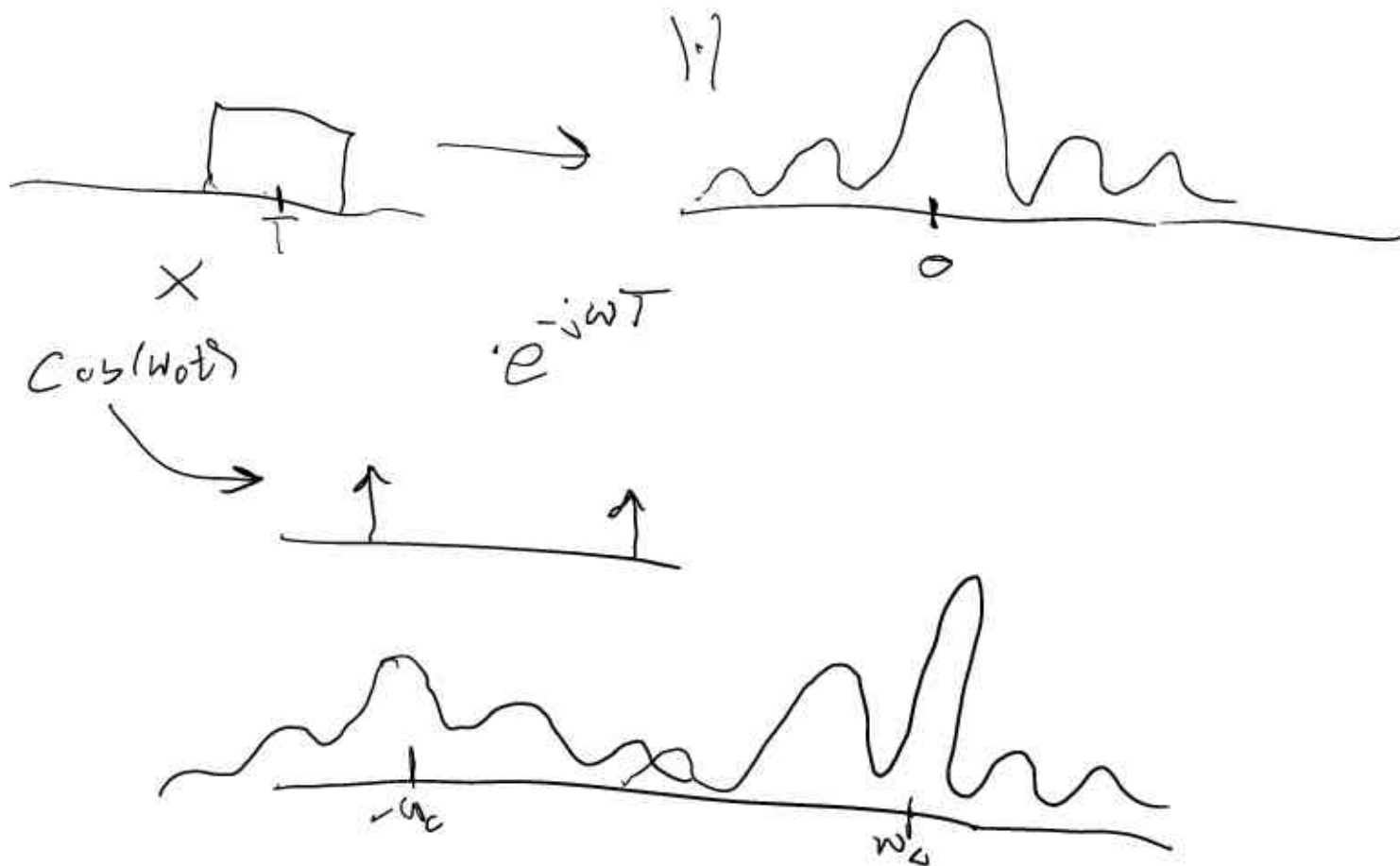


HW REVIEW

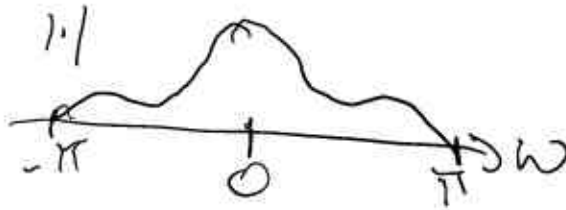
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FFT = fast FT

↳ DFT = Discrete Fourier Transform

$$\text{DTFT: } \sum_n x[n] e^{-j\omega n} \in \mathcal{F}(\mathbb{R}, \mathbb{C})$$



$\sim \sin(x) \rightarrow$ rep. as vector

DFT = sampled DTFT

$x(t) \rightarrow x[n]$ Matlab vector approx $x(t)$

$$\begin{aligned} X(\Omega) &= \int x(t) e^{-j\Omega t} dt \approx \sum x[n] e^{-j\omega n} \Delta t \\ &= \Delta t X(\omega) \\ &\approx \Delta t \text{DFT}\{x\} \end{aligned}$$

$$\approx \underbrace{\Delta t D}_{\text{FT}} \{X\}$$

$$\Delta t = 1/f_s$$

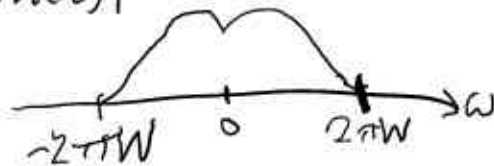
AM

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Setting

Some signal $m(t)$ baseband, $m(t) \in \mathbb{R}$

$|M(\omega)|$



assume finite bandwidth
(BWS, W)

Hermitian

Ex. m audio, then $W \approx 15 \text{ kHz}$

- Not at RF, so can't transmit across channel
- I want many channels, but a bunch of these would interfere

Modulation THM we want to "lift" the audio signal
to a higher freq.

Carrier Signal $c(t) = A_c \cos(\omega_c t)$, $\omega_c = 2\pi f_c$

$m(t)c(t)$? maybe...

