# Ph213 – Section D      Quiz 5

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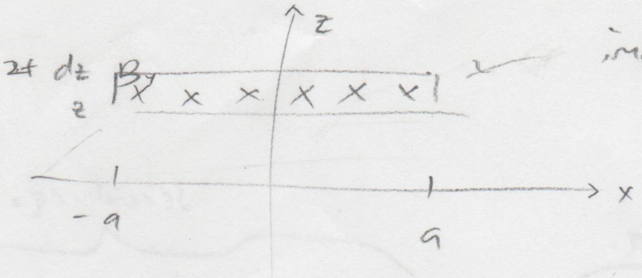
11.0 ± 3.9

Derive the EM wave equations for E & B. Assume propagation in the z direction. Assume the fields don't change except in the z direction (i.e., plane waves).




imaginary path

B field ⊥ to xz plane ($B_y$)

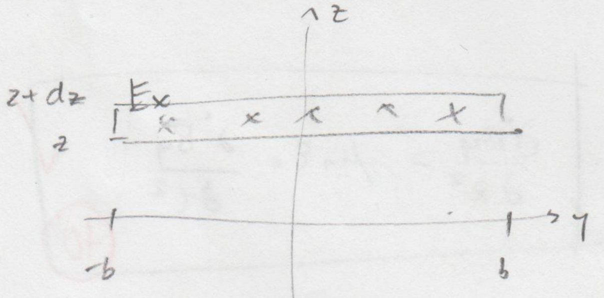
Faraday's Law  $\oint \vec{E}_x \cdot d\vec{s} = \mathcal{E} = -\frac{d\Phi_B}{dt}$

dz contributions to ds are small, ignore

$$(E_{x_{z+dz}})(2a) - (E_{x_z})(2a) = -\frac{d}{dt}(B_y dA)$$

$$\frac{\partial E_x}{\partial z}(dz\ \cancel{2a}) = -\frac{d}{dt}(B_y \cdot \cancel{2a \cdot dz})$$

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$ ✓ +5




Ampere's Law

$$\oint \vec{B}_y \cdot d\vec{s} = \mu_0\left(\cancel{I} + \varepsilon_0 \frac{d\Phi_E}{dt}\right)$$ ✓

real current = 0.

$$(+)(B_{y_z})(2b) \ominus (B_{y_{z+dz}})(2b) = \mu_0\varepsilon_0\left(\frac{d}{dt}(E\,dA)\right)$$

~~$B_y(z+dz) - B_y(z)$~~ ok

$$-\frac{\partial B_y}{\partial z}(\cancel{2b \cdot dz}) = \mu_0\varepsilon_0\left(\frac{\partial E_x}{\partial t}\cancel{2b\,dz}\right)$$

negative b/c of orientation. ok

$$\Rightarrow \frac{\partial B_y}{\partial z} = -\mu_0\varepsilon_0\frac{\partial E_x}{\partial t}$$ ✓ +5

From front side of page:

$$\begin{cases} \dfrac{\partial E_x}{\partial z} = \dfrac{-\partial B_y}{\partial t} \\ \dfrac{\partial B_y}{\partial z} = -\mu_0 \varepsilon_0 \dfrac{\partial E_x}{\partial t} \end{cases}$$

solving for $E(z, t)$:

differentiate:

$$\underbrace{\underbrace{\frac{\partial}{\partial z}\left(\frac{\partial E_x}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{-\partial B_y}{\partial t}\right)}_{\text{first eq.}} \; \underbrace{= -\frac{\partial}{\partial t}\left(\frac{\partial B_y}{\partial t}\right)\right) = \mu_0 \varepsilon_0 \frac{\partial E_x}{\partial t}}_{\text{second eq.}}}_{\text{equal}}$$

$$\Rightarrow \boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

+2

solving for $B(z, t)$:

differentiate:

$$\underbrace{\underbrace{\frac{\partial}{\partial t}\left(\frac{\partial B_y}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial E_x}{\partial z}\right)}_{\text{first eq.}} \; \underbrace{= \frac{1}{\mu_0 \varepsilon_0}\left(\frac{\partial}{\partial z}\left(-\mu_0 \varepsilon_0 \frac{\partial E_x}{\partial t}\right)\right) = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial}{\partial z}\left(\frac{\partial B_y}{\partial z}\right)}_{\text{second eq.}}}_{\text{equal}}$$

$$\Rightarrow \frac{\partial^2 B_y}{\partial t^2} = \frac{1}{\mu_0 \varepsilon_0} \frac{\partial^2 B_y}{\partial z^2} \Rightarrow \boxed{\frac{\partial^2 B_y}{\partial z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_y}{\partial t^2}}$$

+2

Both of these fit the wave equation form,

And may be solved by sinusoidal equations of $z$ and $t$ such as:

$E(z, t) = E_0 \cos(kz - \omega t)$

$B(z, t) = B_0 \cos(kz - \omega t)$