quad (5)$$

$$\rightarrow \frac{1}{2}A_c(am(t)) + n_I(t) \quad (\text{after DC block and mixing with cosine})$$

$$P_y = \frac{a^2 A_c^2}{4} P_m \quad (6)$$

$$P_r = \frac{2N_0 W}{4} \quad (7)$$

$$\text{SNR} = \frac{a^2 A_c^2}{2N_0 W} P_m \quad (8)$$

The calculation for noise power was derived in the analog domain. See 10 for details about the digital domain calculation.

### 2.1.3 Implementation details

The modulation function is fairly straightforward. Notes on demodulation:

```

1 % modulates a signal using the conventional AM scheme
2 % params:
3 % f_c    = frequency of carrier (Hz)
4 % A_c    = carrier amplitude
5 % a      = modulation index
6 % sig_m  = message signal
7 % f_s_m  = message sampling frequency (Hz)
8 % f_s_c  = carrier sampling frequency (Hz)
9 % returns:
10 % sig_c  = conventional AM-modulated signal
11 function sig_c = conv_mod(f_c, A_c, a, sig_m, f_s_m, f_s_c)
12     duration = length(sig_m) / f_s_m;
13     t_m = linspace(0, duration, length(sig_m));
14     t_c = linspace(0, duration, f_s_c * duration);
15
16     % upsample sig_m
17     sig_m_us = interp1(t_m, sig_m, t_c);
18
19     % generate conventional AM-modulated signal
20     sig_c = A_c * (1 + sig_m_us * a) .* cos(2 * pi * f_c * t_c);
21 end

```

Listing 1: Conventional AM modulation

- I perform the latter demodulation scheme because it is more common.
- Half-wave rectification was arbitrarily chosen for my implementation of the demodulation.
- The LPF and HPF are the frequency response of first-order systems.
- A HPF with an arbitrarily low cutoff frequency was used to help alleviate a low-frequency noise caused by rectification.
- An additional “DC block” is performed by subtracting the mean of the filtered signal, since the HPF doesn’t always perfectly center the signal.
- The constant 3.3 was empirically determined to recover the energy lost from the rectification and LPF. This constant scalar seemed to work pretty well for a variety of input samples. I wasn’t able to find a way to do this analytically, especially because rectification is a nonlinear function.
- The original signal amplitude was recovered by dividing by  $A_c a$ .

Note that the SNR equation for conventional AM doesn’t assume that the original amplitude is restored (i.e., by dividing by  $A_c a$ ); thus, when computing SNR experimentally, we have to remultiply the scaled output by  $A_c a$ .

```

1 % demodulates a signal modulated using the conventional AM scheme
2 % params:
3 % sig_c = conventional-AM modulated signal
4 % A_c    = carrier amplitude
5 % a      = modulation index
6 % f_s_c = carrier sampling rate (Hz)
7 % tau   = LPF cutoff (Hz)
8 % returns:
9 % sig_m = demodulated signal at lower frequency
10 function sig_m = conv_demod(sig_c, A_c, a, f_s_m, f_s_c, tau)
11     % half-wave rectification
12     sig_c(sig_c < 0) = 0;
13
14     % low-pass filtering in freq. domain
15     wd = linspace(-pi, pi, length(sig_c));
16     f_c = wd * f_s_c / (2 * pi);
17     lpf = (1 + f_c/tau*1j).^-1;           % LPF frequency response
18     hpf = (1 - 20*f_c.^-1*1j).^-1;       % HPF to get rid of offset
19                                         % and rectifier noise
20     ft_c = fftshift(fft(sig_c)) / f_s_c; % freq. domain rectified signal
21     ft_c = lpf .^ 2 .* hpf .* ft_c;       % apply filter in freq. domain
22
23     sig_c = ifft(ifftshift(ft_c) * f_s_c);
24
25     % downsample
26     duration = length(sig_c) / f_s_c;
27     t_c = linspace(0, duration, f_s_c * duration);
28     t_m = linspace(0, duration, f_s_m * duration);
29     sig_m = real(interp1(t_c, sig_c, t_m));
30
31     % DC block
32     sig_m = sig_m - mean(sig_m);
33
34     % scaling; the extra factor is empirically determined to account for
35     % the power in the carrier lost when filtering
36     sig_m = 3.3 * sig_m / A_c / a;
37 end

```

Listing 2: Conventional AM demodulation

## 2.2 SSB AM modulation

### 2.2.1 Overview

Single-sideband amplitude modulation aims to be more bandwidth efficient than the other AM schemes. It does this by employing the Hilbert transform to encode some of its information into a quadrature component. Given a message signal  $m(t)$ , the transmitted modulated signal looks like (some of the terms used in conventional AM will not be redefined here):

$$u_{(u|l)ssb}(t) = A_c (m(t) \cos(2\pi f_c t) \mp \hat{m}(t) \sin(2\pi f_c t)) + A_p \cos(2\pi f_c t) \quad (9)$$

Both modulation and demodulation occur using an I-Q (de)modulator. A pilot tone is necessary to ensure coherence, and a narrow-band filter and mixer are required in the demodulator to employ the pilot tone. These additions make SSB-AM also power-inefficient (as, like conventional AM, much of the power is in the carrier signal), and expensive (as it requires a mixer circuit and narrow-band filter.)

### 2.2.2 Analysis

Examining the power transmitted by this signal:

$$P_u = P_c + P_y \quad (10)$$

$$P_c = \frac{A_p^2}{2} \quad (11)$$

$$P_y = \frac{A_c^2}{2} P_m + \frac{A_c^2}{2} P_{\hat{m}} = A_c^2 P_m \quad (12)$$

Again, a large portion of the power is transmitted in the carrier signal. Twice as much power is transmitted in the message part of the signal as in conventional AM (assuming that  $a = 1$ ), but there is also twice as much noise power, which contributes to a SNR that is equal to that of DSB SC (and roughly equal to that of conventional AM, assuming that  $a = 1$ ). Let  $r(t)$  again be the received signal,

$$r(t) = (A_c m(t) + n_I(t)) \cos(2\pi f_c t) - (\pm A_c \hat{m}(t) + n_Q(t)) \sin(2\pi f_c t) \quad (13)$$

$$\rightarrow \frac{A_c}{2} m(t) + \frac{1}{2} n_I(t) \quad (\text{after mixing with cosine})$$

$$P_y = \frac{A_c^2}{4} P_m \quad (14)$$

$$P_n = \frac{1}{4} N_0 W \quad (15)$$

$$\text{SNR} = \frac{A_c^2}{N_0 W} P_m \quad (16)$$

Again, the calculation for noise power was derived in the analog domain. See 10 for details about the digital domain calculation.

### 2.2.3 Implementation details

See 9.2 for the implementation of the Hilbert transform. The modulator follows the definition of SSB fairly closely, and easily allows for both USSB and LSSB. Notes on demodulation:

```

1 % modulates a signal using the SSB AM scheme; currently assumes phase
2 % coherence (no pilot tone in modulated signal)
3 % params:
4 % f_c = carrier frequency (Hz)
5 % A_c = carrier amplitude
6 % sig_m = message signal
7 % f_s_m = message sampling frequency (Hz)
8 % f_s_c = carrier sampling frequency (Hz)
9 % ussb = whether it is USSB (true for USSB, false for LSSB)
10 % returns:
11 % sig_c = SSB AM-modulated signal
12 function sig_c = ssb_mod(f_c, A_c, sig_m, f_s_m, f_s_c, is_ussb)
13     duration = length(sig_m) / f_s_m;
14     t_m = linspace(0, duration, length(sig_m));
15     t_c = linspace(0, duration, f_s_c * duration);
16
17     % upsample sig_m
18     sig_m_us = interp1(t_m, sig_m, t_c);
19
20     % generate in-phase component (same as DSB-SC AM-modulated signal)
21     sig_dbsc = sig_m_us .* cos(2 * pi * f_c * t_c);
22
23     % generate quadrature component
24     sig_mhat = hilbert_transform(sig_m_us);
25     sig_quad = sig_mhat .* sin(2 * pi * f_c * t_c);
26
27     if is_ussb
28         sig_c = A_c * (sig_dbsc - sig_quad);
29     else
30         sig_c = A_c * (sig_dbsc + sig_quad);
31     end
32
33     % add pilot tone
34     A_p = 1;
35     sig_c = sig_c + A_p * cos(2 * pi * f_c * t_c);
36 end

```

Listing 3: SSB AM modulation

- A narrow-band filter would be hard to realistically construct in MATLAB, and I did not get very far with my approximations for one (using only first-order filters). Thus, we abstract away these hardware details, assume phase coherence, and use a pure generated cosine and sine with no phase offset for the I-Q demodulation.
- A first-order LPF and HPF were designed for this function.

- The LPF is largely intended to filter out the double-carrier-frequency signal generated from I-Q mixing, and the HPF reduces any low-frequency noise and the DC component caused by the I-Q mixing.

```

1 % demodulates a signal modulated using the SSB AM scheme; currently
2 % assumes phase coherence (no pilot tone in modulated signal)
3 % params:
4 % sig_c = SSB AM-modulated signal
5 % f_c = carrier frequency (Hz)
6 % A_c = carrier amplitude
7 % f_s_m = baseband sampling frequency
8 % f_s_c = carrier sampling frequency (Hz)
9 % tau = LPF cutoff frequency
10 % returns:
11 % sig_m = demodulated signal at baseband sampling frequency
12 function sig_m = ssb_demod(sig_c, f_c, A_c, f_s_m, f_s_c, tau)
13     duration = length(sig_c) / f_s_c;
14     t_c = linspace(0, duration, f_s_c * duration);
15     t_m = linspace(0, duration, f_s_m * duration);

16
17     % demodulate in-phase component
18     sig_c = sig_c .* cos(2 * pi * f_c * t_c);

19
20     % low-pass filtering in freq. domain; set cutoff frequency to carrier
21     % frequency
22     wd = linspace(-pi, pi, length(sig_c));
23     f_c = wd * f_s_c / (2 * pi);
24     lpf = (1 + f_c/tau*1j).^-1;           % LPF frequency response
25     hpf = (1 - 20*f_c.^-1*1j).^-1;       % HPF to remove low frequency
26                                         % distortion from pilot
27     ft_c = fftshift(fft(sig_c)) / f_s_c;   % freq. domain rectified signal
28     ft_c = lpf .* hpf .* ft_c;            % apply filter in freq. domain
29     sig_c = ifft(ifftshift(ft_c) * f_s_c);

30
31     % scale and downsample
32     sig_m = real(interp1(t_c, sig_c, t_m));

33
34     % scale back up to original amplitude
35     sig_m = 2 * sig_m / A_c;
36 end

```

Listing 4: SSB AM demodulation

## 2.3 PM Modulation

### 2.3.1 Overview

Phase modulation involves encoding the message into the phase  $\phi(t)$  of the carrier signal:

$$\phi(t) = k_p m(t) \quad (17)$$

where  $k_p$  is an arbitrary scaling constant. We also define the modulation index,  $\beta_p$ :

$$\beta_p := k_p \max |m(t)| \quad (18)$$

This modulation index indicates the maximum phase deviation,  $\Delta\phi_{max}$ . If we adjust the message signal so that its maximum value has unity amplitude, it makes the maximum deviation simply  $k_p$ . The modulated signal looks like:

$$u_{pm}(t) = A_c \cos(2\pi f_c t + \phi(t)) \quad (19)$$

$$= A_c (\cos \phi \cos(2\pi f_c t) - \sin \phi \sin(2\pi f_c t)) \quad (20)$$

$$\approx A_c (\cos(2\pi f_c t) - \phi \sin(2\pi f_c t)) \quad (\phi \ll 1) \quad (21)$$

These three forms of the equation lend them to three modes of demodulation (and also multiple modes of modulation).

- If we differentiate the form in (19) (e.g., using an inductor), then we get:

$$u'_{pm}(t) = -A_c (2\pi f_c + \phi'(t)) \sin(2\pi f_c t + \phi(t)) \quad (22)$$

This has the same form as conventional AM, where the offset is  $2\pi f_c$  (sufficiently large) and the message signal is  $\phi'(t)$ . By rectifying and LPF-ing, this can produce the envelope,  $\phi'(t)$ . This can then be integrated (e.g., by using a capacitor), then divided by  $k_p$  in order to recover the original message. This method would introduce some rectification noise, but is cheap to implement like conventional AM. This method also does not require sending an additional pilot tone.

- The form in (20) is an I-Q decomposition of the PM-modulated signal, which can be I-Q demodulated to reproduce  $I = \cos \phi$  and  $Q = \sin \phi$ .  $\phi$  can be recovered with  $\arctan(Q/I)$ , and then we can divide by  $k_p$ . I haven't tried implementing this, but I've heard from peers that the arctangent may sometimes introduce numeric instabilities using this method.
- (20) can be approximated as (21) with the assumption that  $|\phi| \ll 1$ . This is called the **narrow band approximation**. When demodulating, we only need recover the Q-component, and don't require the use of an arctangent function, so this is simpler than the second method.

For phase coherence, an additional pilot tone term  $A_p \cos(2\pi f_c t)$  should be added. This is required for the latter two methods (I-Q demodulation), but is not required for the former (envelope detection).

PM-signal generation can be implemented as an I-Q demodulation (one of the two latter equation forms); the last form is easiest to implement, but only can be used when the modulation index is small. However, there are ways to use this method even with a wide band signal, namely by frequency division and multiplication.

Unlike amplitude-modulated signals, angle-modulated signals (PM and FM) take up infinite bandwidth because of their continuous frequency (phase) deviations. This can be seen mathematically as the Fourier transform of angle-modulated signals involves the Bessel functions, and thus have infinite bandwidth. However, Carson's rule states that the majority of an angle-modulated signal's power ( $\geq 98\%$ ) lies in a bandwidth of:

$$W_{carson} = 2(\beta + 1)W \quad (23)$$

where  $\beta = \beta_p$  for phase modulation and  $\beta = \beta_f$  for frequency modulation.

### 2.3.2 Analysis

When assuming the narrow band approximation, we can estimate that both  $A_c + A_p$  is the amplitude of the in-phase carrier, and  $A_c \beta_p$  is the amplitude of the quadrature carrier. Thus:

$$P_u = \frac{(A_c + A_p)^2}{2} + \frac{A_c^2 \beta_p^2 P_m}{2} \quad (24)$$

Since the modulated signal is narrow band (by assumption), then  $\beta_p$  is small and the first term dominates. I defer the derivation of the noise power and SNR to the class notes, or section 5.3 of [2].

$$P_m = k_p^2 P_m \quad (25)$$

$$P_n = \frac{2WN_0}{A_c^2} \quad (26)$$

$$\text{SNR} = \frac{k_p^2 A_c^2}{2} \frac{P_m}{N_0 W} = \frac{A_c^2}{2} \left( \frac{\beta_p}{\max |m(t)|} \right)^2 \frac{P_m}{N_0 W} \quad (27)$$

Again, the calculation for noise power was derived in the analog domain. See 10 for details about the digital domain calculation.

### 2.3.3 Implementation details

The modulation is fairly straightforward. We calculate the phase function, and add this phase pointwise to the carrier signal. (In practice, it would be easier to implement by mixing a narrow band signal with the carrier frequency using the narrow band approximation, but this is MATLAB.) A pilot tone is added to the signal.

