Define Data Import Functions

```
SetDirectory[NotebookDirectory[]];
(*Define the Data Import Functions*)
importdata[link_, length_] :=
  Module[{rawdata, data}, rawdata = Import[link, "Table"];
   data = Select[Select[rawdata, Length[#] == length &], VectorQ[#, NumberQ] &];
   data];
importenergy[link_, pattern_] := ToExpression[
   StringCases[StringCases[Import[link, "Text"], pattern],
    DigitCharacter .. ~~ "." ~~ DigitCharacter ..]];
```

Data Import

```
dataDXS = importdata[
   "https://dl.dropboxusercontent.com/u/61536361/dsigma_dOmega.txt", 4];
dataASY = importdata[
   "https://dl.dropboxusercontent.com/u/61536361/Sigma_asymmetry.txt", 4];
dataT = importdata[
   "https://dl.dropboxusercontent.com/u/61536361/sigmaT_SvenPeter.txt", 4];
```

Creates arrays of the data (Scattering-Angle, Observable, Error) from the raw data

```
diffDXS = Table[{((Pi / 180) * dataDXS[[i, 2]]), dataDXS[[i, 3]], dataDXS[[i, 4]]},
   {i, 1, Length[dataDXS]}];
diffT = Table[{((Pi / 180) * dataT[[i, 1]]), dataT[[i, 2]], dataT[[i, 3]]},
   {i, 1, Length[dataT]}];
```

Patitions the data into the individual energy bands

```
angleDXS = Partition[diffDXS, 20];
angleT = Partition[diffT, 18];

asymmetry = Table[{dataASY[[i, 1]], ArcCos[dataASY[[i, 2]]],
    dataASY[[i, 3]], dataASY[[i, 4]]}, {i, 1, Length[dataASY]}];
asypart = Table[{}, {i, 1, 23}];
For[i = 1;
  j = 1, i < Length[asymmetry], If[asymmetry[[i + 1, 1]] < asymmetry[[i, 1]], j ++];
  i ++, AppendTo[asypart[[j]], Drop[asymmetry[[i]], 1]]];
AppendTo[asypart[[j]], Drop[asymmetry[[i]], 1]];
Clear[i, j]
angleASY = asypart;
```

Imports the energy data into mathematica

```
tmpDXS =
  importenergy["https://dl.dropboxusercontent.com/u/61536361/dsigma_dOmega.txt",
   "W=" ~~ Shortest[___] ~~ "\n"];
tempASY = importenergy[
   "https://dl.dropboxusercontent.com/u/61536361/Sigma_asymmetry.txt",
   "W=" ~~ Shortest[___] ~~ "+/-"];
tmpT = importenergy[
   "https://dl.dropboxusercontent.com/u/61536361/sigmaT_SvenPeter.txt",
   "E =" ~~ Shortest[___] ~~ "\n"];
```

Converts the energy in lab frame to center of mass frame and creates a table

```
energyDXS = Table[Mean[tmpDXS[[i]]], {i, 1, Length[tmpDXS]}];
energyASY = Flatten[tempASY];
energyT = Table[(2 * tmpT[[i, 1]] * 938.28 + 938.28^2)^.5, {i, 1, Length[tmpT]}];
```

