Blue Book

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SUBJECT Signals & Systems

INSTRUCTOR Fontaine

EXAM SEAT NO. SECTION

DATE 3/5/20 GRADE

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100

$h * x =$

1 a) $\int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = y(t)$

b)

$y[n] = h * x = \sum_{m=-\infty}^{\infty} h[m]\, x[n-m]$

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c) $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty.$ (i.e., $h \in L^1$).

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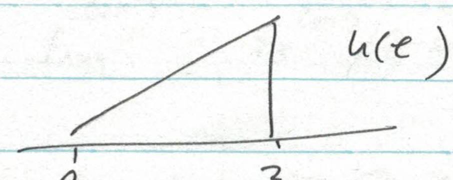
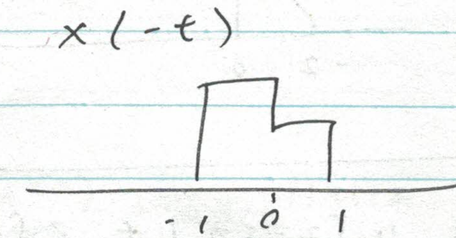
2 a) support: $-1 \le t \le 3$

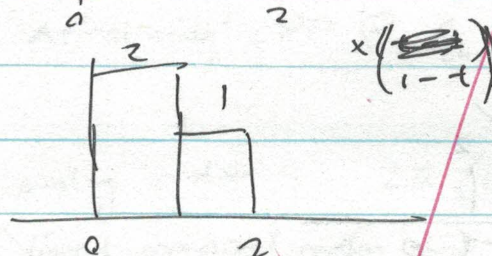
duration: 4.

6

b)



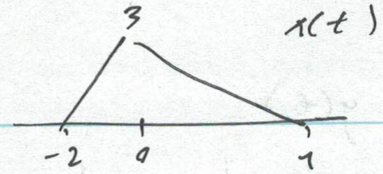




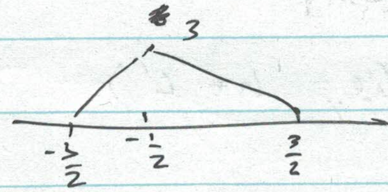
c) $\int_0^1 2t\, dt + \int_1^2 t\, dt$

$= t^2\Big|_0^1 + \frac{t^2}{2}\Big|_1^2 = 1 + \left(2 - \frac{1}{2}\right) = \frac{5}{2}.$

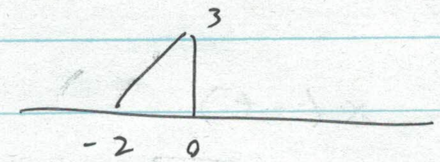
3).



a) $x(2t+1) = x(2(t+\frac{1}{2}))$

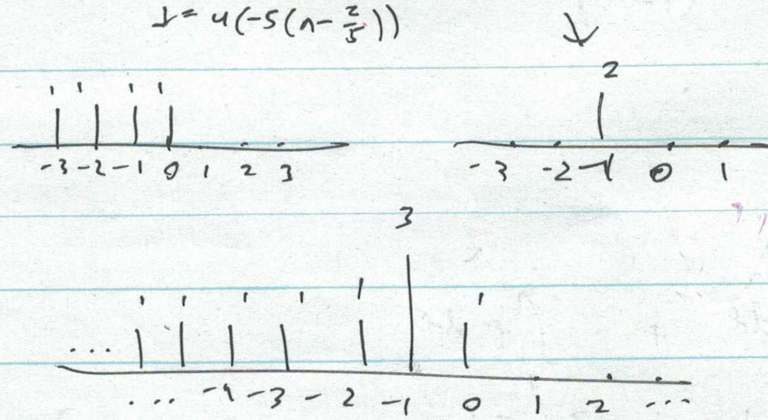


b) $x(t) \cdot u(-3t)$.



4.) $u[2-5n] + 2\delta[n+1]$

$= u(-5(n-\frac{2}{5}))$



5a) length h: 7

length x: 4

support y: $-2 \le n \le 7$

length y: 10

3b) $y[2] = \sum_{m=-\infty}^{\infty} h[m]\, x[n-m] = h[0]x[2] + h[1]x[1] + h[2]x[0] + h[3]x[-1]$

5c) 1) yes (because impulse response is causal)

2) yes (because FIR $\Rightarrow h(x) \in L^1$)

6.) a) $f_s^{(in)} = \cancel{\quad}$. 22kHz

b). 32kHz, 34kHz

c) $f_{nyq}^{(in)} = f_s^{(in)}/2 = 11\,kHz$.

d) antialiasing LPF @ $f_{cutoff} = f_{nyq}^{(in)} = \cancel{\quad}$ 11kHz.

