Jonathan Lam

Circuit Analysis Exam 1

Professor: Brian Frost

Fall 2019

1 Theoretical Questions (3 points each)

1. What can be said about two voltage sources in parallel with one another?
2. What can be said of two current sources in series with one another?
3. Draw a circuit which mesh analysis would fail to solve.
4. The passive sign convention, which we use in class, determines the sign of power across acircuit element. The active sign convention makes the exact opposite choice (all power values under the active sign convention are negatives of those under the passive sign convention). Describe, in words, the active sign convention without mentioning the passive sign convention.
5. With respect to the active sign convention, write an inequality which defines a passive element(including an integral). Is a resistor still passive?
6. Consider a resistor with resistance $R < 0$ in parallel with a voltage source of voltage $V > 0$. Does the resistor generate or dissipate power?
7. Consider a circuit with a complex resistive network, and a second circuit where this networkis replaced by one equivalent resistor. Which of the following are necessarily the same across the network and the single resistor in their respective circuits: Voltage, current, power?
8. Figure 1 shows a branch in a circuit. Write its branch constituent equation such that thebranch current is in terms of the branch voltage.
9. Write a brief argument using only Kirchoff's laws for the equivalent conductance formula for$N$ conductors in parallel.
10. Write an argument for the equivalent resistance formula for N resistors in parallel, either by arguing from your above answer or by deriving it from Kirchoff's laws.
11. Spike failed to pay attention in circuits class, and is now at a loss when building his newestinvention. He has access to only a 5V battery, but wishes to apply 2.5V to a load resistor with resistance 250Ω. Spike can use two resistors to provide this voltage to the load, and he wants to make it as precise as possible. Draw the circuit to "divide" Spike's voltage, and write a small note about the size of resistors Spike should choose.

2 Computational Questions (33 points total)

12. (11 points) Consider the circuit in Figure 2. Using mesh analysis, write a matrix equation which will solve the circuit upon solution.

13. (6 points) Consider the circuit in Figure 2. Count the number of nodes, meshes and branches.

14. (11 points) Consider the circuit in Figure 3. Using modified nodal analysis, write a matrix equation which will solve the circuit upon solution.

15. (5 points) Consider the resistive network in Figure 4. Find the equivalent resistance of the network assuming all resistors have the same value R.

3 Expository Example (34 points total)

16. (2 points) This question will introduce you to the concept of voltage gain. In a circuit which is interpreted as having a voltage input and a voltage output, the voltage gain is defined as V_{OUT}/V_{IN}. For Spike's circuit in question 11, what is the desired voltage gain?

17. (4 points) With only resistors, what is the possible range of voltage gains one can achieve? Briefly explain.

18. (2 points) Theoretically, can one achieve voltage gains outside of this range using only passive elements? Hint: Does the integral definition of passive preclude this?.

19. (4 points) For the circuit in Figure 5, what is the voltage gain?

20. (2 point) There are elements other than resistors in this device – is it still limited to the range determined in question 17?

21. (1 point) The effective voltage gain of Spike's circuit is changed by the addition of a load. If you add a resistive load at the output of the circuit in Figure 5, does the gain change?

22. (3 points) This is related to a concept called output impedance. Considering the input to a circuit as an open circuit, one can compute the equivalent resistance of the network across the output terminals. In Spike's circuit, the output impedance is simply the resistance of the resistor across the output terminals. When a load is added, it is effectively placed in parallel with the output impedance of the circuit. For a load to have minimal effect on the gain, should the output impedance be small or large relative to the load?

23. (4 points) Inspired by your above answers, what do you think the output impedance of a dependent voltage source is? More specifically, what is the output impedance of the circuit in Figure 5.

24. (5 points) What is the voltage gain of the circuit in Figure 6?

25. (2 points) Is this circuit impacted by a load?

26. (2 points) What is this circuit's output impedance?

27. (1 points) What does the sign of gain represent?

