Errors (Chapter 2)

Experimental errors: random and systematic

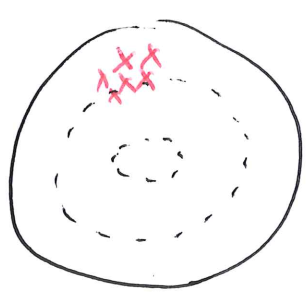
random: not repeatable, lead to fluctuations

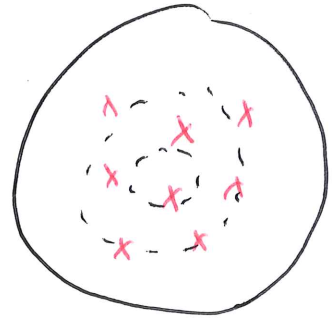
due to: - precision of measuring equipment
  try to measure the width of a human hair with meter stick
- fluctuations in environment (temp)
- true randomness (radioactive decay)

Systematic: repeatable and based on how the experimenter chooses to measure

- operator error
- uncalibrated measuring device
- wrong theory or approximation (small angle for pendulum)

Accuracy and Precision

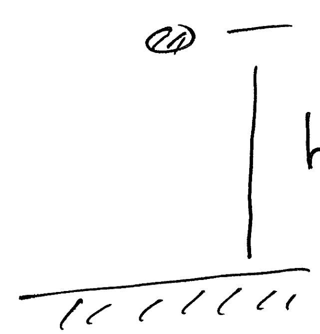




Precision limited by random errors
- closeness of grouping
- may or may not get 'correct' value
- estimate by repetition (std. dev.)

Accuracy measures how close to 'correct' value
includes both random & systematic

Consider simple physics lab to measure
acceleration due to gravity g



Drop ball from height (h)

Measure time to fall (t)

Repeat N times

Assume we <u>know</u> $h = 1$ m exactly

Stopwatch claims accuracy to 5 ms

Data

$t_1 = 30$

$t_2 = 52$

$t_3 = 47$ ms

$t_4 = 41$

$t_5 = 52$

How do we report our answer for g?

We define mean and standard deviation of our sample

Sample mean $\bar{t} = \frac{1}{N} \sum_{i=1}^{N} t_i$ (44.4 ms)

Sample variance $s^2(t_i) = \frac{1}{N-1} \sum_{i=1}^{N} (t_i - \bar{t})^2$

Sample standard deviation: $\sqrt{s^2(t_i)} \equiv s(t_i)$

$s(t_i) = 9.2$ ms ← reasonable?

Theory: $y = 1/2\, g t^2 \Rightarrow g = 2y/t^2$

How could we estimate our 'error' or uncertainty in g based on our measured values t_i?

Method: We measure $t, r, s, \ldots$ and want to compute some value a $a = f(t, r, s)$

[like $g = f(t, h)$ since $g = 2h/t^2$]

We estimate $\bar{g} = 2h/(\bar{t})^2$ or

$\bar{a} = f(\bar{t}, \bar{r}, \bar{s}, \ldots)$

[Not the only way]

Remember: Taylor Series

Write $f(x)$ around a point x_0 as a polynomial

$$f(x) = f(x_0) + (x - x_0) f'(x_0) + \frac{(x-x_0)^2 f''(x_0)}{2!} + \frac{(x-x_0)^3}{3!} f'''(x_0) + \ldots$$

