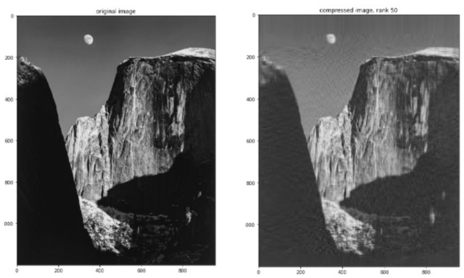
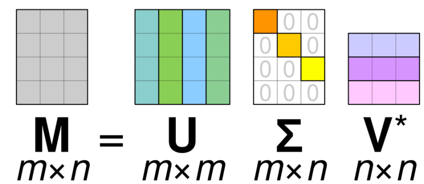


Lecture 13:
Singular Value Decomposition2

CS 111: Intro to Computational Science
Spring 2023



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Administrative

- Current homework due today
- New homework out later today

- Quiz 5 on Wednesday
  - Lectures 11 and 12 (FP, SVD part 1)

- Today's lecture is mostly demonstration and will be recorded

Using **numpy**

```
U, sigma, V = npla.svd(A, full_matrices=False)
                       # Only use that option if A is not a square matrix
```

Example: `A = np.array([[1, 2], [3, 4]])`

Gives you:

- `U =  [[-0.4045 -0.9145 ], [-0.9145   0.4045]]`
- `V = [[-0.5760 -0.8174], [ 0.8174 -0.5760]]` **`# NOTE: This is already transposed`**
- `sigma = [5.4649857  0.36596619]`

Note that this is presented as a vector, but the Σ factor in SVD is a diagonal matrix.

- Checking the decomposition, fina matrix **Anew**, such that: **`Anew = U@np.diag(sigma)@V`**

  And then compare to the original matrix **A** (i.e. do a relative residual)

Singular Value Decomposition on Square Matrix

$$A = U\Sigma V^T$$

$$= \begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \sigma_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \dots \\ v_n \end{bmatrix}$$

$$= u_1 \ \sigma_1 v_1^T + u_2 \sigma_2 v_2^T + \dots + u_n \sigma_n v_n^T$$

Theorems Relating to SVD

1. The *rank* of **A** is the number of *nonzero* singular values (**number of σ_i**)

2. The ***2-norm*** $||\mathbf{A}||_2$ is equal to the largest singular value, i.e. σ_0

3. The ***2-norm condition number*** $\kappa_2(\mathbf{A})$ is equal to the ratio of the largest and smallest singular values. That is, $\kappa_2(\mathbf{A}) = \sigma_0/\sigma_{min(m,n)-1}$

4. The ***Frobenius norm*** $||\mathbf{A}||_F$ is equal to $(\Sigma_i \sigma_i^2)^{1/2}$

5. The ***determinant of a square matrix*** is the product of its singular values, $\Pi_i\sigma_i$

6. Matrix **A** is the sum of rank-1 matrices: $$A_k = \sum_{i=0}^{k-1} \sigma_i u_i v_i^T$$

Quick! To the Python-mobile!

Your TO DOs!

- Assignment 06 due tonight

- Quiz 5 on Wednesday
  - Lectures 11 and 12 (FP, SVD part 1)

</LECTURE>