Functional Definitions of Multipoles

```
T0 = {{1, 0, 0, 0, 0, 0, 0, 0}, {0, 6, 0, 0, 0, 0, 0, 0},
   {0, 0, 18, 0, 0, 0, 0, 0}, {0, 0, 0, 2, 0, 0, 0, 0}, {0, 0, 0, 0, 2, 0, 0, 0},
   {0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 9, 0}, {0, 0, 0, 0, 0, 0, 0, 6}};
T1 = {{0, 3, 0, 0, 1, -1, 0, 0}, {3, 0, 72/5, -3/5, 0, 0, 9/5, -9/5},
   {0, 72/5, 0, 0, 0, 0, 0, 0}, {0, -3/5, 0, 0, 1, -1, 0, 0},
   {1, 0, 0, 1, 0, 0, 27/5, 3/5}, {-1, 0, 0, -1, 0, 0, 0, 3},
   {0, 9/5, 0, 0, 27/5, 0, 0, 0}, {0, -9/5, 0, 0, 3/5, 3, 0, 0}};
T2 = {{0, 0, 6, 1, 0, 0, 3, -3}, {0, 3, 0, 0, 3, -3, 0, 0},
   {6, 0, 108/7, -12/7, 0, 0, 36/7, -36/7}, {1, 0, -12/7, -1, 0, 0, 3, -3},
   {0, 3, 0, 0, -1, -1, 0, 0}, {0, -3, 0, 0, -1, 0, 0, 0},
   {3, 0, 36/7, 3, 0, 0, 36/7, 9/7}, {-3, 0, -36/7, -3, 0, 0, 9/7, 3}};
T3 = {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 18/5, 18/5, 0, 0, 36/5, -36/5},
   {0, 18/5, 0, 0, 6, -6, 0, 0}, {0, 18/5, 0, 0, 0, 0, 0, 0},
   {0, 0, 6, 0, 0, 0, -12/5, -18/5}, {0, 0, -6, 0, 0, 0, -3, 0},
   {0, 36/5, 0, 0, -12/5, -3, 0, 0}, {0, -36/5, 0, 0, -18/5, 0, 0, 0}};
T4 = {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 18/7, 54/7, 0, 0, 90/7, -90/7}, {0, 0, 54/7, 0, 0, 0, 0, 0},
   {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
   {0, 0, 90/7, 0, 0, 0, -36/7, -72/7}, {0, 0, -90/7, 0, 0, 0, -72/7, 0}};
S0 = {{0, 0, 3/2, 3/2, 0, 0, -3/2, 3/2}, {0, -9/2, 0, 0, 3/2, -3/2, 0, 0},
   {3/2, 0, -24, 6, 0, 0, 6, -6}, {3/2, 0, 6, -3/2, 0, 0, -3/2, 3/2},
   {0, 3/2, 0, 0, 3/2, 3/2, 0, 0}, {0, -3/2, 0, 0, 3/2, 0, 0, 0},
   {-3/2, 0, 6, 3/2, 0, 0, 12, 21/2}, {3/2, 0, -6, 3/2, 0, 0, 21/2, 9/2}};
S1 = {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -27/2, 9, 0, 0, 0, 0},
```

```
    {0, -27/2, 0, 0, 15/2, -15/2, 0, 0}, {0, 9, 0, 0, 0, 0, 0, 0},
    {0, 0, 15/2, 0, 0, 0, 6, 9}, {0, 0, -15/2, 0, 0, 0, 15/2, 0},
    {0, 0, 0, 0, 6, 15/2, 0, 0}, {0, 0, 0, 0, 9, 0, 0, 0}};
S2 = {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, -30, 45/2, 0, 0, 15/2, -15/2}, {0, 0, 45/2, 0, 0, 0, 0, 0},
    {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 15/2, 0, 0, 0, 15, 30}, {0, 0, -15/2, 0, 0, 0, 30, 0}};
F0 = {{0, -3/2, 0, 0, 3/2, 0, 0, 0}, {-3/2, 0, -15/2, 0, 0, 0, 15/2, 0},
    {0, -15/2, 0, 0, -5/2, 1, 0, 0}, {0, 0, 0, 0, 0, -3/2, 0, 0},
    {3/2, 0, -5/2, 0, 0, 0, 5/2, 0}, {0, 0, 1, -3/2, 0, 0, -1, -3/2},
    {0, 15/2, 0, 0, 5/2, -1, 0, 0}, {0, 0, 0, 0, 0, -3/2, 0, 0}};
F1 = {{0, 0, -6, 3/2, 0, 0, 6, 3/2}, {0, -9, 0, 0, 3, 3/2, 0, 0},
    {-6, 0, -36, -3/2, 0, 0, 9, 9/2}, {3/2, 0, -3/2, 3, 0, 0, 3/2, -3},
    {0, 3, 0, 0, 3, -3/2, 0, 0}, {0, 3/2, 0, 0, -3/2, 0, 0, 0},
    {6, 0, 9, 3/2, 0, 0, 18, -9/2}, {3/2, 0, 9/2, -3, 0, 0, -9/2, -9}};
F2 = {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -39/2, 3, 0, 0, 6, 9},
    {0, -39/2, 0, 0, 11/2, 5, 0, 0}, {0, 3, 0, 0, 3, 0, 0, 0},
    {0, 0, 11/2, 3, 0, 0, 8, -3}, {0, 0, 5, 0, 0, 0, -5, 0},
    {0, 6, 0, 0, 8, -5, 0, 0}, {0, 9, 0, 0, -3, 0, 0, 0}};
F3 = {{0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -36, 9/2, 0, 0, 9, 45/2},
    {0, 0, 9/2, 0, 0, 0, 9, 0}, {0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0},
    {0, 0, 9, 9, 0, 0, 18, -9}, {0, 0, 45/2, 0, 0, 0, -9, 0}};


mpion = 138.03;
mnucl = 938.27;

ec[w_, theta_] :=
  (F1 =
    1/128. (32. (4. E2minus[W] + 9. Em[4, w] + 4. E0plus[W] + 9. E2plus[W]) + 450. Ep[4, w] +
       8. Cos[2. theta] (60. Em[4, w] + 60. E2plus[W] + 105. Ep[4, w] - 48. M2minus[W] -
           20. Mm[4, w] + 48. M2plus[W] + 20. Mp[4, w]) +
       70. Cos[4. theta] (9. Ep[4, w] + 16. (-Mm[4, w] + Mp[4, w])) +
       2. Cos[theta] (192. Em[3, w] + 360. Em[5, w] + 192. E1plus[W] +
           360. Ep[3, w] + 525. Ep[5, w] - 64. M1minus[W] - 24. Mm[3, w] -
           15. Mm[5, w] + 64. M1plus[W] + 24. Mp[3, w] + 15. Mp[5, w]) +
