Differentiable Manifolds and the Hairy Ball Theorem

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1 Manifolds and Tangent Spaces

Definition 1.1 (Manifold)**.** Let M be a Hausdorff, second countable, locally $\mathbb{R}^n$ topological space. A chart is a pair (U, ϕ) where U is open in M and $\phi : U \to \mathbb{R}^n$ is a homeomorphism onto its image. An atlas $\mathcal{A}$ on M is a collection of charts that cover M such that if $(U, \phi), (V, \psi) \in \mathcal{A}$, the functions $\psi \circ \phi^{-1}$ and $\phi \circ \psi^{-1}$ are smooth on $\phi(U \cap V)$ and $\psi(U \cap V)$ respectively. Each atlas is contained in a unique maximal atlas. A manifold is the space M together with a maximal atlas $\mathcal{A}$.

Definition 1.2 (Smooth maps between manifolds)**.** A continuous map $F : N \to M$ between manifolds is smooth if for all charts (U, ϕ) on N and (V, ψ) on M, the map $\phi \circ F \circ \psi^{-1}$ is smooth.

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