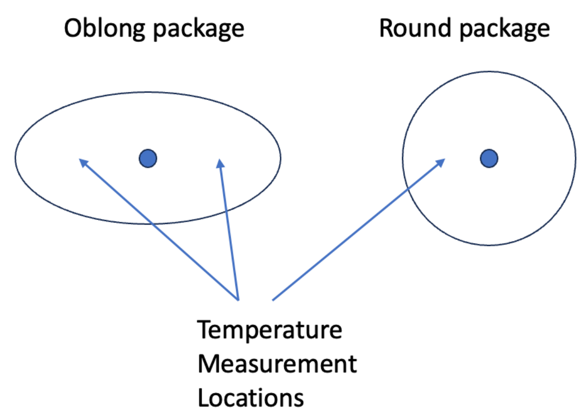
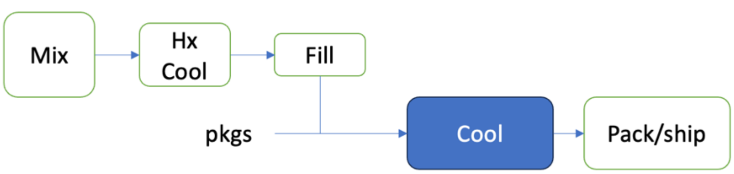
Solids are a convenient form for consumers to apply a product onto their skin. Deodorants, antiperspirants, sunscreens and bug repellants are examples. A common way to formulate and produce them is with an active ingredient suspended in a molten liquid containing a molten gellant ingredient. This is then cooled to close to the gellant's solidification point and poured (aka "filled") into the packgage at high speed. The resulting products are then cooled at a controlled rate in a convection, cooling tunnel having relatively long residence time of 30+ minutes. In addition to solidifying the product so that the active ingredient does not settle, the cooling can affect the final hardness of the product. That is problematic if not controlled. If the product is too soft, it will smear on the consumer's skin. If it is too hard, not enough product will be applied, and it will not work as desired.

The data set **consumer_solid.jmp** contains cooling data from hypothetical lab and plant scale cooling experiments with such products. The temperature data are the interior temperature of the product measured by inserting a temperature probe into the center at various times while products are cooled down from the filling temperature. These measurements are taken at the furthest, interior location away from the package walls and other package parts such as the screw elevator used to dispene the product.





Exercises Using JMP® software:

1. Plot the raw temperature versus time data using color-coding for type of process
2. Add a dimensionless temperature column, Theta (θ), to non-dimensionalize temperature to a 0 to 1 scale ranging from the the initial, filling temperature ($\theta = 1$) to the final, ambient temperature ($\theta = 0$)
3. Make it possible to linearize the data by adding a **- ln(θ)** column
4. Plot **- ln(θ)** versus time and fit straight lines by type of process (exclude initial data points where the product's core temperature has not yet started to react to the cooling process)
5. Calculate each process' characteristic cooling time constant, τ, based on the limiting case theory below.

