Functional Definitions

```
SetDirectory[NotebookDirectory[]];
Needs["ErrorBarPlots`"]

plotGrid[l_List, w_, h_, opts : OptionsPattern[]] :=
 Module[{nx, ny, sidePadding = OptionValue[plotGrid, ImagePadding],
   topPadding = 0, widths, heights, dimensions, positions,
   frameOptions = FilterRules[{opts}, FilterRules[Options[Graphics],
       Except[{ImagePadding, Frame, FrameTicks}]]]}, {ny, nx} = Dimensions[l];
  widths = (w - 2 sidePadding) / nx Table[1, {nx}];
  widths[[1]] = widths[[1]] + sidePadding;
  widths[[-1]] = widths[[-1]] + sidePadding;
  heights = (h - 2 sidePadding) / ny Table[1, {ny}];
  heights[[1]] = heights[[1]] + sidePadding;
  heights[[-1]] = heights[[-1]] + sidePadding;
  positions = Transpose@
    Partition[Tuples[Prepend[Accumulate[Most[#]], 0] & /@ {widths, heights}], ny];
  Graphics[Table[Inset[Show[l[[ny - j + 1, i]],
      ImagePadding → {{If[i == 1, sidePadding, 0], If[i == nx, sidePadding, 0]},
        {If[j == 1, sidePadding, 0], If[j == ny, sidePadding, topPadding]}},
      AspectRatio → Full], positions[[j, i]], {Left, Bottom},
     {widths[[i]], heights[[j]]}], {i, 1, nx}, {j, 1, ny}],
   PlotRange → {{0, w}, {0, h}}, ImageSize → {w, h},
   Evaluate@Apply[Sequence, frameOptions]]]
Options[plotGrid] = {ImagePadding → 60};

importdata[link_] :=
 Module[{rawdata, temp, cosdata, data}, rawdata = Import[link, "Table"];
  cosdata = Select[Select[rawdata, Length[#] == 3 &], VectorQ[#, NumberQ] &];
  data = Table[{ArcCos[cosdata[[i, 1]]], cosdata[[i, 2]], cosdata[[i, 3]]},
    {i, 1, Length[cosdata]}];
  temp = Table[{}, {i, 1, rawdata[[1, 1]]}];
  For[i = 1;
    j = 1;, i < Length[data], If[data[[i + 1, 1]] > data[[i, 1]], j++]
     i++, AppendTo[temp[[j]], data[[i]]]]
   AppendTo[temp[[j]], data[[i]]];
  temp]

importenergy[link_] := Module[{rawdata, temp, data}, rawdata = Import[link, "Table"];
   data = Select[Select[rawdata, Length[#] == 2 &], VectorQ[#, NumberQ] &];
```

```
    temp = Table[data[[i, 2]], {i, 1, Length[data]}];
    temp];

plotter[data_, energy_, function_, fit_, label_, rows_, columns_, width_, height_] :=
 Monitor[Module[{temp, mins, maxes, plot},
    temp = Table[If[4 * i + j <= Length[data], Max[Flatten[data[[4 * i + j]]]], 0],
      {i, 0, rows - 1}, {j, 1, columns}];
    maxes = 1.2 * Flatten[Table[Max[temp[[i]]], {i, 0 + 1, rows}, {j, 1, columns}]];
    temp = Table[If[4 * i + j <= Length[data], Min[Flatten[data[[4 * i + j]]]], 0],
      {i, 0, rows - 1}, {j, 1, columns}];
    mins = 1.2 * Flatten[Table[Min[temp[[i]]], {i, 0 + 1, rows}, {j, 1, columns}]];
    plot = Table[If[4 * i + j > Length[data], Plot[0, {t, 0, Pi},
        FrameLabel → {{label, label}, {"θ [rad]", "θ [rad]"}}, PlotStyle → White,
        Frame → True, AspectRatio → 1, Axes → False, FrameTicks → All,
        PlotRange → {{0, Pi}, {mins[[4 * i + j]] - 4, maxes[[4 * i + j]]}}],
       Show[ErrorListPlot[data[[4 * i + j]],
         PlotRange → {{0, Pi}, {mins[[4 * i + j]] - 4, maxes[[4 * i + j]]}}, Frame → True,
         AspectRatio → 1, Epilog → Text[Style[energy[[i * 4 + j]] "MeV", FontSize → 8],
           Scaled[{0.03, 0.93}], {-1, 0}], Axes → False, FrameTicks → All,
         FrameLabel → {{label, label}, {"θ [rad]", "θ [rad]"}}], ListPlot[
         Table[{θ, function[energy[[4 * i + j]], θ] /. fit[[2]]}, {θ, 0, Pi, 0.1}],
         Joined → True, PlotStyle → Red]]], {i, 0, rows - 1}, {j, 1, columns}];
    plotGrid[plot, width, height]], ProgressIndicator[4 * i + j,
    {1, 4 * (rows - 1) + columns}]]

mpoleplot[fitresult_] :=
  Module[{plot1, plot2, plot3, plot4, plot5, plot6, plot7, plot8, plot9, plot10},
    plot1 = Plot[{Re[S01half[W]] /. fitresult[[2]], Im[S01half[W]] /. fitresult[[2]]},
      {W, mK + mXi, 3150}, Frame → True, PlotRange → {{mK + mXi, 3150}, All},
      FrameLabel → {{"S01", "S01"}, {"W [MeV]", "W [MeV]"}},
      PlotStyle → {{Thick, Dashed}, Thick}, LabelStyle → 12, AspectRatio → 1,
      Axes → False, PlotRange → {{mK + mXi, 3150}, {-.09, .09}}, FrameTicks → All];
    plot2 = Plot[{Re[S11half[W]] /. fitresult[[2]], Im[S11half[W]] /. fitresult[[2]]},
      {W, mK + mXi, 3265}, Frame → True, PlotRange → {{mK + mXi, 3265}, All}, FrameLabel →
       {{"S11", "S11"}, {"W [MeV]", "W [MeV]"}}, PlotStyle → {{Thick, Dashed}, Thick},
      LabelStyle → 12, AspectRatio → 1, Axes → False, FrameTicks → All];
    plot3 = Plot[{Re[P01half[W]] /. fitresult[[2]], Im[P01half[W]] /. fitresult[[2]]},
      {W, mK + mXi, 3150}, Frame → True, PlotRange → {{mK + mXi, 3150}, All}, FrameLabel →
       {{"P01", "P01"}, {"W [MeV]", "W [MeV]"}}, PlotStyle → {{Thick, Dashed}, Thick},
      LabelStyle → 12, AspectRatio → 1, Axes → False, FrameTicks → All];
    plot4 = Plot[{Re[P11half[W]] /. fitresult[[2]], Im[P11half[W]] /. fitresult[[2]]},
      {W, mK + mXi, 3265}, Frame → True, PlotRange → {{mK + mXi, 3265}, All}, FrameLabel →
       {{"P11", "P11"}, {"W [MeV]", "W [MeV]"}}, PlotStyle → {{Thick, Dashed}, Thick},
      LabelStyle → 12, AspectRatio → 1, Axes → False, FrameTicks → All];
```

```
plot5 = Plot[{Re[P03half[W]] /. fitresult[[2]], Im[P03half[W]] /. fitresult[[2]]},
  {W, mK + mXi, 3150}, Frame → True, PlotRange → {{mK + mXi, 3150}, {-.09, .09}},
  FrameLabel → {{"P03", "P03"}, {"W [MeV]", "W [MeV]"}},
  PlotStyle → {{Thick, Dashed}, Thick}, LabelStyle → 12,
  AspectRatio → 1, Axes → False, FrameTicks → All];
plot6 = Plot[{Re[P13half[W]] /. fitresult[[2]], Im[P13half[W]] /. fitresult[[2]]},
  {W, mK + mXi, 3265}, Frame → True, PlotRange → {{mK + mXi, 3265}, All}, FrameLabel →
   {{"P13", "P13"}, {"W [MeV]", "W [MeV]"}}, PlotStyle → {{Thick, Dashed}, Thick},
  LabelStyle → 12, AspectRatio → 1, Axes → False, FrameTicks → All];
plot7 = Plot[{Re[D03half[W]] /. fitresult[[2]], Im[D03half[W]] /. fitresult[[2]]},
  {W, mK + mXi, 3150}, Frame → True, PlotRange → {{mK + mXi, 3150}, All},
  FrameLabel → {{"D03", "D03"}, {"W [MeV]", "W [MeV]"}}, PlotStyle →
   {{Thick, Dashed}, Thick}, LabelStyle → 12, AspectRatio → 1, Axes → False];
plot8 = Plot[{Re[D13half[W]] /. fitresult[[2]], Im[D13half[W]] /. fitresult[[2]]},
  {W, mK + mXi, 3265}, Frame → True, PlotRange → {{mK + mXi, 3265}, All}, FrameLabel →
   {{"D13", "D13"}, {"W [MeV]", "W [MeV]"}}, PlotStyle → {{Thick, Dashed}, Thick},
  LabelStyle → 12, AspectRatio → 1, Axes → False, FrameTicks → All];
plot9 = Plot[{Re[D05half[W]] /. fitresult[[2]], Im[D05half[W]] /. fitresult[[2]]},
  {W, mK + mXi, 3150}, Frame → True, PlotRange → {{mK + mXi, 3150}, All},
  FrameLabel → {{"D05", "D05"}, {"W [MeV]", "W [MeV]"}},
  PlotStyle → {{Thick, Dashed}, Thick}, LabelStyle → 12,
  AspectRatio → 1, Axes → False, FrameTicks → All];
plot10 = Plot[{Re[D15half[W]] /. fitresult[[2]],
   Im[D15half[W]] /. fitresult[[2]]}, {W, mK + mXi, 3265},
  Frame → True, PlotRange → {{mK + mXi, 3265}, All}, FrameLabel →
   {{"D15", "D15"}, {"W [MeV]", "W [MeV]"}}, PlotStyle → {{Thick, Dashed}, Thick},
  LabelStyle → 12, AspectRatio → 1, Axes → False, FrameTicks → All];
plotGrid[{{plot1, plot2}, {plot3, plot4}, {plot5, plot6},
  {plot7, plot8}, {plot9, plot10}}, 600, 1000]];
mKp = 493.677; (*mass of K+*)
mKm = 493.677;(*mass of K-*)
mK0 = 497.648;(*mass of K0*)
mK = (mKp + mKm + mK0)/3;
mXi0 = 1314.86; (*mass of Ξ0*)
mXim = 1321.71;(*mass of Ξ-*)
mXi = (mXi0 + mXim)/2;
mP = 938.27; (* mass of proton *)
mN = 939.565;(*mass of nuetron*)
factor = 3.894 × 10^8;  (* unit conversion factor: [MeV]^-2 to [μbarn] *)
Λ = N[10^3]; (*1Gev=10^3Mev*)
```

Data Import

```
Cross1 = Import["Cross1toy.m"];
Cross2 = Import["Cross2toy.m"];
Diff1 = Import["Diff1toy.m"];
Diff2 = Import["Diff2toy.m"];
pol1 = Import["P1toy.m"];
pol2 = Import["P2toy.m"];
Diff1Energy = Table[W, {W, mKp + mXi + 100, 2800, 28}];
Diff2Energy = Table[W, {W, mKp + mXi + 100, 2800, 28}];
pol1Energy = Table[W, {W, mKp + mXi + 100, 2800, 56}];
pol2Energy = Table[W, {W, mKp + mXi + 100, 2800, 56}];
```

Plots of data and their partial waves

$$k[W\_, m1\_, m2\_] := \frac{1}{2 * W}\left(\sqrt{\left(\left(W^2 - (m1 - m2)^2\right) * \left(W^2 - (m1 + m2)^2\right)\right)}\right)$$

```
kf[W_] = k[W, mK, mXi];
ki[W_] = k[W, mK, mP];
jmax = 5 / 2;
ons = 1;
onp = 1;
ond = 1;
nopars = 6 * ons + 12 * onp + 12 * ond;
num = If[nopars == 6, 1, If[nopars == 18, 5, 9]];
pars =
  Flatten[Join[Table[{s[i], α[i]}, {i, 0, num}], Table[{ϕ[i]}, {i, 0, num - 1}]]];
```

$$S01half[W\_] := ons * \left(\left(\left(s[0] + \frac{ss[0]}{10}\,\frac{kf[W]}{\Lambda}\right)\text{Exp}\left[-\alpha[0]\text{^}2 * \frac{kf[W]^2}{\Lambda^2}\right] - x[0]\,\text{Exp}[I * ph[0]]\,\frac{\Gamma[0]}{2\left(W - M[0] + I\,\frac{\Gamma[0]}{2}\right)}\right)\text{Exp}[I * \phi[0]]\right) * \left(\frac{kf[W]}{\Lambda}\right)^{0+1/2}$$

$$S11half[W\_] := ons * \left(\left(s[1] + \frac{ss[1]}{10}\,\frac{kf[W]}{\Lambda}\right)\text{Exp}[I * \phi[1]]\right) * \left(\frac{kf[W]}{\Lambda}\right)^{0+1/2}\text{Exp}\left[-\alpha[1]\text{^}2 * \frac{kf[W]^2}{\Lambda^2}\right]$$

$$P01half[W\_] := onp * \left(\left(s[2] + \frac{ss[2]}{10}\,\frac{kf[W]}{\Lambda}\right)\text{Exp}[I * \phi[2]]\right) *$$

$$\left(\frac{kf[W]}{\Lambda}\right)^{1+1/2} \text{Exp}\left[-\alpha[2]\wedge 2 * \frac{kf[W]^2}{\Lambda^2}\right]$$

$$P03half[W\_] := onp * \left(\left(s[3] + \frac{ss[3]}{10}\ \frac{kf[W]}{\Lambda}\right) \text{Exp}[I * \phi[3]]\right) *$$
$$\left(\frac{kf[W]}{\Lambda}\right)^{1+1/2} \text{Exp}\left[-\alpha[3]\wedge 2 * \frac{kf[W]^2}{\Lambda^2}\right]$$