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e) analog radian: $2\pi(10kHz) = 20000\pi$ rad/s.

digital normalized radian freq: $= \omega T = 2\pi\left(\frac{10kHz}{22kHz}\right) = \frac{10\pi}{11}$.

as frac of sampling rate: $\frac{10kHz}{22kHz} = \frac{5}{11}$

f). $f_s^{(out)} = 2f_s^{(in)} = 44kHz$.

cutoff freq $= f_{nyq}^{(out)} = 22kHz$.

antiimaging filter.

g). 20kHz, 24kHz, 64kHz, 68kHz.

7.) $e^{jk\frac{\pi}{4}} = e^{j(\frac{19\pi}{4})} = e^{j(4\pi + \frac{3\pi}{4})}$

$\Rightarrow k = 3.$

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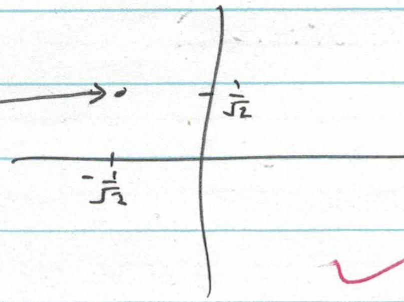
$e^{j\frac{3\pi}{4}}$

$= \frac{1}{\sqrt{2}}(-1+j)$.



8.) $\sqrt{2}\cos(\omega_0 t - \frac{\pi}{4}) + 5\cos\omega_0 t \quad - 4\sin\omega_0 t$

10 $= \sqrt{2}e^{j(-\frac{\pi}{4})} + (5+4j)e^{0j}$

$= \sqrt{2}e^{-j\frac{\pi}{4}} + (5+4j)$

9.) $x(t) = A\cos(2\pi f_0 t + \alpha \sin 2\pi f_m t + \phi_0)$

a) $f_{inst}(t) = \frac{1}{2\pi}\frac{d\phi}{dt} = \frac{1}{\cancel{2\pi}}(\cancel{2\pi} f_0 + \alpha \cancel{2\pi} f_m \cos 2\pi f_m t)$

$= f_0 + \alpha f_m \cos(2\pi f_m t)$

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b) $f_{min} = f_0 - \alpha f_m$.

$f_{max} = f_0 + \alpha f_m$

For f_{min} to be > 0, then $f_0 > \alpha f_m$.

10.) a) $x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\omega_0 n t}$

write! $\omega_0 = \frac{2\pi}{T}$

$c_n = \frac{1}{T}\int_0^T x(t) e^{-j\omega_0 n t} dt$

b) $c_m = c_{-m}^*$

c) fundamental freq $= f_0 = \frac{1}{T}$ (Hz)

3^{rd} harmonic $= 3f_0 = \frac{3}{T}(Hz)$ ✓

d) DC power $= |c_0|^2$

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e) at third harmonic, power is $|c_3|^2 + |c_{-3}|^2$.

11.) a) $x_m(t) = A\cos(2\pi(f_0 + m\Delta f)t)$, $0 \le t \le T$.

$= Re\left(x_{BB,m}(t) e^{j2\pi f_0 t}\right)$.

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$= Re\left(Ae^{j(2\pi(f_0 + m\Delta f)t)}\right) = Re\left(Ae^{j(2\pi m\Delta f t)} e^{j2\pi f_0 t}\right)$

$\Rightarrow x_{BB,m}(t) = Ae^{j(2\pi m \Delta f t)}$.

JUSTIFY THIS

b) $x_{BB,m}(t)$, $x_{BB,n}(t)$, $m \ne n$ orth. if.

