12 /12

6.6 ± 2.9

Ph213 – Section D

Quiz 1

159 not sure

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The E field produced by a uniform 1-D ring of charge of charge density λ and radius R has magnitude $k2\pi\lambda Rh/(h^2+R^2)^{3/2}$ at a point along the symmetry (z) axis of the ring, at height h above the plane of the ring. Assume λ is positive, so the field points away from the ring, along the z axis. This expression is a given, you do not need to prove it.

For each of the charge distribution geometries below, use rings as building blocks to **SET UP**, but **DO *NOT* SOLVE**, integrals for the E field produced by the charge distribution on the z axis.

No magic appearance of memorized formulas.

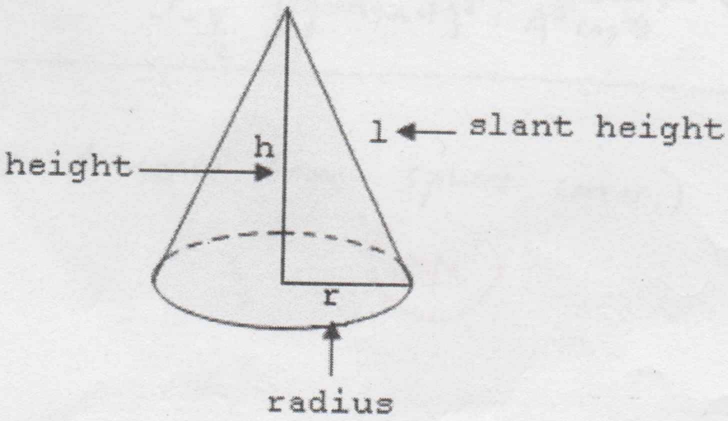
You must show all of the following:
* the proper transition from λ to σ, as we did in class
* a perfectly annotated sketch showing the entire charge distribution as well as the randomly placed building block that contributes its dE
* all steps necessary to write the integrand as a function of a single variable
* proper (explicit) limits of integration.

THEN STOP. DO NOT DO THE INTEGRALS. DO NOT TAKE LIMITS. (4 pts each)

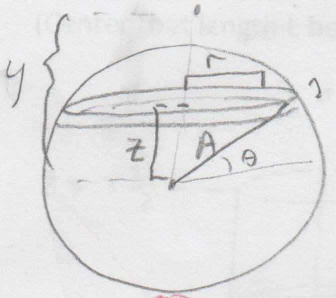
a) A thin conducting spherical shell of uniform charge density σ and radius A centered at the origin.

b) A thin insulating cylinder of uniform surface charge density σ, radius B and length L. (Center that length L between z = +L/2 and z = -L/2).

c) A thin insulating cone of uniform surface charge density σ, with dimensions as shown below. The charge is only on the "body" not on the circular base. If it helps, the surface area of the cone can be written as π r L or as π r $\sqrt{h^2+r^2}$. (L is the 'slant height' shown as "l" below.)



a) A thin conducting spherical shell of uniform σ and radius A centered at the origin.



+.5

+.5

FOR A RING:

$$E = \frac{k\, 2\pi \lambda R h}{\sqrt{h^2+R^2}^{\,3}}$$

$$\lambda = \frac{q}{2\pi R}$$

$$E = \frac{k\, \cancel{2\pi}\left(\frac{q}{\cancel{2\pi R}}\right)\cancel{R} h}{\sqrt{h^2+r^2}^{\,3}} = k\frac{qh}{\sqrt{h^2+r^2}^{\,3}}$$

q is charge of ring.

FOR THE RING WITH HEIGHT $(Ad\theta)$:

$$dq = \sigma(2\pi r)(A\,d\theta) = \sigma\, 2\pi A^2 \cos\theta\, d\theta$$ +.5

$r = A\cos\theta$ +.5 $\qquad R = A$

$z = A\sin\theta$ +.5

$h = y - z = y - A\sin\theta$ +.5

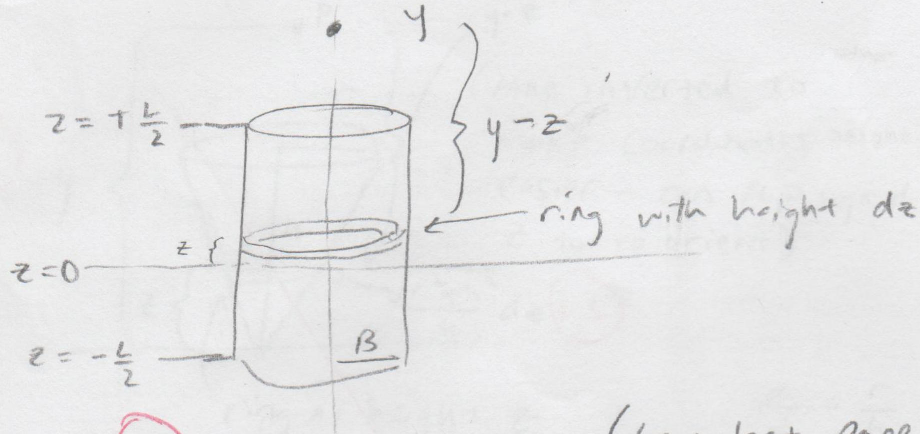
$$dE = \frac{k\,dq\,h}{\sqrt{h^2+R^2}^{\,3}} = \frac{k(\sigma 2\pi A^2\cos\theta)(y - A\sin\theta)}{\sqrt{(y-A\sin\theta)^2+(A\cos\theta)^2}^{\,3}}\,d\theta$$