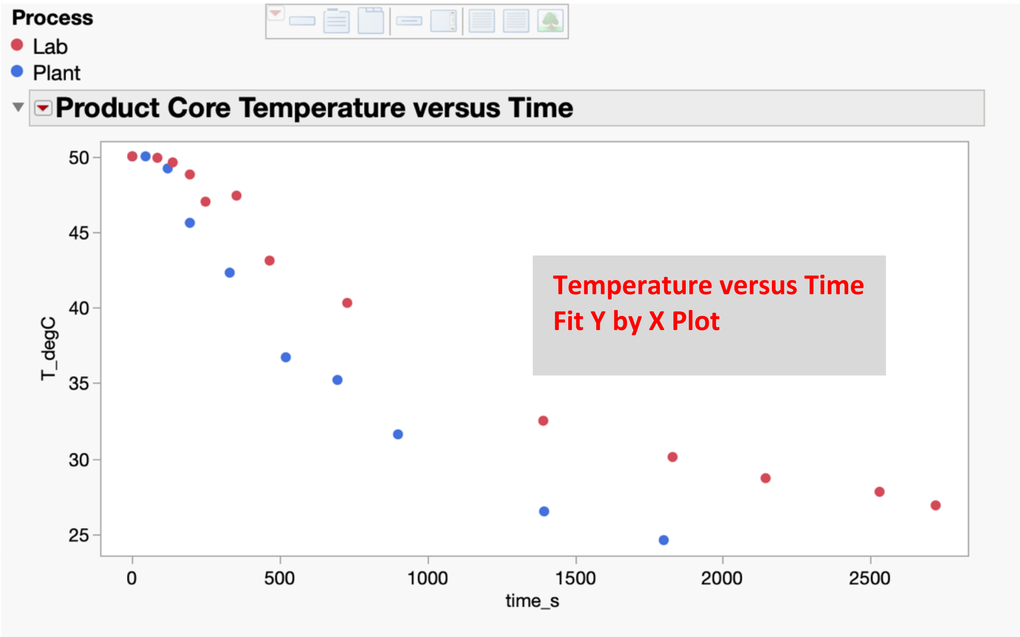
<u>**JMP Step-by-step Guide for Exercises**</u>

The **consumer_solid.jmp** data contain Temperature versus time data for two cooling processes –Lab and Plant. Based on the Theory section below, to quantify/characterize the Lab and Plant processes' cooling, we should convert temperature to a dimensionless temperature and plot the negative natural log of it. If there is a good, linear fit with that plot, we can then calculate a characteristic time constant for each process from the plot's slope.

To explore this in JMP:

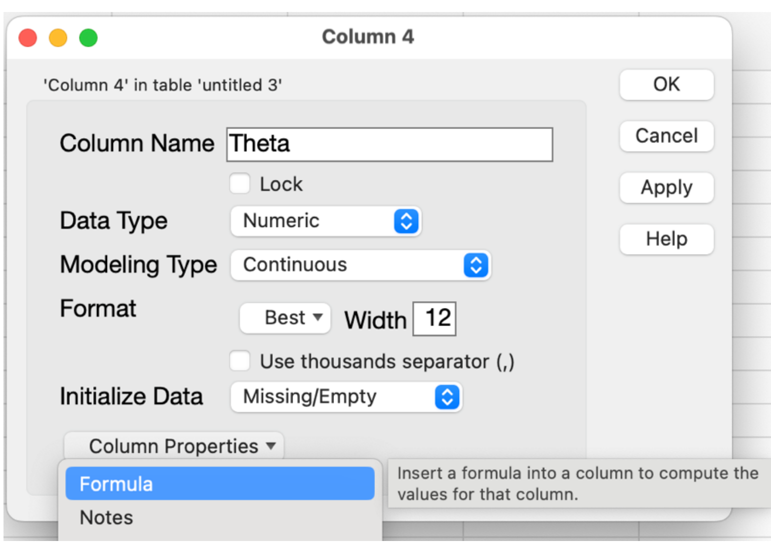
1. Open the data and make a plot (Analyze / Fit Y by X) of Temperature (Y) by time (x)