$m\Delta f - n\Delta f = k(\frac{1}{T})$. Technically I didn't ask so OK

~~→ let $\Delta f = \frac{1}{MT}$ for any particular m.~~

~~If Δf was to be the same for all $x_{BB,m}$, $0 \le m \le M+1$~~,

$\Delta f = \frac{1}{T} \Rightarrow x_{BB,m}(t) = Ae^{j(2\pi(\frac{m}{T})t)}$ mutually orthogonal

12.) $P_{in} = 0.01W$, $P_{out} = 50 dBm$.

a) $P_{in}(dBm) = 10 \log_{10}\left(\frac{0.01W}{0.001W}\right)$

$= 10 \log_{10}(10) = 10(1) = 10 dBm$.

b). dB

c) $\text{gain}(dB) = P_{out} - P_{in} = \underbrace{50 dBm - 10 dBm}_{\text{Valid because of log magic.}} = 40 dB$

d). $40 dB = 20 \log_{10}\left(\frac{V_{out}}{1V}\right)$

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$2 = \log_{10}(V_{out})$

$V_{out} = 10^2 V = 100 V$.

13.) $\langle x, y \rangle = \int_{-\infty}^{\infty} x(t)y(t)\,dt$.

a) $\|\phi_1(t)\| = \int_{-\infty}^{\infty} |\phi_1^2(t)|\,dt = A_1^2(2) = 1$

$\Rightarrow \frac{1}{\sqrt{2}} = A_1$

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$\|\phi_2(t)\| = \int_{-\infty}^{\infty} |\phi_2^2(t)|\,dt = A_2^2(1) + (-A_2)^2(1)$

$= 2A_2^2$

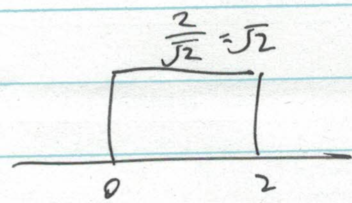
$\Rightarrow \frac{1}{\sqrt{2}} = A_2$.

b.) $\psi_3(t) = s_3 - \frac{\langle s_3, \phi_1 \rangle}{\cancel{\|\phi_1\|^2}} \phi_1 - \frac{\langle s_3, \phi_2 \rangle}{\cancel{\|\phi_2\|^2}} \phi_2.$

$\|\phi_1\| = \|\phi_2\| = 1$

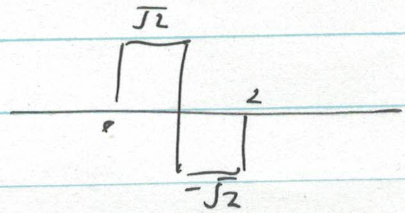
$= s_3 - \langle s_3, \phi_1 \rangle \phi_1 - \langle s_3, \phi_2 \rangle \phi_2.$

$s_3 \phi_1 =$



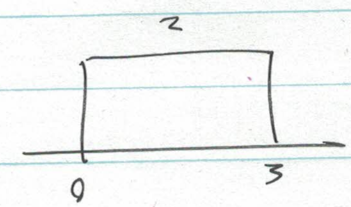
$\Rightarrow \langle s_3, \phi_1 \rangle = 2\sqrt{2}.$

$s_3 \phi_2 =$

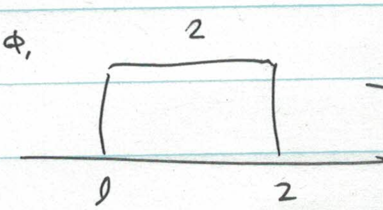


$\Rightarrow \langle s_3, \phi_2 \rangle = 0.$

$\Rightarrow$ s_3

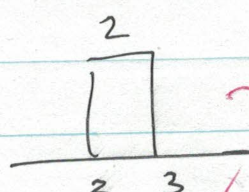


$-$ $\langle s_3, \phi_1 \rangle \phi_1$



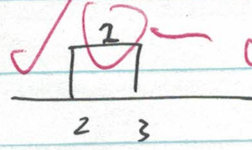
$-$ $\langle s_3, \phi_2 \rangle \phi_2$

$\Rightarrow \psi_3 =$



$\sim \cancel{\|\psi_3\| = 4 \cdot \psi_3}$

$\phi_3 = \frac{\psi_3}{\|\psi_3\|} = \frac{\psi_3}{2} =$



ah ok, your 1 looks like 2

c). $\vec{s}_3 = \begin{pmatrix} 2\sqrt{2} \\ 0 \\ 2 \end{pmatrix}$

d) $\hat{s}_3 = \begin{pmatrix} 2\sqrt{2} \\ 0 \\ 0 \end{pmatrix}$ $\vec{e} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$

e) GRAM SCHMIDT ORTHONORMALIZATION.