ECE211 – Pset 6

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1. *Use the method of partial fractions (residue approach) to find the (right-sided) inverse z-transform of*

$$H(z) = \frac{3z^3+2}{(3z-1)(4z+3)}$$

$$z^{-1}H(z) = \frac{3z^3+2}{12z(z-1/3)(z+3/4)} = A + \frac{B}{z} + \frac{C}{z-1/3} + \frac{D}{z+3/4}$$

$$\begin{aligned}
A & &= \frac{1}{4} \\
B &= \frac{2}{12(-1/3)(3/4)} &= -\frac{2}{3} \\
C &= \frac{3(1/3)^3+2}{12(1/3)(1/3+3/4)} &= \frac{19}{39} \\
D &= \frac{3(-3/4)^3+2}{12(-3/4)(-3/4-1/3)} &= \frac{47}{624}
\end{aligned}$$

$$H(z) = \frac{z}{4} - \frac{2}{3} + \frac{19}{39}\frac{z}{z-1/3} + \frac{47}{624}\frac{z}{z+3/4}$$

$$h[n] = \frac{1}{4}\delta[n+1] - \frac{2}{3}\delta[n] + \frac{19}{39}\left(\frac{1}{3}\right)^n u[n] + \frac{47}{624}\left(-\frac{3}{4}\right)^n u[n]$$

2. *A causal discrete-time signal $x[n]$ has the following poles: 0.4 with multiplicity 3, and $0.3\exp \pm j4\pi/5$, each with multiplicity 2. Assume the underlying sampling rate is 10kHz.*

   (a) *Write a general expression for $x[n]$.*

$$\begin{aligned}
x[n] =& (a_1n^2 + a_2n + a_3)(0.4)^n u[n] \\
&+ (a_4n + a_5)(0.3)^n e^{j4\pi n/5}u[n] + (a_6n + a_7)(0.3)^n e^{-j4\pi n/5}u[n] \\
=& (a_1n^2 + a_2n + a_3)(0.4)^n u[n] + (a_8n + a_9)(0.3)^n \cos\left(\frac{4\pi n}{5} + \theta\right) u[n]
\end{aligned}$$

(b) *For each mode, specify the time constant in seconds and, if there is an oscillation, the frequency of the oscillation in Hertz.*

For the pole at 0.4:

$$\tau = \frac{T}{|\ln |p||} = \frac{10000^{-1}\text{s}}{|\ln 0.4|} = 1.09 \times 10^{-4} s$$

For the pole at $0.3\exp(4\pi/5)$:

$$\tau = \frac{10000^{-1}\text{s}}{|\ln 0.3|} = 8.31 \times 10^{-5} s$$

And this pole oscillates at:

$$f = \frac{\omega_d}{2\pi} \cdot f_s = \frac{4\pi}{5}\frac{1}{2\pi}(10\text{kHz}) = 4\text{kHz}$$

3. *The input to a digital filter $H(z)$ is:*

$$x[n] = 3u[n] + 4(-0.5)^n u[n]$$

The output is

$$y[n] = 6u[n]+5(-0.5)^n u[n]+(6n+3)(0.5)^n u[n]+0.4(0.2)^n \cos(n\pi/3+\pi/4)$$

(a) *Identify the natural and forced responses in y.*

Natural:

$$(6n + 3)(0.5)^n u[n] + 0.4(0.2)^n \cos(n\pi/3 + \pi/4)$$

Forced:

$$6u[n] + 5(-0.5)^n u[n]$$

(b) *Specify the system poles, with multiplicity, assuming no pole-zero cancellation with x.*

- 0.5 (double pole)
- $0.2\exp(\pm j\pi/3)$ (simple poles)

4. *An analog signal is given by:*

$$x(t) = 3\exp(-2t)u(t) + e^{-4t}(t\cos(3t) - \sin(3t))u(t)$$

Specify the poles of x with multiplicity. Also, the above contains two modes; for each mode, specify the time constant in seconds, and if there is oscillation the frequency in Hertz.

Pole at -2 have multiplicity 1
Poles at $-4 \pm j3$: have multiplicity 2 (because of t factor)

Mode for pole at -2: $\tau = 0.5$s, no oscillation
Mode for poles at $-4 \pm j3$: $\tau = 0.25$s, oscillating with frequency $\frac{3}{2\pi}$Hz

5. *The response of an analog filter $H(s)$ with input x is y. x has poles at $s = 0, -1, -4$, all simple. y has poles at $s = 0, -1, -2 \pm j3$ (simple), and $s = -3$ (double).*

   (a) *The fact that the output has no pole at -4 tells you something about $H(s)$. What?*

   There is a pole-zero cancellation with the filter. In particular, $H(s)$ has a zero at 4.

   (b) *Of the output poles, which must be the poles of $H(s)$ (and with what multiplicity)?*

   These would be all of the poles that don't come from the input. In particular: $s = -2 \pm j3$ (single) and $s = -3$ (double). With pole-zero cancellations, it's possible that these multiplicities are higher.

   (c) *At first glance the system appears to be stable – why? However, it turns out we are missing some information, that makes it possible for the system to be unstable! Explain what might be happening that would "disguise" the fact that the system is unstable?*

   It would appear stable because all of the poles of the system (from the previous question) are in the LHP. However, it is possible that there is a pole-zero cancellation (with the zero from the input and the pole from the output) that is hiding a pole of the transfer function in the RHP (or on the $j\omega$ axis).