28. (2 points) In reality, do you think we can build circuits with gains that are not impacted by loads? Explain.

Figures

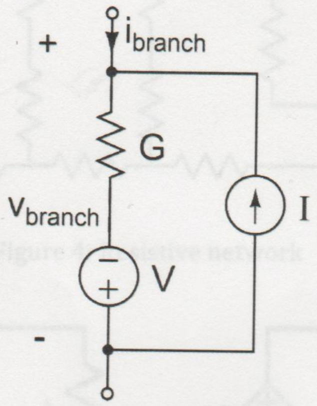


Figure 1: A given branch

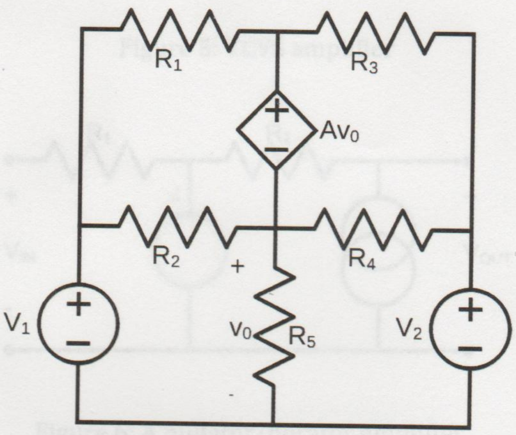


Figure 2: Using Mesh Analyisis

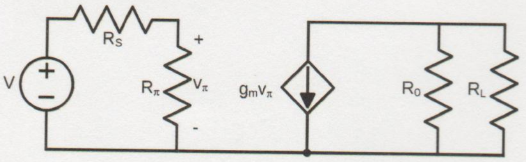


Figure 3: Using MNA

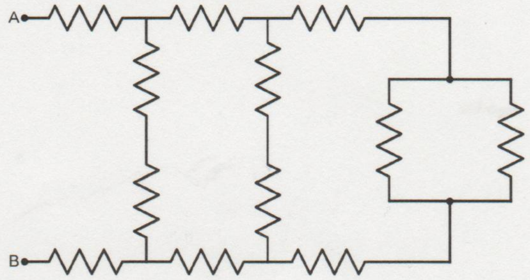


Figure 4: Resistive network

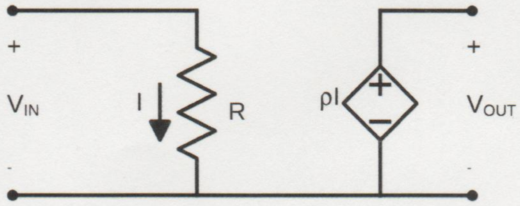


Figure 5: VCVS amplifier

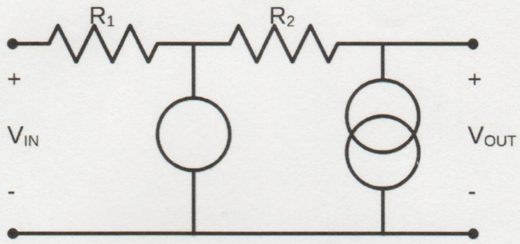
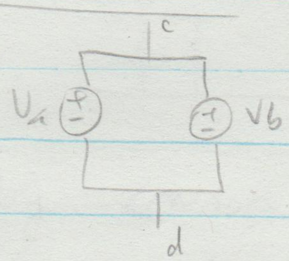


Figure 6: A nullator/norator amplifier

Theoretical Q5.

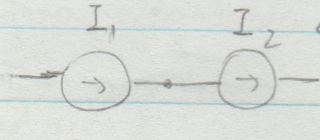
(1)



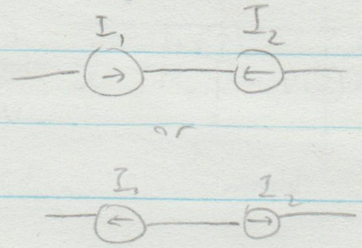
This can only exist if $V_a = V_b$, since the BCEs are:

$V_b = V_{cd} = V_a$.