       63. Cos[5. theta] (11. Ep[5, w] + 25. (-Mm[5, w] + Mp[5, w])) +
       5. Cos[3. theta] (112. Em[5, w] + 112. Ep[3, w] +
           9. (21. Ep[5, w] - 16. Mm[3, w] - 7. Mm[5, w] + 16. Mp[3, w] + 7. Mp[5, w])));
   F2 = 1/64 (64 M1minus[W] + 144 (3 + 5 Cos[2 theta]) Mm[3, w] +
       75 (15 + 28 Cos[2 theta] + 21 Cos[4 theta]) Mm[5, w] +
```

```
        64 (2 M1plus[W] + 3 (3 + 5 Cos[2 theta]) Mp[3, w]) +
        16 Cos[theta] (24 M2minus[W] + 20 (1 + 7 Cos[2 theta]) Mm[4, w] +
           36 M2plus[W] + 25 (1 + 7 Cos[2 theta]) Mp[4, w]) +
        90 (15 + 28 Cos[2 theta] + 21 Cos[4 theta]) Mp[5, w]);
    F3 = 1/64 (192 Em[3, w] + 1200 Em[5, w] + 192 E1plus[W] + 1200 Ep[3, w] +
        3675 Ep[5, w] + 64 M1minus[W] + 624 Mm[3, w] + 2325 Mm[5, w] - 64 M1plus[W] -
        624 Mp[3, w] + 24 Cos[theta] (40 Em[4, w] + 40 E2plus[W] + 175 Ep[4, w] +
           4 (4 M2minus[W] + 25 Mm[4, w] - 4 M2plus[W] - 25 Mp[4, w])) +
        280 Cos[3 theta] (9 Ep[4, w] + 4 Mm[4, w] - 4 Mp[4, w]) +
        60 Cos[2 theta] (28 Em[5, w] + 28 Ep[3, w] + 105 Ep[5, w] +
           12 Mm[3, w] + 63 Mm[5, w] - 12 Mp[3, w] - 63 Mp[5, w]) +
        315 Cos[4 theta] (11 Ep[5, w] + 5 Mm[5, w] - 5 Mp[5, w]) - 2325 Mp[5, w]);
    F4 = 3/8 (-8 E2minus[W] - 50 Em[4, w] - 8 E2plus[W] - 50 Ep[4, w] -
        8 M2minus[W] - 50 Mm[4, w] + 8 M2plus[W] -
        70 Cos[2 theta] (Em[4, w] + Ep[4, w] + Mm[4, w] - Mp[4, w]) + 50 Mp[4, w] -
        5 Cos[theta] (8 Em[3, w] + 35 Em[5, w] + 8 Ep[3, w] + 35 Ep[5, w] +
           8 Mm[3, w] + 35 Mm[5, w] - 8 Mp[3, w] - 35 Mp[5, w]) -
        105 Cos[3 theta] (Em[5, w] + Ep[5, w] + Mm[5, w] - Mp[5, w]));
    F1 = -I * (4. * 3.14159 * w) / Sqrt[mnucl * mnucl] * F1;
    F2 = -I * (4. * 3.14159 * w) / Sqrt[mnucl * mnucl] * F2;
    F3 = -I * (4. * 3.14159 * w) / Sqrt[mnucl * mnucl] * F3;
    F4 = -I * (4. * 3.14159 * w) / Sqrt[mnucl * mnucl] * F4;
    dsdo = Abs[F1] ^ 2 + 0.5 * (Abs[F2] ^ 2 + Abs[F3] ^ 2 + Abs[F4] ^ 2 +
         2. * Re[(F1 + F3 * Cos[theta]) * Conjugate[F4]] ) * (Sin[theta]) ^ 2;
    easymmetry = (Abs[F1] ^ 2. + Re[ Conjugate[F2] * (F3 + F4 * Cos[theta]) +
           Conjugate[F1] * F4 ] * (Sin[theta]) ^ 2.) / dsdo;
    fasymmetry = -Re[Conjugate[F2] * (F1 + F4 * (Sin[theta]) ^ 2.) -
         Conjugate[F1] * (F3 + F4 * Cos[theta])] * Sin[theta] / dsdo;
    tasymmetry = Im[(-F2 + F3 + F4 * Cos[theta]) * Conjugate[F1] +
        (F3 + F4 * Cos[theta]) * Conjugate[F4] * (Sin[theta]) ^ 2.] * Sin[theta] / dsdo;
    sasymmetry = 0.5 * (Abs[F2] ^ 2. - Abs[F3] ^ 2. - Abs[F4] ^ 2. -
        2. * Re[(F1 + F3 * Cos[theta]) * Conjugate[F4]]) * (Sin[theta]) ^ 2. / dsdo;
    qeta = 1. / (2. * w) * Sqrt[(w ^ 2. - (mnucl + mπ0) ^ 2.) * (w ^ 2. - (mnucl - mπ0) ^ 2.)];
    qgamma = 1. / (2. * w) * (w ^ 2. - mnucl ^ 2.);
    dsdotrue =
     mnucl * mnucl / (4. * 3.14159 * w) ^ 2. * Abs[qeta] / Abs[qgamma] * dsdo / 100.;
  {easymmetry, fasymmetry, tasymmetry, sasymmetry, dsdotrue});
```

```
vec = {E0p, E1p, E2p, E2m, M1p, M1m, M2p, M2m};

Wt[θ_, E0p_, E1p_, E2p_, E2m_, M1p_, M1m_, M2p_, M2m_] =
  Re[Conjugate[vec].T0.vec] + Re[Conjugate[vec].T1.vec] * LegendreP[1, Cos[θ]] +
   Re[Conjugate[vec].T2.vec] * LegendreP[2, Cos[θ]] + Re[Conjugate[vec].T3.vec] *
    LegendreP[3, Cos[θ]] + Re[Conjugate[vec].T4.vec] * LegendreP[4, Cos[θ]];

Ws[θ_, E0p_, E1p_, E2p_, E2m_, M1p_, M1m_, M2p_, M2m_] =
  (Re[Conjugate[vec].S0.vec] + Re[Conjugate[vec].S1.vec] * LegendreP[1, Cos[θ]] +
     Re[Conjugate[vec].S2.vec] * LegendreP[2, Cos[θ]]) * Sin[θ]^2;

Wf[θ_, E0p_, E1p_, E2p_, E2m_, M1p_, M1m_, M2p_, M2m_] =
  (Re[Conjugate[vec].F0.vec] + Re[Conjugate[vec].F1.vec] * LegendreP[1, Cos[θ]] +
     Re[Conjugate[vec].F2.vec] * LegendreP[2, Cos[θ]] +
     Re[Conjugate[vec].F3.vec] * LegendreP[3, Cos[θ]]) * Sin[θ];
```

