Let $(\cdot,\cdot), (\cdot,\cdot)_h$ denote the exact and quadrature-based L^2 inner product on the volume, and let $\langle\cdot,\cdot\rangle, \langle\cdot,\cdot\rangle_h$ denote the exact and quadrature-based L^2 inner products on the surface. We will assume that, in $d > 1$ dimensions, the volume and surface quadratures satisfy

$$
\begin{aligned}
\left|(u,v) - (u,v)_h\right| &\lesssim h^{r_{\rm vol}+1+d} \|u\|_{H^{r+1}(D^k)} \|v\|_{H^{r+1}(D^k)} \\
\left|\langle u,v\rangle - \langle u,v\rangle_h\right| &\lesssim h^{r_{\rm surf}+d} \|u\|_{H^{r+1}(\partial D^k)} \|v\|_{H^{r+1}(\partial D^k)}.
\end{aligned}
$$

We wish to show error estimates for certain finite difference approximations of derivatives. More precisely, we will show that, for sufficiently regular f, the weak derivative $D_h f$ satisfies

$$
\left\|\frac{\partial f}{\partial x} - D_h f\right\| \leq C h^{p+1} \|f\|_{H^{p+1}}.
$$

More precisely, we will show that the L^2 projection satisfies

$$
\left\|\Pi_N \frac{\partial f}{\partial x} - D_h f\right\| \leq C h^{\alpha} \|f\|_{H^{p+1}}, \qquad \alpha = \min\left(r_{\rm vol} + 1 - p, r_{\rm surf} - p\right).
$$

Main notes: if $\boldsymbol{Q}_N$ does not satisfy the SBP property, then the error estimate is limited by the accuracy of the weak derivative. The strong derivative is always $O(h^p)$, but the weak derivative is $O\left(h^{\lfloor \alpha/2 \rfloor}\right)$, where α is defined above.