We want to calculate the coefficients for the following values:

$$\langle {}^2P_{3/2} F' = [3,4,5,6], m_f = \mathrm{All} | r | {}^2S_{1/2} F = [4,5], m_f = \mathrm{All} \rangle$$

for all values of q.

Thus we need these C-G coefficients:

$$\langle F, m_f, 1, q | F', m_f' \rangle$$

for

$$q = -1, 0, 1$$
$$F = 4, 5$$
$$F' = 3, 4, 5, 6$$

In [1]:
```
from sympy.physics.quantum.cg import CG
from sympy import S
import sympy
```

First, we try looking at all the non-zero Clebsch-Gordan coefficients:

In [2]:
```
Fs=[S(4),S(5)]
Fps=[S(3),S(4),S(5),S(6)]
qs=[S(-1),S(0),S(1)]
for q in qs:
    for F in Fs:
        for Fp in Fps:
            for mf in range(-F,F+1):
                for mfp in range(-Fp,Fp+1):
                    if(CG(F,mf,S(1),q,Fp,mfp).doit()<>0):
                        print 'F=',F,',Fp=',Fp,',mf=',mf,',mfp=',mfp,',q=',q,'
,    CG COEFFICIENT =',CG(F,mf,S(1),q,Fp,mfp).doit()
```

```
F= 4 ,Fp= 3 ,mf= -2 ,mfp= -3 ,q= -1 ,    CG COEFFICIENT = 1/6
F= 4 ,Fp= 3 ,mf= -1 ,mfp= -2 ,q= -1 ,    CG COEFFICIENT = sqrt(3)/6
F= 4 ,Fp= 3 ,mf= 0 ,mfp= -1 ,q= -1 ,    CG COEFFICIENT = sqrt(6)/6
F= 4 ,Fp= 3 ,mf= 1 ,mfp= 0 ,q= -1 ,    CG COEFFICIENT = sqrt(10)/6
F= 4 ,Fp= 3 ,mf= 2 ,mfp= 1 ,q= -1 ,    CG COEFFICIENT = sqrt(15)/6
F= 4 ,Fp= 3 ,mf= 3 ,mfp= 2 ,q= -1 ,    CG COEFFICIENT = sqrt(21)/6
F= 4 ,Fp= 3 ,mf= 4 ,mfp= 3 ,q= -1 ,    CG COEFFICIENT = sqrt(7)/3
F= 4 ,Fp= 4 ,mf= -3 ,mfp= -4 ,q= -1 ,    CG COEFFICIENT = sqrt(5)/5
F= 4 ,Fp= 4 ,mf= -2 ,mfp= -3 ,q= -1 ,    CG COEFFICIENT = sqrt(35)/10
F= 4 ,Fp= 4 ,mf= -1 ,mfp= -2 ,q= -1 ,    CG COEFFICIENT = 3*sqrt(5)/10
F= 4 ,Fp= 4 ,mf= 0 ,mfp= -1 ,q= -1 ,    CG COEFFICIENT = sqrt(2)/2
F= 4 ,Fp= 4 ,mf= 1 ,mfp= 0 ,q= -1 ,    CG COEFFICIENT = sqrt(2)/2
F= 4 ,Fp= 4 ,mf= 2 ,mfp= 1 ,q= -1 ,    CG COEFFICIENT = 3*sqrt(5)/10
F= 4 ,Fp= 4 ,mf= 3 ,mfp= 2 ,q= -1 ,    CG COEFFICIENT = sqrt(35)/10
F= 4 ,Fp= 4 ,mf= 4 ,mfp= 3 ,q= -1 ,    CG COEFFICIENT = sqrt(5)/5
F= 4 ,Fp= 5 ,mf= -4 ,mfp= -5 ,q= -1 ,    CG COEFFICIENT = 1
F= 4 ,Fp= 5 ,mf= -3 ,mfp= -4 ,q= -1 ,    CG COEFFICIENT = 2*sqrt(5)/5
F= 4 ,Fp= 5 ,mf= -2 ,mfp= -3 ,q= -1 ,    CG COEFFICIENT = 2*sqrt(35)/15
F= 4 ,Fp= 5 ,mf= -1 ,mfp= -2 ,q= -1 ,    CG COEFFICIENT = sqrt(105)/15
F= 4 ,Fp= 5 ,mf= 0 ,mfp= -1 ,q= -1 ,    CG COEFFICIENT = sqrt(3)/3
F= 4 ,Fp= 5 ,mf= 1 ,mfp= 0 ,q= -1 ,    CG COEFFICIENT = sqrt(2)/3
F= 4 ,Fp= 5 ,mf= 2 ,mfp= 1 ,q= -1 ,    CG COEFFICIENT = sqrt(30)/15
F= 4 ,Fp= 5 ,mf= 3 ,mfp= 2 ,q= -1 ,    CG COEFFICIENT = sqrt(15)/15
F= 4 ,Fp= 5 ,mf= 4 ,mfp= 3 ,q= -1 ,    CG COEFFICIENT = sqrt(5)/15
F= 5 ,Fp= 4 ,mf= -3 ,mfp= -4 ,q= -1 ,    CG COEFFICIENT = sqrt(55)/55