Parameters and constants

```
mπp = 139.57; (* mass of π+ *)
mπ0 = 134.98; (* mass of π0 *)
Mp = 938.27; (* mass of proton *)
Mn = 939.565;
β = 3.35/(1000 * mπp); (* cusp parameter *)
factor = 3.894 × 10^8;
(* unit conversion factor: [MeV]^-2 to [μbarn] *)
```

Define the Multipole Functions

```
nmaxs = 4;
nmaxp = 4;
nmaxd = 4;
nmaxsi = 2;
nmaxpi = 0;
onsi = 1;
onpi = 1;
ond = 1;
q[W_, m_] = 1/(2 * W) (√((W^2 - (m - Mp)^2) * (W^2 - (m + Mp)^2)));
(* center of mass frame momentum *)
qn[W_] = 1/(2 * W) (√((W^2 - (mπp - Mn)^2) * (W^2 - (mπp + Mn)^2)));
ω[W_] = √(q[W, mπ0]^2 + mπ0^2); (* pion energy *);
```

$$\mathrm{E0plus}[W\_] = \frac{1}{1000}\left(\frac{1}{\mathrm{m\pi p}}\right) * \left(\mathrm{Sum}\left[\frac{\mathrm{e}[i]}{10^{-(i)}} * \left(\frac{\omega[W]-\mathrm{m\pi 0}}{\mathrm{m\pi p}}\right)^{i}, \{i, 0, \mathrm{nmaxs}\}\right]\right) + \mathrm{onsi} * I * \beta * \frac{\mathrm{qn}[W]}{\mathrm{m\pi p}} * \left(\mathrm{Sum}\left[\frac{\mathrm{ei}[i]}{10^{-(i)}} * \left(\frac{\mathrm{qn}[W]}{\mathrm{m\pi p}}\right)^{2*i}, \{i, 0, \mathrm{nmaxsi}\}\right]\right);$$

$$\mathrm{P1}[W\_] = \frac{1}{1000}\left(\frac{1}{\mathrm{m\pi p}}\right) * \left(\frac{q[W, \mathrm{m\pi 0}]}{\mathrm{m\pi p}} * \mathrm{Sum}\left[\frac{\mathrm{p1}[i]}{10^{-(i)}} * \left(\frac{\omega[W]-\mathrm{m\pi 0}}{\mathrm{m\pi p}}\right)^{i}, \{i, 0, \mathrm{nmaxp}\}\right] + \mathrm{onpi} * I * \frac{q[W, \mathrm{m\pi 0}]^4}{\mathrm{m\pi p}^4}\,\mathrm{Sum}\left[\frac{\mathrm{p1i}[i]}{10^{-(i)}} * \left(\frac{\omega[W]-\mathrm{m\pi 0}}{\mathrm{m\pi p}}\right)^{i}, \{i, 0, 0\}\right]\right);$$