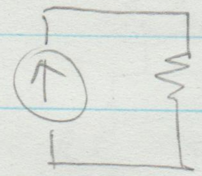
(2)



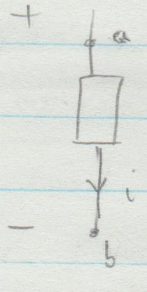
Since current is equal for 2 circuit elements in series, only possible if $I_1 = I_2$ (if facing same direction) or $I_1 = -I_2$ (if facing opposite directions)



(3)



(4)



If, in the diagram on the left, we specify $P = iV_{ab}$ as negative then the active sign convention is in use. ie, this convention means a drop in voltage (and therefore power dissipated) is negative power.

perfect. first great answer I've seen for this problem cannot generate power, so $P \not> 0$. Thus

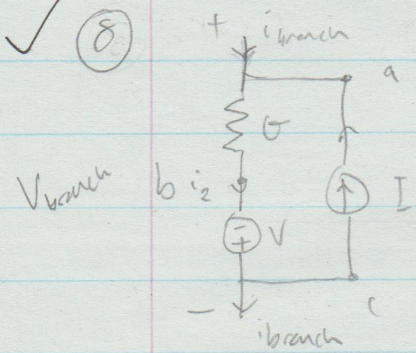
⑤. $\int_0^T P\,dt \le 0$ (i.e., reverse sign from passive sign convention)

Yes, a resistor is still passive

⑥. It generates power (and is thus active)

⑦. Between the start and end node of the more complex resistive circuit, the power, voltage, and current would all be equal to the single resistor

⑧.



node a: $i_{branch} + I = i_2$

for conductor: $i_2 = (V_a - V_b)G$

voltage source: $V = V_c - V_b$

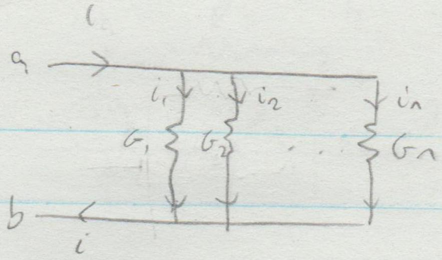
branch voltage: $V_{branch} = V_a - V_c$

Solve for i_{branch} in terms of V_{branch}

$$i_{branch} = i_2 - I$$
$$= (V_a - V_b)G - I$$
$$= ((V_{branch} + V_c) - V_b)G - I$$
$$= (V_{branch} + (V_c - V_b))G - I$$

$$i_{branch} = (V_{branch} + V)G - I$$

(9)



by KCL at node a, $i = i_1 + i_2 + \ldots + i_n$
(same at node b)

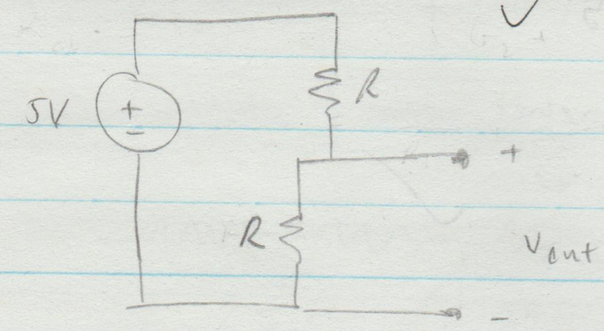
since $i_1 = V_{ab} G_1$ (by BCE of conductor),

$i = V_{ab}G_1 + V_{ab}G_2 + \ldots + V_{ab} G_n$

divide both sides by V_{ab}

$$\frac{i}{V_{ab}} = G_1 + G_2 + \ldots + G_n = G_{eq} \quad \left(\text{since } G = \frac{i}{V}\right)$$

(10) $$R_{eq} = \frac{1}{G_{eq}} = \frac{1}{G_1 + G_2 + \ldots + G_n} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}}$$