Notes on demodulation:

- We use the narrow band approximation here in order to simplify the calculation. This means an I-Q demodulation (the third PM demodulation method described) by only mixing with the quadrature carrier.
- Similar to in SSB, we generated a pilot tone in the modulation “for show”: this would be necessary for coherence with the I-Q demodulation, but actually extracting this tone in MATLAB with a realistic filter would be hard. Thus, again we don't use this pilot tone in the demodulator, but we generate a quadrature carrier (sine wave) and assume coherence.

```

1 % modulates a signal using the PM scheme
2 % params:
3 % f_c = frequency of carrier
4 % A_c = amplitude of carrier
5 % sig_m = message signal
6 % k = k_p, phase deviation coefficient
7 % f_s_m = downsampled sampling rate
8 % f_s_c = upsampled sampling rate
9 % returns:
10 % sig_c = PM-modulated signal at upsampled rate
11 function sig_c = pm_mod(f_c, A_c, sig_m, k, f_s_m, f_s_c)
12     duration = length(sig_m) / f_s_m;
13     t_m = linspace(0, duration, length(sig_m));
14     t_c = linspace(0, duration, f_s_c * duration);
15
16     % upsample sig_m, multiply by k to get phase
17     sig_phi = k * interp1(t_m, sig_m, t_c);
18
19     % generate phase-modulated signal w/ pilot tone
20     A_p = 1;
21     sig_c = A_c * cos(2 * pi * f_c * t_c + sig_phi) ...
22             + A_p * cos(2 * pi * f_c * t_c);
23 end

```

Listing 5: PM modulation

- As with the previous modulation schemes, a first-order LPF and HPF were used to filter the components. The LPF was applied twice to get rid of some extra noise at higher frequencies.
- To regain the original amplitude, we multiply by a factor of 2. This can be explained by the power losses in the demodulation. Firstly, half of the power should be in the I component, which we completely disregard. Another half of the power is lost when mixing with the carrier. Losing power by a factor of 4 means an amplitude attenuation by a factor of 2.

```

1 % demodulates a signal modulated using the PM scheme
2 % params:
3 % f_c = frequency of carrier
4 % A_c = amplitude of carrier
5 % sig_c = PM-modulated signal
6 % k = k_p, phase deviation coefficient
7 % f_s_m = downsampled sampling rate
8 % f_s_c = upsampled sampling rate
9 % tau = LPF cutoff frequency
10 % returns:
11 % sig_m = demodulated signal at downsampled rate
12 function sig_m = pm_demod(f_c, A_c, sig_c, k, f_s_m, f_s_c, tau)
13     duration = length(sig_c) / f_s_c;
14     t_c = linspace(0, duration, f_s_c * duration);
15     t_m = linspace(0, duration, f_s_m * duration);

16
17     % mix with sine; negative because quadrature component is negative
18     sig_c = -sig_c .* sin(2 * pi * f_c * t_c);

19
20     % LPF and HPF
21     wd = linspace(-pi, pi, length(sig_c));
22     f_c = wd * f_s_c / (2 * pi);
23     hpf = (1 - 20*f_c.^-1*1j).^-1;           % HPF
24     lpf = (1 + f_c/tau*1j).^-1;               % LPF frequency response
25     ft_c = fftshift(fft(sig_c));              % freq. domain rectified signal
26     ft_c = lpf .^ 2 .* hpf .* ft_c;          % apply filter in freq. domain
27     sig_c = ifft(ifftshift(ft_c));

28
29     % downsample
30     sig_m = real(interp1(t_c, sig_c, t_m) / (k * A_c));
31
32     % restore amplitude
33     sig_m = sig_m * 2;
34 end

```

Listing 6: PM demodulation

## 2.4 FM Modulation

### 2.4.1 Overview

Frequency modulation involves encoding the message into the frequency of the carrier signal. This first requires the concept of instantaneous frequency, which can be defined as a function of the phase  $\phi$ :

$$f_{inst} = \frac{1}{2\pi} \frac{d\phi}{dt} \quad (28)$$

Now, say that we wish to encode the message signal as  $a$  in the frequency, i.e.:

$$f_{inst} = k_f m(t) \quad (29)$$

Here,  $k_f$  is the frequency deviation coefficient from the carrier (I.e., if the message has an amplitude of  $a$ , then the instantaneous frequency of the modulated signal will be  $f_c + a$  at that moment; intuitively,  $k_f \max |m(t)| = \Delta f_{max}$ ) Then we can express the phase as a function of the message signal:

$$\phi(t) = 2\pi k_f \int_{-\infty}^t m(t) dt \quad (30)$$

Then, we can apply the same principles behind PM to modulate this phase. This generates the modulated signal:

$$u_{fm}(t) = A_c \cos \left( 2\pi f_{ct} t + 2\pi k_f \int_{-\infty}^t m(t) dt \right) \quad (31)$$

Similar to PM, we can define a modulation factor  $k_f$  that indicates the phase shift:

$$\beta_f := \frac{k_f \max |m(t)|}{W} = \frac{\Delta f_{max}}{f_{m,max}} \quad (32)$$

where  $W$  is the bandwidth of  $m(t)$  (i.e., the highest frequency of the baseband signal  $m(t)$ ). This can intuitively be interpreted to mean the bandwidth efficiency of a modulated signal: a higher  $\beta_f$  indicates a higher carrier modulation for the same input bandwidth (and thus lower bandwidth efficiency).  $\beta_f$  can also be roughly interpreted as the maximum phase deviation (just like  $\beta_p$  for PM), since, for the pure tone  $m_{pure}(t) = \cos(2\pi f_0 t)$  ( $\max |m(t)| = 1$ ,  $\beta = k_f/f_0$ ):

$$\begin{aligned} u(t) &= A_c \cos \left( 2\pi f_{ct} t + 2\pi k_p \int_{-\infty}^{\infty} \cos(2\pi f_0 t) dt \right) \\ &= A_c \cos \left( 2\pi f_{ct} t + \frac{2\pi k_p}{2\pi f_0} \sin(2\pi f_0 t) \right) = A_c \cos(2\pi f_{ct} t + \beta_f \sin(2\pi f_0 t)) \end{aligned} \quad (33)$$

Clearly,  $\beta_f = \Delta\phi_{max}$  in this case.

Demodulation of an FM-modulated signal can be performed identically to a PM demodulator, except that it would require an additional division by  $2\pi k_f$  at the conclusion and a differentiation stage. Note that the first demodulation scheme involves a differentiation, envelope detection, and an integration stage; if we add another differentiation stage at the end of this, it would cancel out the integration stage and make it a simple two-step process very similar to conventional AM. Since differentiation and envelope detection are both easy to implement in hardware, this makes conventional AM and FM the most commonly implemented modulation schemes.

### 2.4.2 Analysis

As with PM, I defer the derivation of the noise power and SNR to the class notes, or section 5.3 of [2].

$$P_m = k_f^2 P_m \quad (34)$$

$$P_n = \frac{2W^3 N_0}{3A_c^2} \quad (35)$$

$$\text{SNR} = \frac{3k_f^2 A_c^2}{2W^2} \frac{P_m}{N_0 W} = \frac{3A_c^2}{2} \left( \frac{\beta_f}{\max |m(t)|} \right)^2 \frac{P_m}{N_0 W} \quad (36)$$

Again, the calculation for noise power was derived in the analog domain. See 10 for details about the digital domain calculation. What is a little confusing is that the bandwidth  $W$  in the first term in (36) can be in analog units (it should be the same value used when defining  $k_f$ ), whereas the  $W$  in the second term indicates the cutoff frequency and should be in digital radian units when working in a sampled space. The equivalent expression using  $\beta_f$  is a little nicer because  $\beta_f$  is invariant to the domain (digital or analog), and so it hides this ugliness.

### 2.4.3 Implementation details

This uses the cheap method involving differentiation and conventional AM demodulation (envelope detection). The modulation implementation is pretty straightforward. The only design decision was to use `cumsum` as a simple running integral.

Notes about demodulation:

- Differentiation is performed in the time domain using a difference equation approximation. It is padded with a single zero to maintain the length.
- Like in the conventional AM case, half-wave rectification is arbitrarily chosen (as opposed to full-wave rectification).
- A first-order LPF and HPF were used to filter the circuit. As in the previous examples, the LPF is primarily to filter out the double frequency carrier and the DC components created by the carrier mixing.
- The LPF was arbitrarily applied multiple times, similar to the SSB case, in order to reduce additional noise at higher frequencies.

```

1 % modulates a signal using the FM scheme
2 % params:
3 % f_c = frequency of carrier
4 % A_c = amplitude of carrier
5 % sig_m = message signal
6 % k = k_f, frequency deviation coefficient
7 % f_s_m = downsampled sampling rate
8 % f_s_c = upsampled sampling rate
9 % returns:
10 % sig_c = FM-modulated signal at upsampled rate
11 function sig_c = fm_mod(f_c, A_c, sig_m, k, f_s_m, f_s_c)
12     duration = length(sig_m) / f_s_m;
13     t_m = linspace(0, duration, length(sig_m));
14     t_c = linspace(0, duration, f_s_c * duration);
15
16     % upsample sig_m, multiply by k, integrate and multiply by 2pi to get
17     % phase (in rad)
18     sig_phi = 2 * pi * k * interp1(t_m, cumsum(sig_m) / f_s_m, t_c);
19
20     % generate phase-modulated signal (since FM is basically PM)
21     sig_c = A_c * cos(2 * pi * f_c * t_c + sig_phi);
22 end

```

Listing 7: FM modulation

```

1 % demodulates a signal modulated using the FM scheme
2 % params:
3 % sig_m = message signal
4 % A_c = amplitude of carrier
5 % f_s_m = downsampled sampling rate
6 % f_s_c = upsampled sampling rate
7 % k = k_f, frequency deviation coefficient
8 % tau = LPF cutoff frequency
9 % returns:
10 % sig_m = FM-demodulated signal at downsampled rate
11 function sig_m = fm_demod(sig_c, A_c, f_s_m, f_s_c, k, tau)
12     % differentiate (same as multiplying by jw in freq domain)
13     sig_c = [(diff(sig_c) * f_s_c) 0];
14
15     % rectify signal
16     sig_c(sig_c < 0) = 0;
17
18     % lpf and hpf
19     wd = linspace(-pi, pi, length(sig_c));
20     f_c = wd * f_s_c / (2 * pi);
21     hpf = (1 - 20*f_c.^-1*1j).^-1;           % HPF
22     lpf = (1 + f_c/tau*1j).^-1;               % LPF frequency response
23     ft_c = fftshift(fft(sig_c));              % freq. domain rectified signal
24
25     % apply LPF multiple times in a row to completely get rid of carrier
26     % frequency
27     ft_c = lpf .^ 2 .* hpf .* ft_c;          % apply filter in freq. domain
28     sig_c = ifft(ifftshift(ft_c));
29
30     % factor of 3.4 found empirically (similar to conventional AM demod)
31     sig_c = 3.4 * real(sig_c) / (2 * pi * k * A_c);
32
33     % downsample sig_c
34     duration = length(sig_c) / f_s_c;
35     t_m = linspace(0, duration, f_s_m * duration);
36     t_c = linspace(0, duration, length(sig_c));
37     sig_m = interp1(t_c, sig_c, t_m);
38 end

```

Listing 8: FM demodulation

## 2.5 Modulation schemes comparison

See Table 1 for a general comparison between modulation schemes.

	CONV	SSB	FM
PM	PM more robust to noise CONV cheaper	Similar I-Q demod scheme SSB more bandwidth efficient PM more robust to noise	FM cheaper FM more common
FM	FM more robust to noise Similar cheap demod scheme	FM cheaper SSB more bandwidth efficient PM more robust to noise	
SSB	CONV cheaper SSB use half bandwidth SSB transmits double signal power		

Table 1: Comparison of modulation schemes

**Power** Conventional AM uses half the power of SSB. Angle-modulation schemes are usually more power-efficient because they can increase  $\beta$  to increase SNR instead of increasing  $A_c$ .

**Bandwidth** Conventional AM uses double the bandwidth of SSB. Angle-modulation schemes are usually less bandwidth-efficient because they use a larger bandwidth (larger  $\beta$ ) to increase SNR without increasing  $A_c$ .

**SNR (and modulation index)** In all of the schemes, increasing the carrier amplitude increases the SNR (but is less power efficient). In SSB AM, this is the only way to affect the SNR (assuming the message power is fixed). In conventional AM, increasing  $a$  also increases the SNR, but this also decreases power efficiency, and it is limited by the fact that  $0 \leq a \leq 1$  so that no clipping occurs at demodulation. In the angle modulation schemes, in addition to the option of increasing the carrier amplitude, increasing the modulation index  $\beta$  increases SNR at no power cost, and is not as limited as  $a$  is; however, this uses a larger bandwidth.

## 2.6 “Fairly” comparing different modulation schemes

The “design problem” of this assignment is that there are multiple parameters to vary, and most do not affect the noiseless demodulated output (but affect the SNR when noise is added to the modulated signal). For example, the demodulated signal of a no-noise modulated signal is invariant to increases in  $A_c$ , but the SNR is directly related. In other words, if we set the criterion for choosing parameters to be such that the demodulated signal has the same power, there are infinitely-many possible combinations of parameters to choose from. This can be seen to make the estimated SNRs exactly the same in the simulations below (see Table 5), and the experimental SNR values are very close.

The assignment asks us to “ensure that the signal power will be equal at the output in each [modulated] scheme.” This means that the criterion for choosing parameters is such that the *power of the component of the modulated signal encoding the message* is equal between each modulated scheme. This is easily done with conventional AM and SSB AM, where the power of the message

signal component  $P_y$  are given by (derived in their respective sections):

$$P_{y,conv} = \frac{A_c^2 a^2}{2} P_m \quad (37)$$

$$P_{y,ssb} = A_c^2 P_m \quad (38)$$

In this case, the modulation index  $a$  conventional AM is arbitrarily set to 0.5 (which allows us to double it later without worrying about clipping), and the carrier amplitude  $A_c$  is arbitrarily set to 4. To match this, in the SSB case  $A_{c,ssb} \leftarrow A_{c,conv}/\sqrt{8}$ .

For the angle-modulated cases, I am not sure how to solve it because of the complex nature of the Fourier transform and PSD (i.e., with the Bessel functions). Thus we give up on this idea of fair comparison and just give the FM and PM the same  $A_c$  as the conventional AM case.

The assumption that signal power will be equal in each modulation scheme is unfair because this is (probably) not what happens in the real world: different modulation schemes probably get transmitted at vastly different signal powers and different transmitted powers. In other words, I give up on the idea of directly trying to compare the different modulation schemes to each other with some absolute measure (e.g., comparing their SNRs at some given noise variance level to determine which is overall more resistant to noise), but rather observe the general trends w.r.t. changes in their parameters.

### 3 Results

	CONV	SSB	PM	FM
modulation index	$a = 0.5$	—	$k_p = 0.1$	$k_f = 10W \approx 60650\text{Hz}$
carrier amplitude $A_c$	4	$\sqrt{2}$	4	4
carrier frequency $f_c$	500000	500000	500000	400000
sampling rate of modulated signal $f_{s,c}$	2000000	2000000	5000000	5000000
USSB/LSSB	—	LSSB	—	—
noise variance (when varying modulation index)	1	—	1	0.5

Table 2: Parameters for modulation schemes

		CONV	SSB	PM	FM
Total transmitted power	Experimental	8.1044	0.6031	12.4990	8.000
	Theoretical	8.1042	0.6044	12.5000	8.000
	% error	0.00%	0.22%	0.01%	0.00%
Demodulated power	Experimental	0.0546	0.0513	0.0498	0.0553
	Theoretical	0.0522	0.0522	0.0522	0.0522
	% error	4.60%	1.72%	4.60%	5.94%

Table 3: Expected vs. theoretical power of modulated (transmitted) and demodulated signals

	PM	FM
% power of signal in Carson's bandwidth	100.00%	100.00%

Table 4: Percent power of signal in Carson's bandwidth for angle-modulated signals

		$\sigma = 0.1$	$\sigma = 0.5$	$\sigma = 1$
CONV SNR (dB)	Experimental	22.52	15.12	9.83
	Theoretical	31.99 (28.98 $^\dagger$ )	18.01 (15.00 $^\dagger$ )	11.99 (8.98 $^\dagger$ )
SSB SNR (dB)	Experimental	21.40	13.55	8.11
	Theoretical	31.99 (28.98 $^\dagger$ )	18.01 (15.00 $^\dagger$ )	11.99 (8.98 $^\dagger$ )
PM SNR (dB)	Experimental	20.93	9.66	3.79
	Theoretical	21.99	8.01	1.99
FM SNR (dB)	Experimental	19.46	9.97	2.32
	Theoretical	66.76	52.78	47.76

Table 5: Effect of varying noise variance on SNR.  $^\dagger$ using an extra scaling factor of 2

		Modulation index ( $a$ , $\beta_p$ , or $\beta_f$ )		
		0.5*original	original	2*original
CONV SNR (dB)	Experimental	4.06	9.83	14.79
	Theoretical	5.97 (2.96 $^\dagger$ )	11.99 (8.98 $^\dagger$ )	18.01 (15.00 $^\dagger$ )
PM SNR (dB)	Experimental	-2.26	3.72	9.64
	Theoretical	-4.03	1.98	8.01
FM SNR (dB)	Experimental	3.43	9.97	13.54
	Theoretical	45.93	51.95	57.97

Table 6: Effect of varying modulation index on SNR.  $^\dagger$ using an extra scaling factor of 2