Example: $f(x) = (1-x)^{-1/2}$

expand around $x = 0$

$$f(0) = (1-0)^{-1/2} = 1$$

$$f'(x) = -\tfrac{1}{2}(1-x)^{-3/2}(-1) \qquad f'(0) = 1/2$$

$$f''(x) = -3/4(1-x)^{-5/2}(-1) \qquad f''(0) = 3/4$$

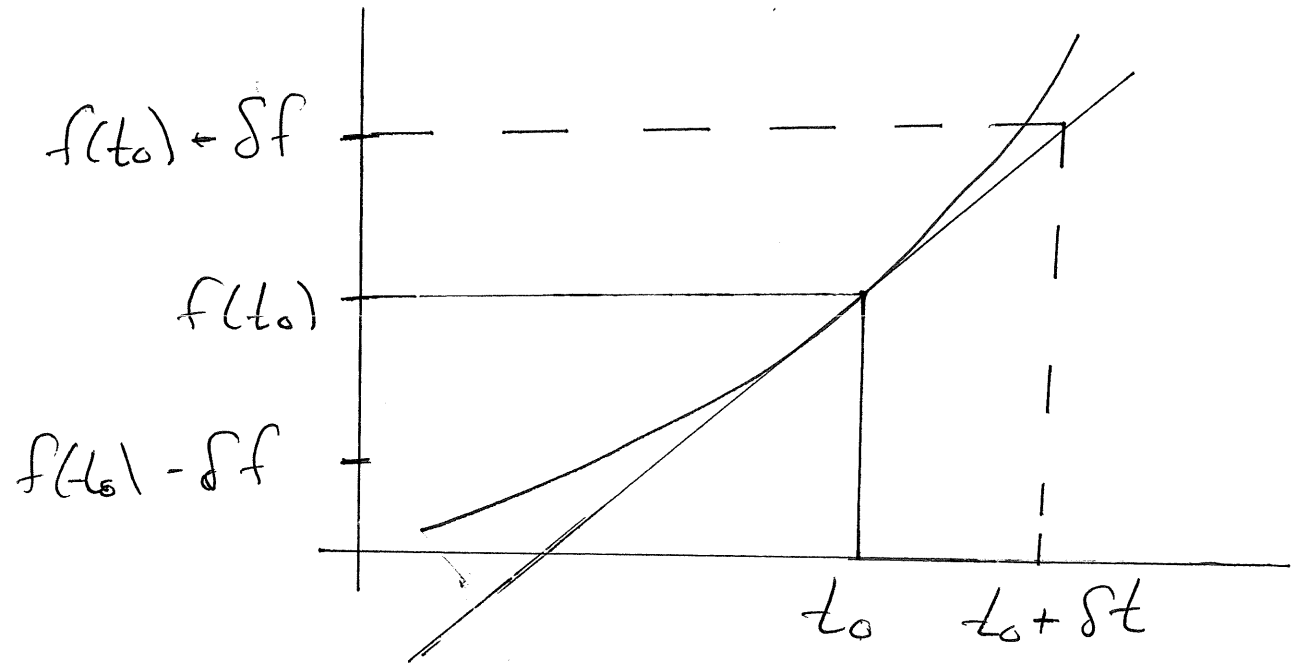
$$f(x) \approx 1 + x(1/2) + \frac{x^2}{2!}(3/4) = 1 + \frac{x}{2} + \frac{3x^2}{8}$$

[See plot]

Uncertainty Propogation

We measure t with some uncertainty δt

We want uncertainty in $f(t)$ [like $^{2y}/t^2$]

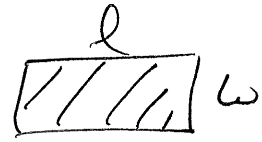


$$\delta f \simeq f'(t_0)\,\delta t$$

For many variables: $f(t, r, s, \ldots)$

$$\delta f^2 \simeq \left(\frac{\partial f}{\partial t}\right)^2 \delta t^2 + \left(\frac{\partial f}{\partial r}\right)^2 \delta r^2 + \left(\frac{\partial f}{\partial s}\right)^2 \delta s^2 + \ldots$$

Ex: Area $= l\,w$



Measure $l_0 \pm \delta l$, $w_0 \pm \delta w$ $>$ Area $= l_0 w_0 \pm$

$$\delta A^2 = \left(\frac{\partial A}{\partial l}\right)^2 \delta l^2 + \left(\frac{\partial A}{\partial w}\right)^2 \delta w^2$$

$$\delta A^2 = w^2 \delta l^2 + l^2 \delta w^2$$

More useful: $\left(\frac{\delta A}{A}\right)^2 = \left(\frac{\delta l}{l}\right)^2 + \left(\frac{\delta w}{w}\right)^2$

Let's say $l_0 = 100 \pm 1$ mm
$w_0 = 10 \pm 1$ mm $\quad A_0 = 1000\ \text{mm}^2$

$$\left(\frac{\delta A}{A}\right)^2 = \left(\frac{1}{100}\right)^2 + (1/10)^2 = (1\%)^2 + (10\%)^2$$
$$\cong (10\%)^2$$

$\delta A/A \cong 10\%$

Gravity lab: What is our uncertainty in $\bar{t}$?

Is it sample standard deviation $s(t_i)$?

$$\bar{t} = \frac{1}{5}[t_1 + t_2 + \ldots t_5]$$

$$(\delta \bar{t})^2 = \left(\frac{\partial \bar{t}}{\partial t_1}\right)^2 \delta t_1^2 + \left(\frac{\partial \bar{t}}{\partial t_2}\right)^2 \delta t_2^2 + \ldots \left(\frac{\partial \bar{t}}{\partial t_5}\right)^2 \delta t_5^2$$

<u>Assume</u> $\delta t_1 = \delta t_2 = \ldots = \delta t_5 = \delta t$

We estimate δt by $s = \left[\frac{1}{N-1} \sum (t_i - \bar{t})^2\right]^{1/2}$

$$(\delta \bar{t})^2 = (1/5)^2 s^2 + (1/5)^2 s^2 + \ldots (1/5)^2 s^2$$

$$(\delta \bar{t})^2 = 5\,(1/5)^2\, S^2 = S^2/5$$

Generally $\delta \bar{t} = \dfrac{S(t)}{\sqrt{N}}$ uncertainty in mean

Numerical computing 'errors'

- approximation (we cut off taylor series after finite terms)

- roundoff
  computer doesn't have infinite memory to store $\sqrt{2}$
  uses floating point representation

python floats are double precision (doubles)

$$X_{double} = (-1)^s \times 1.f \times 2^{n-1023}$$

$s \equiv$ sign, 1 bit

$f \equiv$ mantissa, 52 bits, precision

$n \equiv$ exponent, 11 bits, range

range of doubles $\pm 2^{-1074} \longleftrightarrow \pm 2^{1024}$

$\pm 4.9e-324 \longleftrightarrow \pm 1.8e308$