RA-MIRI Assignment 1: Galton box

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Source code: https://github.com/jlapeyra/RA-assignments/tree/main/assignment-1/src

1 Implementation

I implemented the Galton box in Python. The program goes as follows. Given $n, m \in \mathbb{Z}^+$, drop a ball into the box m times. Each drop consists of moving the ball n times. Each move is a random choice between left and right with equal probability, namely $1/2$. I used the same notation as in the statement, where the position of a ball is encoded in a pair (i, j), the ball starts at $(0, 0)$, a move to the left is $(i + 1, j)$ and a move to the right is $(i, j + 1)$. For each ball, I store the final position i (j is redundant because $i + j = n$). Then I plot using `matplotlib` the empirical probability of each i, i.e. the rate of balls that fell on position $(i, n - i)$. Over this bar plot, I print the normal distribution $\mathcal{N}(\mu = \frac{n}{2}, \sigma^2 = \frac{n}{4})$ using `scipy`.

2 Results

The central limit theorem explains that the sum of a large number of independent random variables (like each step) will tend to form a normal distribution, regardless of the underlying distribution of the individual events (in this case, left or right).

In a Galton box, as each ball goes through numerous steps and encounters many random binary choices, the sum of these independent decisions creates the bell-shaped curve of a normal distribution.

In figure 1 we see different plots of a Galton box with 6 steps. As expected, the larger the sample (number of balls thrown), the closer the random binomial distribution is to a normal probability density function. In addition, the smaller the sample, the more random is the distribution, i.e. the more different are different runs of the same experiment.

In figures 2, 3 and 4, we see the same plots for Galton boxes of 12, 20 and 40 steps. The behaviour is very similar: the random distribution tends to a normal probability density function with the sample size.