(resistor values are equal)

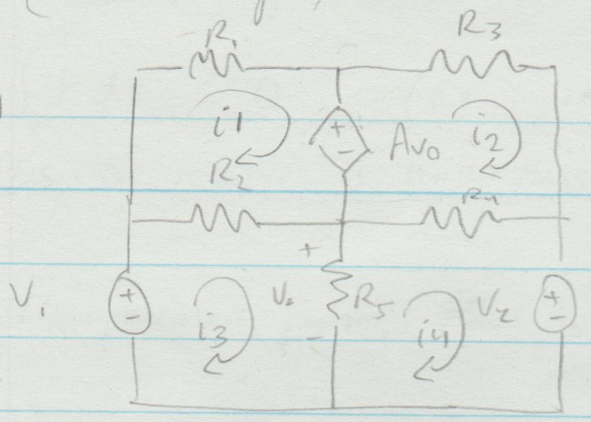
If spike connects his circuit to the +/- pins of Vout, then he can power his circuit with roughly 2.5 V.

Note that to be more accurate means choosing smaller Rs (i.e., R should be much smaller than resistive load), but choosing too small an R could mean a very high current.

PERFECT
theoretical

(mesh analysis)

(12)



$V_0 = (i_3 - i_4) R_5$

mesh 1: $R_1 i_1 + A(i_3 - i_4)R_5 + (i_1 - i_3)R_2 = 0$

" 2: $R_3 i_2 + (i_2 - i_4)R_4 - A(i_3 - i_4)R_5$

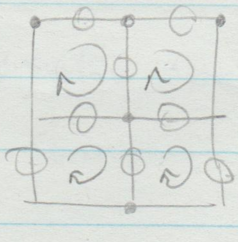
" 3: $(i_3 - i_1)R_2 + (i_3 - i_4)R_5 - V_1 = 0$

" 4: $(i_4 - i_2)R_4 + V_2 + (i_4 - i_3)R_5 = 0.$

$$\begin{pmatrix} R_1+R_2 & 0 & AR_5 - R_2 & -AR_5 \\ 0 & R_3+R_4 & -AR_5 & -R_4 + AR_5 \\ -R_2 & 0 & R_2+R_5 & -R_5 \\ 0 & -R_4 & -R_5 & R_4+R_5 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ V_1 \\ -V_2 \end{pmatrix}$$

-0.5

(13)