2. Use Rows / Color or Mark by Column to color code the data by Process



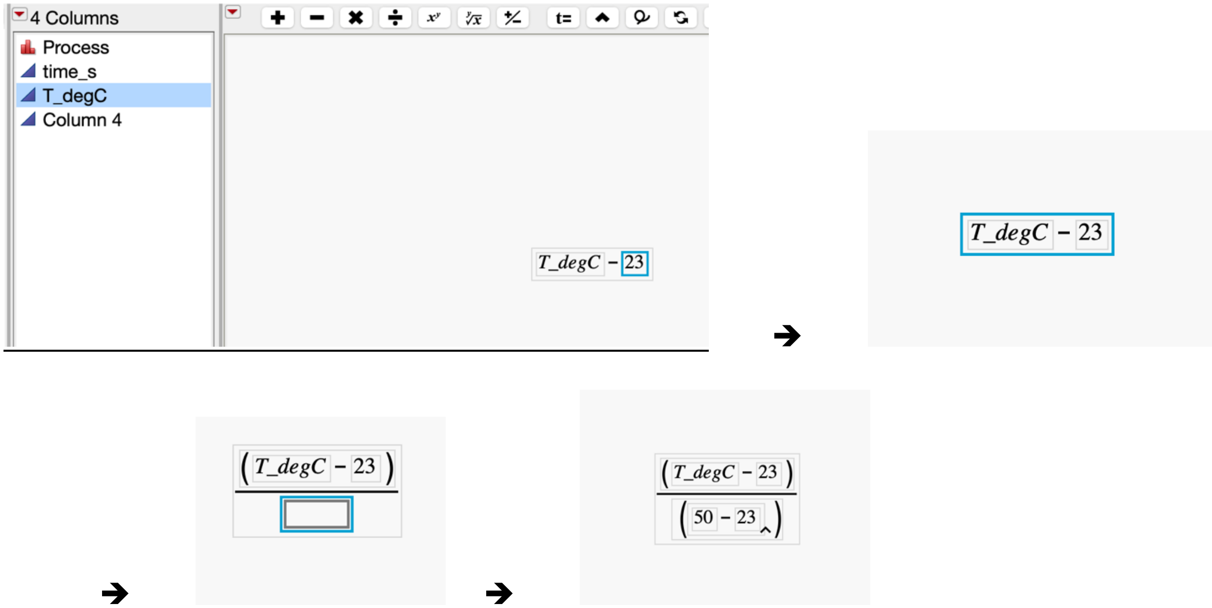
- Which process has faster cooling? Lab or Plant?
- What is the initial behavior of the core temperature? Does this make sense physically based on the product or process?

3. Add a calculated dimensionless temperature, Theta, column
   a. For these data, the initial temperature, T_0 is 50 C, and the ambient temperature is 23.0 C. Realistically, these might differ between lab and plant, but we keep them the same here to simplify.
   b. To add a new column to the JMP data table, either double-click in the data table's column header region or choose Cols / New Column menu to add a new blank column
   c. In its Column Info (double click on header or choose Cols / Col Info menu), set its name to "Theta" and select "Formula" under Column Properties as shown



   d. To enter the formula $(T - T_a)/(T_0 - T_a)$, double-click on the T_degC column name to use that column's values. Type a minus sign, 23 and Enter. This creates the numerator

e. Once the numerator is entered, click on the bigger rectangle surrounding the equation to select it (blue highlight) and type a slash "/" or the divide-by operator
f. Type "50.0", Enter, "23.0", Enter to input the denominator