DSB-SC AM

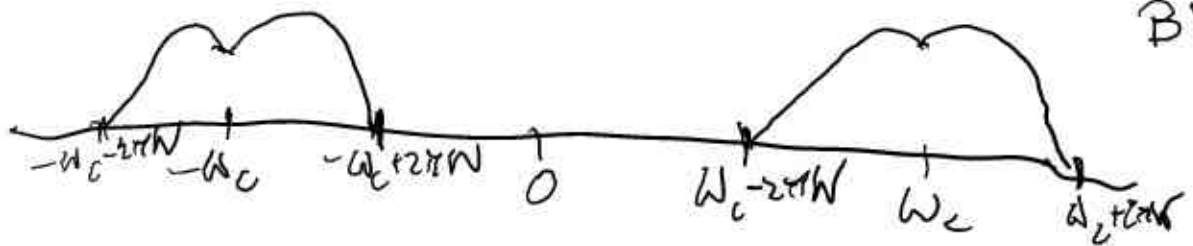
Tuesday, September 8, 2020 6:27 PM

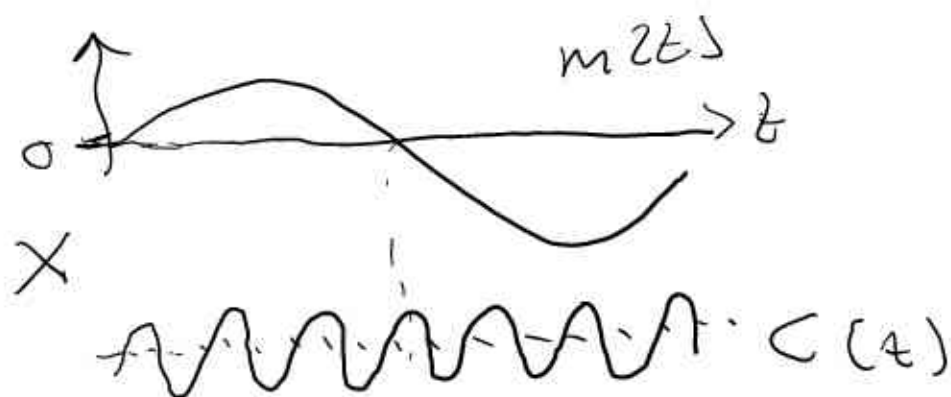
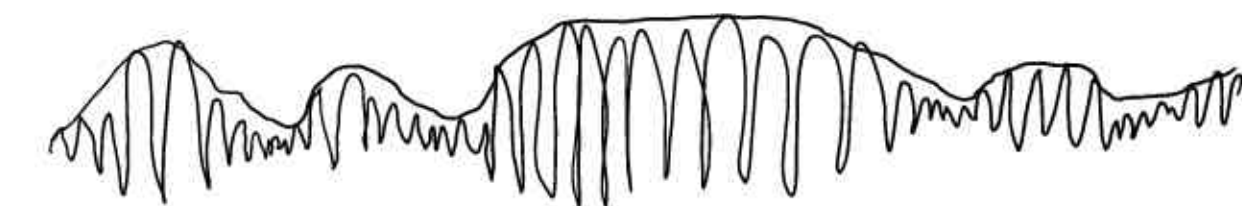
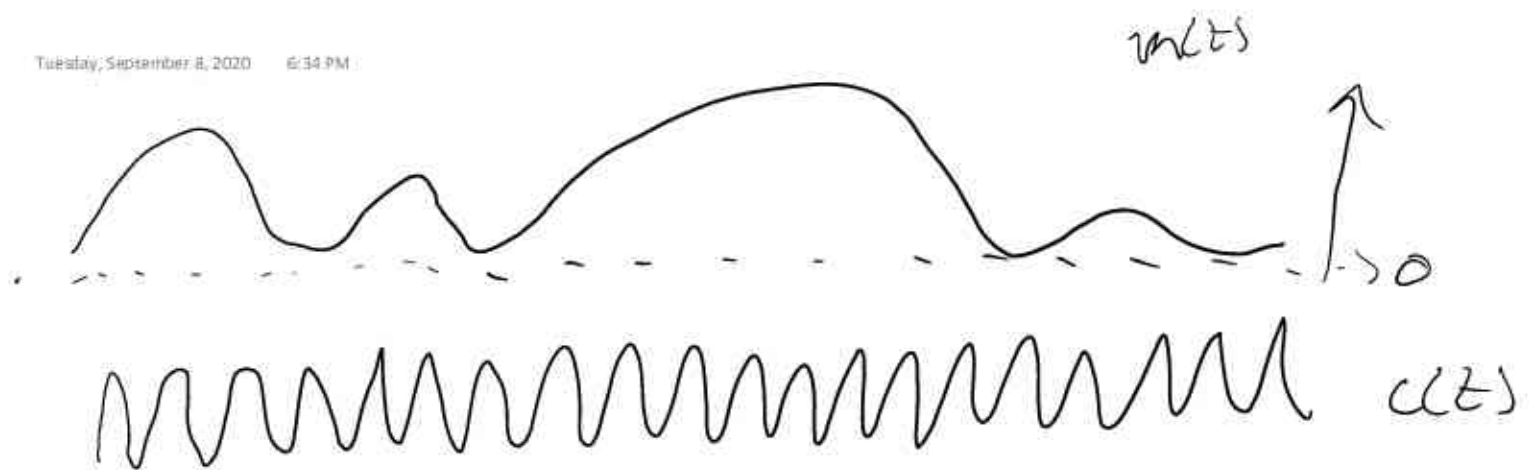
D Double Sideband Suppressed Carrier Amplitude modulation

$$\begin{aligned} u(t) &= m(t)c(t) = A_c m(t) \cos(\omega_c t) \\ &= A_c m(t) \left(\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2} \right) \end{aligned}$$

