Simultaneous Localization And Mapping Sparse Extended Information Filters

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Problem

The Simultaneous Localization And Mapping (SLAM) problem consists in mapping an unknown environment while localizing the agent based on the agent's observations. We limit the mapping problem of the environment to the position of the landmarks y. Thus, neither the positions of the landmarks nor of the agent are directly observed. What we do observe though, comes from the sensor measurements z of the agent and the controls u, both with noise. So, at time t, the position x_t of the agent needs to be determined : $p(x_t|z_t, ..., z_0, u_t, ..., u_0)$ and the positions of the landmarks too : $p(y_t|z_t, ..., z_0, u_t, ..., u_0)$. This is a filtering problem.

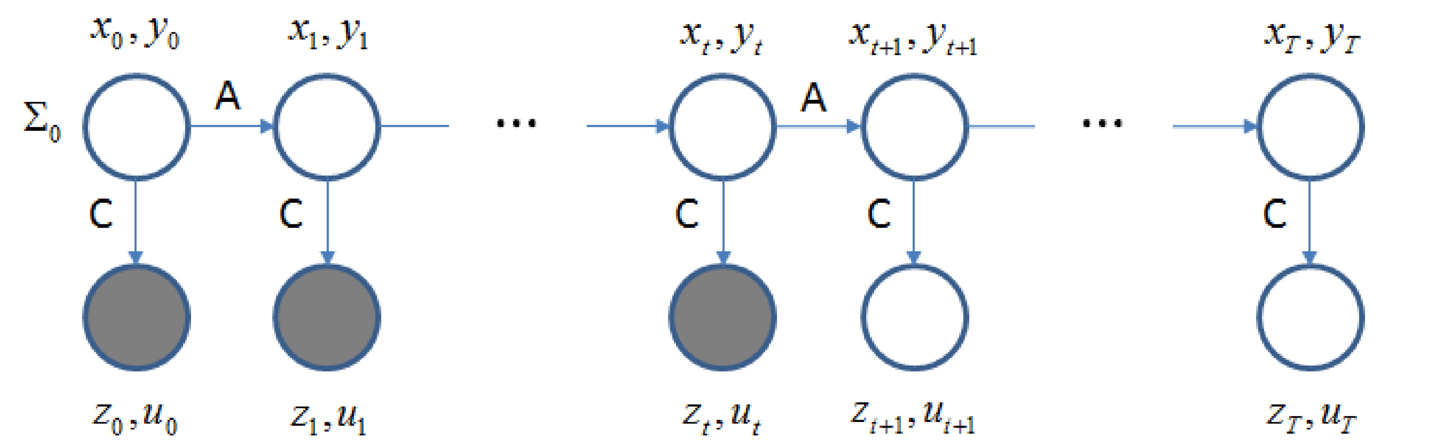

FIGURE – Graphical representation of the SLAM problem at time t. (Dark for observed variables)

Implementation

U_t, Z the covariance matrices of the error terms

$C_t = (\frac{\partial h}{\partial x_t}, 0, ..., 0, \frac{\partial h}{\partial y_i}, 0, ..., 0)^T$

$A_t = \nabla_\xi g(\mu_{t-1}, u_t)$

Extended Kalman Filter

Motion update

$\bar{\mu}_t = g(u_t, \mu_{t-1})$; $\bar{\mathbf{\Sigma}}_t = A_t \mathbf{\Sigma}_{t-1} A_t^T + U_t$

Measurement update

$K_t = \bar{\mathbf{\Sigma}}_t C_t^T (C_t \bar{\mathbf{\Sigma}}_t C_t^T + Z)^{-1}$

$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$; $\mathbf{\Sigma}_t = (I - K_t C_t)\bar{\mathbf{\Sigma}}_t$