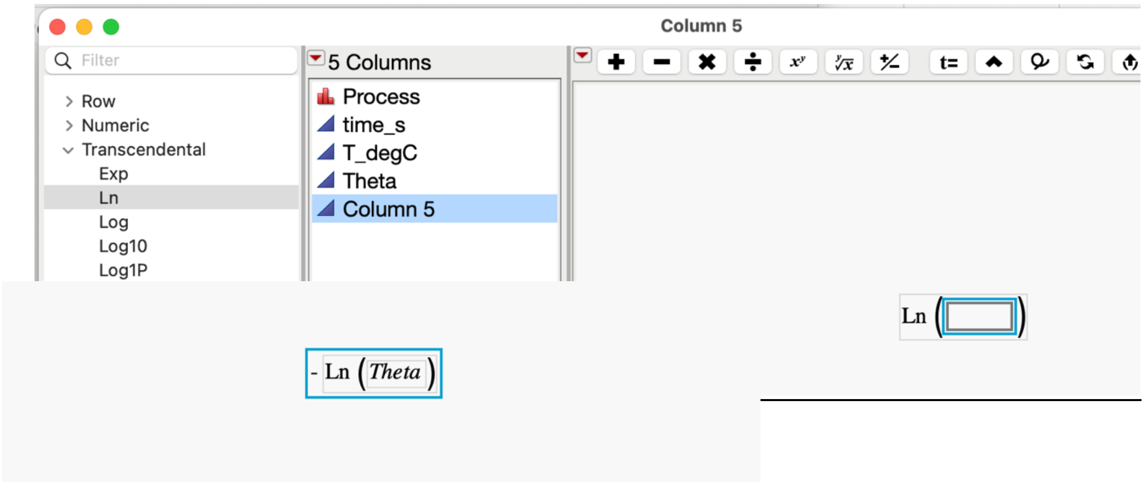


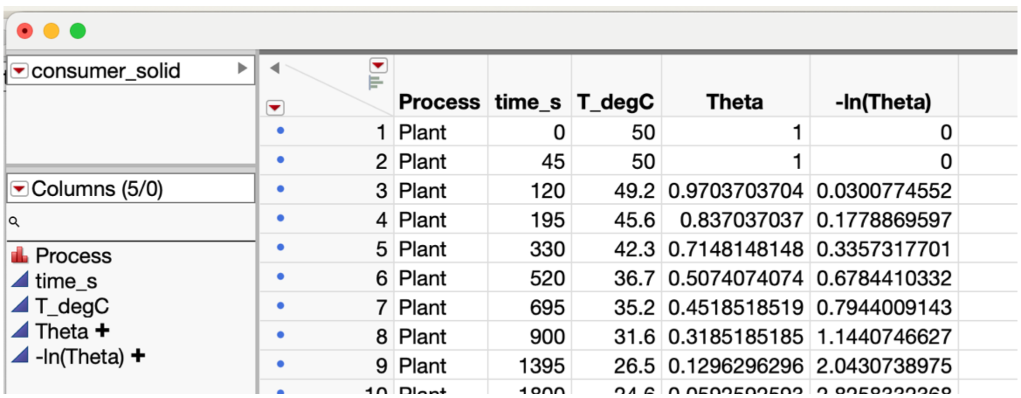
g. Click OK twice to apply the formula. Notice that, because Theta is a calculated column, JMP prohibits manual changes to its values

	Process	time_s	T_degC	Theta
1	Plant	0	50	1
2	Plant	45	50	1
3	Plant	120	49.2	0.9703703704
4	Plant	195	45.6	0.837037037
5	Plant	330	42.3	0.7148148148
6	Plant	520	36.7	0.5074074074
7	Plant	695	35.2	0.4518518519
8	Plant	900	31.6	0.3185185185
9	Plant	1395	26.5	0.1296296296
10	Plant	1800	24.6	0.0592592593
11	Lab	0	50	1
12	Lab	85	49.9	0.9962962963
13	Lab	137	49.6	0.9851851852
14	Lab	195	48.8	0.9555555556
15	Lab	248	47	0.8888888889
16	Lab	353	47.4	0.9037037037
17	Lab	465	43.1	0.7444444444
18	Lab	728	40.3	0.6407407407
19	Lab	1392	32.5	0.3518518519
20	Lab	1830	30.1	0.262962963
21	Lab	2145	28.7	0.2111111111
22	Lab	2531	27.8	0.1777777778
23	Lab	2721	26.9	0.1444444444

4. Add a "-ln(Theta)" column to calculate the negtive log of dimensionless Temperature, Theta.
   a. Add another new column and select Formula / Edit Formula in its Col Info as before
   b. The "ln" function is under the Transcendental grouping in the formula entry dialog
   c. Use the "+/-" button at the top of the formula entry to flip the equation's sign to minus