$$U(\omega) = \frac{A_c}{2} (M(\omega - \omega_c) + M(\omega + \omega_c))$$

as long as $\omega_c > 2\pi W$

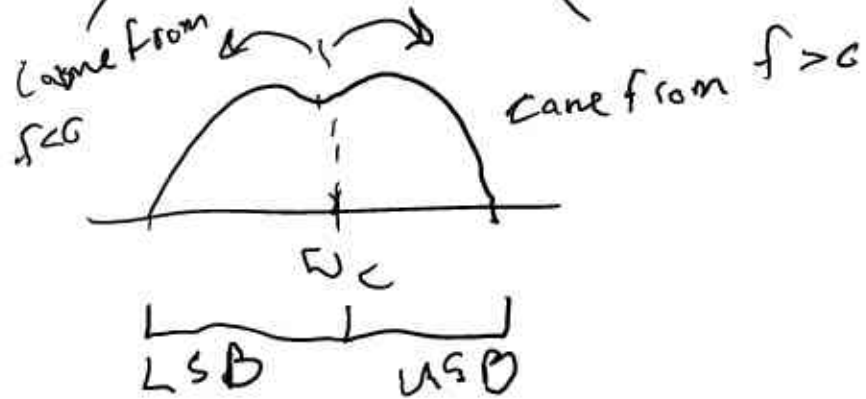




Why is it called DSB-SC?



DSB:



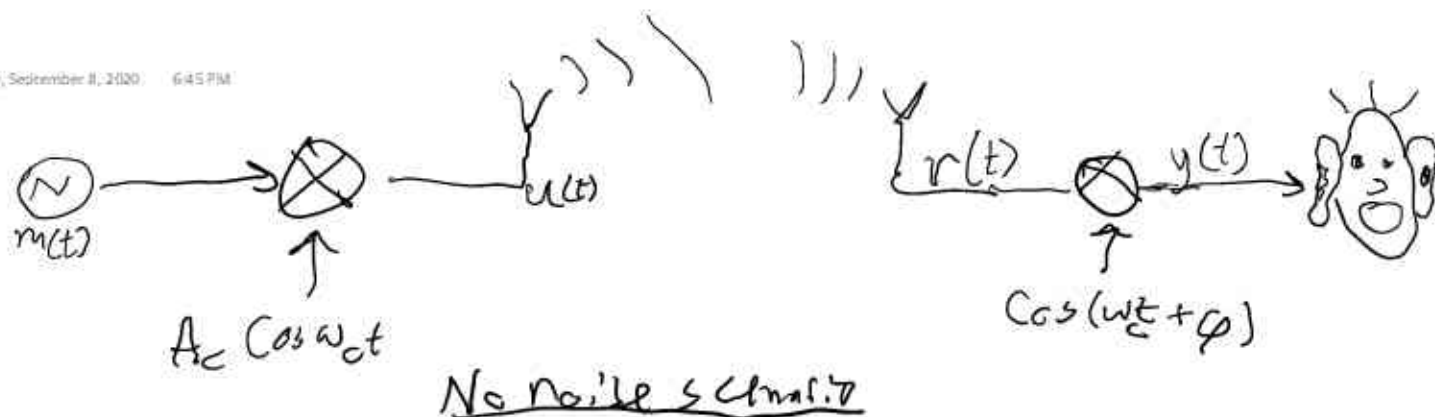
$$P_u = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} A_c^2 m^2(t) \cos^2(\omega_c t) dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{A_c^2}{2} m^2(t) (1 + \cos(2\omega_c t)) dt$$

$$= \lim_{T \rightarrow \infty} \frac{A_c^2}{2T} \left(\int_{-T/2}^{T/2} m^2(t) dt + \int_{-T/2}^{T/2} m^2(t) \cos(2\omega_c t) dt \right)$$

$$= \frac{A_c^2}{2} P_m$$

No power coming from the "carrier component"

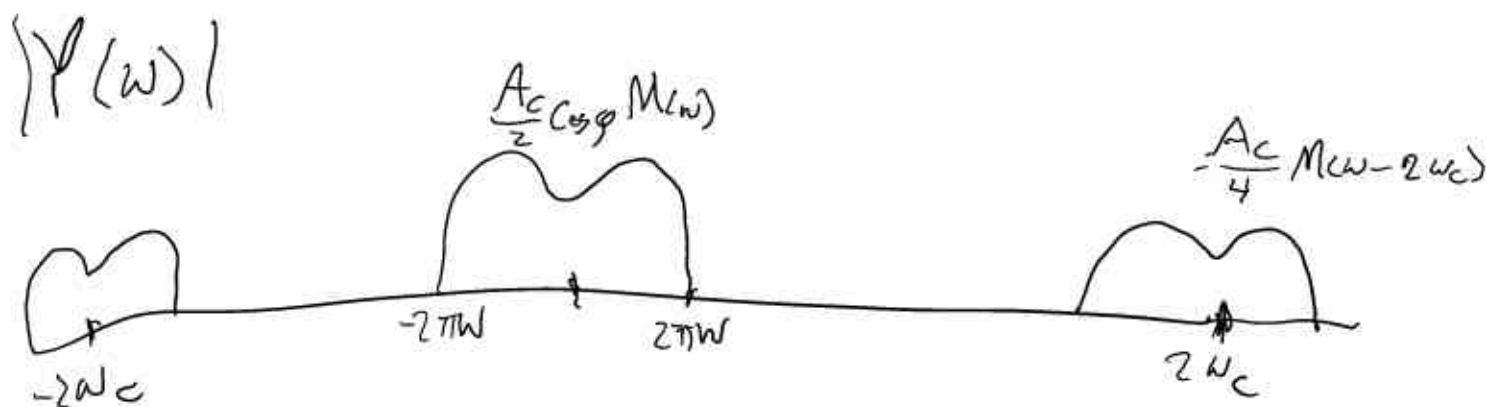
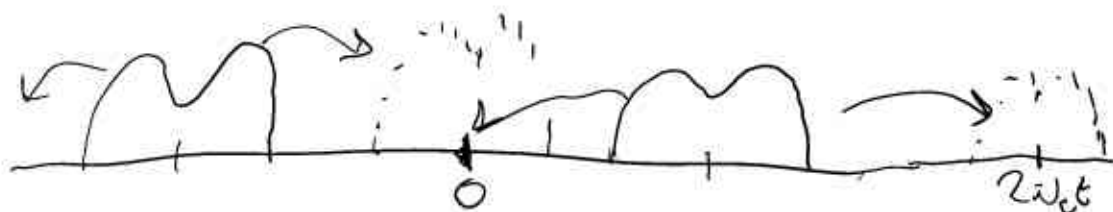