$$P11half[W\_] := onp * \left(\left(\left(s[4] + \frac{ss[4]}{10}\ \frac{kf[W]}{\Lambda}\right) \text{Exp}\left[-\alpha[4]\wedge 2 * \frac{kf[W]^2}{\Lambda^2}\right] -\right.\right.$$
$$\left.\left. x[4]\ \text{Exp}[I * ph[4]]\ \frac{\Gamma[4]}{2\left(W - M[4] + I\ \frac{\Gamma[4]}{2}\right)}\right) \text{Exp}[I * \phi[4]]\right) * \left(\frac{kf[W]}{\Lambda}\right)^{1+1/2}$$

$$P13half[W\_] := onp * \left(\left(s[5] + \frac{ss[5]}{10}\ \frac{kf[W]}{\Lambda}\right) \text{Exp}[I * \phi[5]]\right) *$$
$$\left(\frac{kf[W]}{\Lambda}\right)^{1+1/2} \text{Exp}\left[-\alpha[5]\wedge 2 * \frac{kf[W]^2}{\Lambda^2}\right]$$

$$D03half[W\_] := ond * \left(\left(s[6] + \frac{ss[6]}{10}\ \frac{kf[W]}{\Lambda}\right) \text{Exp}[I * \phi[6]]\right) *$$
$$\left(\frac{kf[W]}{\Lambda}\right)^{2+1/2} \text{Exp}\left[-\alpha[6]\wedge 2 * \frac{kf[W]^2}{\Lambda^2}\right]$$

$$D05half[W\_] := ond * \left(\left(\left(s[7] + \frac{ss[7]}{10}\ \frac{kf[W]}{\Lambda}\right) \text{Exp}\left[-\alpha[7]\wedge 2 * \frac{kf[W]^2}{\Lambda^2}\right] -\right.\right.$$
$$\left.\left. x[7]\ \text{Exp}[I * ph[7]]\ \frac{\Gamma[7]}{2\left(W - M[7] + I\ \frac{\Gamma[7]}{2}\right)}\right) \text{Exp}[I * \phi[7]]\right) * \left(\frac{kf[W]}{\Lambda}\right)^{2+1/2}$$

$$D13half[W\_] := ond * \left(\left(s[8] + \frac{ss[8]}{10}\ \frac{kf[W]}{\Lambda}\right) \text{Exp}[I * \phi[8]]\right)$$
$$\left(\frac{kf[W]}{\Lambda}\right)^{2+1/2} \text{Exp}\left[-\alpha[8]\wedge 2 * \frac{kf[W]^2}{\Lambda^2}\right]$$

$$D15half[W\_] := ond * \left(\left(\left(s[9] + \frac{ss[9]}{10}\ \frac{kf[W]}{\Lambda}\right) \text{Exp}\left[-\alpha[9]\wedge 2 * \frac{kf[W]^2}{\Lambda^2}\right] - x[9]\ \text{Exp}[I * ph[9]]\right.\right.$$
$$\left.\left. \frac{\Gamma[9]}{2\left(W - M[9] + I\ \frac{\Gamma[9]}{2}\right)}\right) \text{Exp}[I * \phi[9]]\right) * \left(\frac{kf[W]}{\Lambda}\right)^{2+1/2} /.\ \{\phi[9] \to 0\}$$

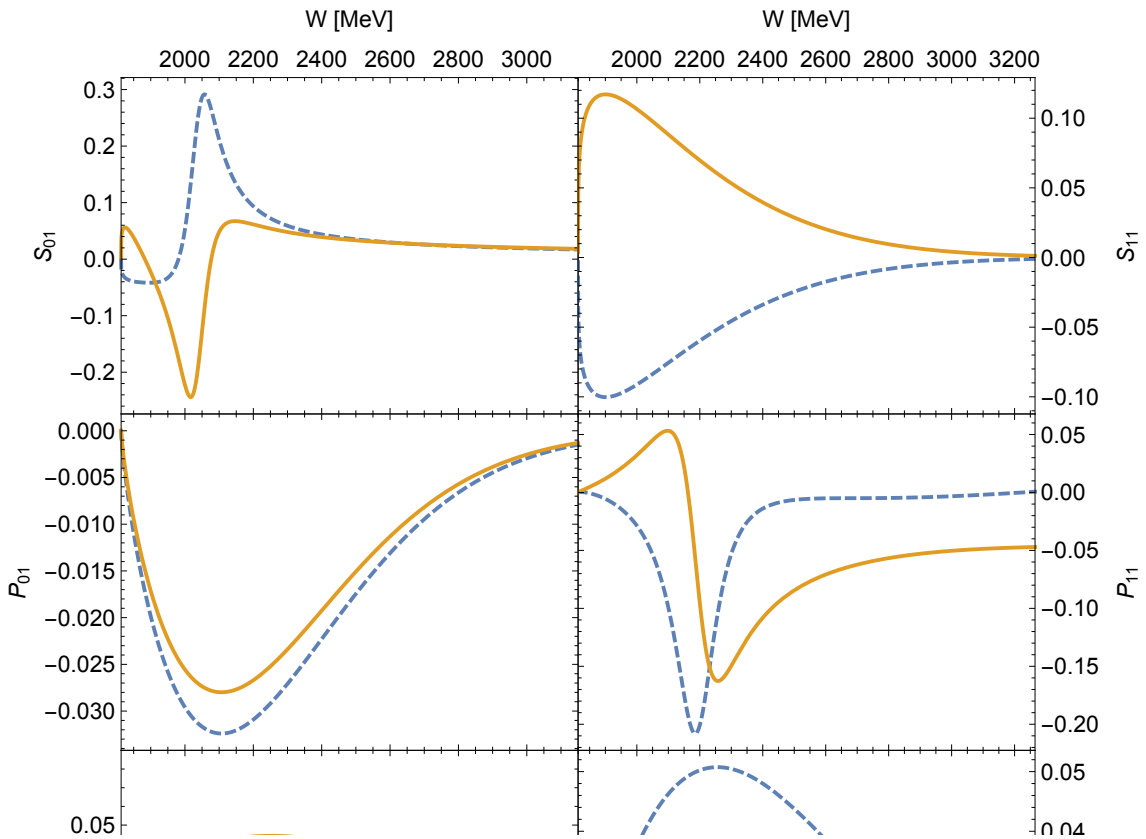
$$\tau 0_{-1/2} = \{S01half[W], P03half[W], D05half[W]\};$$
$$\tau 0_{+1/2} = \{P01half[W], D03half[W], 0\};$$
$$\tau 1_{-1/2} = \{S11half[W], P13half[W], D15half[W]\};$$
$$\tau 1_{+1/2} = \{P11half[W], D13half[W], 0\};$$

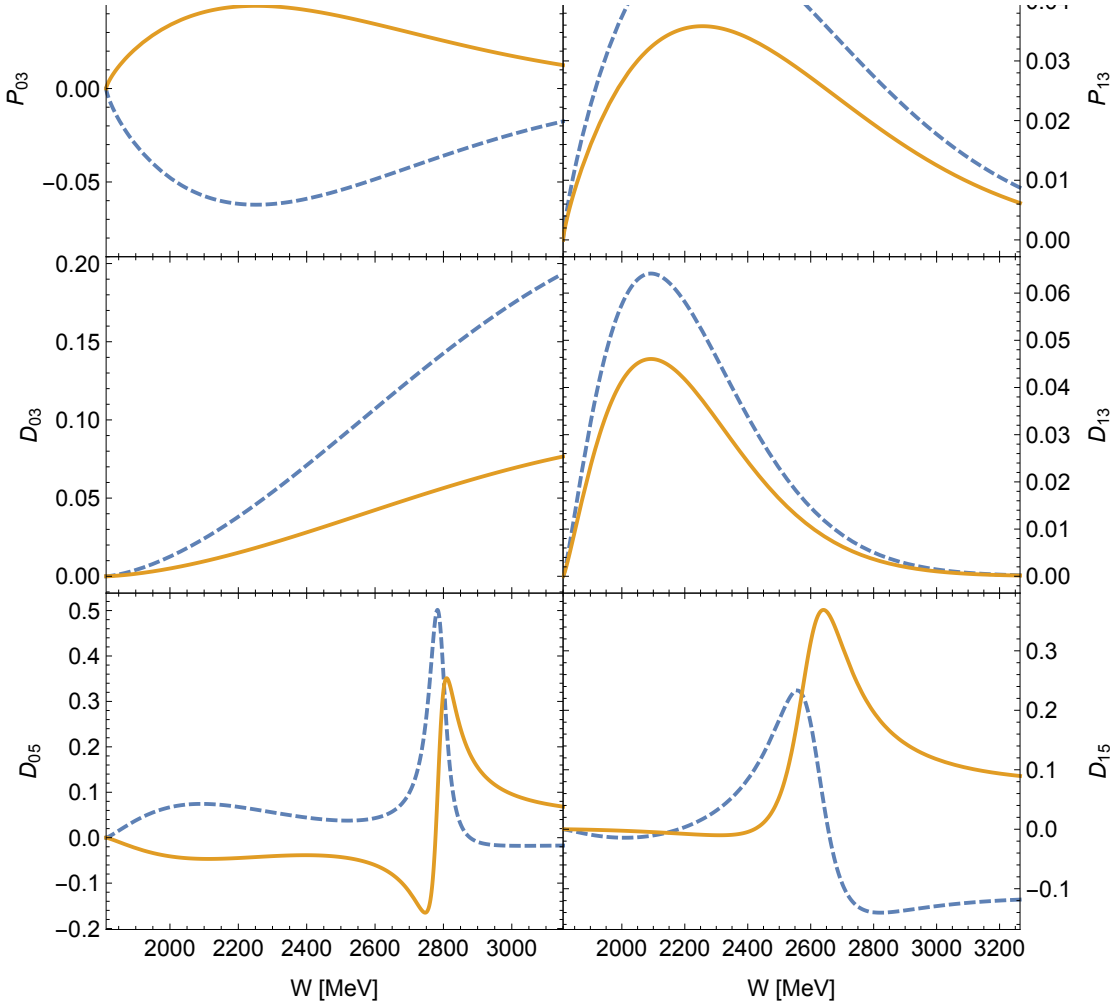
$$g0[\theta\_, W\_] = \frac{1}{2\ \text{Sqrt}[kf[W] * ki[W]]} *$$
$$\text{Sum}\left[(2J+1) * \left(\text{WignerD}\left[\{J, 1/2, 1/2\}, \theta\right] * \left(\tau 0_{-1/2}\left[\left[J+1/2\right]\right] + \tau 0_{+1/2}\left[\left[J+1/2\right]\right]\right)\right.\right.$$
$$\text{Cos}\left[\theta/2\right] + \text{WignerD}\left[\{J, -1/2, 1/2\}, \theta\right]$$
$$\left.\left.\left(\tau 0_{-1/2}\left[\left[J+1/2\right]\right] - \tau 0_{+1/2}\left[\left[J+1/2\right]\right]\right) \text{Sin}\left[\theta/2\right]\right), \{J, 1/2, jmax, 1\}\right];$$

$$h0[\theta\_, W\_] = \frac{-I}{2\ \text{Sqrt}[kf[W] * ki[W]]} * \text{Sum}\left[(2J+1) * \left(\text{WignerD}\left[\{J, 1/2, 1/2\}, \theta\right] *\right.\right.$$
$$\left(\tau 0_{-1/2}\left[\left[J+1/2\right]\right] + \tau 0_{+1/2}\left[\left[J+1/2\right]\right]\right) \text{Sin}\left[\theta/2\right] - \text{WignerD}\left[\{J, -1/2, 1/2\}, \theta\right]$$
$$\left.\left.\left(\tau 0_{-1/2}\left[\left[J+1/2\right]\right] - \tau 0_{+1/2}\left[\left[J+1/2\right]\right]\right) \text{Cos}\left[\theta/2\right]\right), \{J, 1/2, jmax, 1\}\right];$$

```
g1[θ_, W_] = 1/(2 Sqrt[kf[W] * ki[W]]) * Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] *
        (τ1_{-1/2}[[J + 1/2]] + τ1_{+1/2}[[J + 1/2]]) Cos[θ/2] + WignerD[{J, -1/2, 1/2}, θ]
        (τ1_{-1/2}[[J + 1/2]] - τ1_{+1/2}[[J + 1/2]]) Sin[θ/2]), {J, 1/2, jmax, 1}];
h1[θ_, W_] = -I/(2 Sqrt[kf[W] * ki[W]]) * Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] *
        (τ1_{-1/2}[[J + 1/2]] + τ1_{+1/2}[[J + 1/2]]) Sin[θ/2] - WignerD[{J, -1/2, 1/2}, θ]
        (τ1_{-1/2}[[J + 1/2]] - τ1_{+1/2}[[J + 1/2]]) Cos[θ/2]), {J, 1/2, jmax, 1}];


gre1[θ_, W_] := -1/2 g1[θ, W] + 1/2 g0[θ, W] (*<Ξ⁻k⁺| p K⁻ *)
hre1[θ_, W_] := -1/2 h1[θ, W] + 1/2 h0[θ, W] (*<Ξ⁻k⁺| p K⁻ *)

gre2[θ_, W_] := -1/2 g1[θ, W] - 1/2 g0[θ, W] (*<Ξ⁰k⁰| p K⁻ *)
hre2[θ_, W_] := -1/2 h1[θ, W] - 1/2 h0[θ, W] (*<Ξ⁰k⁰| p K⁻ *)


dσdΩ1[W_, θ_] :=
 factor * (Conjugate[gre1[θ, W]] gre1[θ, W] + Conjugate[hre1[θ, W]] * hre1[θ, W]) * kf[W]/ki[W]
 (*<Ξ⁻k⁺| p K⁻ *)
P1[W_, θ_] := 2 Re[gre1[θ, W] * Conjugate[hre1[θ, W]]]/(Abs[gre1[θ, W]]^2 + Abs[hre1[θ, W]]^2) * dσdΩ1[W, θ] (*<Ξ⁻k⁺| p K⁻ *)


dσdΩ2[W_, θ_] :=
 factor * (Conjugate[gre2[θ, W]] gre2[θ, W] + Conjugate[hre2[θ, W]] * hre2[θ, W]) * kf[W]/ki[W]
 (*<Ξ⁰k⁰| p K⁻ *)
P2[W_, θ_] := 2 Re[gre2[θ, W] * Conjugate[hre2[θ, W]]]/(Abs[gre2[θ, W]]^2 + Abs[hre2[θ, W]]^2) * dσdΩ2[W, θ] (*<Ξ⁰k⁰| p K⁻ *)
σ1[W_] =
  factor * 1/2 * 4 Pi/ki[W]^2 Sum[(2 J + 1) (Abs[-1/2 (τ1_{-1/2}[[J + 1/2]])] + 1/2 (τ0_{-1/2}[[J + 1/2]]))^2 +
      Abs[-1/2 (τ1_{1/2}[[J + 1/2]]) + 1/2 (τ0_{1/2}[[J + 1/2]])]^2), {J, 1/2, jmax, 1}];
σ2[W_] = factor * 1/2 * 4 Pi/ki[W]^2 Sum[(2 J + 1)
     (Abs[-1/2 (τ1_{-1/2}[[J + 1/2]]) - 1/2 (τ0_{-1/2}[[J + 1/2]])]^2 +
      Abs[-1/2 (τ1_{1/2}[[J + 1/2]]) - 1/2 (τ0_{1/2}[[J + 1/2]])]^2), {J, 1/2, jmax, 1}];
```

