NOTES ON A PARTIAL ORTHOGONALIZATION OF DPG BASIS FUNCTIONS

JESSE CHAN

Can we take advantage of the inner product structure of DPG?

- $b(u_i, v_j) = (v_i, v_j)_V$, where u_i is a trial basis function and v_j is a corresponding computed optimal test function.
- It is the space spanned by the test space, rather than the test space itself, which is important.

Can we take linear combinations of optimal test functions that span the same space, but are orthogonal with respect to $(\cdot,\cdot)_V$? Suppose we use Gram-Schmidt and orthogonalize v_2 to v_1, and so on, to construct $\{v_i^o\}_{i=1}^n$, where n is size of the trial space over one element. Then,

$$b(u_1, v_j^o) = (v_j^o, v_1)_V = 0.$$

Note that this only works because we set $v_1^o = v_1$, and then for $j > 1$, v_j are orthogonalized with respect to $v_1, \ldots, v_{j-1}$. Noting that the stiffness matrix $K_{ij} = b(u_i, v_j)$, this would imply that our resulting system is upper triangular.

Note: it can only be upper triangular, since

$$b(u_i, v_j^o) = (v_j^o, v_i)_V = 0$$

for $j > i$, but

$$b(u_j, v_i^o) = (v_i^o, v_j)_V \neq 0$$

unless $i > j$. This corresponds roughly to the intuition that taking linear combinations of test functions is equivalent to taking linear combinations of rows; thus, row reduction is just recombining test functions. However, we would need to take linear combinations of our trial functions in order to column-reduce.

The advantage is that, due to the discontinuous nature of DPG, this orthogonalization can be done locally.

REFERENCES