Extended Information Filter

Motion update

$\bar{H}_t = (I + A_t)\mathbf{\Sigma}_{t-1}(I + A_t)^T + S_x U_t S_x^T$

$\bar{b}_t = (\bar{\mu}_{t-1} + g(\mu_{t-1}, u_t))\bar{H}_t$

Measurement update

$H_t = \bar{H}_t + C_t^T Z^{-1} C_t$

$b_t = \bar{b}_t + (z_t - h(\mu_t) + C_t^T \mu_t)^T Z^{-1} C_t^T$

Conversion from canonical parameters to moment parameters

$\mathbf{\Sigma}_t = H_t^{-1}$; $\mu_t = b_t^T \bar{H}_t$

Measurement and Motion Model in a 2D space

Robot Modeling and Motion Model

In our 2D model, the robot is located by its position (x, y) and its orientation α. It has a bounded angular $\dot{\alpha}$ and linear v velocity. Therefore, the EIF and EKF motion updates are written using the following rules and Jacobian :

$$\begin{pmatrix} x_t \\ y_t \\ \alpha_t \end{pmatrix} = \begin{pmatrix} x_{t-1} \\ y_{t-1} \\ \alpha_{t-1} \end{pmatrix} + \begin{pmatrix} v_t \cdot \cos(\alpha_{t-1}) \\ v_t \cdot \sin(\alpha_{t-1}) \\ \dot{\alpha} \end{pmatrix} \Rightarrow A_t = \begin{pmatrix} 0 & 0 & -v_t \cdot \sin(\alpha_{t-1}) \\ 0 & 0 & v_t \cdot \cos(\alpha_{t-1}) \\ 0 & 0 & 0 \end{pmatrix}$$

Environment Modeling and Measurement Model

Features in the simulated world are located thanks to their position (x, y). The robot we simulated has a detection radius r and a detection angle γ, ie. a detection cone. When a environment feature enters the robot vision cone, the robot estimate the distance d and the angle $\theta - \alpha$ between himself and the feature. Thus, we have the feature estimate and the measurement Jacobian :