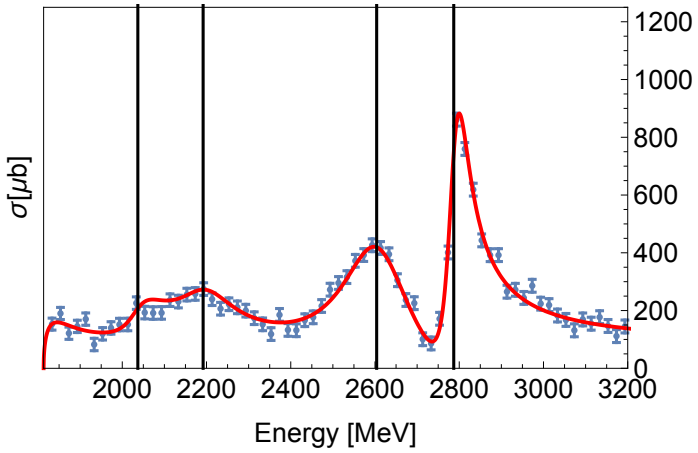
```
Fitresults = {s[0] → -0.34824198024328586`, α[0] → 3.763438455711533`,
    s[1] → -0.4153619899748145`, α[1] → -1.6504395877254527`,
    s[2] → 0.20791938959137438`, α[2] → 1.932359556897292`,
    s[3] → 0.3140263682123946`, α[3] → 1.4353226934402328`,
    s[4] → -0.09335240870654254`, α[4] → 1.2586482592270365`,
    s[5] → 0.19804403841470758`, α[5] → 1.5543752980111297`,
    s[6] → 0.08564949843750701`, α[6] → 0.8114266359183674`,
    s[7] → 1.8696909108351594`, α[7] → 2.38042861587167`, s[8] → 1.6784049993618166`,
    α[8] → 2.3546081341506007`, s[9] → -0.4910616507553648`,
    α[9] → 1.976612736930773`, ϕ[0] → -1.3759095905974634`,
    ϕ[1] → -.8628696533673115`, ϕ[2] → -2.4292106595489664`,
    ϕ[3] → -3.7601064302536518`, ϕ[4] → -3.5505168556001405`,
    ϕ[5] → 0.6146668029672369`, ϕ[6] → 0.37669927068590553`,
    ϕ[7] → -6.818425121493386`, ϕ[8] → 0.6225800989122515`, ss[0] → 2, ss[1] → 2,
    ss[2] → 2, ss[3] → 1/2, ss[4] → 2, ss[5] → 2, ss[6] → 2, ss[7] → 2, ss[8] → 2,
    ss[9] → 2, x[0] → .5, Γ[0] → 100, M[0] → 2036.962`, ph[0] → -.9, x[4] → .5,
    Γ[4] → 160, M[4] → 2191.96, ph[4] → -.8, x[7] → .5, Γ[7] → 60, M[7] → 2786.962`,
    ph[7] → -.7, x[9] → .5, Γ[9] → 200, M[9] → 2603.962`, ph[9] → -.5};
Options[plotGrid] = {ImagePadding → 62};
generator = mpoleplot[{1, Fitresults}]
```



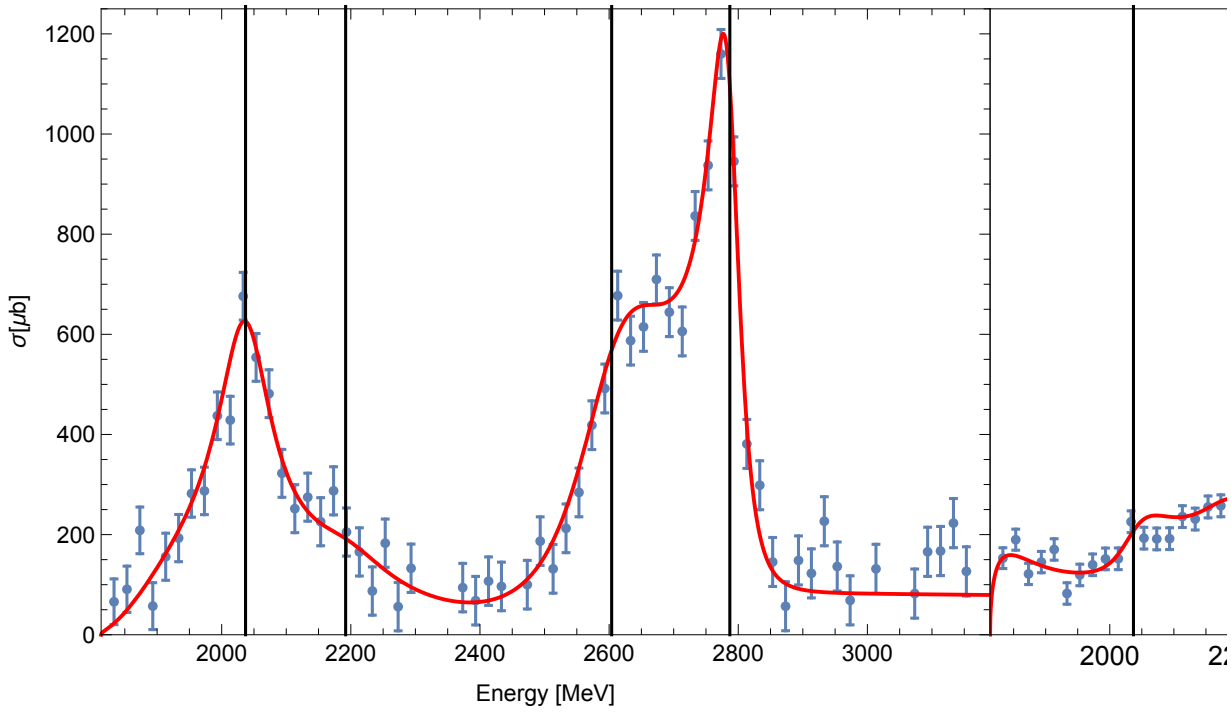

P_{03}
P_{13}
D_{03}
D_{13}
D_{05}
D_{15}
W [MeV]
W [MeV]

```
fig1 = plotter[Diff1, Diff1Energy,
  dσdΩ1, {1, Fitresults}, "dσ/dΩ [μb/sr]", 8, 4, 850, 2 * 650]
fig2 = plotter[Diff2, Diff2Energy, dσdΩ2, {1, Fitresults},
  "dσ/dΩ [μb/sr]", 8, 4, 850, 2 * 650]
fig3 = plotter[pol1, pol1Energy, P1, {1, Fitresults}, "dσ/dΩ [μb/sr]", 4, 4, 850, 2 * 550]
fig4 = plotter[pol2, pol2Energy, P2, {1, Fitresults}, "dσ/dΩ [μb/sr]", 4, 4, 850, 2 * 550]
fig5 =
 Show[ErrorListPlot[Cross1, Frame → True, FrameLabel → {"Energy [MeV]", "σ[μb]"}],
  Plot[σ1[W] /. Fitresults, {W, mK + mXi, 3229}, PlotStyle → Red,
   PlotRange → {{mK + mXi, 3229}, All}, LabelStyle → 12]]
fig6 = Show[ErrorListPlot[Cross2, Frame → True,
   FrameLabel → {"Energy [MeV]", "σ[μb]"}],
  Plot[σ2[W] /. Fitresults, {W, mK + mXi, 3229}, PlotStyle → Red,
   PlotRange → {{mK + mXi, 3229}, All}, LabelStyle → 12]]

fig5 =
  Show[ErrorListPlot[Cross1, Frame → True, FrameLabel → {"Energy [MeV]", "σ[μb]"},
    LabelStyle → 12, PlotRange → {{mK + mXi, 3190}, {0, 1250}}],
   Plot[σ1[W] /. Fitresults, {W, mK + mXi, 3229}, PlotStyle → {Red, Thick},
    PlotRange → {{mK + mXi, 3190}, All}, LabelStyle → 12]];

hm = ListPlot[{Table[{2036.96, i}, {i, 0, 1300, 100}],
    Table[{2191.96, i}, {i, 0, 1300, 100}], Table[{2786.96, i}, {i, 0, 1300, 100}],
    Table[{2603.96, i}, {i, 0, 1300, 100}]}, Joined → True, PlotStyle → Black];
```

```
fig5 =
  Show[ErrorListPlot[Cross1, Frame → True, FrameLabel → {"Energy [MeV]", "σ[μb]"},
    LabelStyle → 12, PlotRange → {{mK + mXi, 3190}, {0, 1250}}],
   Plot[σ1[W] /. Fitresults, {W, mK + mXi, 3229}, PlotStyle → {Red, Thick},
    PlotRange → {{mK + mXi, 3190}, All}, LabelStyle → 14], hm];
fig6 = Show[ErrorListPlot[Cross2, Frame → True,
   FrameLabel → {"Energy [MeV]", "σ[μb]"}, PlotRange → {{mK + mXi, 3200}, {0, 1250}},
   LabelStyle → 14, FrameTicks → {{None, All}, {All, None}}],
  Plot[σ2[W] /. Fitresults, {W, mK + mXi, 3229}, PlotStyle → {Red, Thick},
   PlotRange → {{mK + mXi, 3200}, {0, 1250}}, LabelStyle → 14], hm]
```



```
plotGrid[{{fig5, fig6}}, 1.5 * 700, 1.5 * 300]
```



LASSO

First we run from larger to smaller lambdas

```
k[W_, m1_, m2_] := 1/(2 * W) (√((W^2 - (m1 - m2)^2) * (W^2 - (m1 + m2)^2)))
kf[W_] = k[W, mK, mXi];
ki[W_] = k[W, mK, mP];
jmax = 5 / 2;
ons = 1;
onp = 1;
ond = 1;
nopars = 6 * ons + 12 * onp + 12 * ond;
num = If[nopars == 6, 1, If[nopars == 18, 5, 9]];
pars =
  Flatten[Join[Table[{s[i], α[i]}, {i, 0, num}], Table[{ϕ[i]}, {i, 0, num - 1}]]];
S01half[W_] := ons *
```

$$\left(\left(s[0]\,\text{Exp}\left[-\alpha[0]\text{^}2*\frac{kf[W]^2}{\Lambda^2}\right]-x[0]\,\text{Exp}[I*ph[0]]\,\frac{\Gamma[0]}{2\left(W-M[0]+I\,\frac{\Gamma[0]}{2}\right)}\right)\text{Exp}[I*\phi[0]]\right)*\left(\frac{kf[W]}{\Lambda}\right)^{0+1/2}$$

```
S11half[W_] := ons *
```

$$\left(\left(s[1]\,\text{Exp}\left[-\alpha[1]\text{^}2*\frac{kf[W]^2}{\Lambda^2}\right]-x[1]\,\text{Exp}[I*ph[1]]\,\frac{\Gamma[1]}{2\left(W-M[1]+I\,\frac{\Gamma[1]}{2}\right)}\right)\text{Exp}[I*\phi[1]]\right)*\left(\frac{kf[W]}{\Lambda}\right)^{0+1/2}$$

```
P01half[W_] := onp *
```

$$\left(\left(s[2]\,\text{Exp}\left[-\alpha[2]\text{^}2*\frac{kf[W]^2}{\Lambda^2}\right]-x[2]\,\text{Exp}[I*ph[2]]\,\frac{\Gamma[2]}{2\left(W-M[2]+I\,\frac{\Gamma[2]}{2}\right)}\right)\text{Exp}[I*\phi[2]]\right)*\left(\frac{kf[W]}{\Lambda}\right)^{1+1/2}$$

```
P03half[W_] := onp *
```

$$\left(\left(s[3]\,\text{Exp}\left[-\alpha[3]\text{^}2*\frac{kf[W]^2}{\Lambda^2}\right]-x[3]\,\text{Exp}[I*ph[3]]\,\frac{\Gamma[3]}{2\left(W-M[3]+I\,\frac{\Gamma[3]}{2}\right)}\right)\text{Exp}[I*\phi[3]]\right)*\left(\frac{kf[W]}{\Lambda}\right)^{1+1/2}$$

```
P11half[W_] := onp *
```

$$\left(\left(s[4]\,\text{Exp}\left[-\alpha[4]\text{^}2*\frac{kf[W]^2}{\Lambda^2}\right]-x[4]\,\text{Exp}[I*ph[4]]\,\frac{\Gamma[4]}{2\left(W-M[4]+I\,\frac{\Gamma[4]}{2}\right)}\right)\text{Exp}[I*\phi[4]]\right)*\left(\frac{kf[W]}{\Lambda}\right)^{1+1/2}$$

```
P13half[W_] := onp *
```

$$\left(\left(s[5]\,\text{Exp}\left[-\alpha[5]\text{^}2*\frac{kf[W]^2}{\Lambda^2}\right]-x[5]\,\text{Exp}[I*ph[5]]\,\frac{\Gamma[5]}{2\left(W-M[5]+I\,\frac{\Gamma[5]}{2}\right)}\right)\text{Exp}[I*\phi[5]]\right)*\left(\frac{kf[W]}{\Lambda}\right)^{1+1/2}$$

```
D03half[W_] := ond *
```

$$\left(\left(s[6]\,\text{Exp}\left[-\alpha[6]\text{^}2*\frac{kf[W]^2}{\Lambda^2}\right]-x[6]\,\text{Exp}[I*ph[6]]\,\frac{\Gamma[6]}{2\left(W-M[6]+I\,\frac{\Gamma[6]}{2}\right)}\right)\text{Exp}[I*\phi[6]]\right)*\left(\frac{kf[W]}{\Lambda}\right)^{2+1/2}$$

```
D05half[W_] := ond *
```

$$\left(\left(s[7]\,\text{Exp}\left[-\alpha[7]\text{^}2*\frac{kf[W]^2}{\Lambda^2}\right]-x[7]\,\text{Exp}[I*ph[7]]\,\frac{\Gamma[7]}{2\left(W-M[7]+I\,\frac{\Gamma[7]}{2}\right)}\right)\text{Exp}[I*\phi[7]]\right)*\left(\frac{kf[W]}{\Lambda}\right)^{2+1/2}$$

```
D13half[W_] := ond *
```