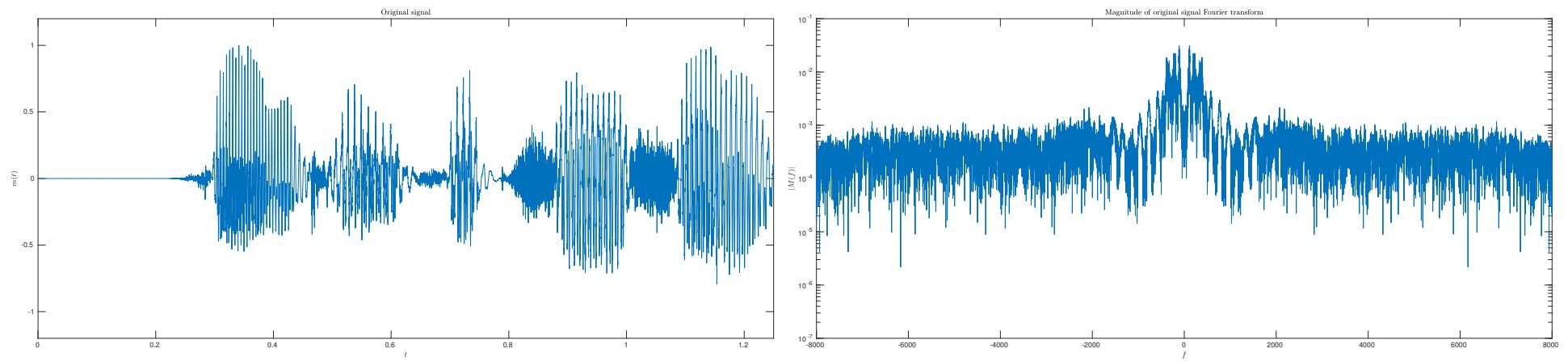


Figure 1: Original message signal

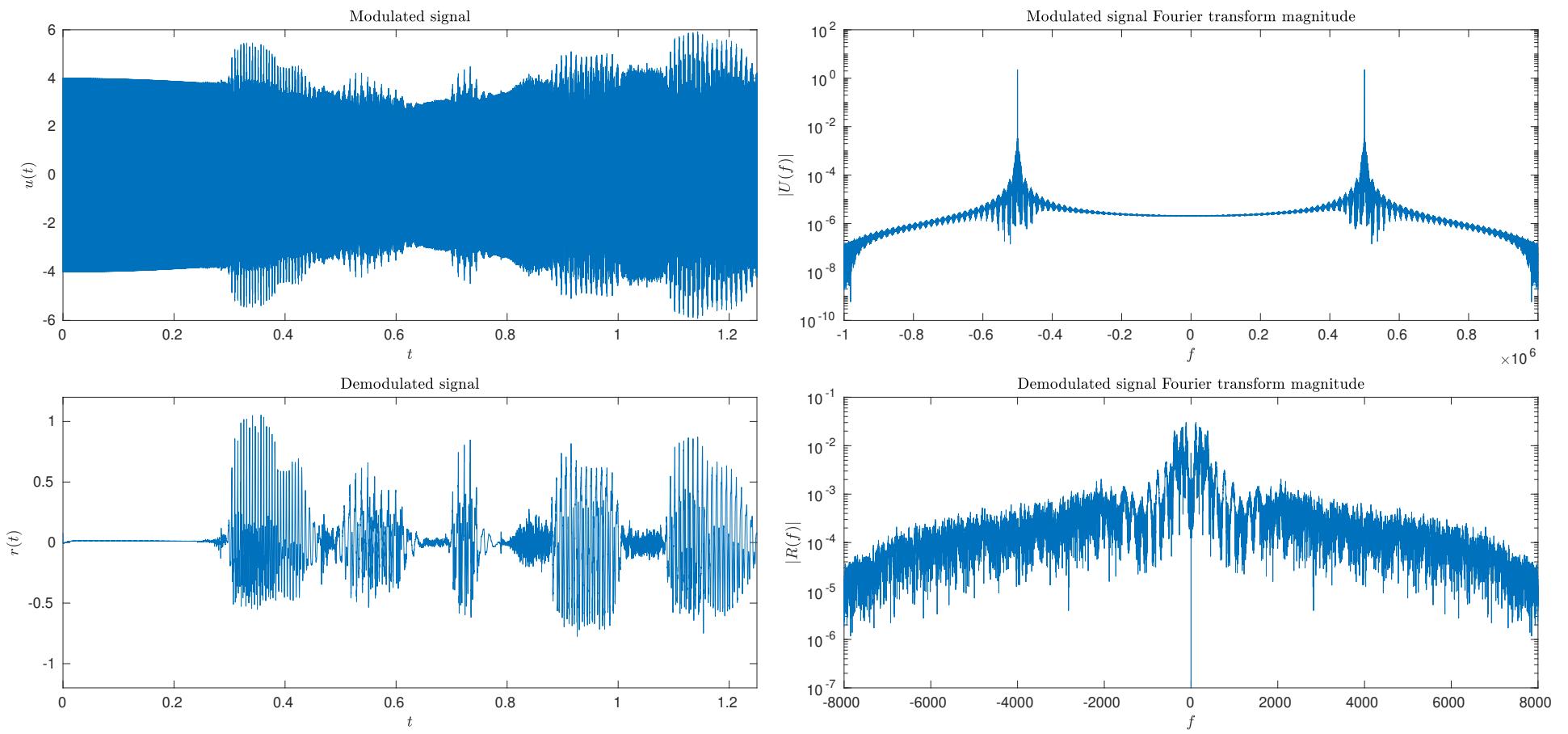


Figure 2: Conventional AM modulated and demodulated signals

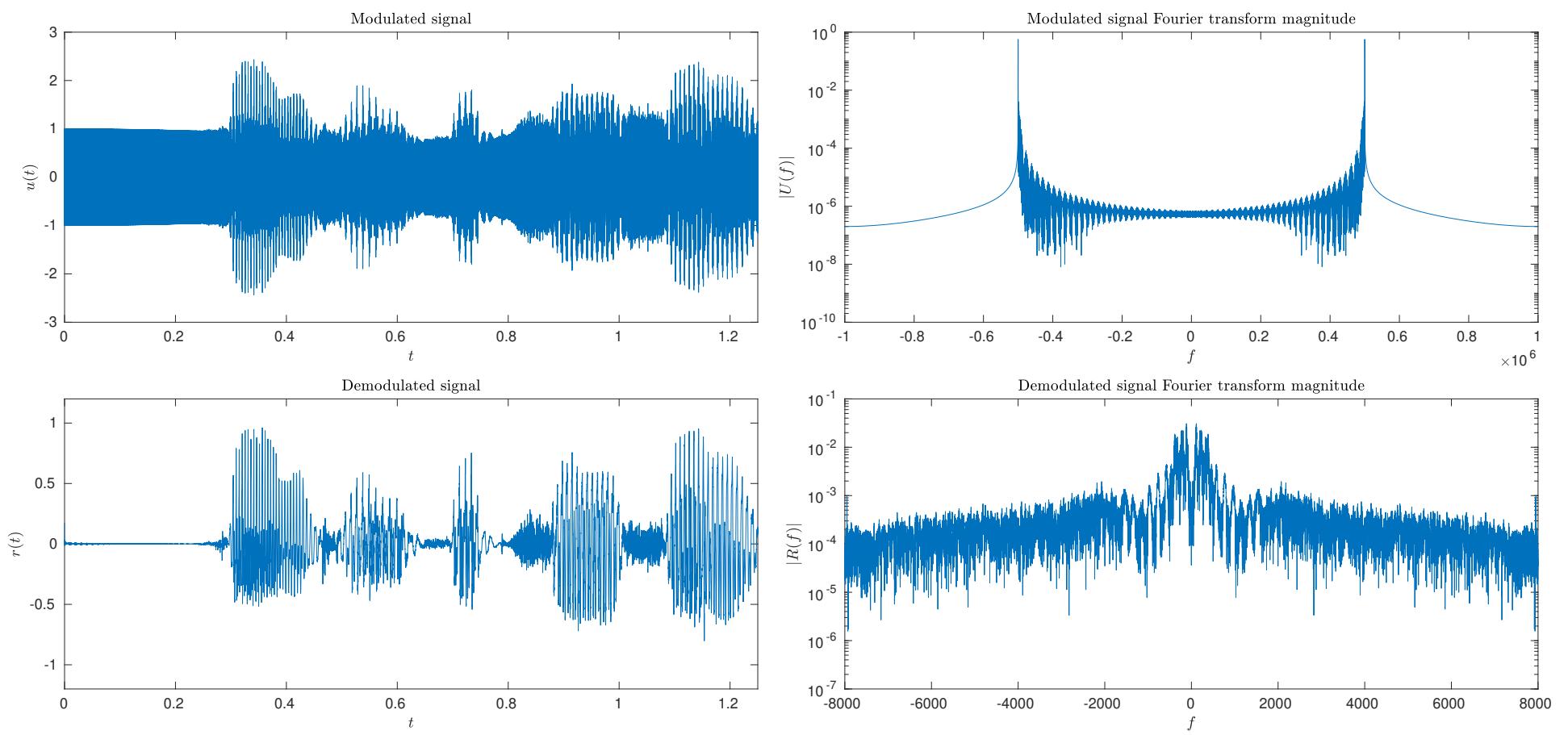


Figure 3: SSB AM modulated and demodulated signals

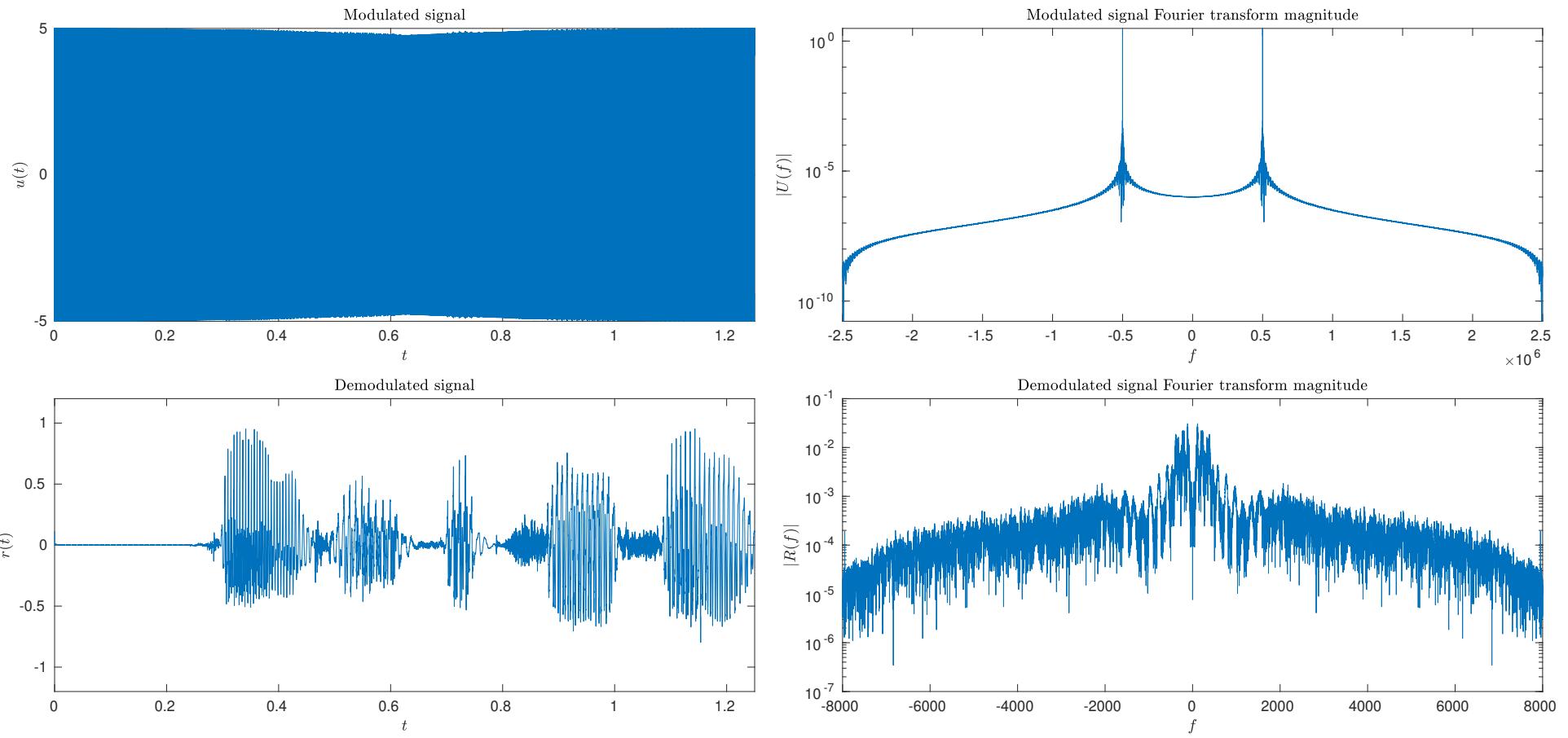


Figure 4: PM modulated and demodulated signals

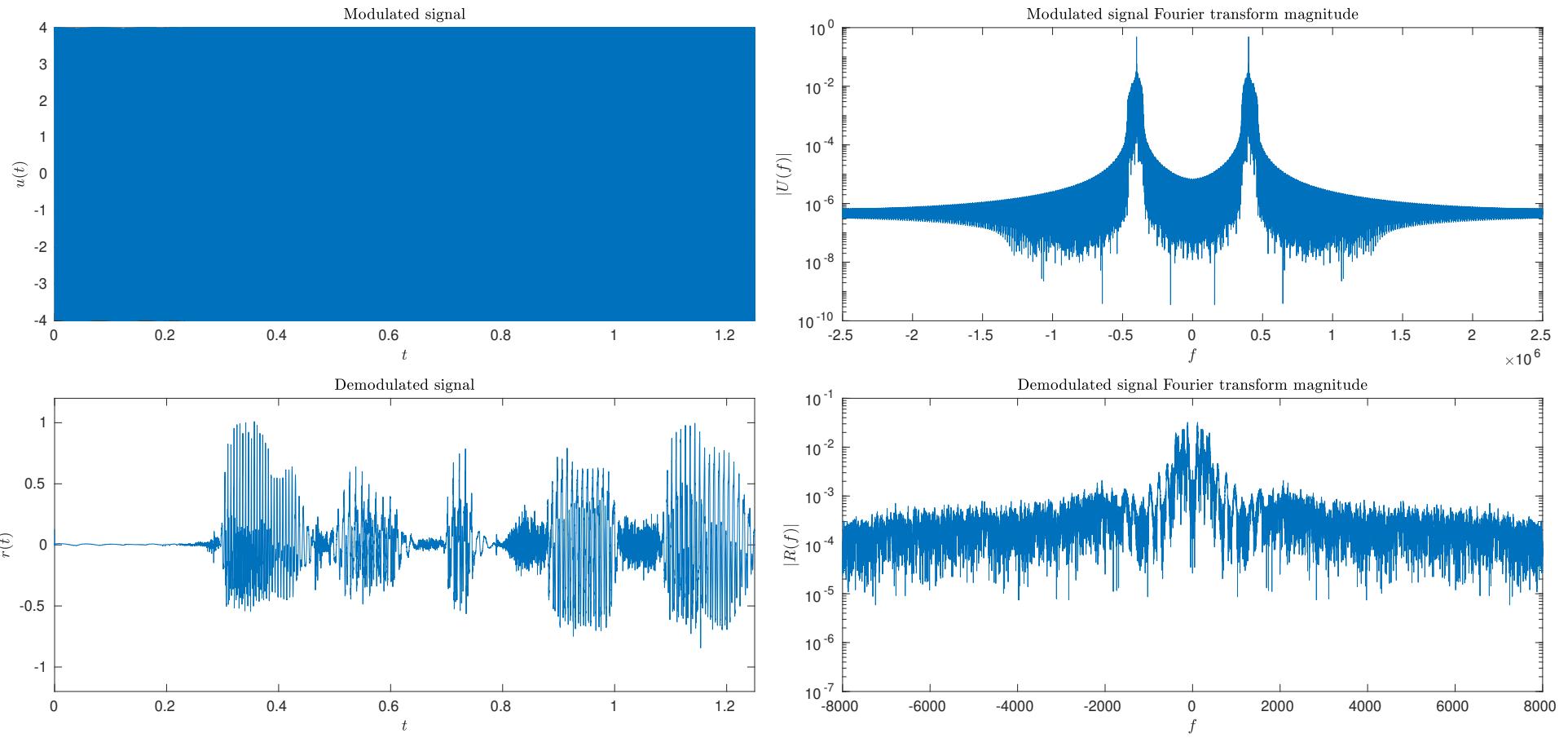


Figure 5: FM modulated and demodulated signals

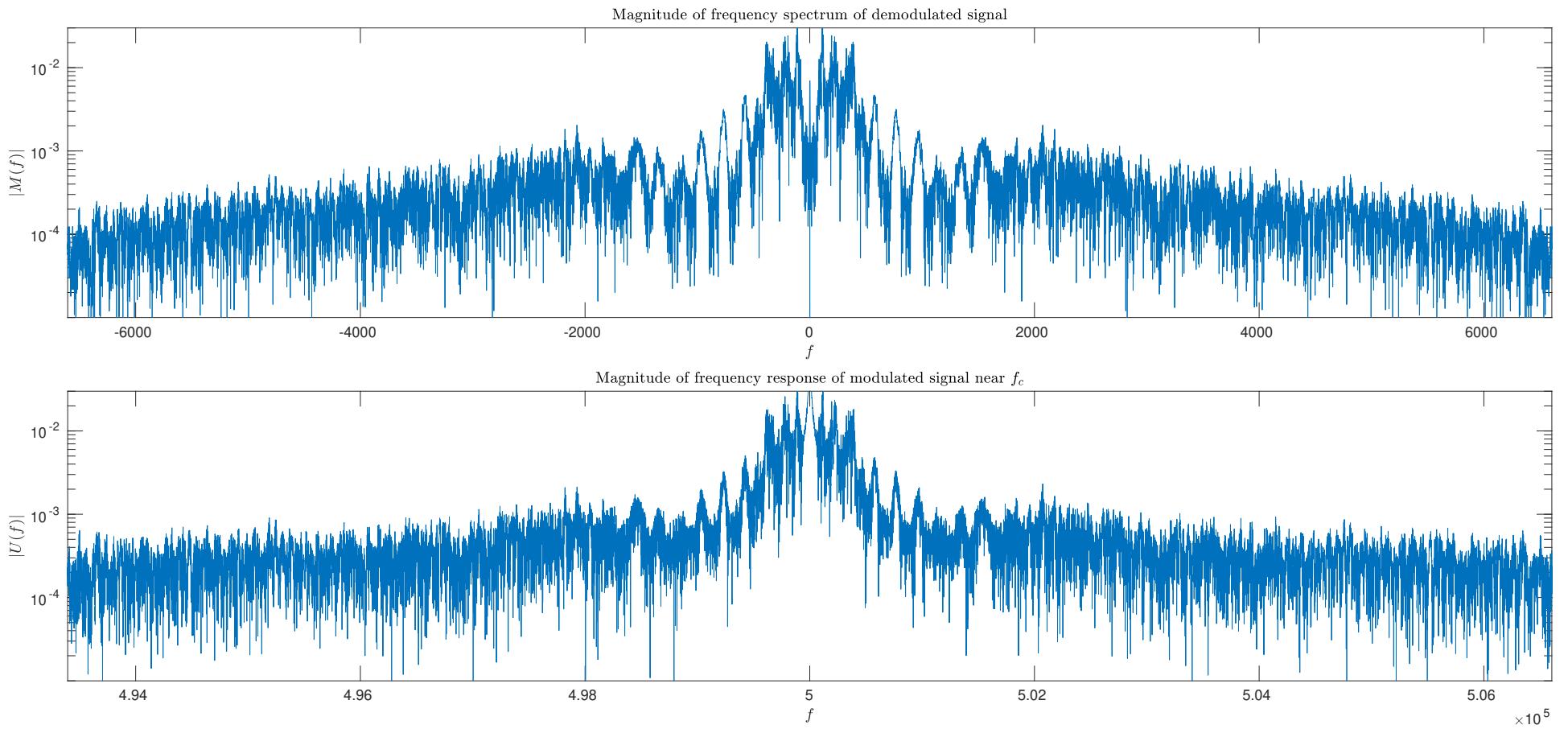


Figure 6: Close-up comparison of conventional AM baseband and modulated spectra

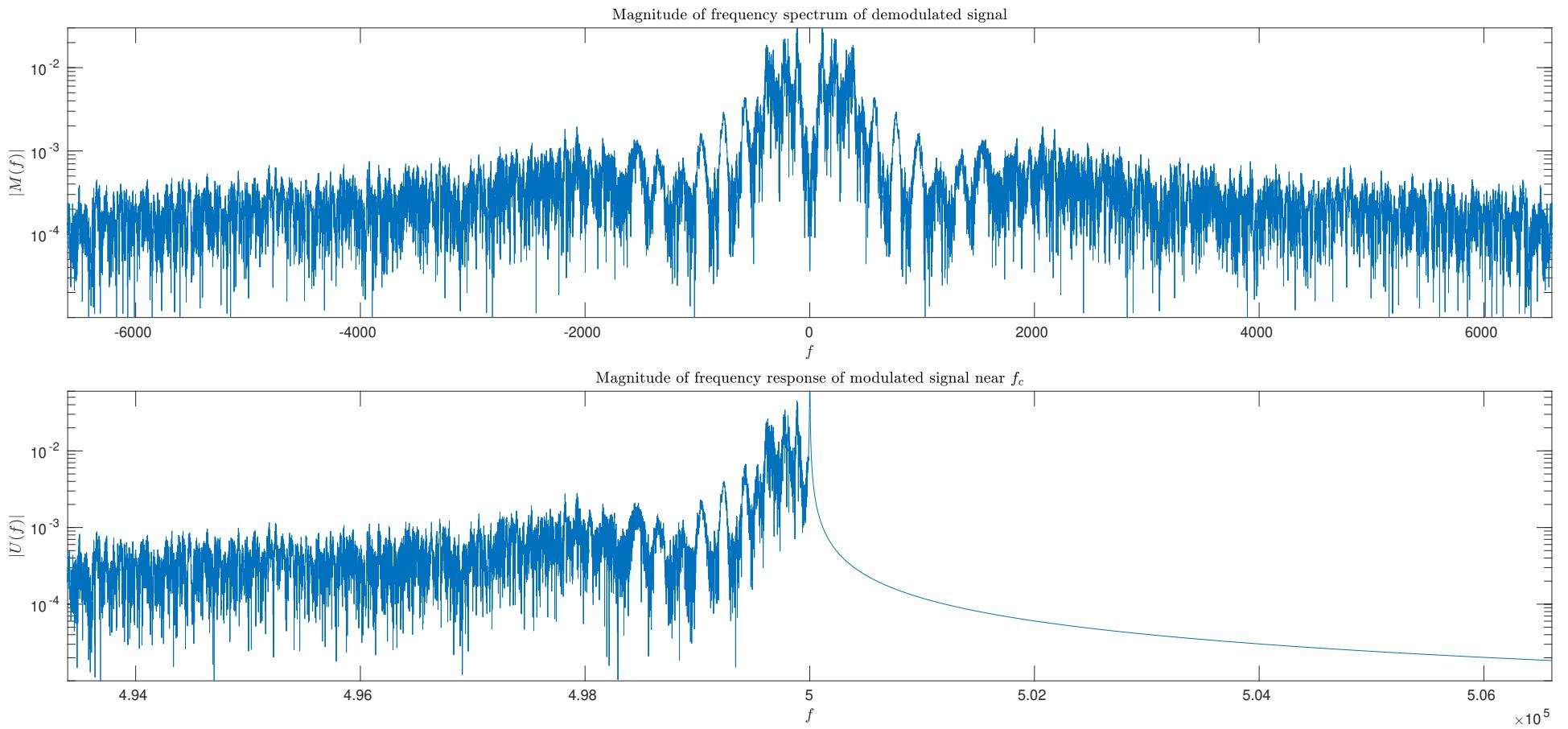


Figure 7: Close-up comparison of SSB AM baseband and modulated spectra

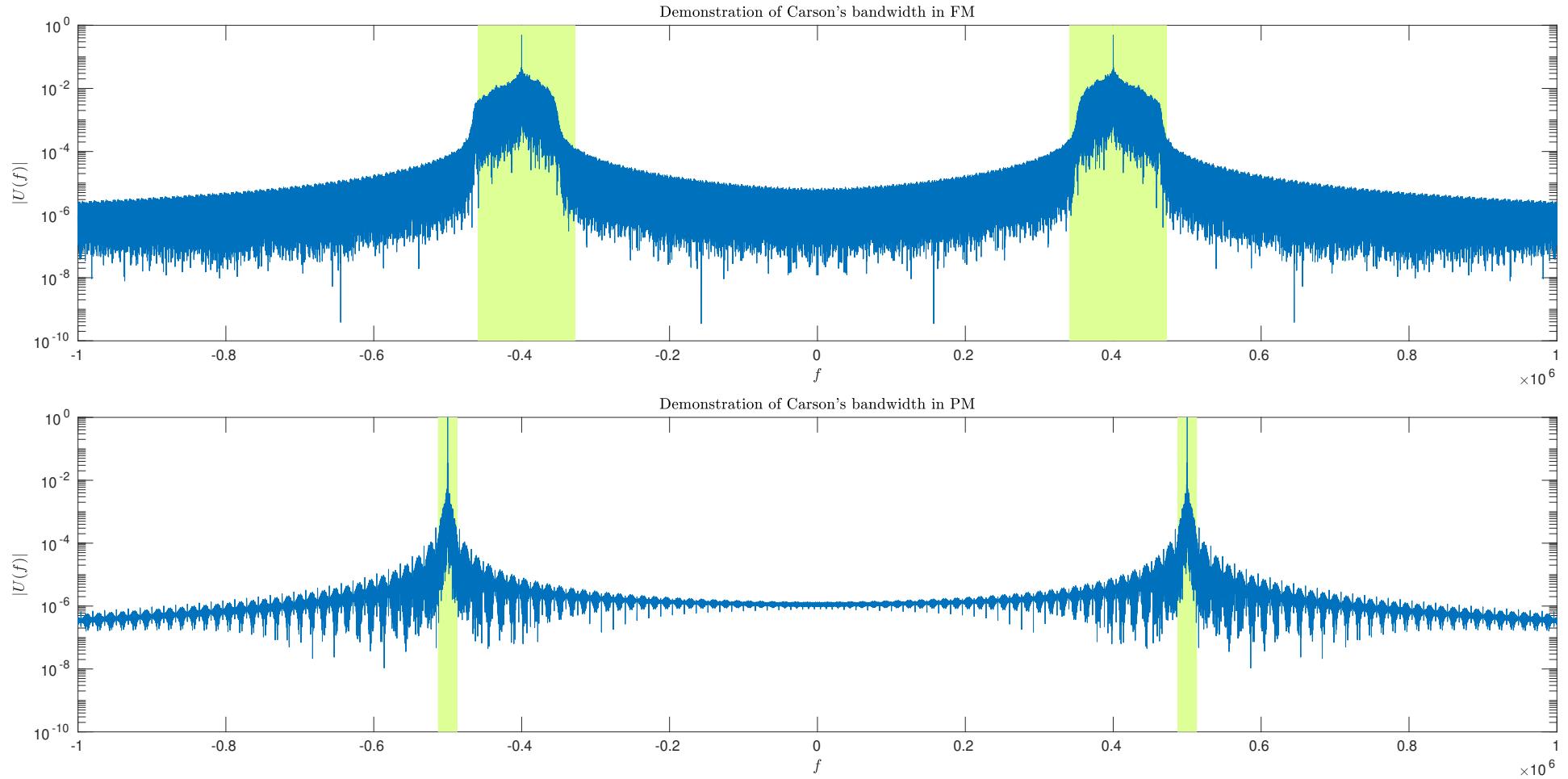


Figure 8: Demonstration of Carson's rule

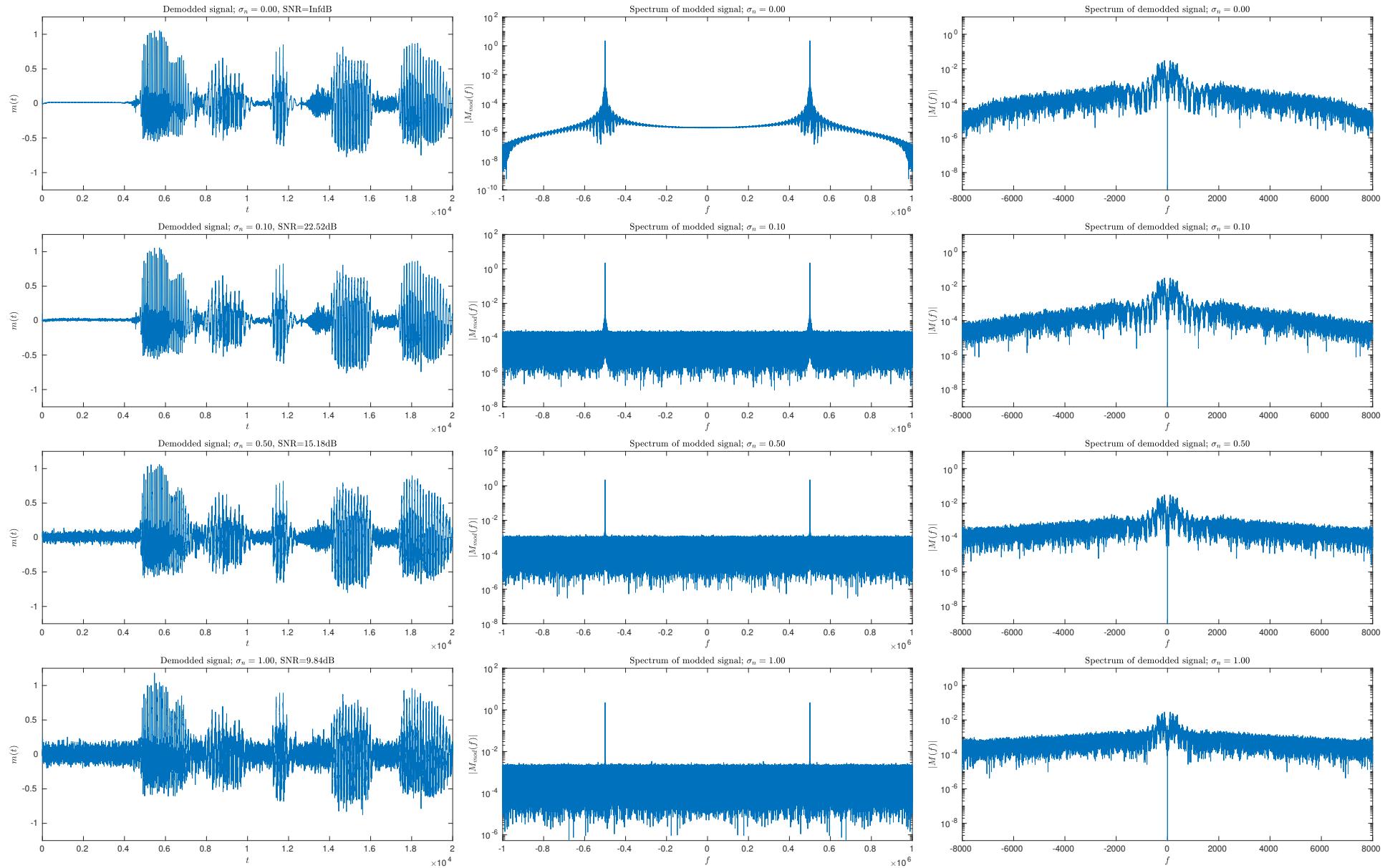


Figure 9: Effect of varying noise variance on conventional AM

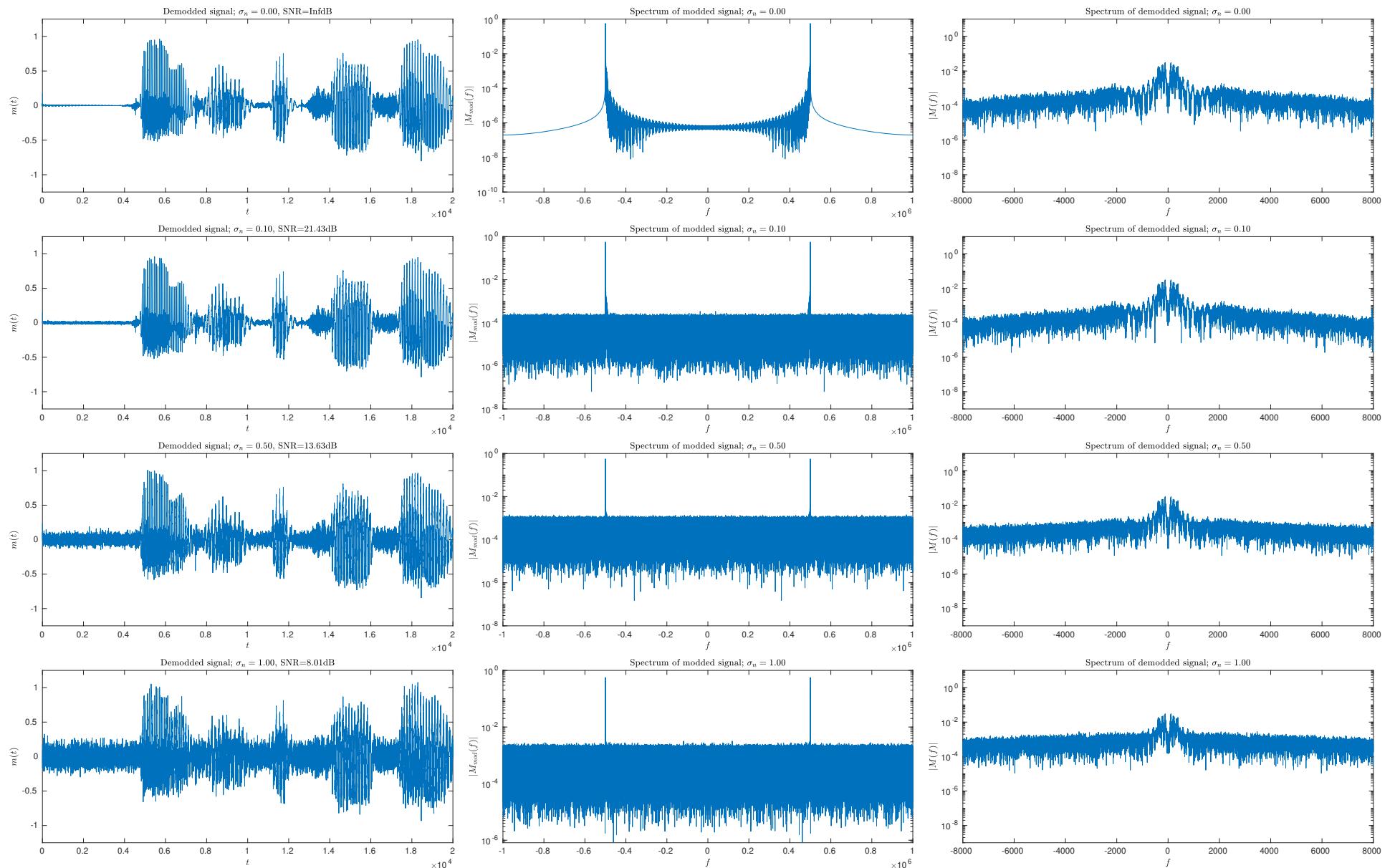


Figure 10: Effect of varying noise variance on SSB AM

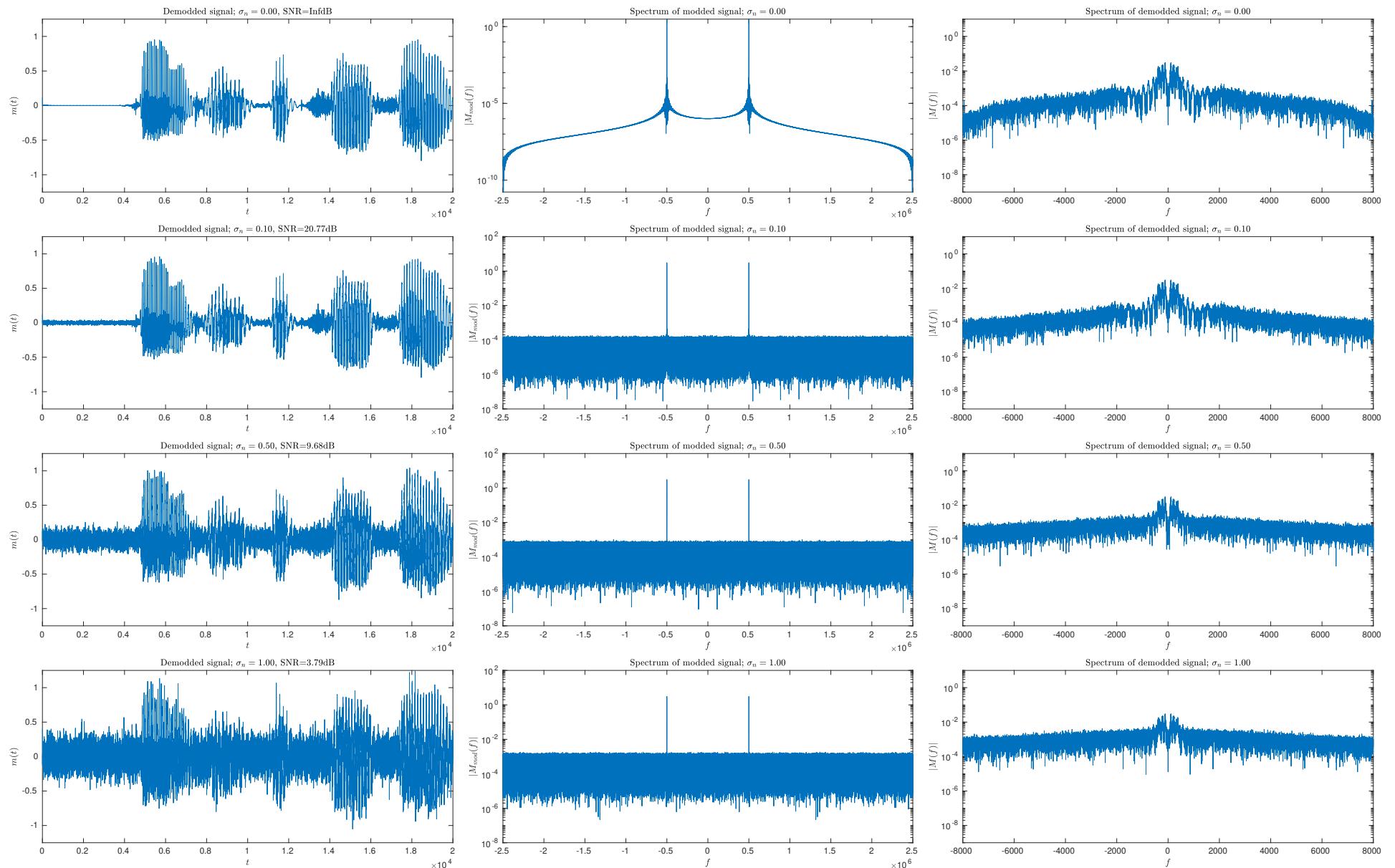


Figure 11: Effect of varying noise variance on PM

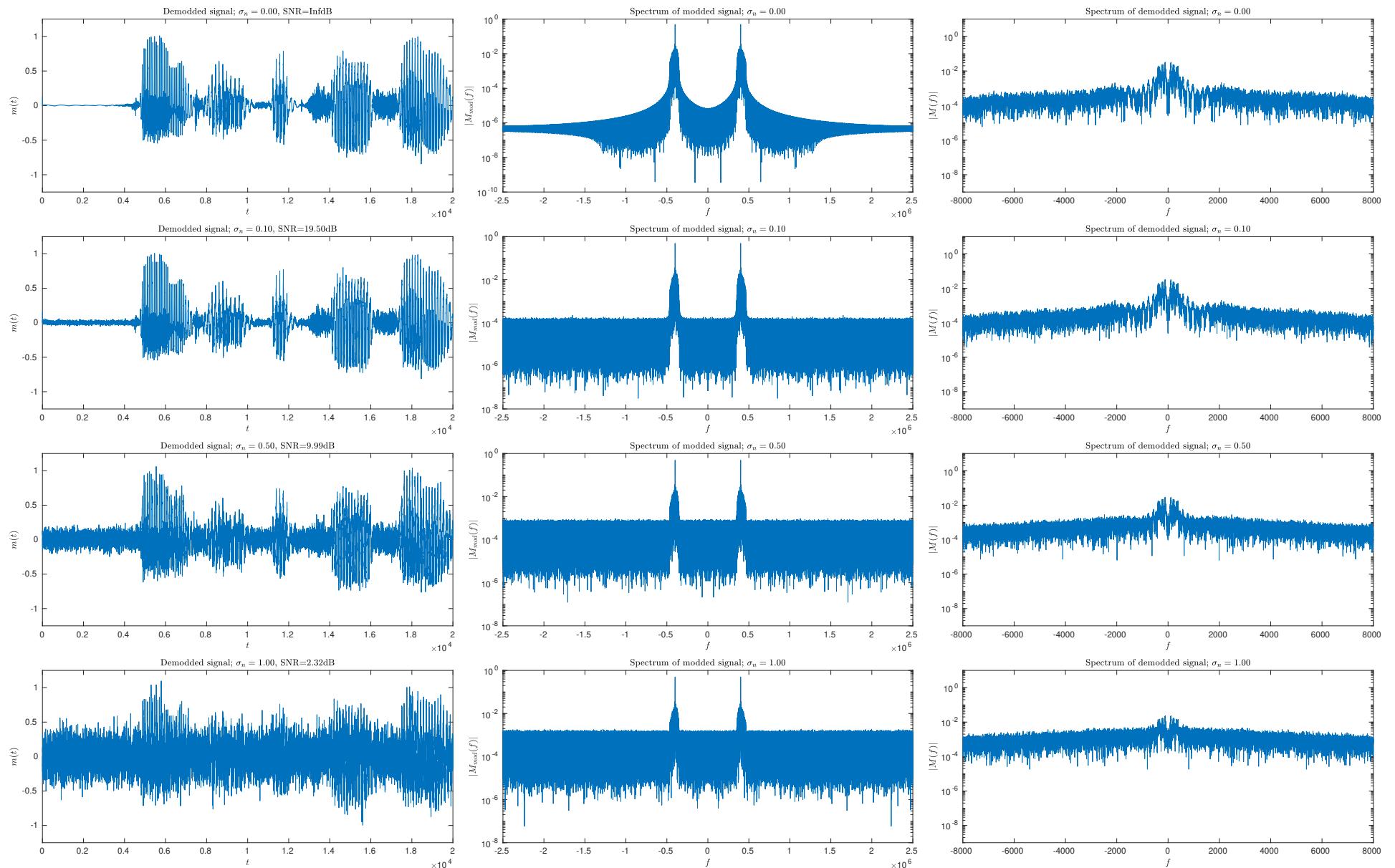


Figure 12: Effect of varying noise variance on FM

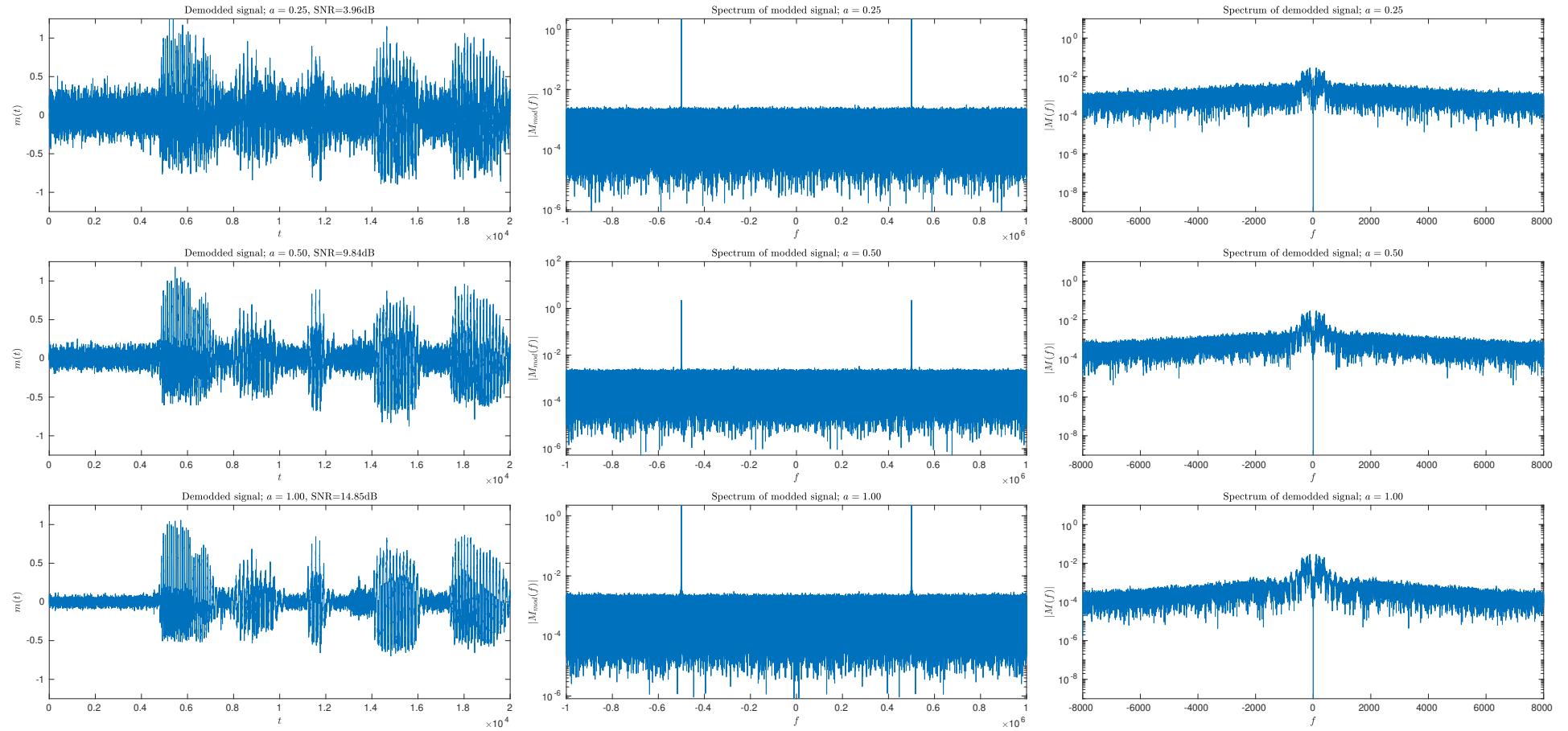


Figure 13: Effect of varying modulation index on conventional AM

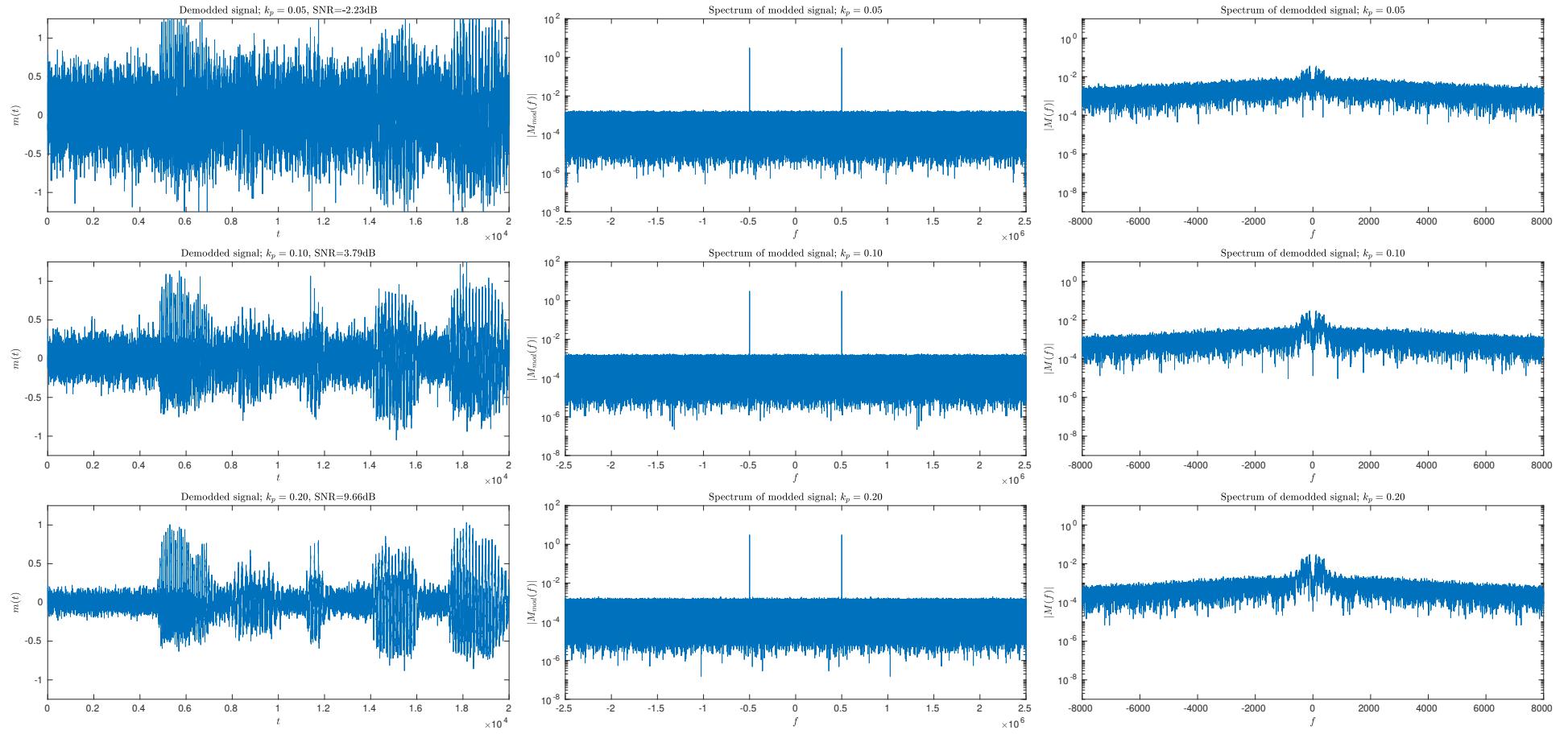


Figure 14: Effect of varying modulation index on PM

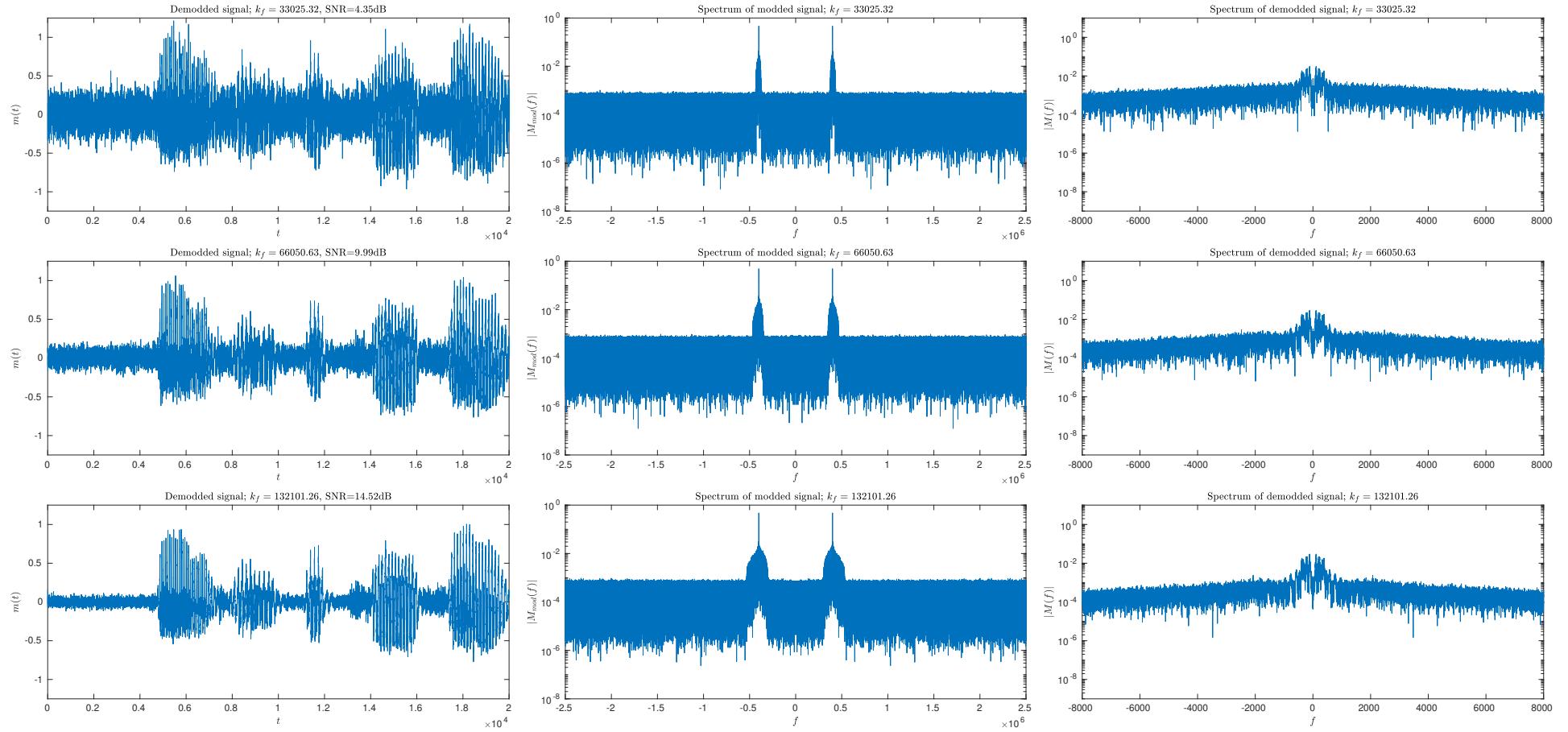


Figure 15: Effect of varying modulation index on FM

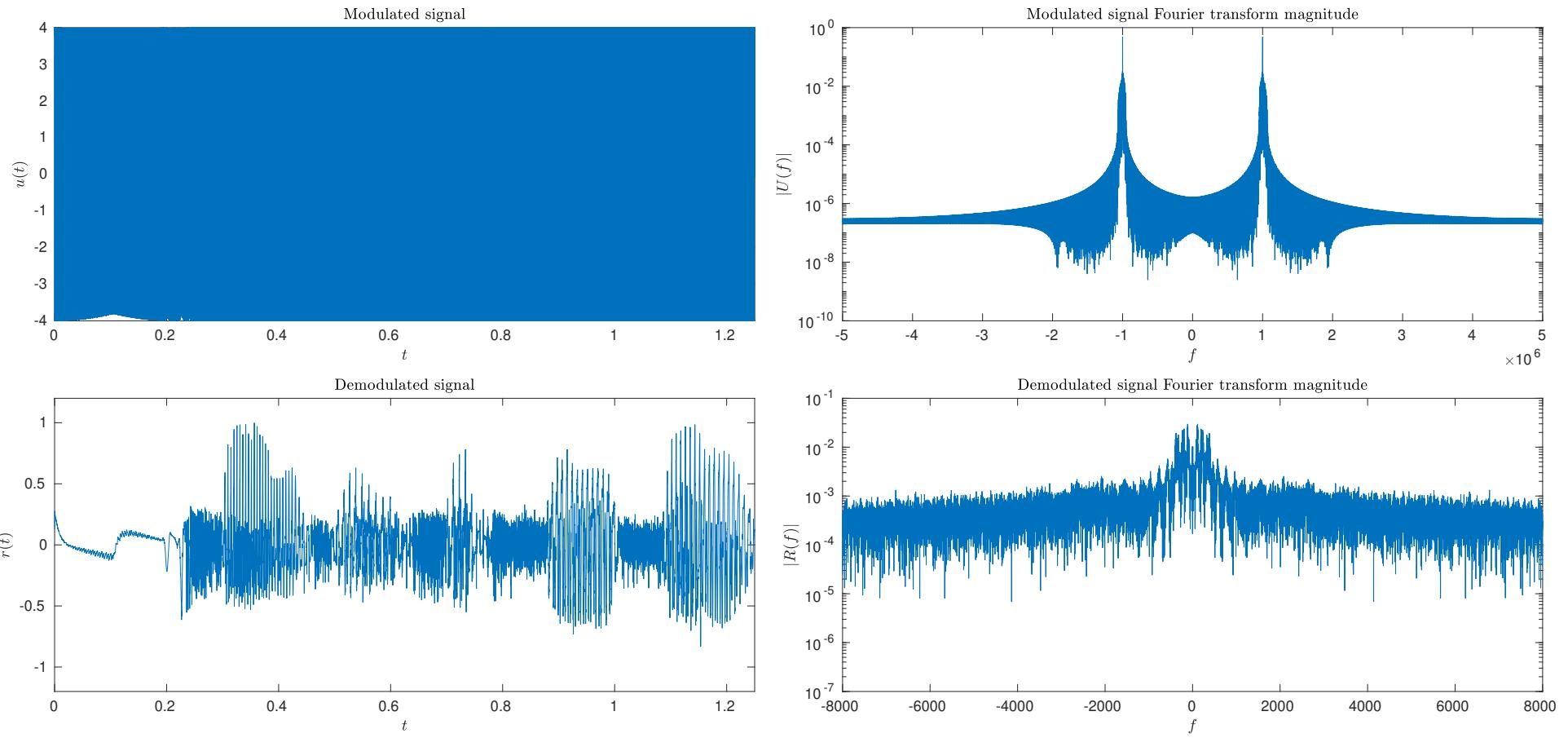


Figure 16: Increased distortion with higher frequencies with FM ( $f_c = 1\text{MHz}$ ,  $f_{s,sc} = 10\text{MHz}$ )

## 4 Discussion

### 4.1 The message signal

The original message signal was scaled to have unity maximum amplitude, and it has a bandwidth (calculated using `obw`) of 6605Hz. Figure 1 is a plot of this signal and its Fourier transform.

### 4.2 Verification of the modulation schemes

The first order of business was to verify that each modulation and demodulation scheme works. The parameters for each of the modulation schemes is shown in Table 2. The modulated and demodulated signals were plotted in the time and frequency domains in Figures 2 (conventional AM), 3 (SSB AM), 4 (PM), and 5 (FM).

Also, the total transmitted power of the modulated signals, and the power of the demodulated signals, for each scheme are given in Table 3. It is clear from this table that the values match the theoretical values. (However, this doesn't mean much in the case of the demodulated conventional AM and demodulated FM cases, because there was an empirically-determined scalar multiplier in the demodulator used to restore the original amplitude.)