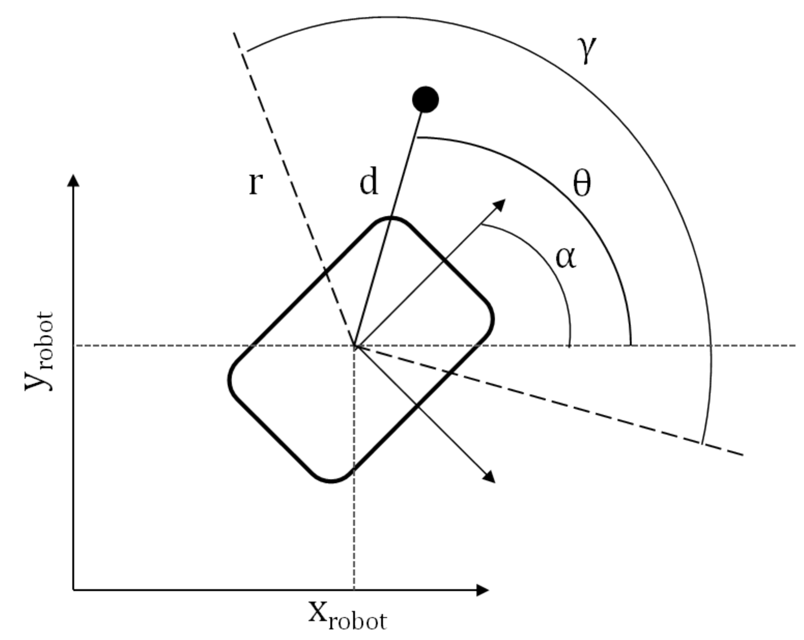

FIGURE – Schematic of the mobile robot

$$h(\xi) = \begin{pmatrix} \sqrt{(x - x_i)^2 + (y - y_i)^2} \\ \arctan(\frac{y - y_i}{x - x_i}) - \alpha \end{pmatrix} \Rightarrow C_t = \begin{pmatrix} \frac{x - x_i}{h_1(\xi)} & \frac{y - y_i}{h_1(\xi)} & 0 & \cdot\cdot 0 \cdot\cdot & \frac{x_i - x}{h_1(\xi)} & \frac{y_i - y}{h_1(\xi)} & \cdot\cdot 0 \cdot\cdot \\ \frac{y_i - y}{h_1(\xi)^2} & \frac{x - x_i}{h_1(\xi)^2} & -1 & \cdot\cdot 0 \cdot\cdot & \frac{y - y_i}{h_1(\xi)^2} & \frac{x_i - x}{h_1(\xi)^2} & \cdot\cdot 0 \cdot\cdot \end{pmatrix}^T$$

Data association

The application to a real case cannot keep the assumption of perfect knowledge of the number of landmarks and their intrinsic identification. Instead, for each measurement of a landmark, we need to determine the index of the landmark or add a new index if it is the first time the landmark is detected.

Choice of the most likely landmark :

$$n_t^* = \arg\max_{n_t} \int (n_t|z_t)\, p(\xi_t|z_1, ..., z_{t-1}, u_1, ..., u_t) d\xi_t$$

Set the landmark as new if

$$\max_{n_t} \int p(n_t|z_t)\, p(\xi_t|z_1, ..., z_{t-1}, u_1, ..., u_t) d\xi_t < \alpha$$

Simulation

In the following experiments $\gamma = 360°$. We perform a simulation of 500 steps of robot's movement. Furthermore, we compare EKF and EIF to a naive estimate that doesn't take the movement and the measurement noise into account.