$$\Rightarrow \boxed{E = k\sigma\, 2\pi A^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos\theta\,(y - A\sin\theta)}{\sqrt{(y-A\sin\theta)^2 + A^2\cos^2\theta}^{\,3}}\,d\theta}$$ +1

(where y is distance from sphere center.)

+4/4

b) thin insulating cylinder of uniform surface charge density σ, radius B and length L. (Center that length L between z = +L/2 and z = -L/2).



+1

(from last page:)

FOR A RING:

$$E = \frac{kqh}{\sqrt{r^2+h^2}^{3}}$$

FOR A RING ON THIS CYLINDER w/ HEIGHT dz:

$dq = \sigma\, 2\pi B\, dz$ +1

$h = y - z$ +.5

$r = B$

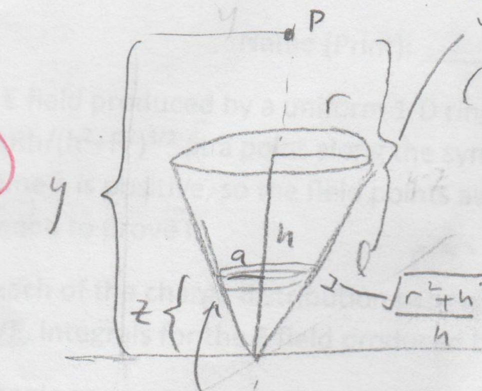
$$dE = k\frac{dq\, h}{\sqrt{r^2+h^2}^{3}} = k\frac{\sigma\, 2\pi B\, dz\,(y-z)}{\sqrt{B^2+(y-z)^2}^{\frac{3}{2}}}$$

$$\Rightarrow \boxed{E = k\sigma 2\pi B \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{(y-z)\,dz}{\sqrt{B^2+(y-z)^2}^{3}}}$$

+.5 +1

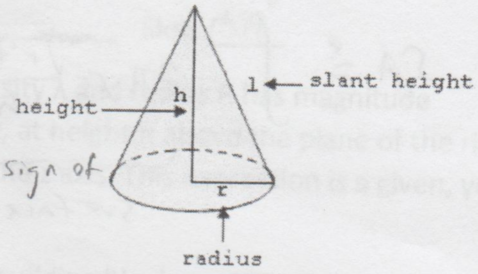
+4/4

c) A thin insulating cone of uniform surface charge density σ, with dimensions as shown below. The charge is only on the "body" not on the circular base. If it helps, the surface area of the cone can be written as $\pi r L$ or as $\pi r\sqrt{h^2+r^2}$. (L is the 'slant height' shown as "l" below.)



(cone inverted to make coordinates easier - can flip sign of z to reorient)

$\frac{\sqrt{r^2+h^2}}{h}dz$ (+.5)



(+1)

ring at height z, with radius $a=\frac{r}{h}z$.

$\frac{a}{z}=\frac{r}{h} \Rightarrow a=\frac{r}{h}z$

FOR ARBITRARY RING: $E=k\frac{qh}{\sqrt{r^2+h^2}^3}$ on z-axis.

HERE: Ring has height $\frac{\sqrt{r^2+h^2}}{h}dz$ (i.e., slant height) and radius a,

$dq=\sigma 2\pi a\left(\frac{\sqrt{r^2+h^2}}{h}\right)dz=\frac{\sigma 2\pi r\sqrt{r^2+h^2}\,z\,dz}{h^2}$ (+.5)

$a=\frac{r}{h}z$ (+.5)

$h=(\text{distance from ring})=y-z$ (+.5)

$r\ (\text{ring radius})=a=\frac{r}{h}z$

$$dE=k\frac{dq\,h}{\sqrt{r^2+h^2}^3}=k\frac{\frac{\sigma 2\pi r\sqrt{r^2+h^2}\,z\,dz\,(y-z)}{h^2}}{\sqrt{\left(\frac{r}{h}z\right)^2+(y-z)^2}^3}$$

$$\Rightarrow \boxed{E=\frac{k\sigma 2\pi r\sqrt{r^2+h^2}}{h^2}\int_0^h\frac{z(y-z)\,dz}{\sqrt{\left(\frac{r}{h}z\right)^2+(y-z)^2}^3}}$$ (+1)

+4/4