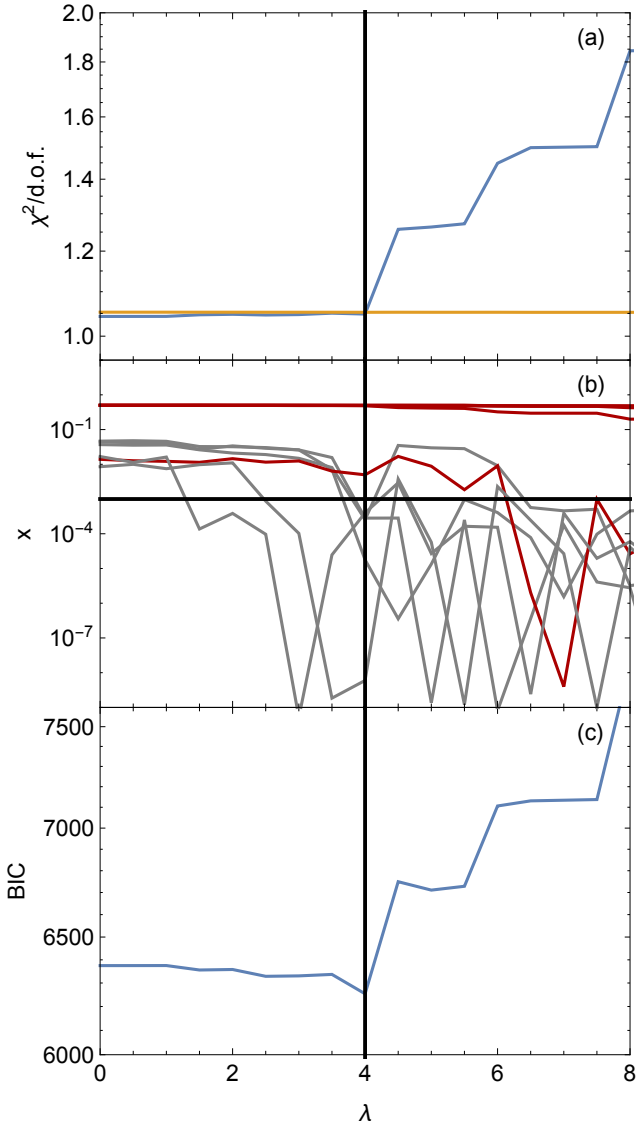
$$\left(\left(s[8]\,\text{Exp}\left[-\alpha[8]\text{^}2 * \frac{kf[W]^2}{\Lambda^2}\right] - x[8]\,\text{Exp}[I * ph[8]]\,\frac{\Gamma[8]}{2\left(W - M[8] + I\,\frac{\Gamma[8]}{2}\right)}\right)\text{Exp}[I * \phi[8]]\right) * \left(\frac{kf[W]}{\Lambda}\right)^{2+1/2}$$

```
D15half[W_] := ond *
   (((s[9] Exp[-α[9]^2 * kf[W]^2/Λ^2] - x[9] Exp[I * ph[9]] Γ[9]/(2 (W - M[9] + I Γ[9]/2))) Exp[I * 0]) *
   (kf[W]/Λ)^(2+1/2)
τ0_{-1/2} = {S01half[W], P03half[W], D05half[W]};
τ0_{+1/2} = {P01half[W], D03half[W], 0};
τ1_{-1/2} = {S11half[W], P13half[W], D15half[W]};
τ1_{+1/2} = {P11half[W], D13half[W], 0};
g0[θ_, W_] = 1/(2 Sqrt[kf[W] * ki[W]]) *
    Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] * (τ0_{-1/2}[[J + 1/2]] + τ0_{+1/2}[[J + 1/2]])
          Cos[θ/2] + WignerD[{J, -1/2, 1/2}, θ]
          (τ0_{-1/2}[[J + 1/2]] - τ0_{+1/2}[[J + 1/2]]) Sin[θ/2]), {J, 1/2, jmax, 1}];
h0[θ_, W_] = -I/(2 Sqrt[kf[W] * ki[W]]) * Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] *
          (τ0_{-1/2}[[J + 1/2]] + τ0_{+1/2}[[J + 1/2]]) Sin[θ/2] - WignerD[{J, -1/2, 1/2}, θ]
          (τ0_{-1/2}[[J + 1/2]] - τ0_{+1/2}[[J + 1/2]]) Cos[θ/2]), {J, 1/2, jmax, 1}];
g1[θ_, W_] = 1/(2 Sqrt[kf[W] * ki[W]]) * Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] *
          (τ1_{-1/2}[[J + 1/2]] + τ1_{+1/2}[[J + 1/2]]) Cos[θ/2] + WignerD[{J, -1/2, 1/2}, θ]
          (τ1_{-1/2}[[J + 1/2]] - τ1_{+1/2}[[J + 1/2]]) Sin[θ/2]), {J, 1/2, jmax, 1}];
h1[θ_, W_] = -I/(2 Sqrt[kf[W] * ki[W]]) * Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] *
          (τ1_{-1/2}[[J + 1/2]] + τ1_{+1/2}[[J + 1/2]]) Sin[θ/2] - WignerD[{J, -1/2, 1/2}, θ]
          (τ1_{-1/2}[[J + 1/2]] - τ1_{+1/2}[[J + 1/2]]) Cos[θ/2]), {J, 1/2, jmax, 1}];


gre1[θ_, W_] := -1/2 g1[θ, W] + 1/2 g0[θ, W](*<Ξ⁻k⁺| p K⁻ *)
hre1[θ_, W_] := -1/2 h1[θ, W] + 1/2 h0[θ, W] (*<Ξ⁻k⁺| p K⁻ *)

gre2[θ_, W_] := -1/2 g1[θ, W] - 1/2 g0[θ, W](*<Ξ⁰k⁰| p K⁻ *)
hre2[θ_, W_] := -1/2 h1[θ, W] - 1/2 h0[θ, W] (*<Ξ⁰k⁰| p K⁻ *)


dσdΩ1[W_, θ_] :=
```

```
  factor * (Conjugate[gre1[θ, W]] gre1[θ, W] + Conjugate[hre1[θ, W]] * hre1[θ, W]) * kf[W] / ki[W]
  (*<Ξ⁻k⁺| p K⁻ *)
P1[W_, θ_] := 2 Re[gre1[θ, W] * Conjugate[hre1[θ, W]]] / (Abs[gre1[θ, W]]^2 + Abs[hre1[θ, W]]^2) * dσdΩ1[W, θ] (*<Ξ⁻k⁺| p K⁻ *)

dσdΩ2[W_, θ_] :=
  factor * (Conjugate[gre2[θ, W]] gre2[θ, W] + Conjugate[hre2[θ, W]] * hre2[θ, W]) * kf[W] / ki[W]
  (*<Ξ⁰k⁰| p K⁻ *)
P2[W_, θ_] := 2 Re[gre2[θ, W] * Conjugate[hre2[θ, W]]] / (Abs[gre2[θ, W]]^2 + Abs[hre2[θ, W]]^2) * dσdΩ2[W, θ] (*<Ξ⁰k⁰| p K⁻ *)
σ1[W_] =
   factor * 1/2 * 4 Pi / ki[W]^2 Sum[(2 J + 1) (Abs[-1/2 (τ1_{-1/2}[[J + 1/2]]) + 1/2 (τ0_{-1/2}[[J + 1/2]])]^2 +
        Abs[-1/2 (τ1_{1/2}[[J + 1/2]]) + 1/2 (τ0_{1/2}[[J + 1/2]])]^2), {J, 1/2, jmax, 1}];
σ2[W_] = factor * 1/2 * 4 Pi / ki[W]^2 Sum[(2 J + 1)
      (Abs[-1/2 (τ1_{-1/2}[[J + 1/2]]) - 1/2 (τ0_{-1/2}[[J + 1/2]])]^2 +
        Abs[-1/2 (τ1_{1/2}[[J + 1/2]]) - 1/2 (τ0_{1/2}[[J + 1/2]])]^2), {J, 1/2, jmax, 1}];

Chisq[λ_] := Sum[(1 / Diff2[[i, j, 3]])^2 *
    (dσdΩ2[Diff2Energy[[i]], Diff2[[i, j, 1]]] - Diff2[[i, j, 2]])^2,
   {i, 1, Length[Diff2]}, {j, 1, Length[Diff2[[i]]]}] + Sum[(1 / Diff1[[i, j, 3]])^2 *
    (dσdΩ1[Diff1Energy[[i]], Diff1[[i, j, 1]]] - Diff1[[i, j, 2]])^2,
   {i, 1, Length[Diff1]}, {j, 1, Length[Diff1[[i]]]}] +
  Sum[(1 / Cross1[[i, 3]])^2 * (σ1[Cross1[[i, 1]]] - Cross1[[i, 2]])^2,
   {i, 1, Length[Cross1]}] + Sum[(1 / Cross2[[i, 3]])^2 *
    (σ2[Cross2[[i, 1]]] - Cross2[[i, 2]])^2, {i, 1, Length[Cross2]}] +
  Sum[(1 / pol1[[i, j, 3]])^2 * (P1[pol1Energy[[i]], pol1[[i, j, 1]]] - pol1[[i, j, 2]])^2,
   {i, 1, Length[pol1]}, {j, 1, Length[pol1[[i]]]}] +
  Sum[(1 / pol2[[i, j, 3]])^2 * (P2[pol2Energy[[i]], pol2[[i, j, 1]]] - pol2[[i, j, 2]])^2,
   {i, 1, Length[pol2]}, {j, 1, Length[pol2[[i]]]}] +
  λ^4 (Abs[x[9]] + Abs[x[8]] + Abs[x[7]] + Abs[x[6]] + Abs[x[5]] +
     Abs[x[4]] + Abs[x[3]] + Abs[x[2]] + Abs[x[1]] + Abs[x[0]])
```

```
guess = {{s[0], -0.34824198024328584`}, {α[0], 3.763438455711533`},
   {s[1], -0.07153619899748145`}, {α[1], -0.5504395877254527`},
   {s[2], 0.20791938959137438`}, {α[2], 1.932359556897292`},
   {s[3], 0.3140263682123946`}, {α[3], 1.4353226934402328`},
   {s[4], -0.09335240870654254`}, {α[4], 1.2586482592270365`},
   {s[5], 0.19804403841470758`}, {α[5], 1.1543752980111297`},
   {s[6], 0.08564949843750701`}, {α[6], 0.8114266359183674`},
   {s[7], 1.8696909108351594`}, {α[7], 2.38042861587167`},
   {s[8], 1.6784049993618166`}, {α[8], 2.3546081341506007`},
   {s[9], -0.4910616507553648`}, {α[9], 1.976612736930773`},
   {ϕ[0], -1.3759095905974634`}, {ϕ[1], -1.8628696533673115`},
   {ϕ[2], -2.4292106595489664`}, {ϕ[3], -3.7601064302536518`},
   {ϕ[4], -3.5505168556001405`}, {ϕ[5], -0.6146668029672369`},
   {ϕ[6], 0.37669927068590553`}, {ϕ[7], -6.818425121493386`},
   {ϕ[8], 0.6225800989122515`}, {x[0], 0.5}, {Γ[0], 100}, {M[0], 2036.96},
   {ph[0], -.9}, {x[1], .25}, {Γ[1], 100}, {M[1], 2000}, {ph[1], -1.2},
   {x[2], .25}, {Γ[2], 100}, {M[2], 2000}, {ph[2], -1}, {x[3], .25}, {Γ[3], 100},
   {M[3], 2000}, {ph[3], -1}, {x[4], 0.5}, {Γ[4], 160}, {M[4], 2191.96},
   {ph[4], -.8}, {x[5], .25}, {Γ[5], 100}, {M[5], 2000}, {ph[5], -.2131},
   {x[6], .25}, {Γ[6], 100}, {M[6], 2000}, {ph[6], -.121}, {x[7], 0.5}, {Γ[7], 60},
   {M[7], 2786.962`}, {ph[7], -.7}, {x[8], .25}, {Γ[8], 100}, {M[8], 2000},
   {ph[8], -1}, {x[9], 0.5}, {Γ[9], 81}, {M[9], 2603.962`}, {ph[9], -0.6}};

chilist = {};
chinoplist = {};
palist = {};
nopalist = {};
Do[
 fitresults = FindMinimum[Re[Chisq[λ]], guess, MaxIterations → 2000] // Quiet;
 chisqwp = Re[fitresults[[1]]];
 pars = fitresults[[2]];
 chisq = Re[Chisq[0]] /. pars;
 AppendTo[chilist, {λ, chisqwp}];
 AppendTo[chinoplist, {λ, chisq}];
 guess = Table[{pars[[i, 1]], pars[[i, 2]]}, {i, 1, Length[pars]}];
 kkcount = 0;
 Do[If[Abs[pars[[i, 2]]] < 10^-3, kkcount = kkcount + 1], {i, 1, Length[guess]}];
 AppendTo[nopalist, {λ, Length[guess] - 4 kkcount}];
 AppendTo[palist, {λ, pars[[ ;; , 2]]}];
 Print[λ];, {λ, 10, 0, -0.5}]
```