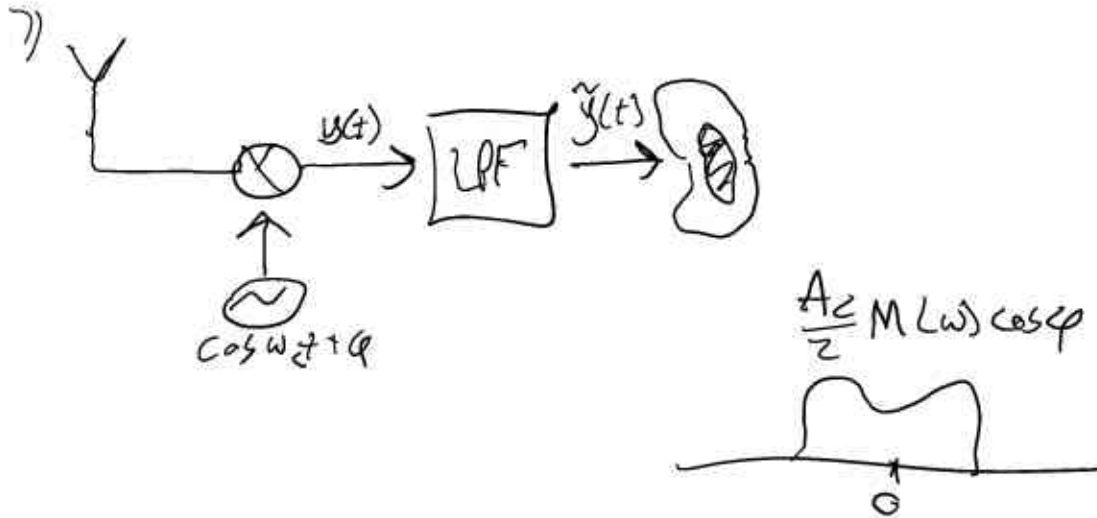
$$r(t) = u(t - \Delta t)$$

First, assume $\Delta t \approx 0$

$$r(t) = u(t) = m(t) c(t)$$

$$\begin{aligned} y(t) &= A_c m(t) \cos(\omega_c t) \cos(\omega_c t + \phi) \\ &= \frac{A_c}{2} m(t) \cos \phi + \frac{A_c}{2} m(t) \cos(2\omega_c t + \phi) \end{aligned}$$





$$\tilde{y}(t) = \frac{A_c}{2} m(t) \cos \phi$$

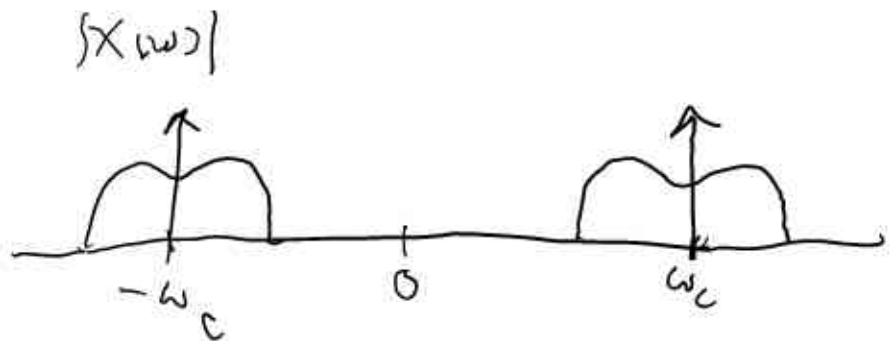
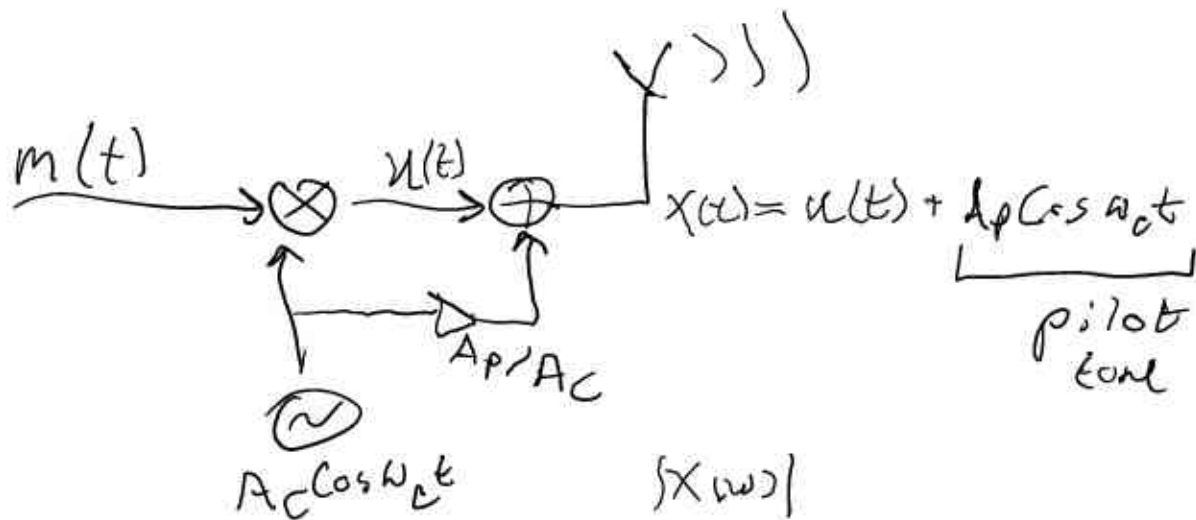
The power received is at worst 0 and dep. on phase