F= 5 ,Fp= 4 ,mf= -2 ,mfp= -3 ,q= -1 ,    CG COEFFICIENT = sqrt(165)/55
F= 5 ,Fp= 4 ,mf= -1 ,mfp= -2 ,q= -1 ,    CG COEFFICIENT = sqrt(330)/55
F= 5 ,Fp= 4 ,mf= 0 ,mfp= -1 ,q= -1 ,    CG COEFFICIENT = sqrt(22)/11
F= 5 ,Fp= 4 ,mf= 1 ,mfp= 0 ,q= -1 ,    CG COEFFICIENT = sqrt(33)/11
F= 5 ,Fp= 4 ,mf= 2 ,mfp= 1 ,q= -1 ,    CG COEFFICIENT = sqrt(1155)/55
F= 5 ,Fp= 4 ,mf= 3 ,mfp= 2 ,q= -1 ,    CG COEFFICIENT = 2*sqrt(385)/55
F= 5 ,Fp= 4 ,mf= 4 ,mfp= 3 ,q= -1 ,    CG COEFFICIENT = 6*sqrt(55)/55
F= 5 ,Fp= 4 ,mf= 5 ,mfp= 4 ,q= -1 ,    CG COEFFICIENT = 3*sqrt(11)/11
F= 5 ,Fp= 5 ,mf= -4 ,mfp= -5 ,q= -1 ,    CG COEFFICIENT = sqrt(6)/6
F= 5 ,Fp= 5 ,mf= -3 ,mfp= -4 ,q= -1 ,    CG COEFFICIENT = sqrt(30)/10
F= 5 ,Fp= 5 ,mf= -2 ,mfp= -3 ,q= -1 ,    CG COEFFICIENT = sqrt(10)/5
F= 5 ,Fp= 5 ,mf= -1 ,mfp= -2 ,q= -1 ,    CG COEFFICIENT = sqrt(105)/15
F= 5 ,Fp= 5 ,mf= 0 ,mfp= -1 ,q= -1 ,    CG COEFFICIENT = sqrt(2)/2
F= 5 ,Fp= 5 ,mf= 1 ,mfp= 0 ,q= -1 ,    CG COEFFICIENT = sqrt(2)/2
F= 5 ,Fp= 5 ,mf= 2 ,mfp= 1 ,q= -1 ,    CG COEFFICIENT = sqrt(105)/15
F= 5 ,Fp= 5 ,mf= 3 ,mfp= 2 ,q= -1 ,    CG COEFFICIENT = sqrt(10)/5
F= 5 ,Fp= 5 ,mf= 4 ,mfp= 3 ,q= -1 ,    CG COEFFICIENT = sqrt(30)/10
F= 5 ,Fp= 5 ,mf= 5 ,mfp= 4 ,q= -1 ,    CG COEFFICIENT = sqrt(6)/6
F= 5 ,Fp= 6 ,mf= -5 ,mfp= -6 ,q= -1 ,    CG COEFFICIENT = 1
F= 5 ,Fp= 6 ,mf= -4 ,mfp= -5 ,q= -1 ,    CG COEFFICIENT = sqrt(30)/6
F= 5 ,Fp= 6 ,mf= -3 ,mfp= -4 ,q= -1 ,    CG COEFFICIENT = sqrt(330)/22
F= 5 ,Fp= 6 ,mf= -2 ,mfp= -3 ,q= -1 ,    CG COEFFICIENT = sqrt(66)/11
F= 5 ,Fp= 6 ,mf= -1 ,mfp= -2 ,q= -1 ,    CG COEFFICIENT = sqrt(462)/33
F= 5 ,Fp= 6 ,mf= 0 ,mfp= -1 ,q= -1 ,    CG COEFFICIENT = sqrt(154)/22
F= 5 ,Fp= 6 ,mf= 1 ,mfp= 0 ,q= -1 ,    CG COEFFICIENT = sqrt(110)/22
F= 5 ,Fp= 6 ,mf= 2 ,mfp= 1 ,q= -1 ,    CG COEFFICIENT = sqrt(165)/33
F= 5 ,Fp= 6 ,mf= 3 ,mfp= 2 ,q= -1 ,    CG COEFFICIENT = sqrt(11)/11
F= 5 ,Fp= 6 ,mf= 4 ,mfp= 3 ,q= -1 ,    CG COEFFICIENT = sqrt(22)/22
F= 5 ,Fp= 6 ,mf= 5 ,mfp= 4 ,q= -1 ,    CG COEFFICIENT = sqrt(66)/66
F= 4 ,Fp= 3 ,mf= -3 ,mfp= -3 ,q= 0 ,    CG COEFFICIENT = -sqrt(7)/6
F= 4 ,Fp= 3 ,mf= -2 ,mfp= -2 ,q= 0 ,    CG COEFFICIENT = -sqrt(3)/3
F= 4 ,Fp= 3 ,mf= -1 ,mfp= -1 ,q= 0 ,    CG COEFFICIENT = -sqrt(15)/6
F= 4 ,Fp= 3 ,mf= 0 ,mfp= 0 ,q= 0 ,    CG COEFFICIENT = -2/3
F= 4 ,Fp= 3 ,mf= 1 ,mfp= 1 ,q= 0 ,    CG COEFFICIENT = -sqrt(15)/6
F= 4 ,Fp= 3 ,mf= 2 ,mfp= 2 ,q= 0 ,    CG COEFFICIENT = -sqrt(3)/3
F= 4 ,Fp= 3 ,mf= 3 ,mfp= 3 ,q= 0 ,    CG COEFFICIENT = -sqrt(7)/6
F= 4 ,Fp= 4 ,mf= -4 ,mfp= -4 ,q= 0 ,    CG COEFFICIENT = -2*sqrt(5)/5
F= 4 ,Fp= 4 ,mf= -3 ,mfp= -3 ,q= 0 ,    CG COEFFICIENT = -3*sqrt(5)/10
```