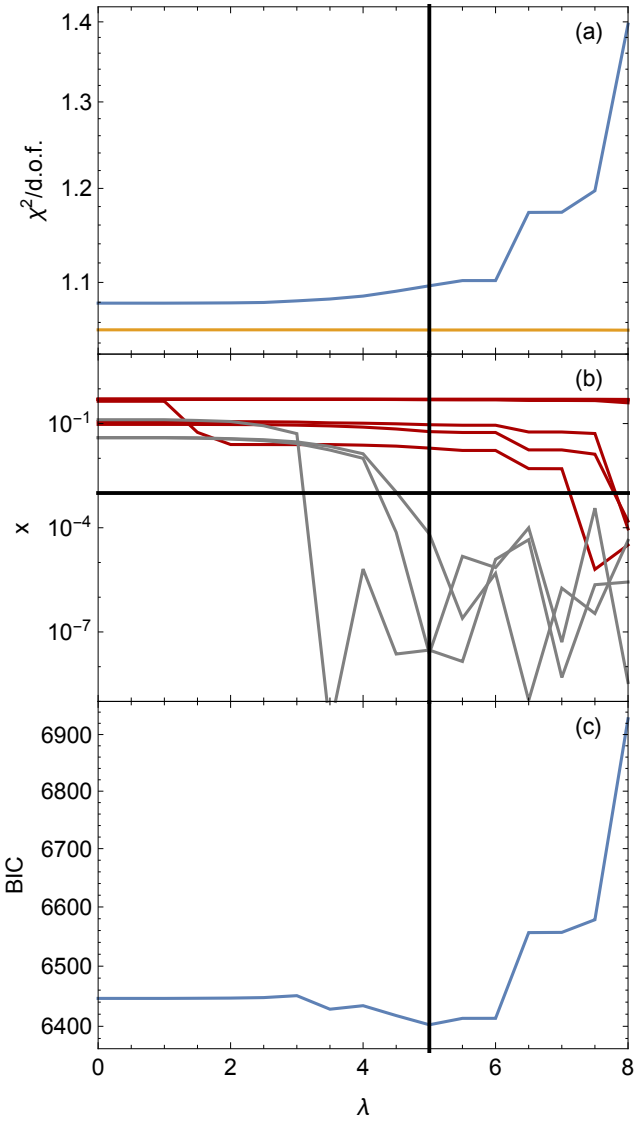
```
nodat =
  Length[Cross1] + Length[Cross2] + Sum[Length[Diff1[[i]]], {i, 1, Length[Diff1]}] +
   Sum[Length[Diff2[[i]]], {i, 1, Length[Diff2]}] + Sum[Length[pol1[[i]]],
    {i, 1, Length[pol1]}] + Sum[Length[pol2[[i]]], {i, 1, Length[pol2]}];
dataPar = Table[{palist[[i, 1]], Abs[palist[[i, 2, j]]]},
   {j, 1, Length[palist[[1, 2]]]}, {i, 1, Length[palist]}];
chisq = chinoplist;
chidof = Table[{chisq[[i, 1]], chisq[[i, 2]] / (nodat - nopalist[[i, 2]])}, {i, 1, Length[chisq]}];
aic = Table[{chisq[[i, 1]], 2 nopalist[[i, 2]] + nodat * Log[2 Pi] + chisq[[i, 2]]},
   {i, 1, Length[chisq]}];
aicc = Table[{chisq[[i, 1]], 2 nopalist[[i, 2]] + nodat * Log[2 Pi] + chisq[[i, 2]] +
     2 * nopalist[[i, 2]] * (nopalist[[i, 2]] + 1) / (nodat - nopalist[[i, 2]] - 1)}, {i, 1, Length[chisq]}];
bic = Table[{chisq[[i, 1]], nodat * Log[2 Pi] + chisq[[i, 2]] +
     nopalist[[i, 2]] Log[nodat]}, {i, 1, Length[chisq]}];

respalist = Table[{palist[[i, 1]], Flatten[Table[palist[[i, 2, 30 + 4 j]], {j, 0, 9}]]},
   {i, 1, Length[palist]}];
respar = Table[{respalist[[i, 1]], Abs[respalist[[i, 2, j]]]},
   {j, 1, Length[respalist[[1, 2]]]}, {i, 1, Length[respalist]}];

plot1 = Show[ListLogPlot[{chidof, chitestdof}, LabelStyle → 12, Joined → True,
    PlotRange → {{0, 8}, {0.95, 2}}, Frame → True, FrameLabel → {"λ", "χ²/d.o.f."}],
   ListPlot[Table[{bic[[Position[bic, Min[bic[[All, 2]]]][[1, 1]]]][[1]], i},
     {i, -1, 10000, 1000}], Joined → True, PlotStyle → {Thick, Black}],
   Epilog → Text[Style["(a)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}]];
plot2 = Show[ListLogPlot[respar, LabelStyle → 12, Joined → True, Frame → True,
    PlotRange → {{0, 8}, {10, 10^-9}}, FrameLabel → {"λ", "x"}, PlotStyle →
     Table[If[Abs[respalist[[Position[bic, Min[bic[[All, 2]]]][[1, 1]], 2, i]]] >
        10^-3, Darker[Red], Gray], {i, 1, Length[respalist[[5, 2]]]}],
    Epilog → Text[Style["(b)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}]],
   LogPlot[10^-3, {x, 0, 8}, PlotStyle → {Thick, Black}],
   ListPlot[Table[{bic[[Position[bic, Min[bic[[All, 2]]]][[1, 1]]]][[1]], i},
     {i, -100, 10, 1}], Joined → True, PlotStyle → {Thick, Black}]];
plot3 = Show[ListLogPlot[bic, LabelStyle → 12,
    Epilog → Text[Style["(c)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}],
    PlotRange → {{0, 8}, {6000, 7599}}, Frame → True,
    Joined → True, FrameLabel → {"λ", "BIC"}],
   ListPlot[Table[{bic[[Position[bic, Min[bic[[All, 2]]]][[1, 1]]]][[1]], i},
     {i, -1, 10000, 1000}], Joined → True, PlotStyle → {Thick, Black}]];
plotGrid[{{plot1}, {plot2}, {plot3}}, 400, 4/3 * 500]
```


(a)
(b)
(c)
χ^2/d.o.f.
x
BIC
λ
2.0
1.8
1.6
1.4
1.2
1.0
10^{-1}
10^{-4}
10^{-7}
7500
7000
6500
6000
0
2
4
6
8

Now we will re run LASSO from small lambdas to large lambdas

```
guess = {{s[0], -0.34824198024328584`}, {α[0], 3.763438455711533`},
    {s[1], -0.07153619899748145`}, {α[1], -0.5504395877254527`},
    {s[2], 0.20791938959137438`}, {α[2], 1.932359556897292`},
    {s[3], 0.3140263682123946`}, {α[3], 1.4353226934402328`},
    {s[4], -0.09335240870654254`}, {α[4], 1.2586482592270365`},
    {s[5], 0.19804403841470758`}, {α[5], 1.1543752980111297`},
    {s[6], 0.08564949843750701`}, {α[6], 0.8114266359183674`},
    {s[7], 1.8696909108351594`}, {α[7], 2.38042861587167`},
    {s[8], 1.6784049993618166`}, {α[8], 2.3546081341506007`},
    {s[9], -0.4910616507553648`}, {α[9], 1.976612736930773`},
    {ϕ[0], -1.3759095905974634`}, {ϕ[1], -1.8628696533673115`},
    {ϕ[2], -2.4292106595489664`}, {ϕ[3], -3.7601064302536518`},
    {ϕ[4], -3.5505168556001405`}, {ϕ[5], -0.6146668029672369`},
    {ϕ[6], 0.37669927068590553`}, {ϕ[7], -6.818425121493386`},
    {ϕ[8], 0.6225800989122515`}, {x[0], 0.5}, {Γ[0], 100}, {M[0], 2036.96},
    {ph[0], -.9}, {x[1], .25}, {Γ[1], 100}, {M[1], 2000}, {ph[1], -1.2},
    {x[2], .25}, {Γ[2], 100}, {M[2], 2000}, {ph[2], -1}, {x[3], .25}, {Γ[3], 100},
    {M[3], 2000}, {ph[3], -1}, {x[4], 0.5}, {Γ[4], 160}, {M[4], 2191.96},
    {ph[4], -.8}, {x[5], .25}, {Γ[5], 100}, {M[5], 2000}, {ph[5], -.2131},
    {x[6], .25}, {Γ[6], 100}, {M[6], 2000}, {ph[6], -.121}, {x[7], 0.5}, {Γ[7], 60},
    {M[7], 2786.962`}, {ph[7], -.7}, {x[8], .25}, {Γ[8], 100}, {M[8], 2000},
    {ph[8], -1}, {x[9], 0.5}, {Γ[9], 81}, {M[9], 2603.962`}, {ph[9], -0.6}};
```

```
chilist = {};
chinoplist = {};
palist = {};
nopalist = {};
Do[
 fitresults = FindMinimum[Re[Chisq[λ]], guess, MaxIterations → 2000] // Quiet;
 chisqwp = Re[fitresults[[1]]];
 pars = fitresults[[2]];
 chisq = Re[Chisq[0]] /. pars;
 AppendTo[chilist, {λ, chisqwp}];
 AppendTo[chinoplist, {λ, chisq}];
 guess = Table[{pars[[i, 1]], pars[[i, 2]]}, {i, 1, Length[pars]}];
 kkcount = 0;
 Do[If[Abs[pars[[i, 2]]] < 10^-3, kkcount = kkcount + 1], {i, 1, Length[guess]}];
 AppendTo[nopalist, {λ, Length[guess] - 4 kkcount}];
 AppendTo[palist, {λ, pars[[ ;; , 2]]}];
 Print[λ];, {λ, 0, 10, 0.5}]


respalist = Table[{palist[[i, 1]], Flatten[Table[palist[[i, 2, 30 + 4 j]], {j, 0, 9}]]},
   {i, 1, Length[palist]}];
respar =  Table[{respalist[[i, 1]], Abs[respalist[[i, 2, j]]]},
   {j, 1, Length[respalist[[1, 2]]]}, {i, 1, Length[respalist]}];
plot1 = Show[ListLogPlot[{chidof, chitestdof}, LabelStyle → 12, Epilog →
     Text[Style["(a)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}], Joined → True,
    PlotRange → {{0, 8}, All}, Frame → True, FrameLabel → {"λ", "χ^2/d.o.f."}],
   ListPlot[Table[{bic[[Position[bic, Min[bic[[All, 2]]]][[1, 1]]]][[1]], i},
     {i, -1, 10 000, 1000}], Joined → True, PlotStyle → {Thick, Black}]];
plot2 = Show[ListLogPlot[respar, LabelStyle → 12,
    Epilog → Text[Style["(b)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}],
    Joined → True, Frame → True, PlotRange → {{0, 8}, {10, 10^-9}},
    FrameLabel → {"λ", "x"}, PlotStyle → Table[
      If[Abs[respalist[[Position[bic, Min[bic[[All, 2]]]][[1, 1]], 2, i]]] > 10^-3,
       Darker[Red], Gray], {i, 1, Length[respalist[[5, 2]]]}]],
   LogPlot[10^-3, {x, 0, 8}, PlotStyle → {Thick, Black}],
   ListPlot[Table[{bic[[Position[bic, Min[bic[[All, 2]]]][[1, 1]]]][[1]], i},
     {i, -100, 10, 1}], Joined → True, PlotStyle → {Thick, Black}]];
plot3 = Show[ListLogPlot[bic, LabelStyle → 12,
    Epilog → Text[Style["(c)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}],
    PlotRange → {{0, 8}, All}, Frame → True, Joined → True, FrameLabel → {"λ", "BIC"}],
   ListPlot[Table[{bic[[Position[bic, Min[bic[[All, 2]]]][[1, 1]]]][[1]], i},
     {i, -1, 10 000, 1000}], Joined → True, PlotStyle → {Thick, Black}]];
plotGrid[{{plot1}, {plot2}, {plot3}}, 400, 4/3 * 500]
```