PHASE can make or break a comm scheme

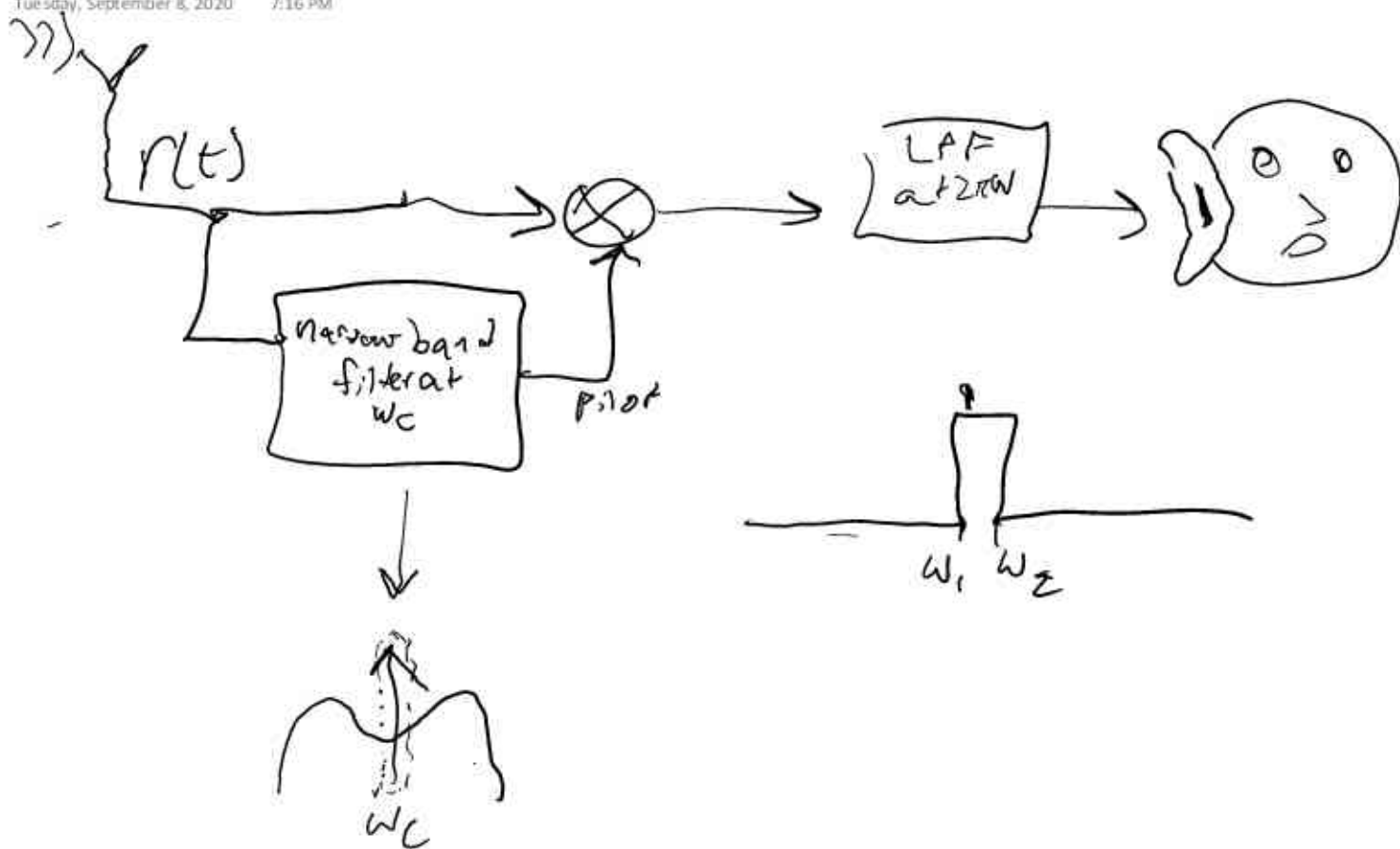
DSB-SC is unpopular due to phase sensitivity.

An Rx which "knows about" the carrier phase is called either "coherent" or "synchronous" (phase-coherent)

Solution Add a "pilot tone", which is a copy of the carrier on top of the message



"DSB AM"



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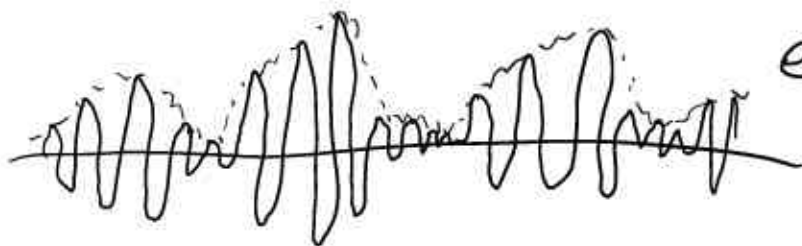
$m(t)$



$1+m(t)$



$u(t)$



envelope is line
 $m(t) + 1$

generally in DSB, envelope is line $|m(t)|$
here, $|1+m(t)| = 1+m(t)$

Convention: $a = \max(|m(t)|)$

write $m(t) = a m_n(t)$ so $\max|m_n(t)| = 1$

if $0 < a \leq 1$, then safe

if $a > 1$, then "overmodulating"

call a the "modulation index"

$$P_u = \lim_{T \rightarrow \infty} \int_{-T/2}^{T/2} [1 + a m_n(t)]^2 A_c^2 \cos^2(\omega_c t) dt$$