$$\mathrm{P2}[W\_] = \frac{1}{1000}\left(\frac{1}{\mathrm{m\pi p}}\right) * \left(\frac{q[W, \mathrm{m\pi 0}]}{\mathrm{m\pi p}} * \mathrm{Sum}\left[\frac{\mathrm{p2}[i]}{10^{-(i)}} * \left(\frac{\omega[W]-\mathrm{m\pi 0}}{\mathrm{m\pi p}}\right)^{i}, \{i, 0, \mathrm{nmaxp}\}\right] + \mathrm{onpi} * I * \frac{q[W, \mathrm{m\pi 0}]^4}{\mathrm{m\pi p}^4}\,\mathrm{Sum}\left[\frac{\mathrm{p2i}[i]}{10^{-(i)}} * \left(\frac{\omega[W]-\mathrm{m\pi 0}}{\mathrm{m\pi p}}\right)^{i}, \{i, 0, 0\}\right]\right);$$

$$\mathrm{P3}[W\_] = \frac{1}{1000}\left(\frac{1}{\mathrm{m\pi p}}\right) * \left(\frac{q[W, \mathrm{m\pi 0}]}{\mathrm{m\pi p}} * \mathrm{Sum}\left[\frac{\mathrm{p3}[i]}{10^{-(i)}} * \left(\frac{\omega[W]-\mathrm{m\pi 0}}{\mathrm{m\pi p}}\right)^{i}, \{i, 0, \mathrm{nmaxp}\}\right] + \mathrm{onpi} * I * \frac{q[W, \mathrm{m\pi 0}]^4}{\mathrm{m\pi p}^4}\,\mathrm{Sum}\left[\frac{\mathrm{p3i}[i]}{10^{-(i)}} * \left(\frac{\omega[W]-\mathrm{m\pi 0}}{\mathrm{m\pi p}}\right)^{i}, \{i, 0, 0\}\right]\right);$$

$$\mathrm{E1plus}[W\_] = \frac{1}{6} * \left(\mathrm{P1}[W] + \mathrm{P2}[W]\right);\ (*\ \text{p-wave}\ *)$$

$$\mathrm{M1plus}[W\_] = \frac{1}{6} * \left(\mathrm{P1}[W] - \mathrm{P2}[W]\right) + \frac{\mathrm{P3}[W]}{3};\ (*\ \text{p-wave}\ *)$$

$$\mathrm{M1minus}[W\_] = \frac{1}{3} * \left(\mathrm{P3}[W] + \mathrm{P2}[W] - \mathrm{P1}[W]\right);\ (*\ \text{p-wave}\ *)$$

$$\mathrm{E2plus}[W\_] = \frac{1}{1000}\left(\frac{1}{\mathrm{m\pi p}}\right) * \left(\frac{q[W, \mathrm{m\pi 0}]^2}{\mathrm{m\pi p}^2} * \left(\mathrm{Sum}\left[\frac{\mathrm{d2p0}[i]}{10^{-(i)}} * \left(\frac{\omega[W]-\mathrm{m\pi 0}}{\mathrm{m\pi p}}\right)^{i}, \{i, 0, \mathrm{nmaxd}\}\right]\right)\right) * \mathrm{ond};$$

$$\mathrm{E2minus}[W\_] = \frac{1}{1000}\left(\frac{1}{\mathrm{m\pi p}}\right) * \left(\frac{q[W, \mathrm{m\pi 0}]^2}{\mathrm{m\pi p}^2} * \left(\mathrm{Sum}\left[\frac{\mathrm{d2m0}[i]}{10^{-(i)}} * \left(\frac{\omega[W]-\mathrm{m\pi 0}}{\mathrm{m\pi p}}\right)^{i}, \{i, 0, \mathrm{nmaxd}\}\right]\right)\right) * \mathrm{ond};$$

$$\mathrm{M2plus}[W\_] = \frac{1}{1000}\left(\frac{1}{\mathrm{m\pi p}}\right) * \left(\frac{q[W, \mathrm{m\pi 0}]^2}{\mathrm{m\pi p}^2} * \left(\mathrm{Sum}\left[\frac{\mathrm{dm2p0}[i]}{10^{-(i)}} * \left(\frac{\omega[W]-\mathrm{m\pi 0}}{\mathrm{m\pi p}}\right)^{i}, \{i, 0, \mathrm{nmaxd}\}\right]\right)\right) * \mathrm{ond};$$

$$\mathrm{M2minus}[W\_] = \frac{1}{1000}\left(\frac{1}{\mathrm{m\pi p}}\right) * \left(\frac{q[W, \mathrm{m\pi 0}]^2}{\mathrm{m\pi p}^2} * \left(\mathrm{Sum}\left[\frac{\mathrm{dm2m0}[i]}{10^{-(i)}} * \left(\frac{\omega[W]-\mathrm{m\pi 0}}{\mathrm{m\pi p}}\right)^{i}, \{i, 0, \mathrm{nmaxd}\}\right]\right)\right) * \mathrm{ond};$$

$$\mathrm{Dxs}[W\_, \theta\_] := \mathrm{factor} * \frac{q[W, \mathrm{m\pi 0}]}{q[W, 0]} * \mathrm{Wt}[\theta, \mathrm{E0plus}[W], \mathrm{E1plus}[W], \mathrm{E2plus}[W], \mathrm{E2minus}[W], \mathrm{M1plus}[W], \mathrm{M1minus}[W], \mathrm{M2plus}[W], \mathrm{M2minus}[W]]$$