Penalize the Second Derivative

```
k[W_, m1_, m2_] := 1/(2*W) (√((W^2 - (m1 - m2)^2) * (W^2 - (m1 + m2)^2)))
kf[W_] = k[W, mK, mXi];
ki[W_] = k[W, mK, mP];
jmax = 5/2;
ons = 1;
```

```
onp = 1;
ond = 1;
nopars = 6 * ons + 12 * onp + 12 * ond;
num = If[nopars == 6, 1, If[nopars == 18, 5, 9]];
pars =
  Flatten[Join[Table[{s[i], α[i]}, {i, 0, num}], Table[{ϕ[i]}, {i, 0, num - 1}]]];
S01half[W_] := ons *
  (((s[0] Exp[-α[0]^2 * kf[W]^2/Λ^2] - x[0] Exp[I * ph[0]]) Γ[0]/(2 (W - M[0] + I Γ[0]/2))) Exp[I * ϕ[0]]) *
  (kf[W]/Λ)^(0+1/2)
S11half[W_] := ons *
  (((s[1] Exp[-α[1]^2 * kf[W]^2/Λ^2] - x[1] Exp[I * ph[1]]) Γ[1]/(2 (W - M[1] + I Γ[1]/2))) Exp[I * ϕ[1]]) *
  (kf[W]/Λ)^(0+1/2)
P01half[W_] := onp *
  (((s[2] Exp[-α[2]^2 * kf[W]^2/Λ^2] - x[2] Exp[I * ph[2]]) Γ[2]/(2 (W - M[2] + I Γ[2]/2))) Exp[I * ϕ[2]]) *
  (kf[W]/Λ)^(1+1/2)
P03half[W_] := onp *
  (((s[3] Exp[-α[3]^2 * kf[W]^2/Λ^2] - x[3] Exp[I * ph[3]]) Γ[3]/(2 (W - M[3] + I Γ[3]/2))) Exp[I * ϕ[3]]) *
  (kf[W]/Λ)^(1+1/2)
P11half[W_] := onp *
  (((s[4] Exp[-α[4]^2 * kf[W]^2/Λ^2] - x[4] Exp[I * ph[4]]) Γ[4]/(2 (W - M[4] + I Γ[4]/2))) Exp[I * ϕ[4]]) *
  (kf[W]/Λ)^(1+1/2)
P13half[W_] := onp *
  (((s[5] Exp[-α[5]^2 * kf[W]^2/Λ^2] - x[5] Exp[I * ph[5]]) Γ[5]/(2 (W - M[5] + I Γ[5]/2))) Exp[I * ϕ[5]]) *
  (kf[W]/Λ)^(1+1/2)
D03half[W_] := ond *
  (((s[6] Exp[-α[6]^2 * kf[W]^2/Λ^2] - x[6] Exp[I * ph[6]]) Γ[6]/(2 (W - M[6] + I Γ[6]/2))) Exp[I * ϕ[6]]) *
  (kf[W]/Λ)^(2+1/2)
```

```
D05half[W_] := ond *
  (((s[7] Exp[-α[7]^2 * kf[W]^2/Λ^2] - x[7] Exp[I * ph[7]] Γ[7]/(2 (W - M[7] + I Γ[7]/2))) Exp[I * ϕ[7]]) *
  (kf[W]/Λ)^(2+1/2)
D13half[W_] := ond *
  (((s[8] Exp[-α[8]^2 * kf[W]^2/Λ^2] - x[8] Exp[I * ph[8]] Γ[8]/(2 (W - M[8] + I Γ[8]/2))) Exp[I * ϕ[8]]) *
  (kf[W]/Λ)^(2+1/2)
D15half[W_] := ond *
  (((s[9] Exp[-α[9]^2 * kf[W]^2/Λ^2] - x[9] Exp[I * ph[9]] Γ[9]/(2 (W - M[9] + I Γ[9]/2))) Exp[I * 0]) *
  (kf[W]/Λ)^(2+1/2)
τ0_-1/2 = {S01half[W], P03half[W], D05half[W]};
τ0_+1/2 = {P01half[W], D03half[W], 0};
τ1_-1/2 = {S11half[W], P13half[W], D15half[W]};
τ1_+1/2 = {P11half[W], D13half[W], 0};
g0[θ_, W_] = 1/(2 Sqrt[kf[W] * ki[W]]) *
    Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] * (τ0_-1/2[[J + 1/2]] + τ0_+1/2[[J + 1/2]])
          Cos[θ/2] + WignerD[{J, -1/2, 1/2}, θ]
          (τ0_-1/2[[J + 1/2]] - τ0_+1/2[[J + 1/2]]) Sin[θ/2]), {J, 1/2, jmax, 1}];
h0[θ_, W_] = -I/(2 Sqrt[kf[W] * ki[W]]) * Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] *
          (τ0_-1/2[[J + 1/2]] + τ0_+1/2[[J + 1/2]]) Sin[θ/2] - WignerD[{J, -1/2, 1/2}, θ]
          (τ0_-1/2[[J + 1/2]] - τ0_+1/2[[J + 1/2]]) Cos[θ/2]), {J, 1/2, jmax, 1}];
g1[θ_, W_] = 1/(2 Sqrt[kf[W] * ki[W]]) * Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] *
          (τ1_-1/2[[J + 1/2]] + τ1_+1/2[[J + 1/2]]) Cos[θ/2] + WignerD[{J, -1/2, 1/2}, θ]
          (τ1_-1/2[[J + 1/2]] - τ1_+1/2[[J + 1/2]]) Sin[θ/2]), {J, 1/2, jmax, 1}];
h1[θ_, W_] = -I/(2 Sqrt[kf[W] * ki[W]]) * Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] *
          (τ1_-1/2[[J + 1/2]] + τ1_+1/2[[J + 1/2]]) Sin[θ/2] - WignerD[{J, -1/2, 1/2}, θ]
          (τ1_-1/2[[J + 1/2]] - τ1_+1/2[[J + 1/2]]) Cos[θ/2]), {J, 1/2, jmax, 1}];


gre1[θ_, W_] := -1/2 g1[θ, W] + 1/2 g0[θ, W] (*<Ξ⁻k⁺| p K⁻ *)
hre1[θ_, W_] := -1/2 h1[θ, W] + 1/2 h0[θ, W] (*<Ξ⁻k⁺| p K⁻ *)
```

```
gre2[θ_, W_] := -1/2 g1[θ, W] - 1/2 g0[θ, W] (*<Ξ⁰k⁰| p K⁻ *)
hre2[θ_, W_] := -1/2 h1[θ, W] - 1/2 h0[θ, W] (*<Ξ⁰k⁰| p K⁻ *)

dσdΩ1[W_, θ_] :=
  factor * (Conjugate[gre1[θ, W]] gre1[θ, W] + Conjugate[hre1[θ, W]] * hre1[θ, W]) * kf[W]/ki[W]
  (*<Ξ⁻k⁺| p K⁻ *)
P1[W_, θ_] := 2 Re[gre1[θ, W] * Conjugate[hre1[θ, W]]]/(Abs[gre1[θ, W]]^2 + Abs[hre1[θ, W]]^2) * dσdΩ1[W, θ] (*<Ξ⁻k⁺| p K⁻ *)

dσdΩ2[W_, θ_] :=
  factor * (Conjugate[gre2[θ, W]] gre2[θ, W] + Conjugate[hre2[θ, W]] * hre2[θ, W]) * kf[W]/ki[W]
  (*<Ξ⁰k⁰| p K⁻ *)
P2[W_, θ_] := 2 Re[gre2[θ, W] * Conjugate[hre2[θ, W]]]/(Abs[gre2[θ, W]]^2 + Abs[hre2[θ, W]]^2) * dσdΩ2[W, θ] (*<Ξ⁰k⁰| p K⁻ *)
σ1[W_] =
   factor * 1/2 * 4 Pi/ki[W]^2 Sum[(2 J + 1) (Abs[-1/2 (τ1_{-1/2}[[J + 1/2]]) + 1/2 (τ0_{-1/2}[[J + 1/2]])]^2 +
        Abs[-1/2 (τ1_{1/2}[[J + 1/2]]) + 1/2 (τ0_{1/2}[[J + 1/2]])]^2), {J, 1/2, jmax, 1}];
σ2[W_] = factor * 1/2 * 4 Pi/ki[W]^2 Sum[(2 J + 1)
      (Abs[-1/2 (τ1_{-1/2}[[J + 1/2]]) - 1/2 (τ0_{-1/2}[[J + 1/2]])]^2 +
        Abs[-1/2 (τ1_{1/2}[[J + 1/2]]) - 1/2 (τ0_{1/2}[[J + 1/2]])]^2), {J, 1/2, jmax, 1}];
```