There are a lot! We can narrow this down by noting that there is a selection rule that says that F can only change by ± 1. Since we need to go both directions, we have to have $F = 4$ or $F = 5$

In [3]:

```
Fs=[S(4),S(5)]
Fps=[S(5),S(4)]
qs=[S(-1),S(0),S(1)]
for q in qs:
    for F in Fs:
        for Fp in Fps:
            for mf in range(-F,F+1):
                for mfp in range(-Fp,Fp+1):
                    if((CG(F,mf,S(1),q,Fp,mfp).doit()<>0)and (mf==0)):
                        print 'F=',F,',Fp=',Fp,',mf=',mf,',mfp=',mfp,',q=',q,'
,    CG COEFFICIENT =',CG(F,mf,S(1),q,Fp,mfp).doit(), ', other CG=',CG(Fp,mfp,
S(1),q,F,mf).doit()
```

```
F= 4 ,Fp= 5 ,mf= 0 ,mfp= -1 ,q= -1 ,    CG COEFFICIENT = sqrt(3)/3 , other CG=
0
F= 4 ,Fp= 4 ,mf= 0 ,mfp= -1 ,q= -1 ,    CG COEFFICIENT = sqrt(2)/2 , other CG=
0
F= 5 ,Fp= 5 ,mf= 0 ,mfp= -1 ,q= -1 ,    CG COEFFICIENT = sqrt(2)/2 , other CG=
0
F= 5 ,Fp= 4 ,mf= 0 ,mfp= -1 ,q= -1 ,    CG COEFFICIENT = sqrt(22)/11 , other CG
= 0
F= 4 ,Fp= 5 ,mf= 0 ,mfp= 0 ,q= 0 ,    CG COEFFICIENT = sqrt(5)/3 , other CG= -s
qrt(55)/11
F= 5 ,Fp= 4 ,mf= 0 ,mfp= 0 ,q= 0 ,    CG COEFFICIENT = -sqrt(55)/11 , other CG=
 sqrt(5)/3
F= 4 ,Fp= 5 ,mf= 0 ,mfp= 1 ,q= 1 ,    CG COEFFICIENT = sqrt(3)/3 , other CG= 0
F= 4 ,Fp= 4 ,mf= 0 ,mfp= 1 ,q= 1 ,    CG COEFFICIENT = -sqrt(2)/2 , other CG= 0
F= 5 ,Fp= 5 ,mf= 0 ,mfp= 1 ,q= 1 ,    CG COEFFICIENT = -sqrt(2)/2 , other CG= 0
F= 5 ,Fp= 4 ,mf= 0 ,mfp= 1 ,q= 1 ,    CG COEFFICIENT = sqrt(22)/11 , other CG=
0
```