8 branches

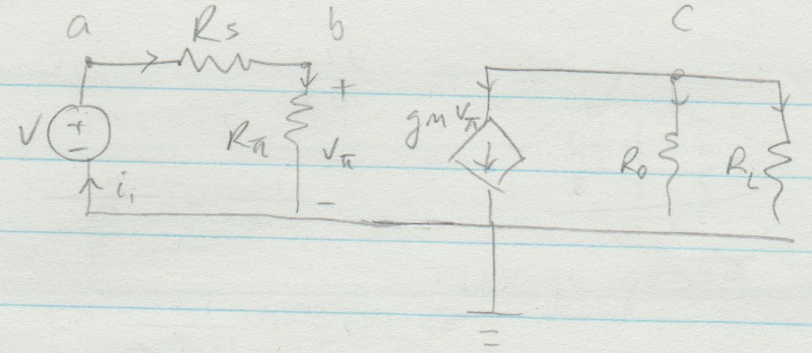
4 meshes

5 nodes

$\boxed{-0.5}$

$\boxed{.5}$

14



$v_\pi = v_b - v_{gnd} = v_b$

a: $i_1 = (v_a - v_b)\, G_s$

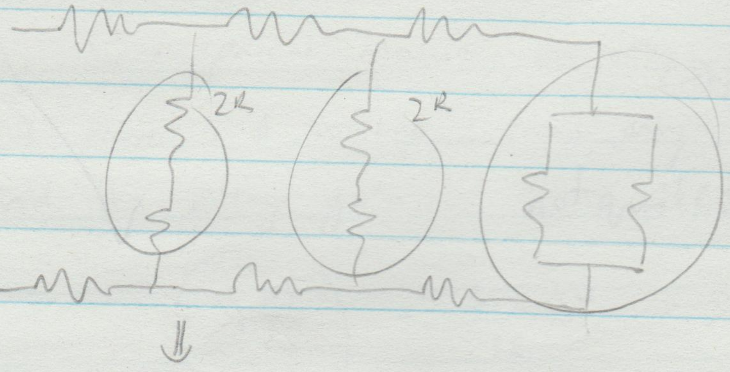
b: $(v_a - v_b)\, G_s = v_b\, G_\pi$

c: $g_m (v_b) + v_c G_o + v_c G_L = 0$

$V = v_a$

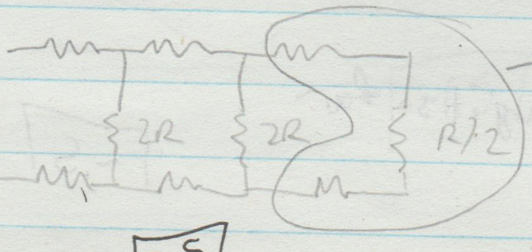
$$\begin{pmatrix} -G_s & G_s & 0 & 1 \\ G_s & -G_s - G_\pi & 0 & 0 \\ 0 & g_m & G_o + G_L & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_a \\ v_b \\ v_c \\ i_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ V \end{pmatrix}$$

15



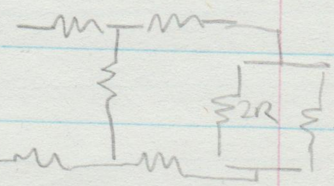
$\frac{R}{2}$

$\Downarrow$



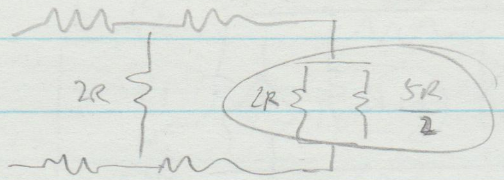
$\rightarrow \frac{5R}{2}$

$\rightarrow$



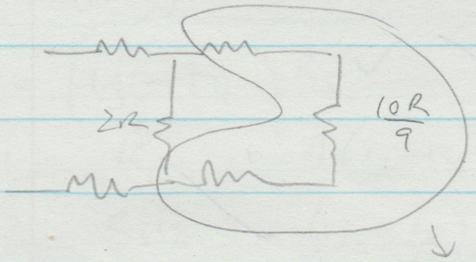
$\frac{5R}{2}$

$\boxed{-.5}$



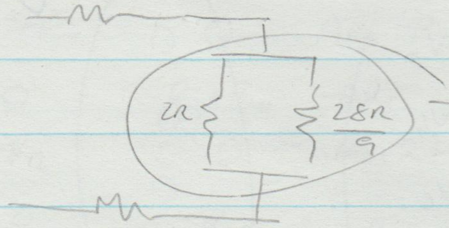
$$\rightarrow \frac{1}{\frac{1}{2R}+\frac{1}{\frac{5R}{2}}} = \frac{1}{\frac{1}{2R}+\frac{2}{5R}} = \frac{1}{\frac{5}{10R}+\frac{4}{10R}} = \frac{1}{\frac{9}{10R}} = \frac{10R}{9}$$

$\Downarrow$



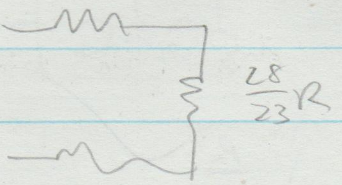
$\frac{28R}{9}$

$\Downarrow$



$$\rightarrow R_{eq} = \frac{(2R)\left(\frac{28R}{9}\right)}{2R+\frac{28R}{9}} = \frac{\frac{56R^2}{\cancel{9}}}{\frac{46R}{\cancel{9}}} = \frac{56}{46} = \frac{28}{23}$$