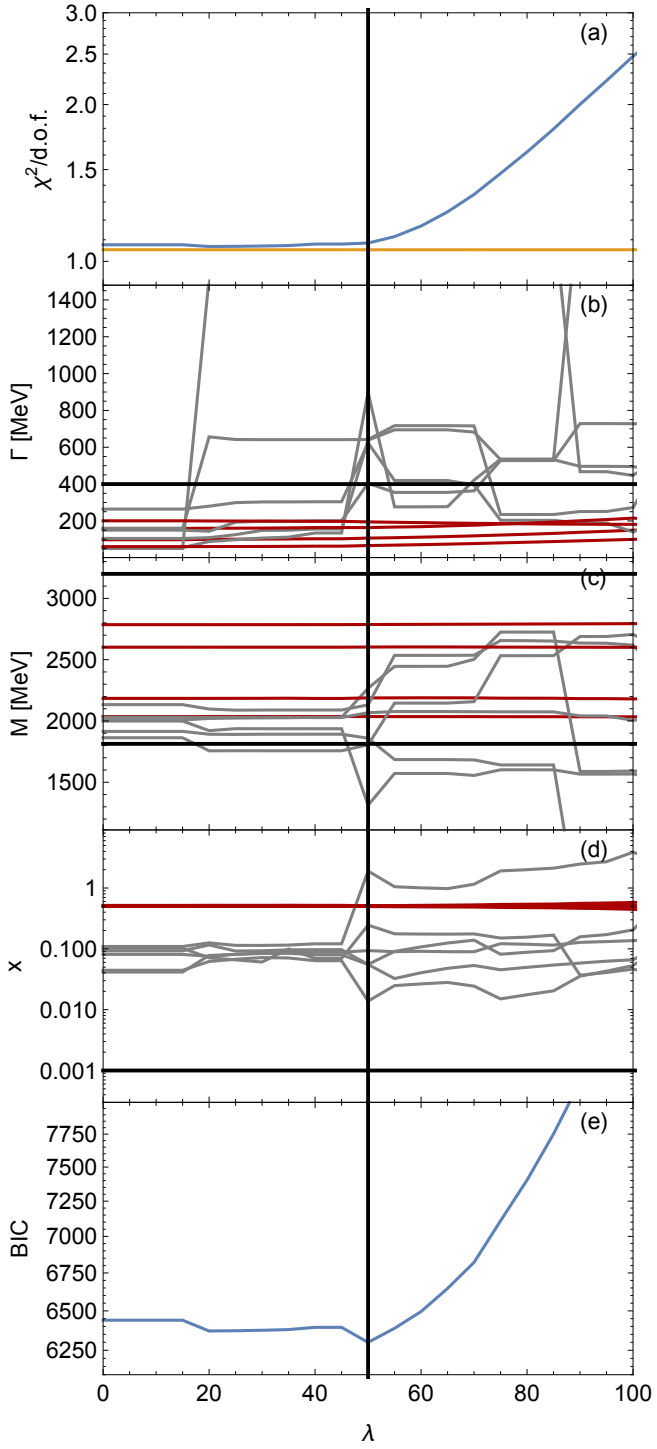
```
Chisq[λ_, S_] := Sum[(1/Diff2[[i, j, 3]])^2 *
    (dσdΩ2[Diff2Energy[[i]], Diff2[[i, j, 1]]] - Diff2[[i, j, 2]])^2,
   {i, 1, Length[Diff2]}, {j, 1, Length[Diff2[[i]]]}] + Sum[(1/Diff1[[i, j, 3]])^2 *
    (dσdΩ1[Diff1Energy[[i]], Diff1[[i, j, 1]]] - Diff1[[i, j, 2]])^2,
   {i, 1, Length[Diff1]}, {j, 1, Length[Diff1[[i]]]}] +
  Sum[(1/Cross1[[i, 3]])^2 * (σ1[Cross1[[i, 1]]] - Cross1[[i, 2]])^2,
   {i, 1, Length[Cross1]}] + Sum[(1/Cross2[[i, 3]])^2 *
    (σ2[Cross2[[i, 1]]] - Cross2[[i, 2]])^2, {i, 1, Length[Cross2]}] +
  Sum[(1/pol1[[i, j, 3]])^2 * (P1[pol1Energy[[i]], pol1[[i, j, 1]]] - pol1[[i, j, 2]])^2,
   {i, 1, Length[pol1]}, {j, 1, Length[pol1[[i]]]}] +
  Sum[(1/pol2[[i, j, 3]])^2 * (P2[pol2Energy[[i]], pol2[[i, j, 1]]] - pol2[[i, j, 2]])^2,
   {i, 1, Length[pol2]}, {j, 1, Length[pol2[[i]]]}] + (λ^5 Sum[
     ((1/Γ[i]^3) 4 x[i]^2 ((2 (3200 - M[i]) Γ[i] (122 880 000 - 76 800 M[i] + 12 M[i]^2 + 5 Γ[i]^2)) /
           (40 960 000 - 25 600 M[i] + 4 M[i]^2 + Γ[i]^2)^2 + 3 ArcTan[(2 (3200 - M[i]))/Γ[i]]) -
       (1/Γ[i]^3) 4 x[i]^2 ((2 (-M[i] + mK + mXi) Γ[i] (12 M[i]^2 - 24 M[i] (mK + mXi) +
             12 (mK + mXi)^2 + 5 Γ[i]^2)) / (4 M[i]^2 - 8 M[i] (mK + mXi) + 4 (mK + mXi)^2 +
           Γ[i]^2)^2 + 3 ArcTan[(2 (-M[i] + mK + mXi))/Γ[i]])) / ((1/2) x[i]^2 Γ[i]
       ArcTan[(2 (3200 - M[i]))/Γ[i]] - (1/2) x[i]^2 Γ[i] ArcTan[(2 (-M[i] + mK + mXi))/Γ[i]]),
    {i, 0, 9}]) + (S * Sum[(10^3/(1 + Exp[(Γ[i]-10)/2])), {i, 0, 9}]);
```

```
guess = {{s[0], -0.34824198024328584`}, {α[0], 3.763438455711533`},
   {s[1], -0.07153619899748145`}, {α[1], -0.5504395877254527`},
   {s[2], 0.20791938959137438`}, {α[2], 1.932359556897292`},
   {s[3], 0.3140263682123946`}, {α[3], 1.4353226934402328`},
   {s[4], -0.09335240870654254`}, {α[4], 1.2586482592270365`},
   {s[5], 0.19804403841470758`}, {α[5], 1.1543752980111297`},
   {s[6], 0.08564949843750701`}, {α[6], 0.8114266359183674`},
   {s[7], 1.8696909108351594`}, {α[7], 2.38042861587167`},
   {s[8], 1.6784049993618166`}, {α[8], 2.3546081341506007`},
   {s[9], -0.4910616507553648`}, {α[9], 1.976612736930773`},
   {ϕ[0], -1.3759095905974634`}, {ϕ[1], -1.8628696533673115`},
   {ϕ[2], -2.4292106595489664`}, {ϕ[3], -3.7601064302536518`},
   {ϕ[4], -3.5505168556001405`}, {ϕ[5], -0.6146668029672369`},
   {ϕ[6], 0.37669927068590553`}, {ϕ[7], -6.818425121493386`},
   {ϕ[8], 0.6225800989122515`}, {x[0], 0.5}, {Γ[0], 100}, {M[0], 2036.96},
   {ph[0], -.9}, {x[1], .25}, {Γ[1], 100}, {M[1], 2000}, {ph[1], -1.2},
   {x[2], .25}, {Γ[2], 100}, {M[2], 2000}, {ph[2], -1}, {x[3], .25}, {Γ[3], 100},
   {M[3], 2000}, {ph[3], -1}, {x[4], 0.5}, {Γ[4], 160}, {M[4], 2191.96},
   {ph[4], -.8}, {x[5], .25}, {Γ[5], 100}, {M[5], 2000}, {ph[5], -.2131},
   {x[6], .25}, {Γ[6], 100}, {M[6], 2000}, {ph[6], -.121}, {x[7], 0.5}, {Γ[7], 60},
   {M[7], 2786.962`}, {ph[7], -.7}, {x[8], .25}, {Γ[8], 100}, {M[8], 2000},
   {ph[8], -1}, {x[9], 0.5}, {Γ[9], 81}, {M[9], 2603.962`}, {ph[9], -0.6}};

chilist = {};
chinoplist = {};
palist = {};
nopalist = {};
Do[
 fitresults = FindMinimum[Chisq[λ, 1], guess, MaxIterations → 4000];
 pars = fitresults[[2]];
 chisqwp = Re[Chisq[λ, 0]] /. pars;
 chisq = Re[Chisq[0, 0]] /. pars;
 AppendTo[chilist, {λ, chisqwp}];
 AppendTo[chinoplist, {λ, chisq}];
 guess = Table[{pars[[i, 1]], pars[[i, 2]]}, {i, 1, Length[pars]}];
 kkcount = 0;
 Do[If[Abs[pars[[i, 2]]] < 10^-3, kkcount = kkcount + 1], {i, 1, Length[guess]}];
 AppendTo[nopalist, {λ, Length[guess] - 4 kkcount}];
 AppendTo[palist, {λ, pars[[ ;; , 2]]}];
 Print[λ];, {λ, 0, 200, 5}]
```

```
nopalist = {};
Do[
 kcount = 0;
 count = 0;
 Do[
  count = If[(palist[[i, 2, 32 + 4 j]] > 3200) ∨ (palist[[i, 2, 32 + 4 j]] < mK + mXi) ∨
      (palist[[i, 2, 31 + 4 j]] > 400) ∨ (Abs[palist[[i, 2, 30 + 4 j]]] < 10^-3), 4, 0];
  kcount = kcount + count;,
  {j, 0, 9}];
 AppendTo[nopalist, {palist[[i, 1]], 69 - kcount}];,
 {i, 1, Length[palist]}]
nodat =
  Length[Cross1] + Length[Cross2] + Sum[Length[Diff1[[i]]], {i, 1, Length[Diff1]}] +
   Sum[Length[Diff2[[i]]], {i, 1, Length[Diff2]}] + Sum[Length[pol1[[i]]],
    {i, 1, Length[pol1]}] + Sum[Length[pol2[[i]]], {i, 1, Length[pol2]}];
dataPar = Table[{palist[[i, 1]], Abs[palist[[i, 2, j]]]},
   {j, 1, Length[palist[[1, 2]]]}, {i, 1, Length[palist]}];
chisq = chinoplist;
chidof = Table[{chisq[[i, 1]], chisq[[i, 2]]/(nodat - nopalist[[i, 2]])}, {i, 1, Length[chisq]}];
aic = Table[{chisq[[i, 1]], 2 nopalist[[i, 2]] + nodat * Log[2 Pi] + chisq[[i, 2]]},
   {i, 1, Length[chisq]}];
aicc = Table[{chisq[[i, 1]], 2 nopalist[[i, 2]] + nodat * Log[2 Pi] + chisq[[i, 2]] +
     2 * nopalist[[i, 2]] * (nopalist[[i, 2]] + 1)/(nodat - nopalist[[i, 2]] - 1)}, {i, 1, Length[chisq]}];
bic = Table[{chisq[[i, 1]], nodat * Log[2 Pi] + chisq[[i, 2]] +
     nopalist[[i, 2]] Log[nodat]}, {i, 1, Length[chisq]}];
chitest = Table[{nopalist[[i, 1]],
    FindRoot[CDF[ChiSquareDistribution[2055 - nopalist[[i, 2]]], x] == 0.95,
      {x, 2000}][[1, 2]]}, {i, 1, Length[chinoplist]}];
chitestdof = Table[{chitest[[i, 1]], chitest[[i, 2]]/(nodat - nopalist[[i, 2]])},
   {i, 1, Length[chisq]}];
```

```
respalist = Table[{palist[[i, 1]], Flatten[Table[palist[[i, 2, 31 + 4 j]], {j, 0, 9}]]},
   {i, 1, Length[palist]}];
respalist2 = Table[{palist[[i, 1]],
    Flatten[Table[palist[[i, 2, 32 + 4 j]], {j, 0, 9}]]}, {i, 1, Length[palist]}];
respalist3 = Table[{palist[[i, 1]], Flatten[
     Table[palist[[i, 2, 30 + 4 j]], {j, 0, 9}]]}, {i, 1, Length[palist]}];
respar = Table[{respalist[[i, 1]], Abs[respalist[[i, 2, j]]]},
   {j, 1, Length[respalist[[1, 2]]]}, {i, 1, Length[respalist]}];
respar2 = Table[{respalist2[[i, 1]], Abs[respalist2[[i, 2, j]]]},
   {j, 1, Length[respalist2[[1, 2]]]}, {i, 1, Length[respalist2]}];
respar3 = Table[{respalist3[[i, 1]], Abs[respalist3[[i, 2, j]]]},
   {j, 1, Length[respalist3[[1, 2]]]}, {i, 1, Length[respalist3]}];
nodat = Length[Cross1] + Length[Cross2] + Sum[Length[Diff1[[i]]],
    {i, 1, Length[Diff1]}] + Sum[Length[Diff2[[i]]], {i, 1, Length[Diff2]}] +
   Sum[Length[pol1[[i]]], {i, 1, Length[pol1]}] +
   Sum[Length[pol2[[i]]], {i, 1, Length[pol2]}];
dataPar = Table[{palist[[i, 1]], Abs[palist[[i, 2, j]]]},
   {j, 1, Length[palist[[1, 2]]]}, {i, 1, Length[palist]}];
chisq = chinoplist;
chidof = Table[{chisq[[i, 1]], chisq[[i, 2]]/(nodat - nopalist[[i, 2]])}, {i, 1, Length[chisq]}];
aic = Table[{chisq[[i, 1]], 2 nopalist[[i, 2]] + nodat * Log[2 Pi] + chisq[[i, 2]]},
   {i, 1, Length[chisq]}];
aicc = Table[{chisq[[i, 1]], 2 nopalist[[i, 2]] + nodat * Log[2 Pi] + chisq[[i, 2]] +
     2 * nopalist[[i, 2]] * (nopalist[[i, 2]] + 1)/(nodat - nopalist[[i, 2]] - 1)}, {i, 1, Length[chisq]}];
bic = Table[{chisq[[i, 1]], nodat * Log[2 Pi] + chisq[[i, 2]] +
     nopalist[[i, 2]] Log[nodat]}, {i, 1, Length[chisq]}];
chitest = Table[{nopalist[[i, 1]],
    FindRoot[CDF[ChiSquareDistribution[2055 - nopalist[[i, 2]]], x] == 0.95,
      {x, 2000}][[1, 2]]}, {i, 1, Length[chinoplist]}];
chitestdof = Table[{chitest[[i, 1]], chitest[[i, 2]]/(nodat - nopalist[[i, 2]])},
   {i, 1, Length[chisq]}];
```

```
plot1 = Show[ListLogPlot[{chidof, chitestdof}, LabelStyle → 12, Epilog →
      Text[Style["(a)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}], Joined → True,
     PlotRange → {{0, 100}, {.9, 3}}, Frame → True, FrameLabel → {"λ", "χ²/d.o.f."}],
    ListPlot[Table[{bic[[Position[bic, Min[bic[[All, 2]]]][[1, 1]]]][[1]], i},
      {i, -1, 10 000, 1000}], Joined → True, PlotStyle → {Thick, Black}]];
plot2 = Show[ListPlot[respar, LabelStyle → 12,
     Epilog → Text[Style["(b)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}],
     Joined → True, Frame → True, PlotRange → {{0, 100}, {1.08, 1480}},
     FrameLabel → {"λ", "Γ [MeV]"}, PlotStyle →
      Table[If[(palist[[11, 2, 32 + 4 j]] > 3200) ∨ (palist[[11, 2, 32 + 4 j]] < mK + mXi) ∨
          (palist[[11, 2, 31 + 4 j]] > 400) ∨ (Abs[palist[[11, 2, 30 + 4 j]]] < 10^-3), Gray,
        Darker[Red]], {j, 0, 9}]], Plot[400, {x, 0, 500}, PlotStyle → {Thick, Black}],
    ListPlot[Table[{bic[[Position[bic, Min[bic[[All, 2]]]][[1, 1]]]][[1]], i},
      {i, -1, 10 000, 1000}], Joined → True, PlotStyle → {Thick, Black}]];
plot3 = Show[ListPlot[respar2, LabelStyle → 12, Epilog →
      Text[Style["(c)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}], Joined → True,
     Frame → True, PlotRange → {{0, 100}, {1111, 3333}}, FrameLabel → {"λ", "M [MeV]"},
     PlotStyle → Table[If[(palist[[11, 2, 32 + 4 j]] > 3200) ∨
          (palist[[11, 2, 32 + 4 j]] < mK + mXi) ∨ (palist[[11, 2, 31 + 4 j]] > 400) ∨
          (Abs[palist[[11, 2, 30 + 4 j]]] < 10^-3), Gray, Darker[Red]], {j, 0, 9}]],
    Plot[mK + mXi, {x, 0, 1000}, PlotStyle → {Thick, Black}],
    Plot[3200, {x, 0, 1000}, PlotStyle → {Thick, Black}],
    ListPlot[Table[{bic[[Position[bic, Min[bic[[All, 2]]]][[1, 1]]]][[1]], i},
      {i, -1, 10 000, 1000}], Joined → True, PlotStyle → {Thick, Black}]];
plot4 = Show[ListLogPlot[respar3, LabelStyle → 12,
     Epilog → Text[Style["(d)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}],
     Joined → True, Frame → True, PlotRange → {{0, 100}, {3 * 10^-4, 9}},
     FrameLabel → {"λ", "x"}, PlotStyle → Table[If[(palist[[11, 2, 32 + 4 j]] > 3200) ∨
          (palist[[11, 2, 32 + 4 j]] < mK + mXi) ∨ (palist[[11, 2, 31 + 4 j]] > 400) ∨
          (Abs[palist[[11, 2, 30 + 4 j]]] < 10^-3), Gray, Darker[Red]], {j, 0, 9}]],
    LogPlot[10^-3, {x, 0, 1000}, PlotStyle → {Thick, Black}],
    ListPlot[Table[{bic[[Position[bic, Min[bic[[All, 2]]]][[1, 1]]]][[1]], i},
      {i, -100, 1000, 1000}], Joined → True, PlotStyle → {Thick, Black}]];
plot5 = Show[ListLogPlot[bic, LabelStyle → 12,
     Epilog → Text[Style["(e)", FontSize → 12], Scaled[{0.9, 0.93}], {-1, 0}],
     PlotRange → {{0, 100}, {6100, 7999}}, Frame → True,
     Joined → True, FrameLabel → {"λ", "BIC"}],
    ListPlot[Table[{bic[[Position[bic, Min[bic[[All, 2]]]][[1, 1]]]][[1]], i},
      {i, -1, 10 000, 1000}], Joined → True, PlotStyle → {Thick, Black}]];
plotGrid[{{plot1}, {plot2}, {plot3}, {plot4}, {plot5}}, 400, 5/3 * 500]
```


(a)
χ^2/d.o.f.
3.0
2.5
2.0
1.5
1.0
(b)
Γ [MeV]
1400
1200
1000
800
600
400
200
(c)
M [MeV]
3000
2500
2000
1500
(d)
x
1
0.100
0.010
0.001
(e)
BIC
7750
7500
7250
7000
6750
6500
6250
0
20
40
60
80
100
λ

