$$(\delta \bar{t})^2 = 5\,(1/5)^2\, s^2 = s^2/5$$

Generally $\delta \bar{t} = \dfrac{s(t)}{\sqrt{N}}$ uncertainty in mean

Numerical computing 'errors'

- approximation (we cut off taylor series after finite terms)

- roundoff
  computer doesn't have infinite memory to store $\sqrt{2}$
  uses floating point representation

python floats are double precision (doubles)

$$X_{double} = (-1)^s \times 1.f \times 2^{n-1023}$$

$s \equiv$ sign
1 bit

$f \equiv$ mantissa
52 bits
precision

$n \equiv$ exponent
11 bits
range

range of doubles $\pm 2^{-1074} \longleftrightarrow \pm 2^{1024}$

$\pm 4.9e-324 \longleftrightarrow \pm 1.8e308$

We get underflow or overflow for trying to store a smaller or larger number

We can't store 324 sig figs

Precision given by $1/2^{52} \approx 2.2e{-16}$

(about 16 decimal digits of precision)

[small jupyter code]

Careful comparing floats

[code]

<u>Computing e^x</u> (an example)

Taylor $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

approximate as $e^x \approx \sum_{n=0}^{n_{max}} x^n/n!$

problem: as n increases (more terms)

x^n and $n!$ can become huge

even though ratio might be small

We can try to be smarter

Note that $\frac{x^n}{n!} = \frac{x}{n} \frac{x^{n-1}}{(n-1)!}$

new term

old term

What should we take for n_{max}? How many terms?

Let's keep adding terms until the sum doesn't change (can't do any better)

[compexp code]

Derivatives

$\vec{v} = d\vec{r}/dt \qquad \vec{E} = -\nabla\phi - \partial\vec{A}/\partial t$

Typical case: We have set of n discrete data points $(x_i, f(x_i))$ $i=0,1,..,n-1$

We want to calculate $f'(x)$ at specific point

recall analytical formula

$$\frac{df(x)}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

We can code this and use a 'small' value for h

→ what uncertainty is in our answer

→ small relative to what

As h ~~gets~~ gets small $f(x+h)$ gets close to $f(x)$ and then it gets divided by small h

Let's try to be more systematic

Note: we have 2 sources of error

approximation due to cutting off Taylor series

roundoff due to subtraction and division

$$\mathcal{E}_{app} = \frac{h}{2} |f''(x)|$$

From Chapter 2: $c = a - b$ $\quad \Delta c \equiv$ uncertainty in c

$$|\Delta c| \le |\Delta a| + |\Delta b|$$

$$|\Delta f(x+h)| \simeq |f(x)| \epsilon_m \qquad |\Delta f(x)| \simeq |f(x)| \epsilon_m$$

$$\mathcal{E}_{ro} \simeq \frac{2|f(x)|\epsilon_m}{h}$$

details not that important

$$\mathcal{E}_{ro} \simeq \alpha / h$$

$$\mathcal{E}_{tot} = \mathcal{E}_{app} + \mathcal{E}_{ro} \simeq \beta h + \alpha/h$$

one term prefers small h for less error

one term prefers large h for less error

We can find optimial by

$$\frac{d\mathcal{E}_{tot}}{dh_*} = 0 = \beta - \alpha/h_*^2 \qquad h_* = \sqrt{\alpha/\beta}$$

$$\mathcal{E}_{tot} = \beta h_* + \alpha/h_* = \beta\sqrt{\alpha/\beta} + \alpha\sqrt{\beta/\alpha}$$

$$= 2\sqrt{\alpha\beta}$$

See Eqn 3.20