$$\Sigma[W\_, \theta\_] := -\mathrm{Ws}[\theta, \mathrm{E0plus}[W], \mathrm{E1plus}[W], \mathrm{E2plus}[W], \mathrm{E2minus}[W], \mathrm{M1plus}[W], \mathrm{M1minus}[W], \mathrm{M2plus}[W], \mathrm{M2minus}[W]] \big/ \mathrm{Wt}[\theta, \mathrm{E0plus}[W], \mathrm{E1plus}[W], \mathrm{E2plus}[W], \mathrm{E2minus}[W], \mathrm{M1plus}[W], \mathrm{M1minus}[W], \mathrm{M2plus}[W], \mathrm{M2minus}[W]]$$

```
A = ec[W, θ] /. Flatten[
    {Table[{Ep[i, W] → 0, Em[i, W] → 0, Mp[i, W] → 0, Mm[i, W] → 0}, {i, 3, 5}]}];
sigmaT[W_, θ_] = A[[3]] * A[[5]] * factor * 10^2;
```

```
parms = Flatten[
   Join[Table[{e[i]}, {i, 0, nmaxs}], Table[{p1[i], p2[i], p3[i]}, {i, 0, nmaxp}],
    Table[{d2p0[i], d2m0[i], dm2p0[i], dm2m0[i]}, {i, 0, nmaxd}], Table[{ei[i]},
     {i, 0, nmaxsi}], Table[{p1i[i], p2i[i], p3i[i]}, {i, 0, nmaxpi}]]];
Parms = Flatten[Join[Table[{e[i]}, {i, 0, nmaxs}],
    Table[{p1[i], p2[i], p3[i]}, {i, 0, nmaxp}],
    Table[{d2p0[i], d2m0[i], dm2p0[i], dm2m0[i]}, {i, 0, nmaxd}], Table[{ei[i]},
     {i, 0, nmaxsi}], Table[{p1i[i], p2i[i], p3i[i]}, {i, 0, nmaxpi}]]];
```

Define χ^2 With the LASSO

```
Chisq[λ_] := Sum[(1/AngleDXS[[i, j, 3]])^2 *
     (Dxs[EnergyDXS[[i]], AngleDXS[[i, j, 1]]] - AngleDXS[[i, j, 2]])^2, {i, 1,
     Length[AngleDXS]}, {j, 1, Length[AngleDXS[[i]]]}] + Sum[(1/angleASY[[i, j, 3]])^2 *
     (Σ[energyASY[[i]], angleASY[[i, j, 1]]] - angleASY[[i, j, 2]])^2, {i, 1,
     Length[angleASY]}, {j, 1, Length[angleASY[[i]]]}] + Sum[(1/AngleT[[i, j, 3]])^2 *
     (sigmaT[EnergyT[[i]], AngleT[[i, j, 1]]] - AngleT[[i, j, 2]])^2,
    {i, 1, Length[AngleT]}, {j, 1, Length[AngleT[[i]]]}] +
   λ^4 * Sum[Abs[parms[[i]]], {i, 1, Length[parms]}];
```