K-Fold Cross Validation

```
guess = {{s[0], -0.34824198024328584`}, {α[0], 3.763438455711533`},
    {s[1], -0.07153619899748145`}, {α[1], -0.5504395877254527`},
    {s[2], 0.20791938959137438`}, {α[2], 1.932359556897292`},
    {s[3], 0.3140263682123946`}, {α[3], 1.4353226934402328`},
    {s[4], -0.09335240870654254`}, {α[4], 1.2586482592270365`},
    {s[5], 0.19804403841470758`}, {α[5], 1.1543752980111297`},
    {s[6], 0.08564949843750701`}, {α[6], 0.8114266359183674`},
    {s[7], 1.8696909108351594`}, {α[7], 2.38042861587167`},
    {s[8], 1.6784049993618166`}, {α[8], 2.3546081341506007`},
    {s[9], -0.4910616507553648`}, {α[9], 1.976612736930773`},
    {ϕ[0], -1.3759095905974634`}, {ϕ[1], -1.8628696533673115`},
    {ϕ[2], -2.4292106595489664`}, {ϕ[3], -3.7601064302536518`},
    {ϕ[4], -3.5505168556001405`}, {ϕ[5], -0.6146668029672369`},
    {ϕ[6], 0.37669927068590553`}, {ϕ[7], -6.818425121493386`},
    {ϕ[8], 0.6225800989122515`}, {x[0], 0.5`}, {Γ[0], 100}, {M[0], 2036.962`},
    {ph[0], -0.9`}, {x[1], 0.25`}, {Γ[1], 100}, {M[1], 2000}, {ph[1], -1.2`},
    {x[2], 0.25`}, {Γ[2], 100}, {M[2], 2000}, {ph[2], -1}, {x[3], 0.25`},
    {Γ[3], 100}, {M[3], 2000}, {ph[3], -1}, {x[4], 0.5`}, {Γ[4], 160},
    {M[4], 2191.96`}, {ph[4], -0.8`}, {x[5], 0.25`}, {Γ[5], 100},
    {M[5], 2000}, {ph[5], -0.2131`}, {x[6], 0.25`}, {Γ[6], 100}, {M[6], 2000},
    {ph[6], -0.121`}, {x[7], 0.5`}, {Γ[7], 60}, {M[7], 2786.962`},
    {ph[7], -0.7`}, {x[8], 0.25`}, {Γ[8], 100}, {M[8], 2000}, {ph[8], -1},
    {x[9], 0.5`}, {Γ[9], 200}, {M[9], 2603.962`}, {ph[9], -0.6`}};

chilist = {};
chinoplist = {};
palist = {};
nopalist = {};
k[W_, m1_, m2_] := 1/(2 * W) (√((W^2 - (m1 - m2)^2) * (W^2 - (m1 + m2)^2)))
kf[W_] = k[W, mK, mXi];
ki[W_] = k[W, mK, mP];
jmax = 5/2;
ons = 1;
onp = 1;
ond = 1;
nopars = 6 * ons + 12 * onp + 12 * ond;
num = If[nopars == 6, 1, If[nopars == 18, 5, 9]];
pars =
  Flatten[Join[Table[{s[i], α[i]}, {i, 0, num}], Table[{ϕ[i]}, {i, 0, num - 1}]]];
```

```
Do[

 Do[on[i] = If[Abs[guess[[4 * i + 30, 2]]] > 10^-3, 1, 0], {i, 0, 9}];
 S01half[W_] :=
  (((s[0] Exp[-α[0]^2 * kf[W]^2/Λ^2] - on[0] x[0] Exp[I * ph[0]] Γ[0]/(2 (W - M[0] + I Γ[0]/2))
      Exp[I * ϕ[0]]) * (kf[W]/Λ)^(0+1/2);
 S11half[W_] := (((s[1] Exp[-α[1]^2 * kf[W]^2/Λ^2] -
        on[1] x[1] Exp[I * ph[1]] Γ[1]/(2 (W - M[1] + I Γ[1]/2))) Exp[I * ϕ[1]]) * (kf[W]/Λ)^(0+1/2);
 P01half[W_] := (((s[2] Exp[-α[2]^2 * kf[W]^2/Λ^2] - on[2] x[2] Exp[I * ph[2]]
          Γ[2]/(2 (W - M[2] + I Γ[2]/2))) Exp[I * ϕ[2]]) * (kf[W]/Λ)^(1+1/2);
 P03half[W_] := (((s[3] Exp[-α[3]^2 * kf[W]^2/Λ^2] - on[3] x[3] Exp[I * ph[3]]
          Γ[3]/(2 (W - M[3] + I Γ[3]/2))) Exp[I * ϕ[3]]) * (kf[W]/Λ)^(1+1/2);
 P11half[W_] := (((s[4] Exp[-α[4]^2 * kf[W]^2/Λ^2] - on[4] x[4] Exp[I * ph[4]]
          Γ[4]/(2 (W - M[4] + I Γ[4]/2))) Exp[I * ϕ[4]]) * (kf[W]/Λ)^(1+1/2);
 P13half[W_] := (((s[5] Exp[-α[5]^2 * kf[W]^2/Λ^2] - on[5] x[5] Exp[I * ph[5]]
          Γ[5]/(2 (W - M[5] + I Γ[5]/2))) Exp[I * ϕ[5]]) * (kf[W]/Λ)^(1+1/2);
 D03half[W_] := (((s[6] Exp[-α[6]^2 * kf[W]^2/Λ^2] - on[6] x[6] Exp[I * ph[6]]
          Γ[6]/(2 (W - M[6] + I Γ[6]/2))) Exp[I * ϕ[6]]) * (kf[W]/Λ)^(2+1/2);
 D05half[W_] := (((s[7] Exp[-α[7]^2 * kf[W]^2/Λ^2] - on[7] x[7] Exp[I * ph[7]]
          Γ[7]/(2 (W - M[7] + I Γ[7]/2))) Exp[I * ϕ[7]]) * (kf[W]/Λ)^(2+1/2);
```

```
D13half[W_] := ((s[8] Exp[-α[8]^2 * kf[W]^2/Λ^2] - on[8] x[8] Exp[I * ph[8]]
      Γ[8]/(2 (W - M[8] + I Γ[8]/2))) Exp[I * ϕ[8]]) * (kf[W]/Λ)^(2+1/2);
D15half[W_] := ((s[9] Exp[-α[9]^2 * kf[W]^2/Λ^2] - on[9] x[9] Exp[I * ph[9]]
      Γ[9]/(2 (W - M[9] + I Γ[9]/2))) Exp[I * 0]) * (kf[W]/Λ)^(2+1/2);
τ0_-1/2 = {S01half[W], P03half[W], D05half[W]};
τ0_+1/2 = {P01half[W], D03half[W], 0};
τ1_-1/2 = {S11half[W], P13half[W], D15half[W]};
τ1_+1/2 = {P11half[W], D13half[W], 0};
g0[θ_, W_] = 1/(2 Sqrt[kf[W] * ki[W]]) *
  Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] * (τ0_-1/2[[J + 1/2]] + τ0_+1/2[[J + 1/2]])
      Cos[θ/2] + WignerD[{J, -1/2, 1/2}, θ]
      (τ0_-1/2[[J + 1/2]] - τ0_+1/2[[J + 1/2]]) Sin[θ/2]), {J, 1/2, jmax, 1}];
h0[θ_, W_] = -I/(2 Sqrt[kf[W] * ki[W]]) * Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] *
      (τ0_-1/2[[J + 1/2]] + τ0_+1/2[[J + 1/2]]) Sin[θ/2] - WignerD[{J, -1/2, 1/2}, θ]
      (τ0_-1/2[[J + 1/2]] - τ0_+1/2[[J + 1/2]]) Cos[θ/2]), {J, 1/2, jmax, 1}];
g1[θ_, W_] = 1/(2 Sqrt[kf[W] * ki[W]]) * Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] *
      (τ1_-1/2[[J + 1/2]] + τ1_+1/2[[J + 1/2]]) Cos[θ/2] + WignerD[{J, -1/2, 1/2}, θ]
      (τ1_-1/2[[J + 1/2]] - τ1_+1/2[[J + 1/2]]) Sin[θ/2]), {J, 1/2, jmax, 1}];
h1[θ_, W_] = -I/(2 Sqrt[kf[W] * ki[W]]) * Sum[(2 J + 1) * (WignerD[{J, 1/2, 1/2}, θ] *
      (τ1_-1/2[[J + 1/2]] + τ1_+1/2[[J + 1/2]]) Sin[θ/2] - WignerD[{J, -1/2, 1/2}, θ]
      (τ1_-1/2[[J + 1/2]] - τ1_+1/2[[J + 1/2]]) Cos[θ/2]), {J, 1/2, jmax, 1}];


gre1[θ_, W_] := -1/2 g1[θ, W] + 1/2 g0[θ, W](*<Ξ⁻k⁺| p K⁻ *);
hre1[θ_, W_] := -1/2 h1[θ, W] + 1/2 h0[θ, W] (*<Ξ⁻k⁺| p K⁻ *);

gre2[θ_, W_] := -1/2 g1[θ, W] - 1/2 g0[θ, W](*<Ξ⁰k⁰| p K⁻ *);
hre2[θ_, W_] := -1/2 h1[θ, W] - 1/2 h0[θ, W] (*<Ξ⁰k⁰| p K⁻ *);


dσdΩ1[W_, θ_] :=
 factor * (Conjugate[gre1[θ, W]] gre1[θ, W] + Conjugate[hre1[θ, W]] * hre1[θ, W]) *
```

```
   kf[W]/ki[W] (*<Ξ⁻k⁺| p K⁻ *);
P1[W_, θ_] := 2 Re[gre1[θ, W] * Conjugate[hre1[θ, W]]] / (Abs[gre1[θ, W]]^2 + Abs[hre1[θ, W]]^2) * dσdΩ1[W, θ]
(*<Ξ⁻k⁺| p K⁻ *);


dσdΩ2[W_, θ_] :=
 factor * (Conjugate[gre2[θ, W]] gre2[θ, W] + Conjugate[hre2[θ, W]] * hre2[θ, W]) *
   kf[W]/ki[W] (*<Ξ⁰k⁰| p K⁻ *);
P2[W_, θ_] := 2 Re[gre2[θ, W] * Conjugate[hre2[θ, W]]] / (Abs[gre2[θ, W]]^2 + Abs[hre2[θ, W]]^2) * dσdΩ2[W, θ]
(*<Ξ⁰k⁰| p K⁻ *);
σ1[W_] = factor * 1/2 * 4 Pi/ki[W]^2
  Sum[(2 J + 1) (Abs[-1/2 (τ1_{-1/2}[[J + 1/2]])]^2 +
      Abs[-1/2 (τ1_{1/2}[[J + 1/2]]) + 1/2 (τ0_{1/2}[[J + 1/2]])]^2), {J, 1/2, jmax, 1}];
σ2[W_] = factor * 1/2 * 4 Pi/ki[W]^2 Sum[(2 J + 1)
    (Abs[-1/2 (τ1_{-1/2}[[J + 1/2]]) - 1/2 (τ0_{-1/2}[[J + 1/2]])]^2 +
      Abs[-1/2 (τ1_{1/2}[[J + 1/2]]) - 1/2 (τ0_{1/2}[[J + 1/2]])]^2), {J, 1/2, jmax, 1}];
Chisq[λ_, data1_, data2_, data3_, data4_, data5_, data6_] := Sum[(1/data1[[i, j, 3]])^2 *
    (dσdΩ1[Diff1Energy[[i]], data1[[i, j, 1]]] - data1[[i, j, 2]])^2,
   {i, 1, Length[data1]}, {j, 1, Length[data1[[i]]]}] +
  Sum[(1/data2[[i, j, 3]])^2 *
    (dσdΩ2[Diff2Energy[[i]], data2[[i, j, 1]]] - data2[[i, j, 2]])^2,
   {i, 1, Length[data2]}, {j, 1, Length[data2[[i]]]}] + Sum[
   (1/data3[[i, j, 3]])^2 * (P1[pol1Energy[[i]], data3[[i, j, 1]]] - data3[[i, j, 2]])^2,
   {i, 1, Length[data3]}, {j, 1, Length[data3[[i]]]}] + Sum[
   (1/data4[[i, j, 3]])^2 * (P2[pol2Energy[[i]], data4[[i, j, 1]]] - data4[[i, j, 2]])^2,
   {i, 1, Length[data4]}, {j, 1, Length[data4[[i]]]}] + Sum[
   (1/data5[[i, 3]])^2 * (σ1[data5[[i, 1]]] - data5[[i, 2]])^2, {i, 1, Length[data5]}] +
  Sum[(1/data6[[i, 3]])^2 * (σ2[data6[[i, 1]]] - data6[[i, 2]])^2,
   {i, 1, Length[data6]}] + +λ^4 (Abs[x[9]] + Abs[x[8]] + Abs[x[7]] + Abs[x[6]] +
     Abs[x[5]] + Abs[x[4]] + Abs[x[3]] + Abs[x[2]] + Abs[x[1]] + Abs[x[0]]);
```

```
fitresults = FindMinimum[Chisq[λ, Diff1t, Diff2t, pol1t, pol2t, Cross1t, Cross2t],
  guess, MaxIterations → 2000];
pars = fitresults[[2]];
chisqwp = Re[Chisq[λ, Diff1v, Diff2v, pol1v, pol2v, Cross1v, Cross2v] /. pars];
chisq = Re[Chisq[0, Diff1v, Diff2v, pol1v, pol2v, Cross1v, Cross2v] /. pars];
AppendTo[chilist, {λ, chisqwp}];
AppendTo[chinoplist, {λ, chisq}];
guess = Table[{pars[[i, 1]], pars[[i, 2]]}, {i, 1, Length[pars]}];
kkcount = 0;
Do[If[Abs[pars[[i, 2]]] < 10^-3, kkcount = kkcount + 1], {i, 1, Length[guess]}];
AppendTo[nopalist, {λ, Length[guess] - 4 kkcount}];
AppendTo[palist, {λ, pars[[ ;; , 2]]}];
Print[λ];, {λ, 0, 10, 0.5}]
```

```
ListLogPlot[chinoplist, LabelStyle → 12, Joined → True,
 PlotRange → {{0, 8}, {680, 701}}, Frame → True, FrameLabel → {"λ", "Validation χ^2"}]
```