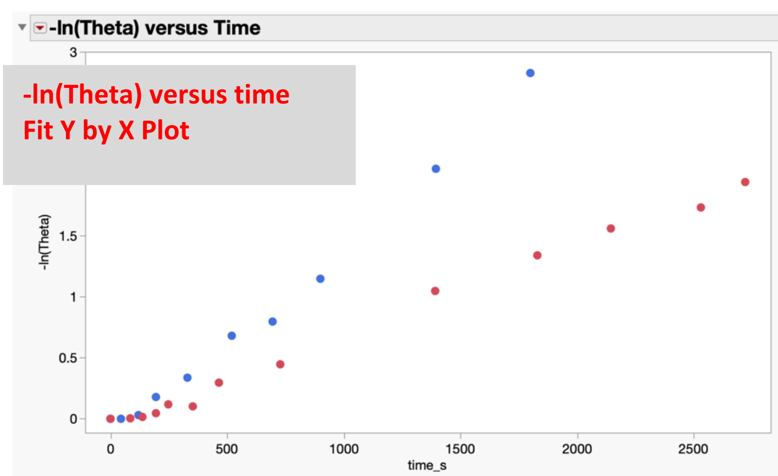
d. Remember to “select the whole equation by clicking on the outer shaded rectangle (blue highlight) so that you end up with **- ln(Theta)** and not **ln(-Theta)**



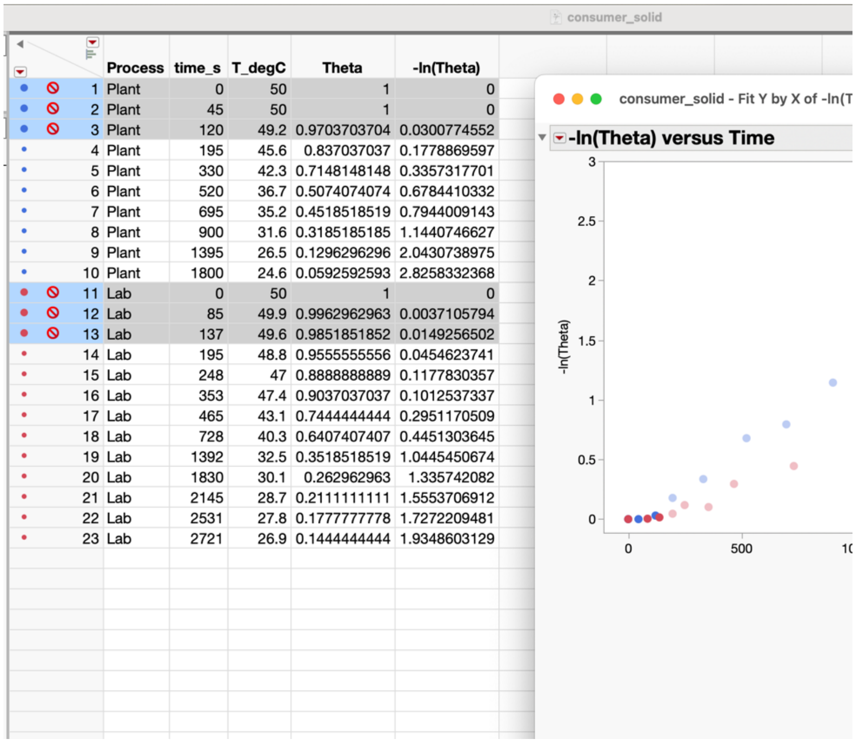


	Process	time_s	T_degC	Theta	-ln(Theta)
1	Plant	0	50	1	0
2	Plant	45	50	1	0
3	Plant	120	49.2	0.9703703704	0.0300774552
4	Plant	195	45.6	0.837037037	0.1778869597
5	Plant	330	42.3	0.7148148148	0.3357317701
6	Plant	520	36.7	0.5074074074	0.6784410332
7	Plant	695	35.2	0.4518518519	0.7944009143
8	Plant	900	31.6	0.3185185185	1.1440746627
9	Plant	1395	26.5	0.1296296296	2.0430738975