Initialized Guesses and Run through the LASSO Routine

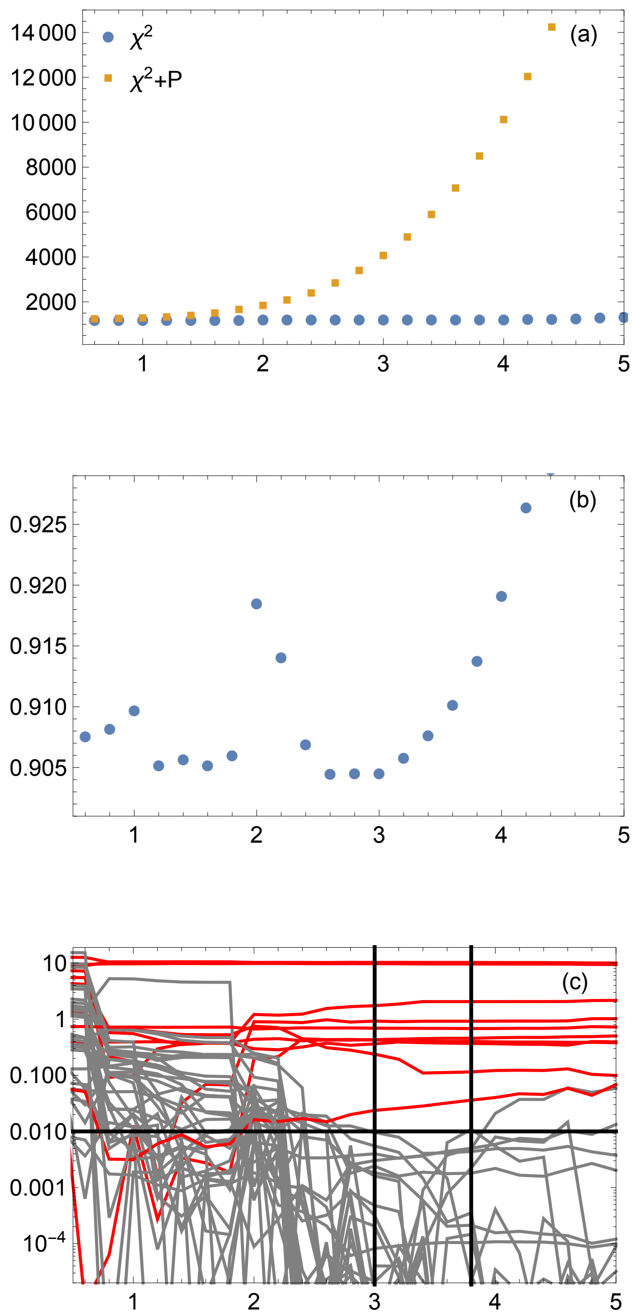
```
Parms = {{e[0], -0.4535575460681614`}, {e[1], -0.39824968299305474`},
   {e[2], -1.2608827224897667`*^-8}, {e[3], -6.747275303446579`*^-9},
   {e[4], -0.00064397827097114994`}, {p1[0], 9.692069521667984`},
   {p2[0], -9.464459025227383`}, {p3[0], 10.370018406489685`},
   {p1[1], 0.4494357050842037`}, {p2[1], -2.355139838724338`},
   {p3[1], 0.8416839776362222`}, {p1[2], -4.941176916514989`*^-9},
   {p2[2], -0.002212688277288109`}, {p3[2], 3.511767442356004`*^-7},
   {p1[3], -8.820031520098282`*^-7}, {p2[3], -1.5694245809466437`*^-8},
   {p3[3], -7.648263268625836`*^-9}, {p1[4], -0.00007739137590943576`},
   {p2[4], 0.0001190951289160686`}, {p3[4], -0.000041940670558400557`},
   {d2p0[0], -0.0007081714535802684`}, {d2m0[0], -1.1770155564894006`*^-9},
   {dm2p0[0], -9.558367398545279`*^-9}, {dm2m0[0], 0.0008562038079498057`},
   {d2p0[1], -1.5446695702253405`*^-8}, {d2m0[1], -5.395676961882261`*^-9},
   {dm2p0[1], 2.3854278673173978`*^-9}, {dm2m0[1], -2.8897424039043888`*^-9},
```

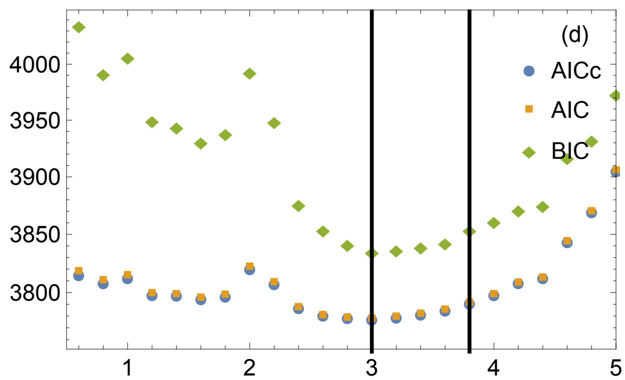
```
    {d2p0[2], -3.975742685623888`*^-9}, {d2m0[2], -5.562342985473301`*^-10},
    {dm2p0[2], 0.0002398870891657044`}, {dm2m0[2], -1.2051601587072032`*^-8},
    {d2p0[3], 0.001967186241741635`}, {d2m0[3], -4.314048753480795`*^-9},
    {dm2p0[3], -2.9261990299821504`*^-9}, {dm2m0[3], -0.0032103149165979505`},
    {d2p0[4], -0.00025598289826616054`}, {d2m0[4], -0.00040079561602610617`},
    {dm2p0[4], -1.967667695755569`*^-8}, {dm2m0[4], -3.596595395325732`*^-9},
    {ei[0], 0.7152191473733729`}, {ei[1], 0.0009588176915899808`},
    {ei[2], -0.00023018581831353046`}, {p1i[0], -5.435735068824109`*^-9},
    {p2i[0], -5.068001881583312`*^-9}, {p3i[0], -6.84327588484269`*^-9}};
init = Parms;

chilist = {};
chinoplist = {};
palist = {};
nopalist = {};
Do[
 fitresults = FindMinimum[Chisq[λ], Parms,
   MaxIterations → Infinity, Gradient → "FiniteDifference"];
 chisqwp = fitresults[[1]];
 pars = fitresults[[2]];
 chisq = Chisq[0] /. pars;
 AppendTo[chilist, {λ, chisqwp}];
 AppendTo[chinoplist, {λ, chisq}];
 Parms = Table[{pars[[i, 1]], pars[[i, 2]]}, {i, 1, Length[pars]}];
 kkcount = 0;
 Do[If[Abs[pars[[i, 2]]] > 10^-2, kkcount = kkcount + 1], {i, 1, Length[parms]}];
 AppendTo[nopalist, {λ, kkcount}];
 AppendTo[palist, {λ, pars[[ ;; , 2]]}];
 Print[λ];
 Print[chisq];, {λ, 0, 5, .2}]
```