The outputs from each scheme are clearly audibly recognizable. Conventional AM and FM both have a very slight staticky sound, which is probably due to rectification noise.

#### 4.2.1 Conventional AM

It is clear from the modulated signal plotted in the time domain that  $A_c = 4$ , and the message signal is clearly visible as the envelope of the modulated signal. In the Fourier transform of the modulated signal, we can see that there are two spikes at the carrier frequency as expected. The demodulated signal looks comparable to the original signal, as well as its Fourier transform; the only difference is that the DC offset is explicitly zero (from the demodulation function), so there is a (nonconsequential) negative spike at  $f = 0$ .

In addition, Figure 6 is a plot of the Fourier transform of the modulated signal shown zoomed horizontally and centered horizontally at  $f = f_c$  and compared to the Fourier transform of the baseband message signal. This clearly demonstrates the function of mixing with a pure tone at the carrier frequency.

#### 4.2.2 SSB AM

Similar to conventional AM, the envelope of the message signal is clearly visible in the carrier. The Fourier transform clearly shows a difference between the lower and upper sidebands, as expected. The output signal looks similar to the message signal in both time and frequency domain.

Similarly to conventional AM, Figure 7 is a plot of the Fourier transform of the modulate signal shown zoomed horizontally and centered horizontally at  $f = f_c$  and compared to the Fourier transform of the baseband message signal. It is clear that all of the power is stored in the lower side band, as expected.

#### 4.2.3 PM

As expected, the modulated signal is hard to see: the amplitude is more or less constant, with only a small visible distortion. The Fourier transform of the modulated signal shows narrow peaks at

the carrier frequency; they aren't quite as narrow as in the AM cases, but since  $\beta_p = k_p = 0.1$  is small, they are fairly narrow. The demodulated output looks as expected.

#### 4.2.4 FM

Much of the same arguments as PM apply. The main difference is that the Fourier transform of the modulated signal looks messier; this is because  $\beta_p = 10$  is larger, so more bandwidth is used in this case.

### 4.3 Verifying Carson's rule for angle modulation

To do this, I simply overlaid a plot of the Carson bandwidth (shaded in green) on top of the plots of the modulated PM and FM signals in the frequency domain in Figure 8. These were calculated using the Carson bandwidth formula:

$$W_{carson} = 2(\beta + 1)W$$

