SUGGESTED EXERCISES: MODERN REGRESSION

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1. Simple linear regression

Homework 1. Prove the following statement

Proposition 1.1 (Prove at home)**.** *Under the conditions that the LINE assumptions are valid, the following partition of errors hold*

$$SSTO = SSR + SSE.$$

Homework 2 (Optional). Try to prove the following relation

Proposition 1.2. *Pearson's correlation coefficient can be computed as*

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}}.$$

2. Multiple linear regression: more dimensions

3. Matrices: what you need to know

Homework 3. What happens to inverses if the matrix A is not square? Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2/3 & -1/3 & 1/3 \\ -1/3 & 2/3 & 1/3 \end{pmatrix}$. Show that BA is some identity (which one?) but AB clearly is not. This shows that we may have left inverses which are not right inverses.

Homework 4. Let A be any square matrix. Show that the matrix $C = \frac{A - A^T}{2}$ is antisymmetric.

Homework 5. Show that the equivalent of Sarrus rule (sum of positive diagonals minus sum of negative diagonals) doesn't work for dimension 4.

Homework 6. Show the following result on inverses of 2 dimensional matrices

Proposition 3.1. *Let* $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathbb{R}^{2\times 2}$. *If A is invertible, is inverse is given by*

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

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