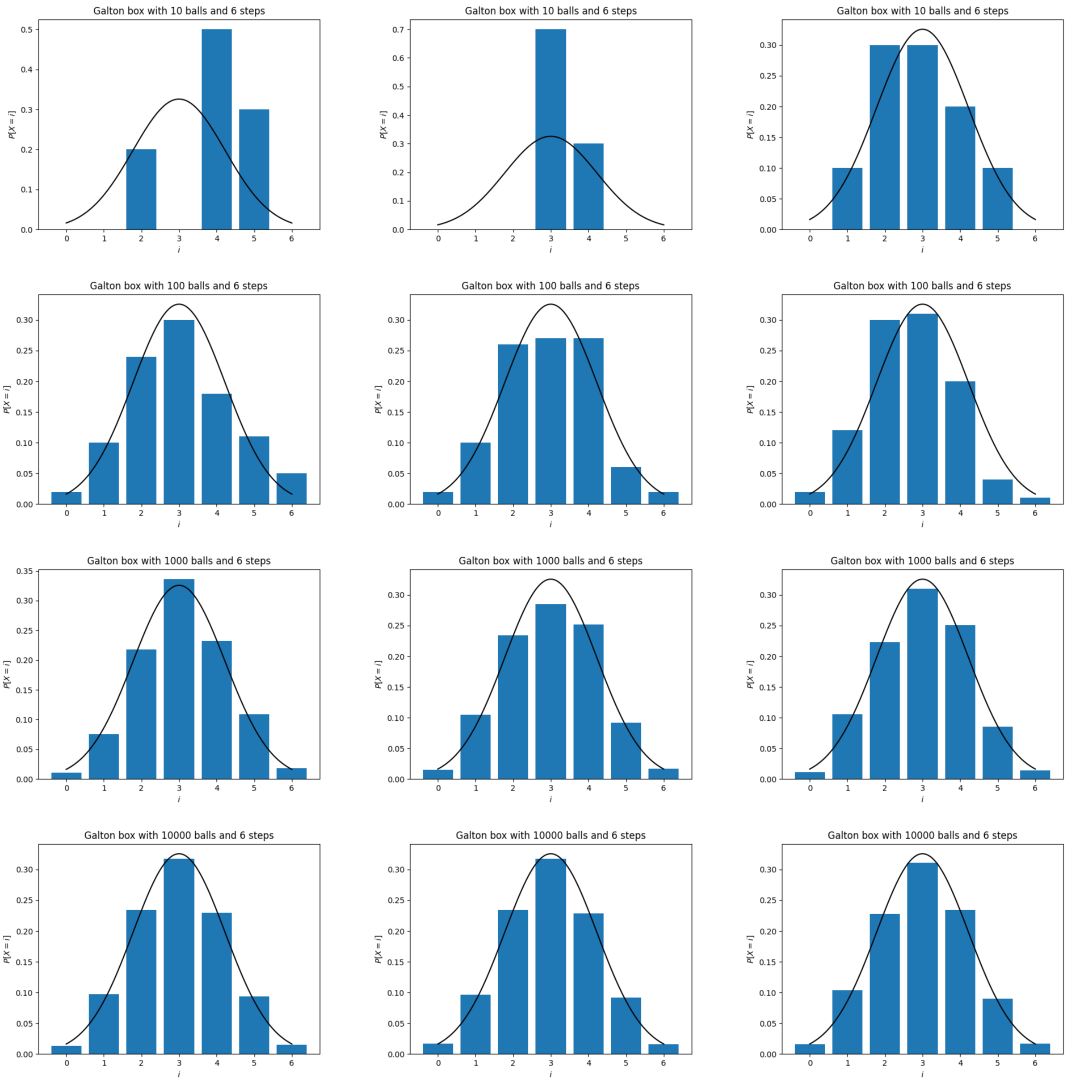

Figure 1: Probability distribution of a Galton box with 6 steps. Each row has 3 runs of a given number of samples (balls thrown): 10, 100, 1,000 and 10,000.