It is visually apparent that most of the power lies in these bounds; to further verify this, I used the following snippet to experimentally calculate the amount of power in both Carson bandwidths (this is for PM, the result for FM is analogous):

```

1 | bandpower(sig_pm_modulated, f_s_c_pm, ...
2 |     [f_c_pm - (beta_p + 1) * W], f_c_pm + (beta_p + 1) * W) ...
3 |     / bandpower(sig_pm_modulated)

```

As shown in Table 4, both calculations give approximately 100% of the power is in the Carson bandwidth.

### 4.4 Effect of varying noise variance on SNR

First, to review the SNR results from 2:

$$\begin{aligned} SNR_{conv} &= \frac{a^2 A_c^2}{2} \frac{P_m}{N_0 W} \\ SNR_{ssb} &= A_c^2 \frac{P_m}{N_0 W} \\ SNR_{pm} &= \frac{A_c^2}{2} \left( \frac{\beta_p}{\max |m(t)|} \right)^2 \frac{P_m}{N_0 W} \\ SNR_{fm} &= \frac{3A_c^2}{2} \left( \frac{\beta_f}{\max |m(t)|} \right)^2 \frac{P_m}{N_0 W} \end{aligned}$$

A few things to note:

- As shown in 10, the noise power  $N_0 W$  in all of these analog domain equations should be replaced with  $N_0 W/f_s$  for use in the digital (sampled) domain.
- In the case of a fixed message signal,  $P_m$  and  $W$  are fixed.

- $a$ ,  $A_c$ ,  $\beta_p$ ,  $\beta_f$ , and  $(1/\sqrt{N_0} = \frac{1}{\sigma\sqrt{2}})$  are all related to the SNR by a quadratic relationship. Thus, changing any of these should have the same result on the SNR (albeit different results on power, bandwidth, clipping, etc.).

Three standard deviations were chosen (arbitrarily) for the noise process: 0.1, 0.5, and 1. The overall results for this section are summarized in Table 5. Plots for the modulation schemes can be found at Figures 9 (conventional AM), 10 (SSB AM), 11 (PM), 12 (FM).

It is visually evident from the plots that increasing the variance of the noise decreases the SNR, in all cases. For conventional AM, SSB AM, and PM, the experimental and the theoretical SNRs agreed fairly well.

Due to the nature of the parameter setup for conventional and SSB AM, which were set up so that the signal power in the modulated signal was the same, incidentally the calculated SNRs were also exactly the same. Experimentally, the SNRs between these two schemes turned out pretty close.

For the AM schemes, the theoretical SNR is systematically higher than the experimental SNR. Dan Kim suggested that dividing the theoretical SNR by a factor of 2 (providing a uniform  $\approx -3$ dB shift) seems to make the experimental and theoretical results significantly closer. While the error doesn't seem to be a simple constant shift, the fact that there's an approximately constant decibel shift means that either the experimental noise power is systematically high or the experimental signal power is systematically low. I was debugging by probing the noise and signal powers at several points through the demodulation function, and found that the signal power was always more or less correct, so I suspect that the noise power was incorrect. I'm not sure where this error comes from, or if it's a constant, but here are a few possibilities:

