Ph213 – Section D Quiz 2

10.5/14

4.9 ± 2.5

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Slot: 159

1) There are two points in 3-D space, point A and point B. There are also various static charge distributions and/or external electric fields in various places. What is the ***meaning*** of $V_B - V_A$? Keep it simple (a minimum of jargon), but complete and accurate. (2)

The amount of energy it takes to move one (hypothetical) coulomb of charge from point A to B through the existing charge distribution

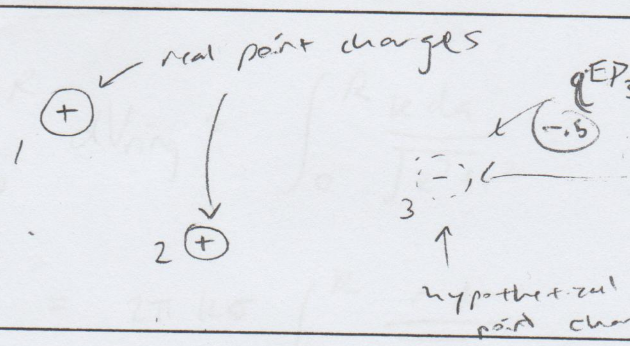
+1/2

2) ***Why*** were we able to use Gauss' Law to find the E field above a randomly-shaped lump of metal, despite the total lack of symmetry of that lump? (3)

We were working at an altitude that was so small, so that the surface was always essentially normal to the field, which is the necessary case for the E-field analysis by Gauss Law.

+1/3

3) What is the relationship between EP and EPE as you complete a charge distribution by adding a final point charge to an existing charge distribution? (Include a simple sketch or two.) (3)



$q EP_3 = EPE_{123} - EPE_{12}$

-.5

+2 .5/3

$Vq + EPE_{12} = EPE_{123}$

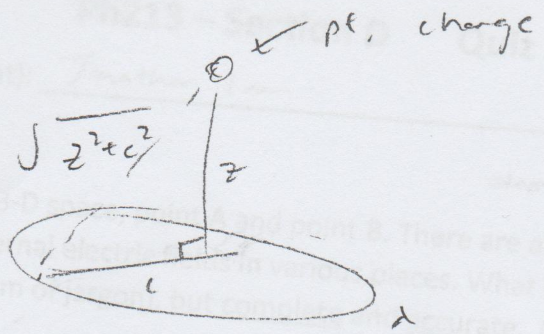
i.e., EP can be thought of as the work applied to get an electrically charged system from one configuration to another, and therefore it is also the change in EPE (per coulomb) when a

4) 1 a) Trivially derive V (a distance z along the symmetry axis) for a 1-D ring of charge of density λ & radius c.

2 b) Using the result of part a, derive V (along that axis) for a 2-D disk of charge of density σ and radius R. Setup and solve. You must make a proper transition from λ to σ (as we did in E field calculations).

3 c) Take all reasonable limits of your answer and show that these results are "expected." Make the appropriate annotated sketch of course. (6)

a)



$dq = \lambda\, ds$ ✓

$$V = \int_0^{2\pi c} \frac{k\,dq}{r} = \frac{k}{\sqrt{z^2+c^2}} \int_0^{2\pi c} \lambda\, ds = \frac{k\lambda}{\sqrt{z^2+c^2}}(2\pi c) ✓$$

Then $q = 2\pi c \lambda$ ✓

so $V = \dfrac{kq}{\sqrt{z^2+c^2}}$ ✓ (+1/1)

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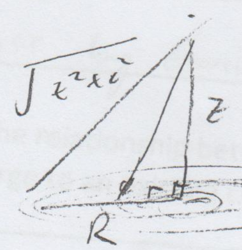
b)



each ring has charge $dq = \sigma 2\pi r\, dr$

$$V = \int_0^R dV_{ring} = \int_0^R \frac{k\,dq}{\sqrt{z^2+r^2}} = \int_0^R \frac{k\sigma 2\pi r\,dr}{\sqrt{z^2+r^2}}$$

$$= 2\pi k\sigma \int_0^R \frac{r\,dr}{\sqrt{z^2+r^2}}$$

let $u = z^2+r^2$, $du = 2r\,dr$

$$= \pi k\sigma \int_{z^2}^{z^2+R^2} u^{-\frac{1}{2}}\, du$$

$$= \pi k\sigma \left(2u^{\frac{1}{2}}\right)\Big|_{z^2}^{z^2+R^2} = 2\pi k\sigma\left(\sqrt{z^2+R^2} - z\right) \quad (+2/2)$$

$$= \frac{\sigma}{2\epsilon}\left(\sqrt{z^2+R^2} - z\right) ✓$$