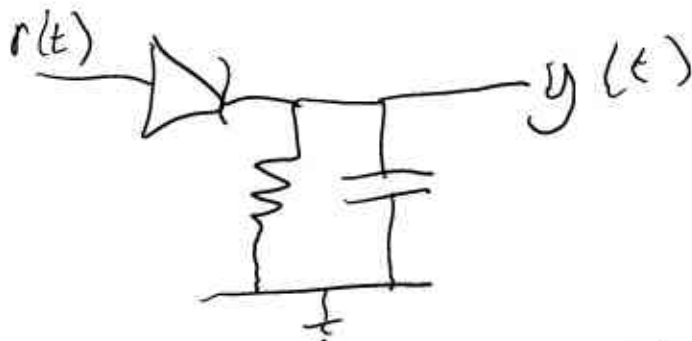
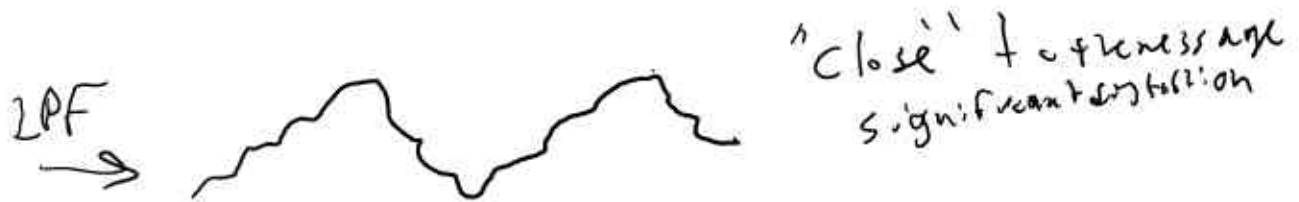
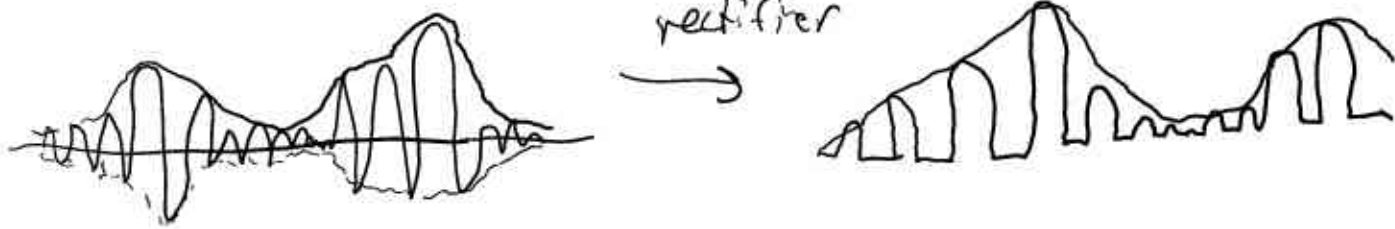
$$P_{m=\cos} \int (1 + a m_n(t))^2 dt = 1 + a^2 P_{m_n}$$

$$P_u = \underbrace{\frac{A_c^2}{2}}_{\sim \text{power in carrier}} + \underbrace{\frac{A_c^2}{2} a^2 P_{m_n}}_{\text{would be power in DSB-SC}}$$

larger power req. than DSB-SC by power in carrier

"Cost" of sending a carrier component
a lot of power, no info

Conventional used entirely due to simple demod



CHEAP AS HELL

Conventional AM has simplest demod implementation
good if many RX

Worse in terms of signal quality

less power efficient

not BW-efficient

SSB AM

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Single sideband AM
 $|V(\omega)|$

BW = W compared to DSB's 2W

~~Want~~



USSB AM



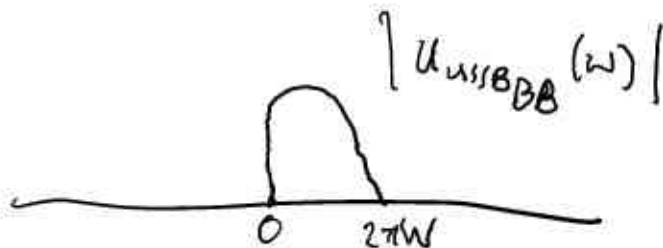
LSSB AM

This is the most BW-efficient + possible

Hilbert Transform

$$H(\omega) = \begin{cases} -j, & \omega > 0 \\ j, & \omega < 0 \end{cases}$$

Look at U at baseband



$$U_{BB} = \tilde{U}(\omega)$$

$$\tilde{U}(t) = I(t) + jQ(t)$$

$$U(t) = I(t) \cos \omega_c t - Q(t) \sin \omega_c t$$

$$M(\omega) = \begin{cases} \tilde{U}(\omega), & \omega > 0 \\ \tilde{U}(-\omega)^*, & \omega < 0 \end{cases}$$

$$\text{So } H(\omega)M(\omega) = \begin{cases} -j\tilde{U}(\omega), & \omega > 0 \\ j\tilde{U}(-\omega)^*, & \omega < 0 \end{cases}$$

$$= \hat{M}(\omega)$$

$$j\hat{M}(\omega) = \begin{cases} \tilde{U}(\omega), & \omega > 0 \\ -j\tilde{U}(-\omega)^*, & \omega < 0 \end{cases}$$

$$jN(\omega) = \begin{cases} u(\omega), \omega > 0 \\ -\tilde{u}(-\omega)^*, \omega < 0 \end{cases}$$

$$M(\omega) + j\hat{M}(\omega) = \begin{cases} 2\tilde{u}(\omega), \omega > 0 \\ 0, \omega < 0 \end{cases}$$