- inherent quantization error
- rectification noise (in the case of conventional AM)
- half-wave rectification, which cuts the signal power in half (in the case of conventional AM)
- the LPF or HPF may cause the retained noise power to be disproportionately higher (w.r.t. signal power) than before filtering

The FM scheme experimental SNR was far from the theoretical value. Theoretically, the modulation index is large and the SNR should be very high. My guess is that there are many sources of numerical and other inherent error, e.g.:

- inherent quantization error
- numerical integration using `cumsum` during modulation
- numerical differentiation using `diff` during demodulation
- half-wave rectification noise during demodulation
- low-pass and high-pass filtering may cause distortions in the ratio of retained noise power to signal power

## 4.5 Effect of varying modulation index on SNR

The same notes from the previous section apply: decreasing  $\sigma$  by a factor of 2 or increasing modulation factor by a factor of 2 (ignoring clipping or other distortion) should cause the same change in SNR. In our specific case, having  $\sigma = 0.5$ ,  $a = 0.5$  in the conventional AM case would have the same SNR as  $\sigma = 1$ ,  $a = 1$ . Indeed, the theoretical calculation produces the same number: 15.00 (with the 2 correction factor).

The overall results for this section are summarized in Table 6. Plots for the modulation schemes can be found at Figures 13 (conventional AM), 14 (PM), and 15 (FM).

Much of the analysis is exactly the same as in the previous section: the conventional AM, SSB AM, and PM had good agreement between experimental and theoretical SNRs. There was a similar systematic offset between the theoretical and experimental values that could be improved by adding the factor of 2.

## 4.6 Effect of higher carrier and sampling frequencies

I originally had a carrier frequency of 1MHz and a sampling frequency of 10MHz, but this caused a lot of distortion, which can be seen in Figure 16. It seems that higher frequencies can cause more errors with quantization errors than it might help by improving resolution. However, for the other modulation methods, I didn't have this problem. Therefore, my FM carrier and sampling frequencies are fairly low.

