Beijing Institute of Technology - School of Mathematics and Statistics - 2020-2021
Wavelet Analysis

Note: this syllabus is still subject to changes. These changes will be expressed clearly during class.

Instructor: Jean-Luc Bouchot, Liangxiang Campus, Room 405, `jlbouchot@bit.edu.cn`
Hours: The class meets twice a week: Monday and Wednesday 14:00-15:35 online. The details of the web-meeting will be provided in due time.
Office hours: Due to the current pandemic, there will be no on-site office hours. Communication may be per email and discussion can be made by appointment.
Ressource: `https://jlbouchot.github.io/teaching.html`. **Textbooks:**

- (Main ressource) *A wavelet tour of signal processing: The sparse way*, 3rd edition, S. Mallat, Elsevier.
- *10 Lectures on Wavelets*, I. Daubechies, SIAM.

Description: Wavelet analysis is the basis of many results in approximation theory and has had great impact in recent applications ranging from scientific computing to image processing... and many more! The goal of this course is to cover the theory behind wavelet and multiresolution analysis and describe how they are applied in other areas of sciences. We will make sure to give a practical understanding of the wavelets both in the theoretical continuous realm and in the applied, discretized domain.

Course format: The class meets twice a week in a lecture format. Given the current global pandemic, in-class exercises are difficult to provide, but I was always try to give a bit of time for you to practice during our sessions before giving out the answers. Moreover, suggested homeworks, which may or may not be corrected in class, will be provided regularly. These exercises will not be graded individually.

Grading policy: Grading will be done through **two compulsory homework assignments** the framework of which will be clarified in due time. They are likely to be due on the 8th week and on the 12th (and last) week of the current course Here is the tentative point distribution:

- 40% for the first assignments. The precise timeline will be made clear in the first weeks of class and this file updated accordingly. You will have 1 week to solve the exercises. You are allowed to think and work the problems in groups, **but the work your turn in should be yours alone**.
- 60% for the second, likely a little bit longer, assignment.

Objectives: By the end of the term, the students are expected (among others)

- to have familiarities with Fourier transforms in both the continuous and discretized setting
- to know how to define and work with (orthogonal) wavelets and their implications in multi-resolution analysis
- to be able to use wavelet decompositions and analysis in various fields of engineering and sciences

Attendance: will not be recorded, but keep in mind that many things will not be available outside the classroom. You are responsible for whatever is being said in class and questions will not be answered if already answered in class. Note that this class is dense and covers a lot of topics. Active participation in the class (both paying attention and working out the in-class exercises) or via the slack/discord channel will also help your understanding of the materials.

While it is okay to mail or ask question outside class, these shall stay unanswered if they have already been answered in class or in any communicated file. All correspondence should be addressed with the [Waves2020] tag in the subject. Failing to follow these conditions will impact your grade negatively.
No homework/exam/project will be accepted later than the due date. Some arrangement might be possible if and only if the students advise <u>before</u> the due date and because of a <u>valid</u> reason (medical, religious).

Tentative schedule: The class will be organised as follows (this is subject to change according to the audience).

- Week 1: Review of linear algebra

  - Groups, fields, vector spaces
  - Norms and inner products
  - Banach and Hilbert spaces
  - Bases and representations
- Week 2: Review of Fourier transforms and functional analysis
  - Fourier series, approximation of functions, Gibbs phenomenon
  - Fourier transform in L^1 and L^2
  - Properties of the Fourier transform
- Week 3: Discretization and computations of the Fourier transform
  - Discrete Fourier transform
  - Fast Fourier Transform
  - Short term Fourier Transform
- Week 4: Time-frequency analysis
  - Windowed Fourier transform
  - Gabor filters
- Intro to wavelets
  - Week 5: The Haar wavelet
  - Wavelet decompositions (Analysis and synthesis operators)
  - Multi-resolution analysis
- Week 6: More wavelets
  - Vanishing moments
  - Construction of wavelets
  - The Daubechies family
  - Function spaces (Besov)
- Week 7: Image denoising 1
  - Image model
  - Noise model
  - Linear filters
- Week 8: Image denoising 2

  - The shrinkage operator and its optimality
  - The wavelet shrinkage operator
  - Image compression: the JPEG2000 format (if time permits)

- Week 9: Wavelets in scientific computing
- Week 10: Advanced linear algebra
  - Riesz bases
  - Frames (and fusion frames?)
  - Discretization of wavelet transforms
  - RBF approximation
- Week 11: More wavelets
  - Wavelet frames
  - Biorthogonal wavelets
- Week 12: To go further (to be in due time)
  - Curvelets
  - Shearlets
  - α molecules
  - Scattering transform