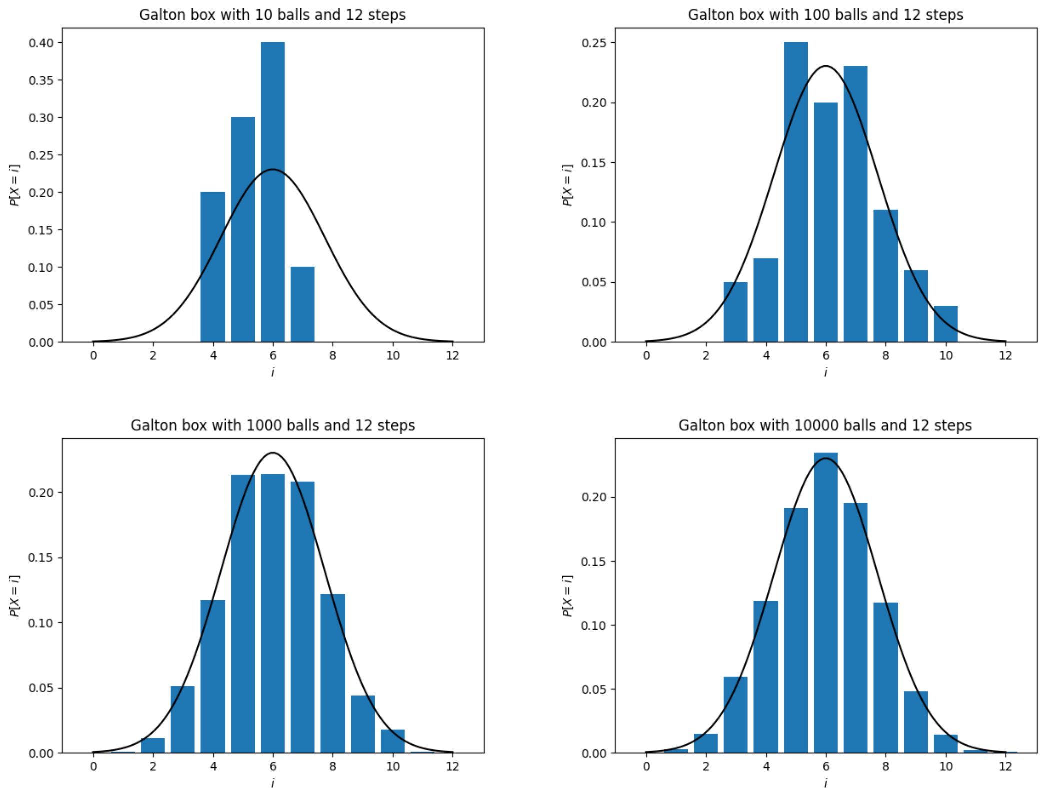

Figure 2: Probability distribution of a Galton box with 12 steps. Each plot has a different number of samples (balls thrown): 10, 100, 1,000 and 10,000.

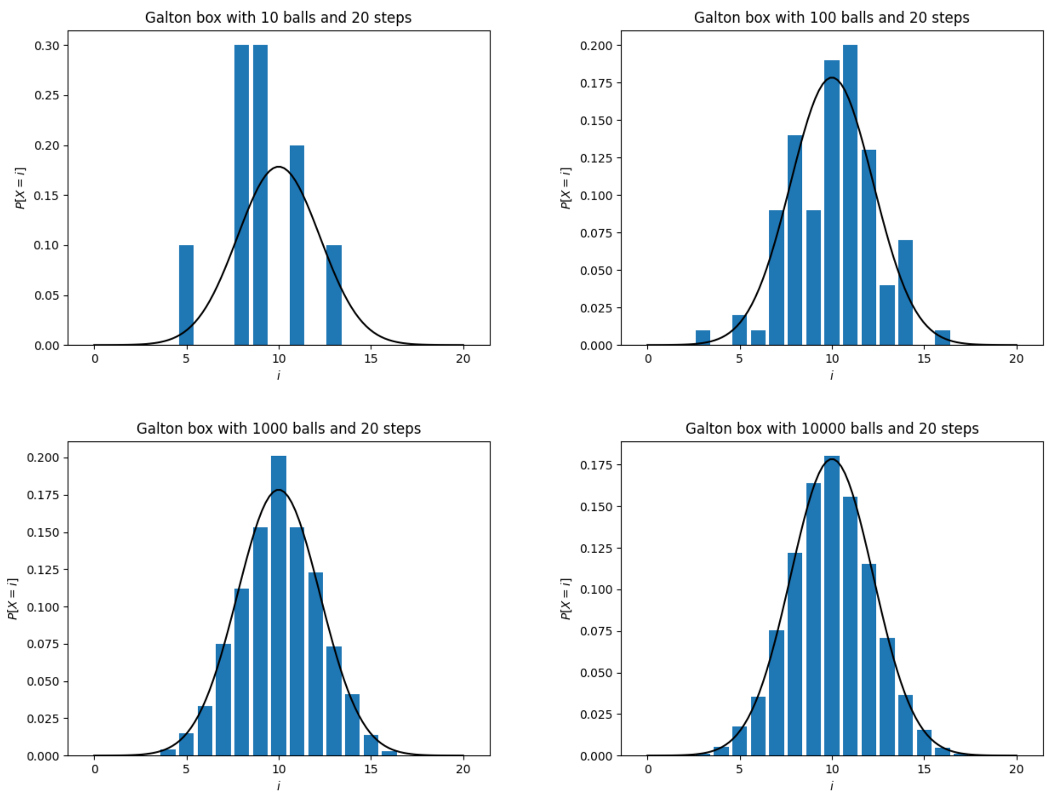

Figure 3: Probability distribution of a Galton box with 20 steps. Each plot has a different number of samples (balls thrown): 10, 100, 1,000 and 10,000.