$\Downarrow$



$\Downarrow$

$$\frac{2(23)+28}{23}R = \frac{46+28}{23} = \frac{74}{23}R$$

✓

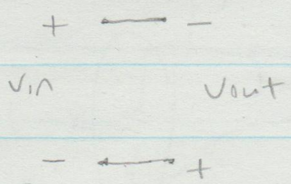
Killing it so far

$\boxed{-.5}$

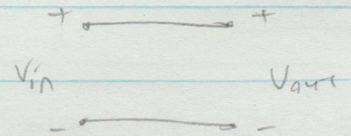
[-8]

(16.) $\frac{V_{out}}{V_{in}} = \frac{2.5V}{5V} = \frac{1}{2}$ ✓

✓ (17)



Voltage gain $= \frac{-V_{in}}{V_{in}} = -1$.



Voltage gain $= \frac{V_{in}}{V_{in}} = 1$.

It is not possible to create energy (and thus a larger voltage potential) using resistors (they only drop voltage) so largest voltage gain is 1 (where output is equal to input voltage). Conversely, since this is the largest possible magnitude for a voltage gain, flipping Vout gives the largest negative voltage gain of −1. Thus the range of voltage gains is [−1, 1]. Any value in between can be achieved with a voltage divider like in question 11, and a voltage gain of 0 can be created by connecting V_{out}^{+} to V_{out}^{-} with a S.C.

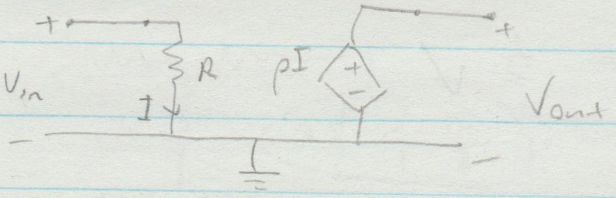
−1 (18) No, since passive elements can only drop (the magnitude of) power and voltage, thus only decreasing Vin and the voltage gain.

not quite! V↗ i↘ ?

[-1.5]

[-1.5]

(19)



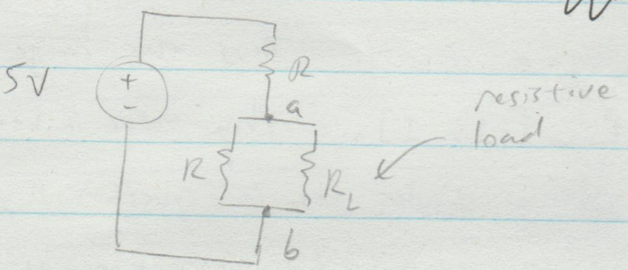
$$I = \frac{V_{in}}{R}, \quad \rho I = \rho\left(\frac{V_{in}}{R}\right) = V_{out}$$

$$\Rightarrow \quad \frac{V_{out}}{V_{in}} = \frac{\rho}{R} \quad \text{is the voltage gain}$$

(20) No, since there is an active element (the voltage source, which is able to supply voltage and thus increase Vout by an indeterminate amount, allowing voltage gain to exceed [-1, 1].

-.5 (21) Yes.

Wrong circuit! Right answer for this one though



$$R_{eq} = R + \frac{RR_L}{R+R_L}$$

$$i = \frac{V}{R_{eq}} = \frac{V}{R + \frac{RR_L}{R+R_L}}$$

$$V_{ab} = iR_{RR_L} = \left(\frac{V}{R+\frac{RR_L}{R+R_L}}\right)\left(\frac{RR_L}{R+R_L}\right) = \frac{V\ (RR_L)}{R(R+R_L)+RR_L}$$