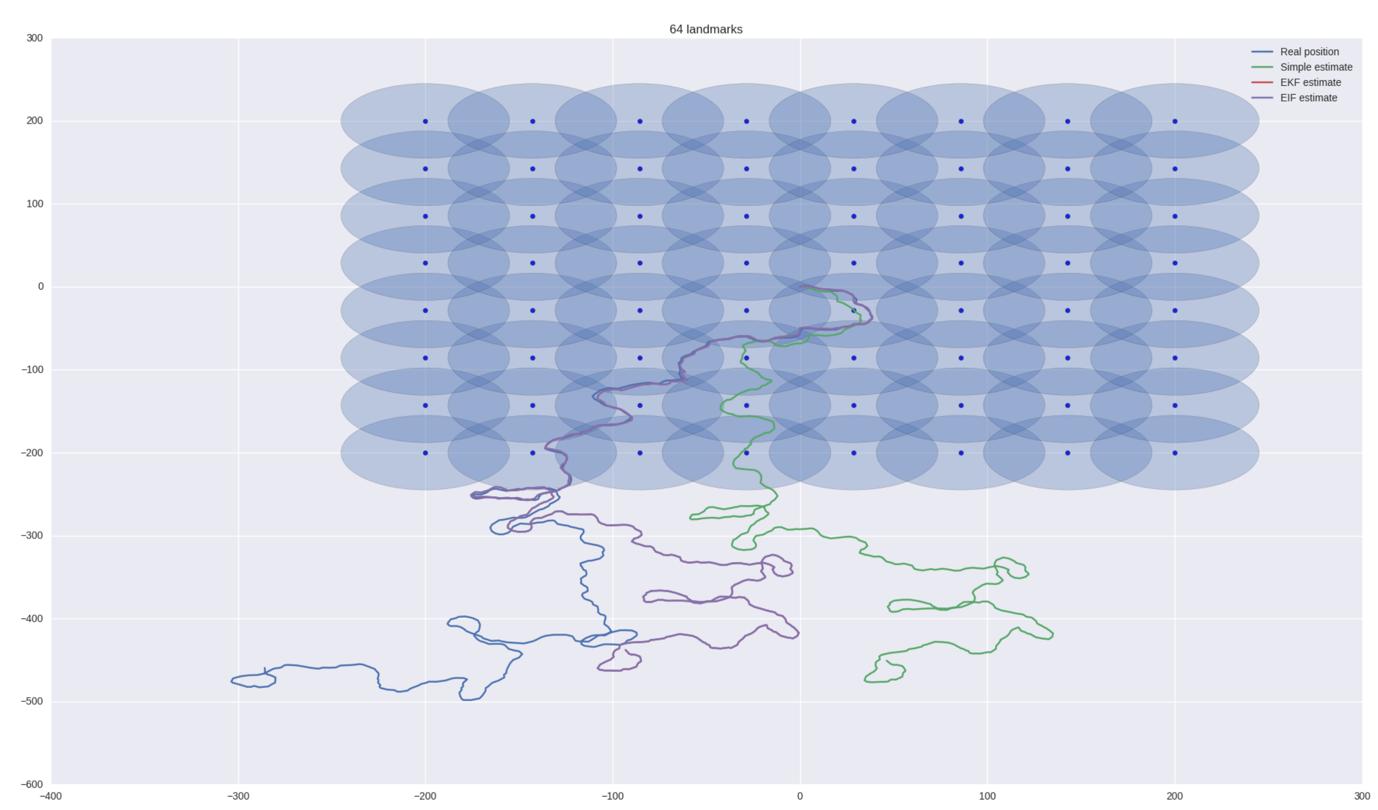

FIGURE – Real and estimated trajectories

Landmarks are denoted by points and form a grid. Observation radius is shown as a circle around each landmark. EKF and EIF produce identical results and fit the real trajectory well inside the grid. Outside the landmark grid filtering behaves similarly to naive estimate and produces the same exact trajectory up to a translation.

The naive estimate behaves like a Brownian movement around the real trajectory and accumulates error over time. An error accumulation by filtering methods at the end of the simulation corresponds to going out of the grid.

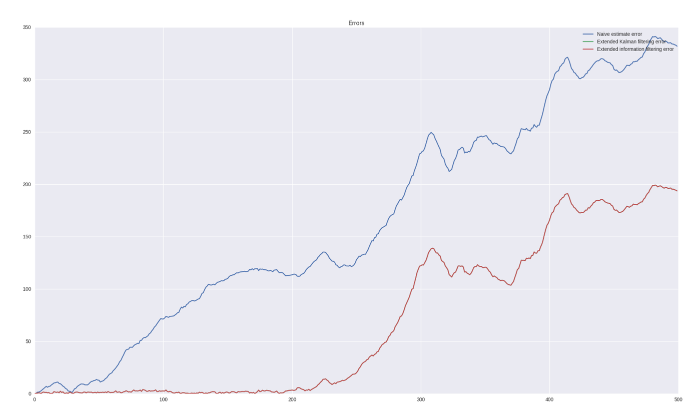

FIGURE – Error at different timesteps

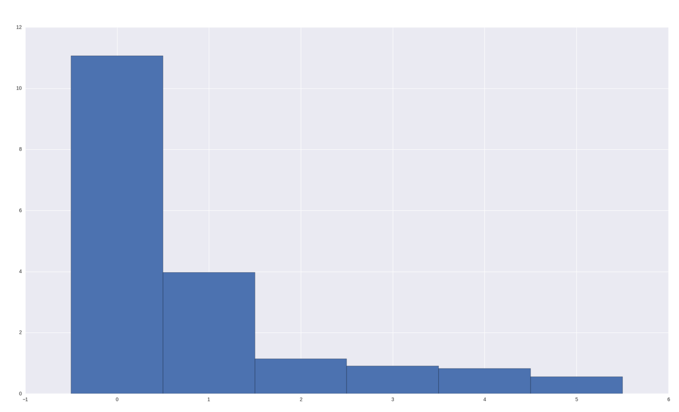

FIGURE – Average error for different number of observed landmarks

Figure on the left illustrates the average error of EIF for different numbers of observed landmarks. As we see landmarks observation is crucial for the good performance of EIF, as it yields an information gain that allows filtering methods to outperform a naive estimation.

References

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- S. Thrun. *Probabilistic robotics*. Communications of the ACM, 45(3), chapter 12, 2002