## 5 Conclusions

A reasonable implementation of multiple analog modulation schemes – conventional AM, SSB AM, FM, and PM – were implemented and simulated in MATLAB. Their operations were verified by a number of methods, including listening to the demodulated signal by ear, examining the modulated signal, checking the power of the modulated and demodulated signals, and comparing the demodulated signal to the original message signal.

These modulation and demodulation functions were used to empirically verify some of the theory. For example, the Carson bandwidth was loosely verified by a visual check and an estimated calculation. The effect of varying the variance of the AWGN noise, and of varying the modulation index, on the SNR was also explored and compared to theoretical values. Most of the calculated SNR values were fairly close to the theoretical values (i.e., most of the errors were less than 10dB); however, the empirical SNRs were systematically lower than the theoretical values. Most of the values were fairly close, but unfortunately I was unable to find a way to make the experimental SNR values for FM match the theoretical values.

These errors may be due to some combination of implementation errors, quantization/rounding errors, and inherent noise in the systems (especially in filtering, rectification, and numerical integration/differentiation stages). More time and effort would be needed to investigate these errors. Despite these errors, the general trends in SNR as a function of the changes in input parameters were correct.

## 6 Acknowledgments

I would like to acknowledge Prof. Brian Frost for answering my (and my peers') numerous late-night questions, hosting extra office hours, and extending the deadline in order to accommodate my peers and I for this project.

I would also like to acknowledge Anthony Belladonna, Dan Kim, Joshua Yoon, Thodoris Kapouranis, Paul Cucchiara, Philip Blumin, Derek Lee, Steven Lee, and other members of a particular Discord channel for discussing many aspects of the project and staying up with me multiple late nights to work on this project.

I would like to acknowledge Dan Kim for giving the suggestion that multiplying by two in the denominator of the conventional AM SNR equation, which gives closer estimates to the experimental values.

I would like to acknowledge Derek Lee for the following illustrious illustration:

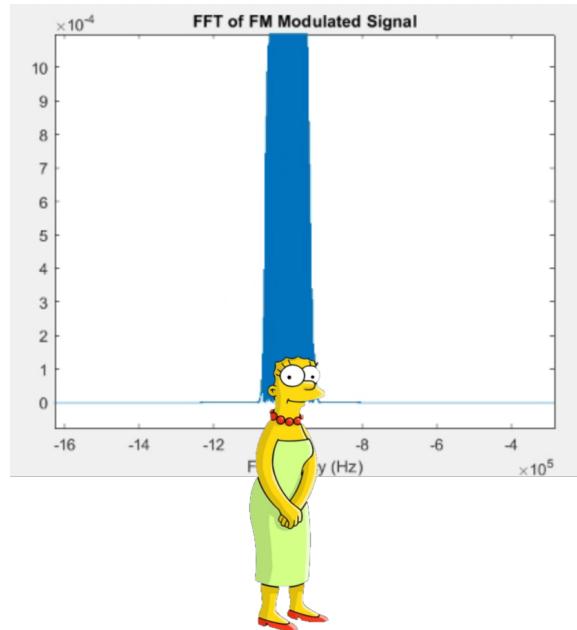


Figure 17: Untitled. Commissioned by Anthony Belladonna.

## 7 References

- [1] "Radio Broadcast Signals." <http://hyperphysics.phy-astr.gsu.edu/hbase/Audio/radio.html>.
- [2] Proakis, J. G., Salehi, M. (2001). *Communication Systems Engineering*. Upper Saddle River, NJ, USA: Prentice-Hall. ISBN: 0130617938
- [3] "Sample Audio Files." <https://www.opdsupport.com/downloads/miscellaneous/sample-audio-files>.

## 8 Appendix I: Audio

The original audio message signal used throughout this report is a speech sample downloaded from Olympus Professional Dictation Support [3]. The audio is truncated to reduce memory and processor strain. The message is also normalized so that its maximum amplitude is 1, which makes calculating modulation indices simpler.

Sampling rate	16kHz
Bandwidth	6.61kHz
Samples	20000 (after truncation)
Duration	1.25s (after truncation)
Signal power	0.0522
$\max  m(t) $	1

Table 7: Audio message signal statistics

```
1 %% Load audio and preprocess
2
3 % audio details
4 audio_filename = 'welcome.wav';
5
6 % truncate to limit memory usage and processing time
7 audio_samples = 20000;
8
9 % loads the message and the sampling rate of the message
10 [sig_m, f_s_m] = audioread(audio_filename);
11 sig_m = sig_m(1:audio_samples)';
12
13 % make maximum magnitude 1 to avoid problems with conventional AM
14 sig_m = sig_m / max(abs(sig_m));
15
16 % get bandwidth
17 W = obw(sig_m, f_s_m);
18
19 % get signal power
20 P_m = bandpower(sig_m);
```

Listing 9: Loading and preprocessing the audio signal

## 9 Appendix II: Auxiliary functions

### 9.1 Estimating signal power in MATLAB

In MATLAB, we only have discrete-time signals, with  $N$  discrete samples, sampling frequency  $F_s$ , and duration  $T = N/F_s$ . We can estimate the power of a signal numerically using the root-mean-squared function (squared) `rms` or `bandpower` functions, where `rms(signal)^2=bandpower(signal)`. To show the equivalence of this mean-squared function to the power of a discrete signal:

$$\begin{aligned}\text{rms}^2\{x[n]\} &= \frac{1}{N} \sum x^2[n] = \frac{1}{TF_s} \sum x^2[n] = \frac{1}{T} \sum x^2[n] \frac{1}{F_s} \\ &= \frac{1}{T} \sum x^2[n] \delta t \approx \frac{1}{T} \int x^2(t) dt = P_x\end{aligned}\quad (39)$$

Thus, the `bandpower` function was used throughout the program to estimate signal power.

### 9.2 Hilbert transform

The Hilbert transform is the LTI, norm-preserving magic sauce behind SSB. In layman's terms, this turns the negative part of the baseband signal into the quadrature part of the signal, which allows for packing the same signal as DSB AM into half of the bandwidth.

```
1 % hilbert transform of a signal; copied from earlier psets; for SSB AM
2 % params:
3 % sig_x = input signal
4 % returns:
5 % res    = hilbert transform of sig_x
6 function res = hilbert_transform(sig_x)
7     ft_x = fft(sig_x);
8     len_x = length(sig_x);
9
10    % get signum of frequency; +1 for 0 to pi, -1 for -pi to 0
11    sgn = [ones(1, floor(len_x/2)), -1*ones(1, ceil(len_x/2))];
12    res = ifft(-1j * sgn .* ft_x);
13 end
```

Listing 10: Hilbert transform function

### 9.3 Lowpass signal

This function is used to lowpass signals by multiplying by the frequency response of a first-order LPF. Used for filtering out noise above the audible range, but can be used with any cutoff frequency.

```

1 % lowpass: set tau to upper range of audible range
2 function sig_c = lowpass_audible(sig, f_s, tau)
3     wd = linspace(-pi, pi, length(sig));
4     f = wd * f_s / (2 * pi);
5     ft = fftshift(fft(sig));
6
7     % first-order LPF with cutoff frequency at tau
8     lpf = (1 + f/tau*1j).^-1;
9     ft = lpf .* ft;
10
11    sig_c = real(ifft(ifftshift(ft)));
12 end

```

Listing 11: Lowpass function

## 9.4 Fourier transform plotter

This function provides the frequency axis and Fourier transform of a signal.

```

1 % helper function to plot the fourier transform of a signal
2 % params:
3 % x      = signal
4 % f_s    = sampling rate
5 % returns:
6 % f      = frequency axis
7 % ft     = fourier transform
8 function [f, ft] = plot_ft(sig, f_s)
9     wd = linspace(-pi, pi, length(sig));
10    f = wd * f_s / (2 * pi);
11    ft = abs(fftshift(fft(sig)) / f_s);
12 end

```

Listing 12: Fourier transform plotting function

## 9.5 Saving figures to PDFs

This function saves figures to PDFs, with arbitrary page dimensions.

```

1 % helper function for printing to PDFs
2 function fig_save(fig_title, paper_size)
3     global pset_name;
4
5     % remove this later
6     pset_name = 'ece300_proj1';
7
8     set(gcf(), ...
9         'Units', 'centimeters', ...
10        'Position', [0 0 paper_size], ...
11        'InnerPosition', [0 0 paper_size], ...
12        'OuterPosition', [0 0 paper_size], ...
13        'PaperPositionMode', 'auto');
14     exportgraphics(gcf(), ...
15                    sprintf('%s_fig_%s.eps', pset_name, fig_title), ...
16                    'ContentType', 'vector');
17 end

```

Listing 13: Figure saver helper function

## 10 Appendix III: Calculating noise power in the digital domain

An AWGN random process  $N(t)$  with variance  $\sigma^2$  has the following autocorrelation function:

$$R_{N,dig}[n] = \sigma^2 \delta[n] \quad (40)$$

By the Weiner-Khinchin theorem, the PSD (in digital radians) of the AWGN noise is given by the DFT of the autocorrelation function:

$$S_{N,dig}(\omega) = \mathcal{F}\{R_N[n]\}(\omega) = \frac{\sigma^2}{2\pi} \quad (41)$$

The total power of the noise process before filtering is the integral of the PSD over its domain:

$$P_{N,dig} = \int_{-\pi}^{\pi} \frac{\sigma^2}{2\pi} d\omega = \sigma^2 \quad (42)$$

This can be confirmed empirically by finding the bandpower of an arbitrary AWGN noise signal. E.g., when  $\sigma^2 = 4$  `bandpower(2 * randn(1, 1000000))` outputs 4. When we want to find the power of the noise in the baseband (demodulated) signal, then we assume a perfect filter with digital cutoff frequency  $\omega_{cut}$  (i.e., this would be set to the bandwidth of the message signal). Then we have a rectangular pulse PSD centered at  $\omega = 0$ , with height  $\sigma^2/(2\pi)$  and width  $2\omega_{cut}$ . The noise power at baseband is the integral (area) under this rectangle, i.e.:

$$P_{N,bb,dig} = \int_{-\pi}^{\pi} (\text{rect}_{\{-\omega_{cut}, \omega_{cut}\}})(S_N(\omega)) d\omega = \frac{\sigma^2}{2\pi} (2\omega_{cut}) \quad (43)$$

Making the conventional substitution for the noise power,  $N_0 = 2\sigma^2$ :

$$P_{N,bb,dig} = \frac{N_0}{2\pi} \omega_{cut} = \frac{N_0}{2\pi} \frac{2\pi f_{cut}}{f_s} = N_0 \frac{f_{cut}}{f_s} \quad (44)$$

where  $f_s$  is the sampling frequency. The final expression here is the most intuitive: it is the product of the noise power with the ratio of the cutoff frequency to the sampling frequency. This contrasts with the analog case, which was covered in class:

$$R_{N,ana}(t) = \sigma^2 \delta(n) \quad (45)$$

$$S_{N,ana}(f) = \sigma^2 \quad (46)$$

$$P_{N,ana} = \int_{-\infty}^{\infty} S_{N,ana}(f) df = \infty \quad (47)$$

$$P_{N,bb,ana} = \int_{-f_{cut}}^{f_{cut}} S_{N,ana}(f) df = N_0 f_{cut} \quad (48)$$

(If analog angular frequency was used instead of analog linear frequency, then there would also be the additional factor of  $1/(2\pi)$ .) Both cases involve clipping a flat noise PSD to that within the bandwidth specified by the cutoff frequency.

**In summary: a noise power (i.e., in SNR equations) of  $N_0 f_{cut}$  (or, equivalently,  $N_0 W$ ) in the analog domain should be replaced with  $N_0 f_{cut}/f_s = N_0 \omega_{cut}/(2\pi)$ .**

Note that most of the demodulation methods involve a LPF at the upsampled rate (i.e., the sampling rate used to sample the modulated signal), and the signal is passed through an additional LPF at the downsampled rate (i.e., the sampling rate used for the baseband audio) to filter out inaudible high-frequency noise. Note that downsampling changes the magnitude of the PSD.

## 11 Appendix IV: Source code

The core functions have already been defined. Here is the rest of the code used to produce the plots and results in this document.

```

1 close all; clear; clc;
2 set(0, 'defaultTextInterpreter', 'latex');
3
4 % Project details
5 global pset_name;
6 pset_name = 'ece300_proj1';
7
8 % audio details
9 audio_filename = 'welcome.wav';
10 audio_samples = 20000;
11
12 % Load audio
13
14 % loads y, Fs
15 [sig_m, f_s_m] = audioread(audio_filename);
16 sig_m = sig_m(1:audio_samples)';
17
18 % make maximum magnitude 1 to avoid problems with conventional AM
19 sig_m = sig_m / max(abs(sig_m));
20
21 % plot audio
22 figure('visible', 'off');
23 tiledlayout(1, 2, 'TileSpacing', 'Compact', 'Padding', 'Compact');
24
25 duration = length(sig_m) / f_s_m;
26 t_m = linspace(0, duration, length(sig_m));
27
28 nexttile();
29 plot(t_m, sig_m);
30 title('Original signal');
31 ylabel('$\mathbf{S}(t)$');
32 xlabel('$\mathbf{t}$');
33 xlim([0, duration]);
34 ylim([-1.2, 1.2]);
35
36 [f, ft] = plot_ft(sig_m, f_s_m);
37 nexttile();
38 semilogy(f, ft);
39 title('Magnitude of original signal Fourier transform');
40 ylabel('$|\mathbf{S}(f)|$');
41 xlabel('$\mathbf{f}$');
42 ylim([10e-8, 10e-2]);
43
44 fig_save('sig_m', [80 20]);
45
46 % get bandwidth
47 W = obw(sig_m, f_s_m);
48
49 % get signal power
50 P_m = bandpower(sig_m);
51
52 % write original signal to file
53 audiowrite('original.wav', sig_m, f_s_m);
54
55 % Modulation configuration
56
57 % conventional AM
58 a_conv = 0.5;
59 A_c_conv = 4;
60 f_c_conv = 500000;
61 f_s_c_conv = 2000000;
62
63 % SSB AM
64 f_c_ss = 500000;
65 A_c_ss = A_c_conv / sqrt(8);
66 f_s_c_ss = 2000000;
67 is_ussb = false;
68
69 % PM
70 f_c_pm = 500000;
71
72 A_c_pm = A_c_conv;
73 f_s_c_pm = 5000000;
74 k_p = 0.1;
75
76 % FM
77 f_c_fm = 400000;
78 A_c_fm = A_c_conv;
79 f_s_c_fm = 5000000;
80 k_f = 10 * W;
81
82 % for when viewing the effect of varying noise std.
83 noise_stds = [0 0.1 0.5 1];
84
85 % for when viewing the effect of varying modulation index
86 noise_std_conv = 1;
87 noise_std_fm = 0.5;
88 noise_std_pm = 1;
89
90 % for storing the theoretical snrs
91 var_noise_theo = zeros(4,4);
92 var_noise_emp = zeros(4,4);
93 var_mod_ind_theo = zeros(3,3);
94 var_mod_ind_emp = zeros(3, 3);
95
96 % conventional AM modulation
97 sig_conv_modded = conv_mod(f_c_conv, A_c_conv, a_conv, ...
98 sig_m, f_s_m, f_s_c_conv);
99 sig_conv_demodded = conv_demod(sig_conv_modded, A_c_conv, ...
100 a_conv, f_s_m, f_s_c_conv, W);
101 plot_modded_demodded(sig_conv_modded, sig_conv_demodded, ...
102 f_s_m, f_s_c_conv, 'conv');
103 audiowrite('conv_demodded.wav', sig_conv_demodded, f_s_m);
104
105 % SSB AM modulation
106 sig_ss_modded = ssb_mod(f_c_ss, A_c_ss, sig_m, f_s_m, f_s_c_ss, ...
107 is_ussb);
108 sig_ss_demodded = ssb_demod(sig_ss_modded, f_c_ss, A_c_ss, ...
109 f_s_m, f_s_c_ss, W);
110 plot_modded_demodded(sig_ss_modded, sig_ss_demodded, f_s_m, ...
111 f_s_c_ss, 'ssb');
112 audiowrite('ssb_demodded.wav', sig_ss_demodded, f_s_m);
113
114 % PM Modulation
115 sig_pm_modded = pm_mod(f_c_pm, A_c_pm, sig_m, k_p, f_s_m, f_s_c_pm, ...
116 f_s_m, f_s_c_pm, W);
117 plot_modded_demodded(sig_pm_modded, sig_pm_demodded, f_s_m, ...
118 f_s_c_pm, 'pm');
119 audiowrite('sig_pm_demodded.wav', sig_pm_demodded, f_s_m);
120
121 % FM Modulation
122 sig_fm_modded = fm_mod(f_c_fm, A_c_fm, sig_m, k_f, f_s_m, f_s_c_fm);
123 sig_fm_demodded = fm_demod(sig_fm_modded, A_c_fm, f_s_m, f_s_c_fm, ...
124 k_f, W);
125 plot_modded_demodded(sig_fm_modded, sig_fm_demodded, f_s_m, ...
126 f_s_c_fm, 'fm');
127 audiowrite('sig_fm_demodded.wav', sig_fm_demodded, f_s_m);
128
129 % Conventional AM with noise
130 demod_fn = @(sig_mod) conv_demod(sig_mod, A_c_conv, a_conv, f_s_m, ...
131 f_s_c_conv, W);
132 [sig_conv_modded, sig_conv_demodded, snrs] = apply_noise(noise_stds, ...
133 demod_fn, sig_conv_modded, sig_conv_demodded, f_s_m, ...
134 f_s_c_conv, W, 'conv');
135 var_noise_emp(1,:) = snrs;
136
137 % calculate conventional AM theoretical SNR
138 var_noise_theo(1,:) = 10 * log10(a_conv^2 * A_c_conv^2 * ...
139 bandpower(sig_m) * (2 * (2 * noise_stds.^2) * W / f_s_c_conv).^(-1));
140