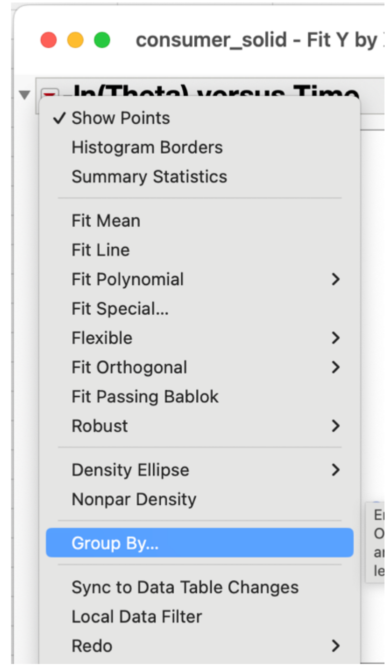
5. Plot -ln(Theta) versus time. Based on the theory, we expect this to be linear for each process –except for the initial times where the center of the product has not yet started to cool.
6. Before doing the linear fit, we need to do two actions:
   a. Lasso and exclude points in the initial “induction” period where the data are not linear. It is a judgement call how many to exclude, but the cooling appears to reach steady state below $\theta \approx 0.97$
   b. Tell JMP to use Process as a Group By variable. This selection is under the red triangle menu as shown[1]



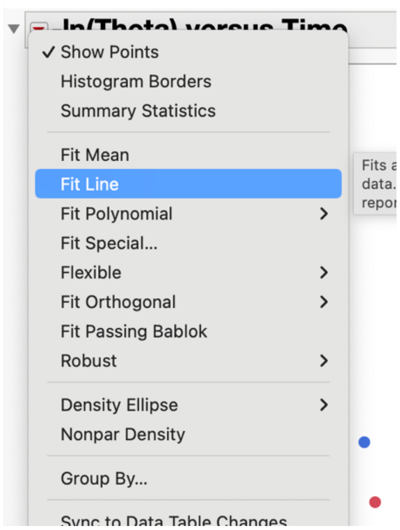
[1] Note that an alternate way to “Group” is to assign the Process variable to the Fit Y by X dialog’s “By” listbox when making the plot. This will result in separate plots for Lab and Process, and you can then perform the linear fit for each.