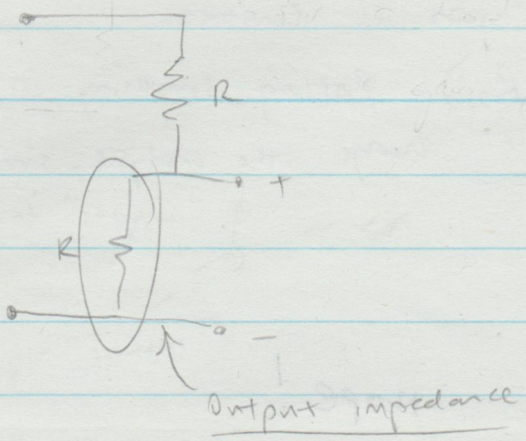
[-2]

$$= V\left(\frac{RR_L}{R^2+2RR_L}\right) < V\left(\frac{1}{2}\right), \text{ since}$$

$RR_L < \frac{1}{2}(R^2+2RR_L)$. Therefore Vout is lower than $\frac{1}{2}$, so the gain drops.

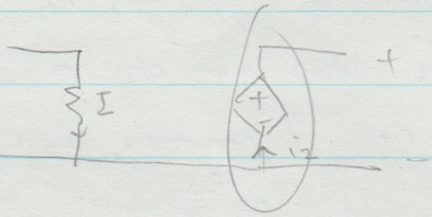
[-2]

✓ (22)



Similar to HW #12, if the output impedance (R) is small, then the voltage gain remains close to the desired value. Therefore the output impedance should be small relative to the load.

-1 (23)



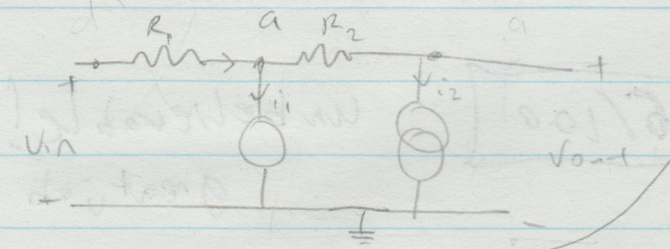
equivalent resistance across a dependent voltage source is

$R = \frac{V}{i_2} = \frac{\rho I}{i_2}$ by Ohm's Law i_2 arbitrary. It's zero

"output impedance" here

✓ (24)

yes!



$(V_{in} - V_a) G_1 = (V_a - V_{out}) G_2$

$V_a = 0$

$(V_a - V_{out}) G_2 = i_2$

$\frac{G_1}{G_2}(V_{in} - V_a) = (V_a - V_{out}) \Rightarrow V_{out} = 0 - \frac{G_1}{G_2}(V_{in} - 0) = -\frac{G_1}{G_2}$

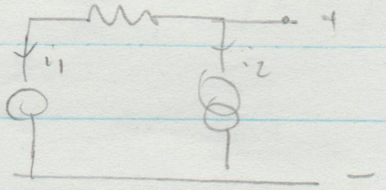
[-3]

$\Rightarrow$ voltage gain $= \dfrac{-\frac{G_1}{G_2} V_{in}}{V_{in}} = \dfrac{-G_1}{G_2} = \dfrac{-R_2}{R_1}$

$\boxed{-3}$

✓ (25) No, since its voltage gain is constant (dependent only on R_1, R_2). Putting something in parallel to V_{out} wouldn't change the analysis, since the norator "adapts" to the new resistance

✓1 (26)



nope!

0 again

$i_1 = 0, \quad i_2 = -V_{out} G_1$

$R = \frac{V}{i} = \frac{V_{out}}{-V_{out}G_1} = -\frac{1}{G_1}$

↑ by ohm's Law

✓ (27) Sign of gain represents whether output +/- is oriented same way as input +/-, i.e., if V_{in} and V_{out} have same sign, then sign of gain is positive; if 0, then $V_{out} = 0$; if negative, then V_{out} has opposite sign of V_{in}

✓ (28) I don't think so. A load always changes the constraints of a real system (e.g. equivalent resistance) and thus should change the circuit's parameters.

$\boxed{-4}$

raw score: $\boxed{96/100}$ unbelievable! great job