19 January 2021 11:38

Ma 347
Lec 1

Text: Lang – Undergraduate Alg.
Will cover upto Ch 4

1. Integers
2 - Groups
3 - Rings
4 – Polynomials

No Ma 326: Read Ch 6 (Matrices)

Midterm – Mar 25 } Take home
Final – May 13 } 2 hrs.

Homeworks – 2 Prob every time we meet.
Due next

Send to my email.

Notation
$\mathbb{N}$ = the set of natural numbers
$= \{1, 2, 3, \ldots\}$

$\mathbb{Z}_+ = \{0, 1, 2, \ldots\} = \{0\} \cup \mathbb{N}$

– the set of nonnegative integers

= the set of nonnegative integers

$\mathbb{Z}$ = the set of integers

$= \{\ldots -2, -1, 0, 1, 2, \ldots\}$

$= -\mathbb{N} \cup \{0\} \cup \mathbb{N}$

where $-\mathbb{N} = \{\ldots -2, -1\}$, negative integers

(a) $m, n \in \mathbb{N} \Rightarrow m+n \in \mathbb{N}$, sum

$\underline{m < n}$, $m.n \in \mathbb{N} \Rightarrow \underline{n = m+k \text{ for}}$ some $k \in \mathbb{N}$

(b) $n, m \in \mathbb{N} \Rightarrow nm \in \mathbb{N}$ product

Part: $\mathbb{Z}$, $m, n \in \mathbb{Z} \Rightarrow$ (1) $m+n \in \mathbb{Z}$

(2) $m.n \in \mathbb{Z}$

(3) $\underline{m < n} \Rightarrow n = m+k$ for some $k \in \mathbb{N}$

$m < n \iff n-m \in \mathbb{N}$

$n - m > 0$

Well ordering Principle (WOP)

$\phi \neq S \subseteq \mathbb{N}$, then $\exists\, n_0 \in S$ s.t.

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$n_0 \le s \quad \forall s \in S$, n_0 is called a smallest or least element of S

FACT: n_0, n_1 are two smallest elements of S then $n_0 = n_1$

Induction 1st form:

For each $n \in \mathbb{N}$, let $A(n)$ be a statement.

Assume (1) $A(1)$ is true

(2) For $n \in \mathbb{N}$, $A(n)$ is true $\Rightarrow$ $A(n+1)$ is true

then $A(n)$ is true for all $n \in \mathbb{N}$

($T \subseteq \mathbb{N}$ s.t. (i) $1 \in T$

(ii) $n \in T \Rightarrow n+1 \in T$

then $T = \mathbb{N}$)

Induction: 2nd form

For each $n \in \mathbb{Z}_+$ let $A(n)$ be a statement.

Suppose (i) $A(0)$ is true

(ii) For $n \in \mathbb{N}$ $A(k)$ is for $0 \le k < n$ then $A(n)$ is true.

(I) $A(n)$ is true for

then A(n) ...

then $A(n)$ is true for all $n \in \mathbb{Z}_+$

Thm (1) WOP $\Rightarrow$ (2) Induction, 1st form

$\Rightarrow$ (3) Induction 2nd form

$\Rightarrow$ (4) WOP

Homework (1) Prove that there is no integer bet 0 and 1

(Hint: use WOP)

(2) Prove that

#4 P5 of Lang.

$$\prod_{k=1}^{n}\left(1+\frac{1}{k}\right)^k = \frac{(n+1)^n}{n!}$$

where $m! = 1.2.3 \cdots m$.

gcd, lcm, Fundamental Thm of arithmetic.

Proof of ① WOP $\Rightarrow$ ② Induction. 1st $\Rightarrow$ ③ Ind. 2nd

$\Downarrow$

(1) WOP

Let $A(n)$ be a statement for $n \in \mathbb{N}$

Assume (1) $A(1)$ is true

(2) For $n \in \mathbb{N}$, $A(n)$ is true $\Rightarrow$ $A(n+1)$ is true

To prove $A(n)$ is true for all n

Let $S = \{n \in \mathbb{N} : A(n) \text{ is false}\}$

To show $S = \phi$,

Suppose $S \neq \phi$, By WOP $\exists\, n_0 \in S$ s.t. $n_0 \leq n \;\; \forall n \in S$

Then $n_0 > 1$ as $1 \notin S$

$n_0 - 1 \notin S$ (otherwise $n_0 \leq n_0 - 1$ which is false

$A(n_0 - 1)$ is true as $n_0 - 1 \notin S$

$\Rightarrow A(n_0) = A(n_0 - 1 + 1)$ is true by prop (ii)

of $A(n)$

by prop ... of $A(n)$

$\Rightarrow n_0 \notin S$ a contradiction.

Hence $S = \phi$.

(2) $\Rightarrow$ (3)

Let $A(n)$ be a statement for $n \in \mathbb{Z}_+$

Suppose (i) $A(0)$ is true

(ii) For $n \geq 1$ integer $A(k)$ is true for $0 \leq k \leq n-1$ then $A(n)$ is true

To prove $A(n)$ is true $\forall\ n \geq 0$ integer.

Let $B(n)$, for $n \in \mathbb{N}$ be the statement:

$A(k)$ is true for $0 \leq k \leq n-1$

To prove $B(n)$ is true $\forall\ n \in \mathbb{N}$

Now $B(1)$: $A(k)$ is true for $0 \leq k \leq 1-1$

$A(0)$ true

$\Rightarrow B(1)$ is true

Suppose $B(n)$ is true

$A(k)$ is true for

$0 \le k \le n-1$

$\Downarrow$

$A(n)$ is true

i.e. $A(k)$ is true

for $0 \le k \le (n+1)-1$

$B(n+1)$ is true

I.I. $\Rightarrow$ $B(n)$ is true for all n

$\Rightarrow$ $A(n)$ is true for all n.