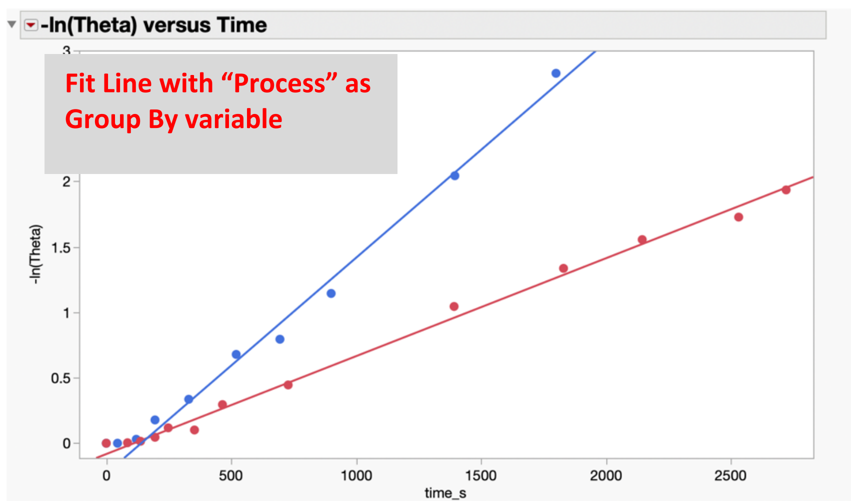


	Process	time_s	T_degC	Theta	-ln(Theta)
1	Plant	0	50	1	0
2	Plant	45	50	1	0
3	Plant	120	49.2	0.9703703704	0.0300774552
4	Plant	195	45.6	0.837037037	0.1778869597
5	Plant	330	42.3	0.7148148148	0.3357317701
6	Plant	520	36.7	0.5074074074	0.6784410332
7	Plant	695	35.2	0.4518518519	0.7944009143
8	Plant	900	31.6	0.3185185185	1.1440746627
9	Plant	1395	26.5	0.1296296296	2.0430738975
10	Plant	1800	24.6	0.0592592593	2.8258332368
11	Lab	0	50	1	0
12	Lab	85	49.9	0.9962962963	0.0037105794
13	Lab	137	49.6	0.9851851852	0.0149256502
14	Lab	195	48.8	0.9555555556	0.0454623741
15	Lab	248	47	0.8888888889	0.1177830357
16	Lab	353	47.4	0.9037037037	0.1012537337
17	Lab	465	43.1	0.7444444444	0.2951170509
18	Lab	728	40.3	0.6407407407	0.4451303645
19	Lab	1392	32.5	0.3518518519	1.0445450674
20	Lab	1830	30.1	0.262962963	1.335742082
21	Lab	2145	28.7	0.2111111111	1.5553706912
22	Lab	2531	27.8	0.1777777778	1.7272209481
23	Lab	2721	26.9	0.1444444444	1.9348603129



7. Fit lines to the data by choosing Fit Line from the Plot's red triangle menu. The result in a good linear fit: R^2 = 0.995 for Lab and 0.992 for Plant





8. Calculate the time constants, τ_c as 1/slope for each cooling process. Do they make sense? Since this is a first-order equation, cooling should be approximately complete (e.g. to ambient) in $3\tau_c$

9. What do you suggest changing about the lab process to reduce τ_c to more closely match the Plant cooling behavior?

Background Theory

The raw, cooling data are not linear. We could just fit a non-linear model to the data, but that will not simplify things or give us a single number to use to quantify what each process is doing.

From heat transfer theory, two, limiting cases guide on how to transform the data based on heat transfer theory[2]:

1. What if the product and its packaging have no (or negligible) conduction resistance relative to the convection rate? This would be the case if the product and package were made of a high conduction solid such as copper, silver or gold.
   - In this case, all heat removed by the cooling instantly shows up at the center of the product in the form of reduced temperature.
   - There is a first-order, ordinary differential equation solution for that unsteady state problem
   - The time constant, τ_C, is a function of the solid's mass, heat capacity, surface area and the convection heat transfer coefficient, so it is a constant for a given solid in a given cooling environment

$$-\ln\theta = \frac{t}{\tau_C}$$

Where:

$\theta = (T - T_a) / (T_0 - T_a)$
$\tau_c = mC_p / hA$

h convection heat transfer coefficient
A effective surface area of the solid
T time-dependent temperature of the solid
T_0 the solid's initial temperature
T_a ambient temperature
M mass of the solid
C_p heat capacity of the solid

2. What if the product and package are a homogeneous sphere, and the convection to conduction ratio (Biot number) is relatively high?
   - There is an infinite series solution for time-dependent conduction in an ideal solid
   - The infinite series solution captures initial behavior where there is a time lag before heat transferred to the outside of the solid shows up as temperature change at the center. The first term captures the temperature change at the center following this induction period

$$-\ln\theta_0 = -\ln C_1 + \left[\zeta_1^2 k / \rho C_p r^2\right] t$$

   - The bracketed quantity is a constant based on the solid's properties and the cooling process (e.g. convection rate)
   - C_1 and ζ_1 are functions of Biot number aka ratio of convection rate and properties of the solid
   - Additional infinite series terms serve to "round off" dog leg but don't change the fundamental shape of the –ln(θ) versus time graph.

These limiting cases both guide to plotting -ln(θ) versus time and treating the inverse slope as a characteristic time constant for the product + process combination. The limiting cases suggest using a dimensionless temperature.

---

[2] D.P. DeWitt and F. P. Incropera, Introduction to Heat Transfer (3rd Edition), John Wiley and Sons, New York, 1996, pp. 225-236