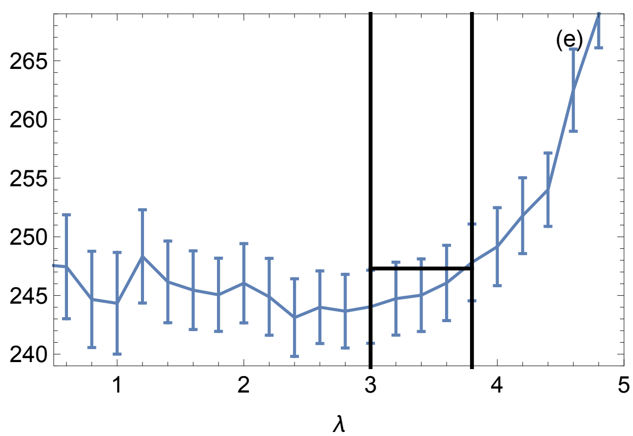
Plots

```
Needs["ErrorBarPlots`"]
```

```
chisqr = Table[
    {chisq[[i, 1]], chisq[[i, 2]] / (nodat - nopalist[[i, 2]])}, {i, 1, Length[chisq]}];
plot1 = ListPlot[{chisq, chisqwp}, Frame → True, PlotRange → {{0.5, 5}, {100, 15 000}},
   PlotLegends → Placed[{"χ²", "χ²+P"}, {Left, Top}],
   FrameLabel → {{"", ""}, {"", ""}}, LabelStyle → 12, PlotMarkers → Automatic,
   Epilog → Text[Style["(a)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}]]
plot2 = ListPlot[chisqr, Frame → True, PlotRange → {{0.5, 5}, {.901, .929}},
   FrameLabel → {{"", ""}, {"", ""}}, LabelStyle → 12, PlotMarkers → Automatic,
   Epilog → Text[Style["(b)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}]]
tab1 = Table[{3.8, i}, {i, -1000, 4500}];
tab2 = Table[{3, i}, {i, -1000, 4500}];
plot3 = Show[ListLogPlot[dataPar,
    Joined → True, PlotRange → {{0.5, 5}, {2 * 10^-5, 19}}, Frame → True,
    Epilog → Text[Style["(c)", FontSize → 12], Scaled[{0.9, 0.9}], {-1, 0}],
    PlotStyle → Table[If[Abs[palist[[16, 2, i]]] > 10^-2, Red, Gray],
      {i, 1, Length[palist[[20, 2]]]}], FrameLabel → {{"", ""}, {"", ""}}],
   LogPlot[10^-2, {x, 0.5, 5}, PlotStyle → {Thick, Black},
    PlotRange → {{0.5, 5}, {2 * 10^-5, 19}}],
   ListPlot[tab1, Joined → True, PlotStyle → {Thick, Black}],
   ListPlot[tab2, Joined → True, PlotStyle → {Thick, Black}], LabelStyle → 12]
plot4 = Show[ListLogPlot[{aic, aicc, bic},
    Epilog → Text[Style["(d)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}],
    Frame → True, PlotRange → {{0.5, 5}, {3752, 4049}},
    PlotLegends → Placed[{"AICc", "AIC", "BIC"}, {0.9, 0.7}],
    FrameLabel → {{"", ""}, {"", ""}}, LabelStyle → 12, PlotMarkers → Automatic],
   ListPlot[tab1, Joined → True, PlotStyle → {Thick, Black}],
   ListPlot[tab2, Joined → True, PlotStyle → {Thick, Black}]]
tab = Table[{3, i}, {i, 230, 280, .1}];
plot5 = Show[ErrorListPlot[crossplot, Frame → True, Joined → True,
    Epilog → Text[Style["(e)", FontSize → 12], Scaled[{0.88, 0.93}], {-1, 0}],
    PlotRange → {{0.5, 5}, {239, 269}}, FrameLabel → {{"", ""}, {"λ", ""}},
    LabelStyle → 12], Plot[247.3, {x, 3, 3.8}, PlotStyle → {Thick, Black}],
   ListPlot[tab2, Joined → True, PlotStyle → {Thick, Black}],
   ListPlot[tab1, Joined → True, PlotStyle → {Thick, Black}]]
```


14 000
12 000
10 000
8000
6000
4000
2000
1
2
3
4
5
χ^2
χ^2+P
(a)
0.925
0.920
0.915
0.910
0.905
(b)
10
1
0.100
0.010
0.001
10^{-4}
(c)


(d)
AICc
AIC
BIC
4000
3950
3900
3850
3800
1
2
3
4
5


(e)
265
260
255
250
245
240
1
2
3
4
5
λ