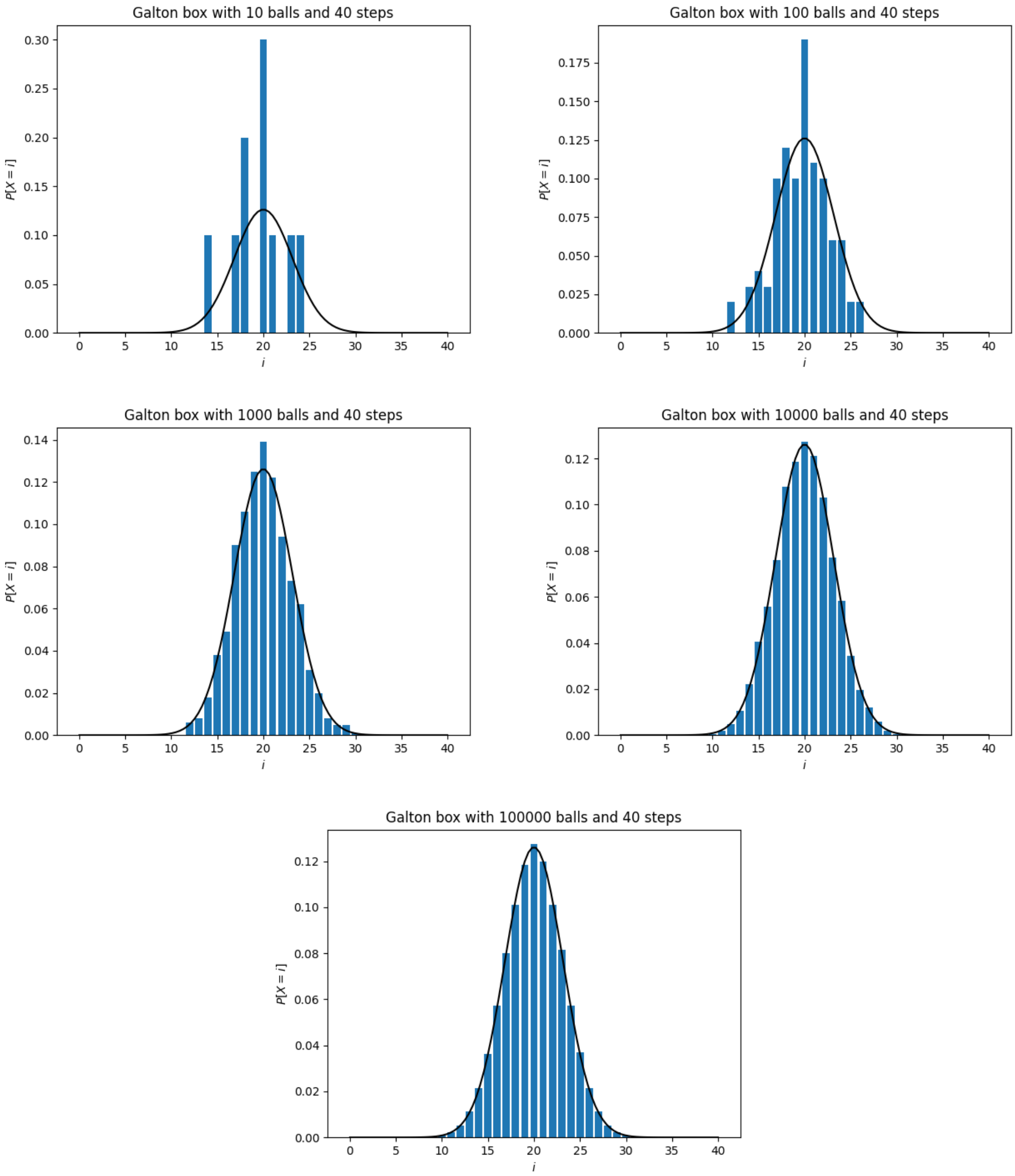

Figure 4: Probability distribution of a Galton box with 40 steps. Each plot has a different number of samples (balls thrown): 10, 100, 1,000, 10,000 and 100,000.