```

```

141 %% SSB AM with noise
142 demod_fn = @(sig_mod) ssb_demod(sig_mod, f_c_ssbb, A_c_ssbb, f_s_m, ...
143     f_s_c_ssbb, W);
144 [sigs_ssbb_modded, sigs_ssbb_demodded, snrs] = apply_noise(noise_stds, ...
145     demod_fn, sig_ssbb_modded, sig_ssbb_demodded, f_s_m, f_s_c_ssbb, ...
146     W, 'ssb');
147 var_noise_emp(2,:) = snrs;
148
149 %% calculate SSB AM theoretical SNR
150 var_noise_theo(2,:) = 10 * log10(A_c_ssbb^2 * bandpower(sig_m) * ...
151     ((2 * noise_stds.^2) * W / f_s_c_ssbb).^.-1);
152
153 %% PM with noise
154 demod_fn = @(sig_mod) pm_demod(f_c_pm, A_c_pm, sig_mod, k_p, f_s_m, ...
155     f_s_c_pm, W);
156 [sigs_pm_modded, sigs_pm_demodded, snrs] = apply_noise(noise_stds, ...
157     demod_fn, sig_pm_modded, sig_pm_demodded, f_s_m, f_s_c_pm, W, 'pm');
158 var_noise_emp(3,:) = snrs;
159
160 %% calculate PM theoretical SNR
161 var_noise_theo(3,:) = 10 * log10(k_p.^2 * A_c_pm.^2 * bandpower(sig_m) * ...
162     (2 * (2 * noise_stds.^2) * W / f_s_c_pm).^.-1);
163
164 %% FM with noise
165 demod_fn = @(sig_mod) fm_demod(sig_mod, A_c_fm, f_s_m, f_s_c_fm, ...
166     k_f, W);
167 [sigs_fm_modded, sigs_fm_demodded, snrs] = apply_noise(noise_stds, ...
168     demod_fn, sig_fm_modded, sig_fm_demodded, f_s_m, f_s_c_fm, W, 'fm');
169 var_noise_emp(4,:) = snrs;
170
171 %% calculate FM theoretical SNR
172 beta_f = k_f / W;
173 var_noise_theo(4,:) = 10 * log10(3 * beta_f.^2 * A_c_fm.^2 * ...
174     bandpower(sig_m) * (2 * (2 * noise_stds.^2) * W / f_s_c_fm).^.-1);
175
176 %% Changing modulation index of Conventional AM
177 mod_fn = @(sig_m, a) conv_mod(f_c_conv, A_c_conv, a, sig_m, f_s_m, ...
178     f_s_c_conv);
179 demod_fn = @(sig_mod, a) conv_demod(sig_mod, A_c_conv, a, f_s_m, ...
180     f_s_c_conv, W);
181 var_mod_ind_emp(1,:) = apply_modulation_factor(...
182     a.conv, noise_std_conv, mod_fn, demod_fn, sig_m, ...
183     sigs_conv_modded(4,:), sigs_conv_demodded(4,:), f_s_m, ...
184     f_s_c_conv, W, sig_conv_demodded, 'a', 'conv');
185
186 %% calculate conventional AM theoretical SNR
187 var_mod_ind_theo(1,:) = 10 * log10([a_conv/2 a_conv a_conv*2].^2 * ...
188     A_c_conv.^2 * bandpower(sig_m) * (2 * (2 * noise_std_conv) * W / ...
189     f_s_c_conv).^.-1);
190
191 %% Changing the modulation index of PM
192 mod_fn = @(sig_m, k_p) pm_mod(f_c_pm, A_c_pm, sig_m, k_p, f_s_m, f_s_c_pm);
193 demod_fn = @(sig_mod, k_p) pm_demod(f_c_pm, A_c_pm, sig_mod, k_p, ...
194     f_s_m, f_s_c_pm, W);
195 var_mod_ind_emp(2,:) = apply_modulation_factor(...
196     k_p, noise_std_pm, mod_fn, demod_fn, sig_m, sigs_pm_modded(4,:), ...
197     sigs_pm_demodded(4,:), f_s_m, f_s_c_pm, W, sig_pm_demodded, ...
198     'k_p', 'pm');
199
200 %% calculate PM theoretical SNR
201 var_mod_ind_theo(2,:) = 10 * log10([k_p/2 k_p k_p*2].^2 * A_c_pm.^2 * ...
202     bandpower(sig_m) * (2 * (2 * noise_std_pm.^2) * W / f_s_c_pm).^.-1);
203
204 %% Changing the modulation index of FM
205 mod_fn = @(sig_m, k_f) fm_mod(f_c_fm, A_c_fm, sig_m, k_f, ...
206     f_s_m, f_s_c_fm);
207 demod_fn = @(sig_mod, k_f) fm_demod(sig_mod, A_c_fm, f_s_m, ...
208     f_s_c_fm, k_f, W);
209 var_mod_ind_emp(3,:) = apply_modulation_factor(...
210     k_f, noise_std_fm, mod_fn, demod_fn, sig_m, sigs_fm_modded(3,:), ...
211     sigs_fm_demodded(3,:), f_s_m, f_s_c_fm, W, sig_fm_demodded, ...
212     'k_f', 'fm');
213
214 %% calculate FM theoretical SNR
215 beta_f = [k_f/2 k_f k_f*2] / W;
216 var_mod_ind_theo(3,:) = 10 * log10(3 * beta_f.^2 * A_c_fm.^2 * ...
217     bandpower(sig_m) * (2 * (2 * noise_std_fm.^2) * W / f_s_c_fm).^.-1);
218
219 %% Print out results
220 fprintf('Theoretical SNRs from varying noise variance');
221 var_noise_theo(:,2:4)
222 fprintf('Empirical SNRs from varying noise variance');
223 var_noise_emp(:,2:4)
224 fprintf('Theoretical SNRs from varying modulation indices');
225 var_mod_ind_theo
226 fprintf('Empirical SNRs from varying modulation indices');
227 var_mod_ind_emp
228
229 %% plot modulation modded and demodded signals and their FTs
230 %% params:
231 %% sig_modded = modulated signal
232 %% sig_demodded = demodulated signal
233
234 %% f_s_m = baseband sampling frequency
235 %% f_s_c = modulated signal sampling frequency
236 %% fname = name of figure
237
238 function plot_modded_demodded(sig_modded, sig_demodded, f_s_m, f_s_c, ...
239     fname)
240
241 figure('visible', 'off');
242 tiledlayout(2, 2, 'TileSpacing', 'Compact', 'Padding', 'Compact');
243
244 duration = length(sig_modded) / f_s_c;
245 t_c = linspace(0, duration, length(sig_modded));
246 t_m = linspace(0, duration, length(sig_demodded));
247
248 %% plot modded signal in time domain
249 nexttile();
250 plot(t_c, sig_modded);
251 title('Modulated signal');
252 ylabel('$u(t)$');
253 xlabel('$t$');
254 xlim([0 duration]);
255
256 %% plot modded signal in frequency domain
257 nexttile();
258 [f, ft] = plot_ft(sig_modded, f_s_c);
259 semilogy(f, ft);
260 title('Modulated signal Fourier transform magnitude');
261 ylabel('$|U(f)|$');
262 xlabel('$f$');
263
264 %% plot demodded signal in time domain
265 nexttile();
266 plot(t_m, sig_demodded);
267 title('Demodulated signal');
268 ylabel('$r(t)$');
269 xlabel('$t$');
270 ylim([-1.2 1.2]);
271 xlim([0 duration]);
272
273 %% plot demodded signal in frequency domain
274 nexttile();
275 [f, ft] = plot_ft(sig_demodded, f_s_m);
276 semilogy(f, ft);
277 title('Demodulated signal Fourier transform magnitude');
278 ylabel('$|R(f)|$');
279 xlabel('$f$');
280
281 fig_save(fname, [40 20]);
282 end
283
284 %% apply noise to signals modulated with different modulation indices
285 %% params:
286 %% mod_index = modulation index values
287 %% noise_std = standard deviation of noise to add
288 %% mod_fn = lambda to modulate a signal
289 %% demod_fn = lambda to demod a noisy modulated signal
290 %% sig_m = message signal
291 %% sig_modded = reference modded signal
292 %% sig_demodded = reference demodded signal
293 %% f_s_m = baseband sampling frequency
294 %% f_s_c = modulated signal sampling frequency
295 %% tau = post-demodulation LPF cutoff frequency
296 %% sig_demodded_no_noise = reference no-noise demodulated signal
297 %% variant = name of modulation index
298 %% mod_type = modulation scheme
299 %% returns:
300 %% snrs = SNRs of noisy modulated signals
301
302 function snrs = apply_modulation_factor(...
303     mod_index, noise_std, mod_fn, demod_fn, sig_m, sig_modded, ...
304     sig_demodded, f_s_m, f_s_c, tau, sig_demodded_no_noise, variant, ...
305     mod_type)
306
307     sig_modded = zeros(3, length(sig_modded));
308     sigs_modded = zeros(3, length(sig_modded));
309
310     %% half the modulation index
311     sigs_modded(1,:) = mod_fn(sig_m, mod_index/2) + ...
312         noise_std * randn(1, length(sig_modded));
313     sigs_modded(2,:) = lowpass_audible(mod_fn(sigs_modded(1,:), ...
314         mod_index/2), f_s_m, tau);
315
316     %% original modulation index
317     sigs_modded(2,:) = sig_modded;
318     sigs_modded(3,:) = sig_demodded;
319
320     %% double the modulation index
321     sigs_modded(3,:) = mod_fn(sig_m, mod_index*2) + ...
322         noise_std * randn(1, length(sig_modded));
323     sigs_modded(3,:) = lowpass_audible(mod_fn(sigs_modded(3,:), ...
324         mod_index*2), f_s_m, tau);
325
326 %% plot

```

```

325 snrs = plot_with_noise([mod_index/2, mod_index, mod_index*2], ...      386 % demodded signal for a series of related signals with one varying
326     sigs_modded, sigs_demodded, f_s_m, f_s_c, ...                      387 % variable; also calculates SNRs and returns them
327     sig_demodded_no_noise, variant, mod_type);                           388 %
328 end
329 %
330 % apply different noise to a modded signal
331 %
332 % noise_stds = list of standard deviations of the noise
333 % demod_fn = lambda to demod a noisy signal
334 % sig_modded_no_noise = no-noise modded signal reference
335 % sig_demodded_no_noise = no-noise demodded signal reference
336 % f_s_m = baseband sampling frequency
337 % f_s_c = modulated signal sampling frequency
338 % tau = cutoff frequency
339 % mod_type = modulation scheme
340 %
341 % returns:
342 % sigs_modded = modulated signals with applied noise
343 % sigs_demodded = demodulated noisy modded signals
344 % snrs = snrs for demodded signals
345 function [sigs_modded, sigs_demodded, snrs] = apply_noise(noise_stds, ... 405
346     demod_fn, sig_modded_no_noise, sig_demodded_no_noise, f_s_m, f_s_c, ... 406
347     tau, mod_type)                                                       407
348
349     sigs_modded = zeros(length(noise_stds), ...                          408
350         length(sig_modded_no_noise));                                     409
351     sigs_modded = zeros(length(noise_stds), ...                          410
352         length(sig_modded_no_noise));                                     411
353 %
354 % first element will be the no-noise variant
355 sigs_modded(1,:) = sig_modded_no_noise;
356 sigs_demodded(1,:) = sig_demodded_no_noise;
357 %
358 % loop through each noise process
359 for i = 2:length(noise_stds)
360     %
361     % modulate, add noise
362     sigs_modded(i,:) = sig_modded_no_noise + ...
363         noise_stds(i) * randn(1, length(sig_modded_no_noise));
364
365     %
366     % demod
367     sigs_demodded(i,:) = lowpass_audible(...                         421
368         demod_fn(sigs_modded(i,:)), f_s_m, tau);                      422
369
370     %
371     % plot
372     snrs = plot_with_noise(noise_stds, sigs_modded, sigs_demodded, ... 430
373         f_s_m, f_s_c, sig_demodded_no_noise, '\sigma_n', mod_type);    431
374 end
375 %
376 % approximates SNR in dB, given the pristine demodded signal and a noisy
377 % variant; approximates noise as the difference between the two
378 %
379 % sig_no_noise = demodded signal with no noise
380 % sig_noisy = noisy signal to calculate the SNR of
381 % returns:
382 % res = SNR (in dB)
383 function res = snr(sig_no_noise, sig_noisy)                            440
384     res = 10 * log10(bandpower(sig_no_noise) / ...                     441
385         bandpower(sig_no_noise - sig_noisy));                           442
386 end
387 %
388 % plots the demodded signal, spectrum of modded signal, and spectrum of
389 % demodded signal for a series of related signals with one varying
390 % variable; also calculates SNRs and returns them
391 %
392 % variants = list of varying parameter values
393 % sigs_modded = modded signals
394 % f_s_m = sample rate of baseband signal
395 % f_s_c = sample rate of modulated signal
396 % sig_snr_ref = reference for SNR calculation (i.e., no-noise demodded)
397 % variant = name of varying parameter
398 % mod_type = modulation type (e.g., CONV, FM)
399 % returns:
400 % snrs = calculated SNRs of each demodded signal wrt. sig_snr_ref
401 function [snrs] = plot_with_noise(variants, sigs_modded, sigs_demodded, ... 405
402     f_s_m, f_s_c, sig_snr_ref, variant, mod_type)                       406
403
404 N = length(variants);
405 snrs = zeros(N, 1);
406
407 figure('visible', 'off');
408 tiledlayout(N, 3, 'TileSpacing', 'Compact', 'Padding', 'Compact');
409
410 for i = 1:N
411
412     % plot demodded signal
413     nexttile();
414     plot(sigs_modded(i,:));
415     ylim([-1.25, 1.25]);
416     snr_db = snr(sig_snr_ref, sigs_demodded(i,:));
417     snrs(i) = snr_db;
418     title(sprintf('Demodded signal; $%s=%0.2f$', SNR=%0.2dB', ...
419         variant, variants(i), snr_db));
420     ylabel('$\sigma(t)$');
421     xlabel('$t$');
422
423     % plot spectrum of modded signal
424     nexttile();
425     [f, ft] = plot_ft(sigs_modded(i,:), f_s_c);
426     semilogy(f, ft);
427     title(sprintf('Spectrum of modded signal; $%s=%0.2f$', ...
428         variant, variants(i)));
429     ylabel('$M(f)$');
430     xlabel('$f$');
431
432     % plot spectrum of demodded signal
433     nexttile();
434     [f, ft] = plot_ft(sigs_demodded(i,:), f_s_m);
435     semilogy(f, ft);
436     ylim([10e-10, 10e0]);
437     title(sprintf('Spectrum of demodded signal; $%s=%0.2f$', ...
438         variant, variants(i)));
439     ylabel('$M(f)$');
440     xlabel('$f$');
441
442 if strcmp(variant, '\sigma_n')
443     variant = 'noise_std';
444 end
445 fig_save(sprintf('%s_var.%s', mod_type, variant), [60, 10 * N]);
446 end

```