$$= 2\hat{u}(\omega)$$

$$\hat{u}(\omega) = \frac{M(\omega)}{2} + j \frac{\hat{M}(\omega)}{2}$$

$$\downarrow$$

$$\tilde{u}(t) = \frac{m(t)}{2} + j \frac{\hat{m}(t)}{2} = I(t) + jQ(t)$$

$$I(t) = \frac{m(t)}{2}, \quad Q(t) = \frac{\hat{m}(t)}{2}$$

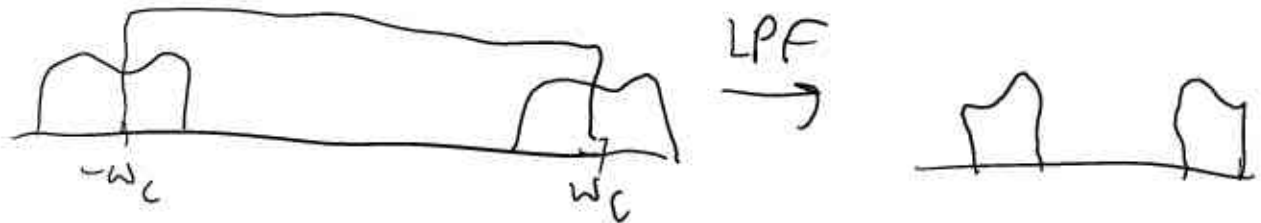
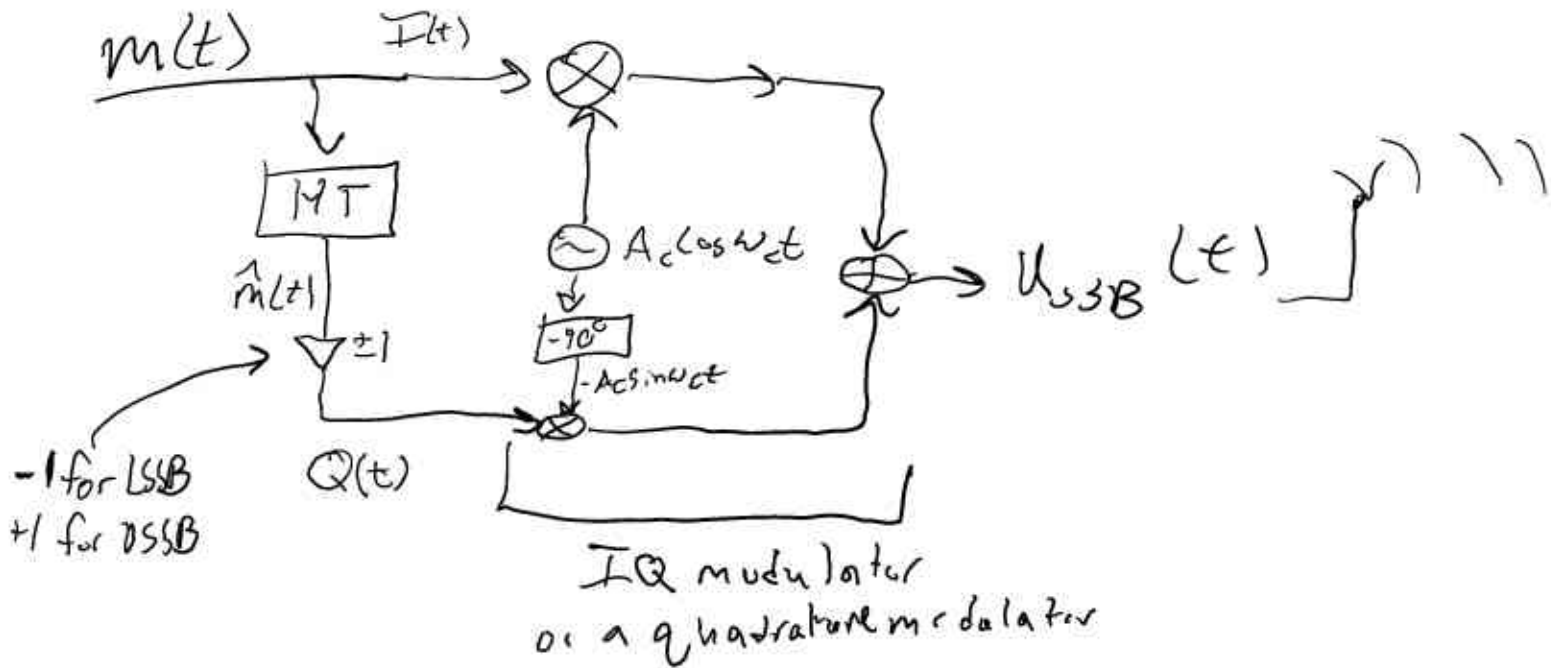
$$u(t) = \frac{m(t)}{2} \cos \omega_c t - \frac{\hat{m}(t)}{2} \sin \omega_c t$$

$$I_{LSSB} = I_{USSB} = \frac{m(t)}{2}$$

$$Q_{LSSB} = -Q_{USSB} = \frac{\hat{m}(t)}{2}$$

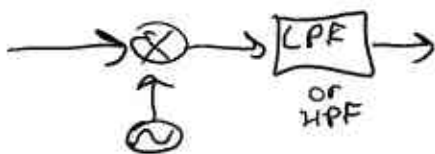
$$u_{LSSB}(t) = A_c m(t) \cos \omega_c t + A_c \hat{m}(t) \sin \omega_c t$$

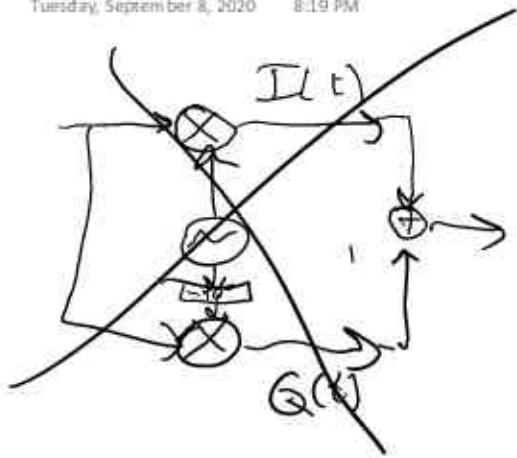
$$u_{USSB}(t) = A_c m(t) \cos \omega_c t - A_c \hat{m}(t) \sin \omega_c t$$



Alternate method uses LPF (LSSB)
or HPF (VSSB)

but requires really tight filter





need phase coherence

So send pilot tone

$$u_{SSB}(t) + A_p \cos \omega_c t + \text{transmitted}$$



SSB is more BW efficient than DSB, then conventional
not necessarily power efficient compared to an SC
method

