EXERCISE SHEET: COMPRESSED SENSING

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1. Introduction, hands-on

Exercise 1. Prove the following:
Given an index set $S \subset \{1, \cdots, N, \}$, if

$$\mathbf{v} := \mathrm{argmin}_{\mathbf{z} \in \mathbb{R}^N} \{\|\mathbf{y} - A\mathbf{z}\|, \mathrm{supp}(\mathbf{z}) \subset S\}$$

then

$$\langle \mathbf{y} - A\mathbf{v}, a_i \rangle = 0,$$

for all $i \in S$.

Exercise 2. Prove the following
Let $A \in \mathbb{C}^{m \times N}$ be a matrix with ℓ_2-normalized columns. Given $S \subset \{1, \cdots, N\}$ and $\mathbf{v}$ supported on S, and $1 \leq j \leq N$, if

$$\mathbf{w} := \mathrm{argmin}_{\mathbf{z} \in \mathbb{C}^N} \{\|A\mathbf{z} - \mathbf{y}\|_2, \mathrm{supp}(\mathbf{z}) \subset S \cup \{j\}\}$$

then

$$\|\mathbf{y} - A\mathbf{w}\|_2^2 \leq \|\mathbf{y} - A\mathbf{v}\|_2^2 - |(A^*(\mathbf{y} - A\mathbf{v}))_j|^2.$$

Exercise 3. Prove the alternate form for the restricted isometry constant:

$$\delta_s := \max_{|S| \leq s} \|A_S^* A_S - I\|_{2 \to 2}.$$

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Date: December 12, 2022.