I want to be able to go from F=4 to F=5. Therefore, if A is the transition operator, then I need to find some state $|i\rangle$ that satisfies

$$\langle i|A|g\rangle \neq 0$$

and

$$\langle e|A|i\rangle \neq 0.$$

In [4]:

```
F4=S(4)
F5=S(5)
Fps=[S(5),S(4)]
qs=[-1,0,1]
for q in qs:
    print "q is ",q
    for Fp in Fps:
        for mf in [S(0)]:
            for mfp in range(-Fp,Fp+1):
                if((CG(F5,mf,S(1),-q,Fp,mfp).doit()<>0)and(CG(Fp,mfp,S(1),q,F4
,mf).doit()<>0)):
                    print 'F=',F5,',Fp=',Fp,',mf=',mf,',mfp=',mfp,',q=',q,',
  CG COEFFICIENT =',CG(F5,mf,S(1),-q,Fp,mfp).doit(), ', other CG=',CG(Fp,mfp,S
(1),q,F4,mf).doit()
                    print 'F=',F4,',Fp=',Fp,',mf=',mf,',mfp=',mfp,',q=',q,',
  CG COEFFICIENT =',CG(F4,mf,S(1),-q,Fp,mfp).doit(), ', other CG=',CG(Fp,mfp,S
(1),q,F5,mf).doit()
```

```
q is  -1
F= 5 ,Fp= 5 ,mf= 0 ,mfp= 1 ,q= -1 ,    CG COEFFICIENT = -sqrt(2)/2 , other CG=
sqrt(33)/11
F= 4 ,Fp= 5 ,mf= 0 ,mfp= 1 ,q= -1 ,    CG COEFFICIENT = sqrt(3)/3 , other CG= s
qrt(2)/2
F= 5 ,Fp= 4 ,mf= 0 ,mfp= 1 ,q= -1 ,    CG COEFFICIENT = sqrt(22)/11 , other CG=
 sqrt(2)/2
F= 4 ,Fp= 4 ,mf= 0 ,mfp= 1 ,q= -1 ,    CG COEFFICIENT = -sqrt(2)/2 , other CG=
sqrt(2)/3
q is  0
q is  1
F= 5 ,Fp= 5 ,mf= 0 ,mfp= -1 ,q= 1 ,    CG COEFFICIENT = sqrt(2)/2 , other CG= s
qrt(33)/11
F= 4 ,Fp= 5 ,mf= 0 ,mfp= -1 ,q= 1 ,    CG COEFFICIENT = sqrt(3)/3 , other CG= -
sqrt(2)/2
F= 5 ,Fp= 4 ,mf= 0 ,mfp= -1 ,q= 1 ,    CG COEFFICIENT = sqrt(22)/11 , other CG=
 -sqrt(2)/2
F= 4 ,Fp= 4 ,mf= 0 ,mfp= -1 ,q= 1 ,    CG COEFFICIENT = sqrt(2)/2 , other CG= s
qrt(2)/3
```

Clearly, the light must have a "magnetic quantum number" (q) of either 1 or -1, while the intermediate state $|i\rangle$ must have $m_f' =$ 1 or -1 (note that m_f' is called mfp in the code).

We are finally ready to make our table. We calculate the Clebsch-Gordan coefficients for values of q_1, q_2, and m_{fi} that allow the transition probability to be nonzero for the transition from $|g\rangle$ to $|i\rangle$ and from $|i\rangle$ to $|e\rangle$. This is pretty similar to before except slightly more neatly organized.

Also, I put in the characters to make a ready-made $\LaTeX$ table.

In [5]:
```
Fi=[4,5]
mfi=[-5,-4,-3,-2,-1,0,1,2,3,4,5]
q1=[-1,0,1]
q2=[-1,0,1]

for Fi_ in Fi:
    for q1_ in q1:
        for q2_ in q2:
            for mfi_ in mfi:
                CG1=CG(5,0,1,q1_,Fi_,mfi_).doit()
                CG2=CG(Fi_,mfi_,1,q2_,4,0).doit()
                if ((CG1<>0) and (CG2<>0)):
                    print Fi_,'&',q1_,'&',q2_,'&',mfi_,'&$',sympy.latex(CG1),'
$&$',sympy.latex(CG2),'$ \\\\ \n',
```

```
4 & -1 & 1 & -1 &$ \frac{\sqrt{22}}{11} $&$ - \frac{\sqrt{2}}{2} $ \\
4 & 1 & -1 & 1 &$ \frac{\sqrt{22}}{11} $&$ \frac{\sqrt{2}}{2} $ \\
5 & -1 & 1 & -1 &$ \frac{\sqrt{2}}{2} $&$ \frac{\sqrt{33}}{11} $ \\
5 & 1 & -1 & 1 &$ - \frac{\sqrt{2}}{2} $&$ \frac{\sqrt{33}}{11} $ \\
```