X doesn't matter

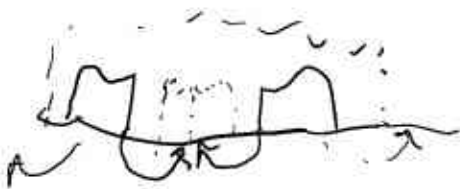
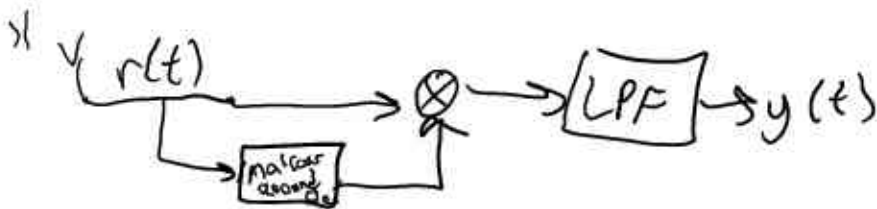
So actually, SSB is pretty power efficient - transmits
~ half the power in the message that DSB does

$$DSB \sim P_m + P_c$$

$$SSB \sim \frac{1}{2}P_m + P_c > \frac{1}{2}P_{DSB} \\ < P_{DSB}$$

more BW efficient, power efficient but complex TX
and can't just use a rectifier RX

as an RX, just the same as DSB (not conv)

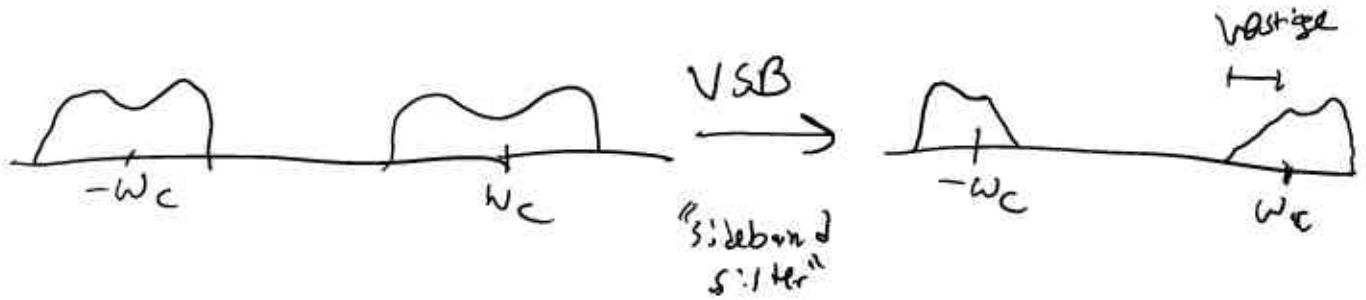


Vestigial Sideband AM

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VSB AM

like SSB, but allows "vestige" of the other sideband



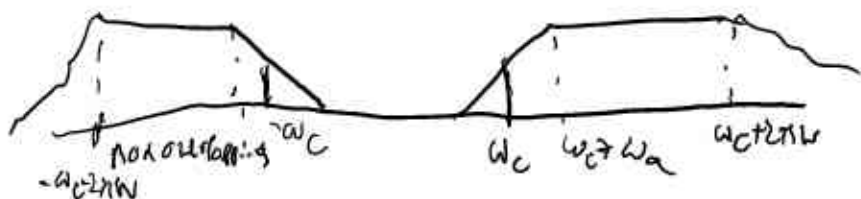
What you want to happen is when you demod,
you want vestiges to give you the "right
signal"
after $\otimes \cos \omega_c t$



$$V(\omega) = (M(\omega)H(\omega - \omega_c) + M(\omega)H(\omega + \omega_c))A$$

$$\propto M(\omega)$$

$$H(\omega - \omega_c) + H(\omega + \omega_c) = C \quad \text{for } |\omega| \leq 2\pi W$$



"H is flat in no overlapping range" $\omega_c < |\omega| < \omega_c + 2\pi W$
linear symm. in overlap

Vestigial "realistic" imp. of the simpler SSB scheme
w/ filter

in simpler SSB scheme only viable if signal has no
low-freq. components

- good enough for music
- = bad for TV

Vestigial works even for signals w/
large low-freq. components

more BW/power efficient than DSB
less so than SSB

and so it's kind of application specific

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SSB - best on power and BW and no rect. distortion

Conv - cheap RX

VSB - good where SSB fails to be simple (large low freq. carrier)

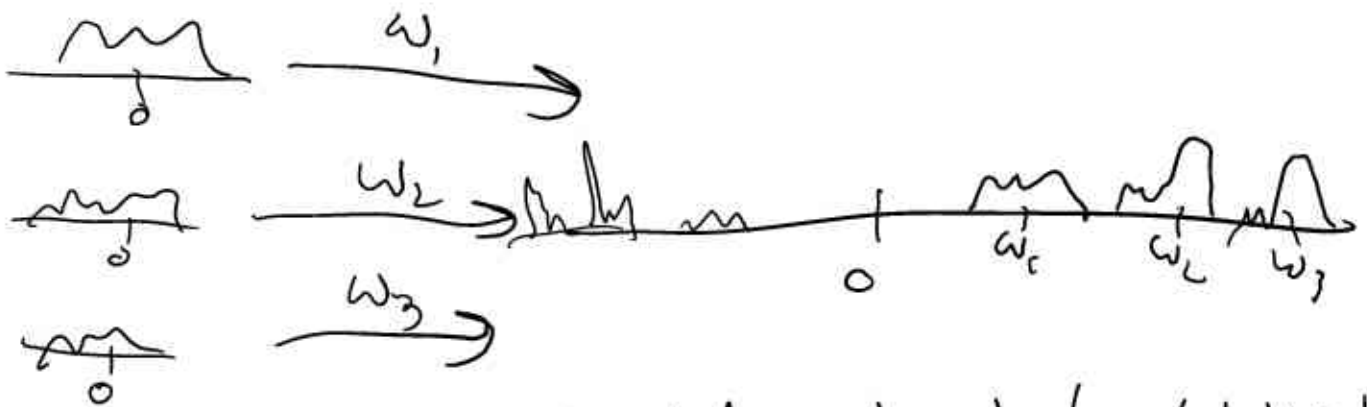
DSSC - BAD but natural pedagogically

FDM

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Frequency Division Multiplexing

means by which we send multiple AM signals at once



signals overlap at base bands but don't interfere at bandpass.

