Identifying Economic Regimes:
Reducing Downside Risks for University Endowments and Foundations

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Abstract

One of the most durable patterns in market behavior involves contagion - increases in correlation and volatility -- during crash periods such as 2008. This condition can cause major problems for an investor when markets severely contract and anticipated diversification benefits vanish. To address contagion, we implement a machine-learning algorithm, trend filtering, to capture distinctive economic conditions. Over long horizons, a multi-regime simulation provides more accurate estimates of downside risk as compared with traditional static portfolio models, and can help in the process of evaluating strategies for reducing the worst-case outcomes. The approach readily applies to non-profit institutions that depend upon their endowment capital to fund liabilities and meet goals.

The dynamic nature of financial markets presents a major challenge for long-term investors. One of the most durable stylized facts involves contagion during severe economic conditions. Not only does the volatility of most securities increase, but also the correlation of returns can approach unity, causing securities to move together downward in a tight pack. The contagion environment has several causes, including plunging liquidity, a spike in systemic risk premium due to bank inter-linkages and related issues, and panic by investors who will often unload liquid securities to protect themselves against further losses. Goldstein and Razin [2015], and Kyle and Xiong [2001] discuss alternative theories of financial crises. The empirical evidence during crash periods, such as 2008, supports these suppositions. Importantly, for portfolio construction, contagion disrupts the anticipated diversification built into traditional models, and accordingly the investor's portfolio performs much worse than would be expected. In turn, drawdown can be severe, leading to a major setback for long-term investors who depend upon capital to achieve their goals and pay future liabilities.

We build on the hypothesis that asset performance over long-time periods can be separated into distinctive classes, called regimes, which display common characteristics. This supposition is not new; there have been numerous studies and methods for addressing regime identification. For examples, see Bilgili [2009], Hamilton and Susmel [1994], and Timmermann [2000].

A multi-regime analysis applies readily to non-profit institutions, such as universities and foundations, for which the endowment supports a substantial portion of the organization's operating budget. For viability, it is critical that these institutions protect their endowment capital from severe erosion over long-time horizons. Accordingly, there must be special attention to the downside risks. The institutions have several levers to improve their long -term viability. They can implement dynamic asset allocation strategies. They can adjust spending rules. And they might be able to seek additional funds from outside sources such as alumni and other benefactors. There has been vast research on adjusting asset allocation based on changing economic conditions and regimes, such as Ang and Bekaert [2002], Bauer et al. [2004], Graflund and Nilsson [2003], Guidonlin and Timmermann [2007, 2008], Nystrup et al. [2015], van Norden and Schaller [1997], and Tu [2010]. In contrast, the latter two levers are taken up herein.

The next section describes an algorithm from the area of machine learning to identify economic regimes. The method, called trend filtering, is one of the non-parametric approaches that have become common in the machine-learning domain. The trend-filtering algorithm categorizes regimes and in turn helps in the modeling of downside risk. The concepts are especially relevant for long-term investors who depend upon their endowment capital to provide a steady source of income. To improve the worst-case outcomes, we show the advantages of adjustable spending rules during drawdown periods. As we will see, the two-regime approach assists in evaluating strategies to protect the long-term viability of an endowment for a non-profit institution. The downside risks are closer to historical patterns than would be the case with a traditional static (single-regime) portfolio model.

Applying Machine Learning to Detect Regimes

At its core, machine learning involves the classification of a "population" into classes possessing relatively homogeneous properties. Hastie et al. [2009] gives an excellent overview. In medicine, for example, there is intense interest in separating individuals into classes that have more or less likelihood of experiencing a disease such as coronary heart failure. To this point, apps and websites are available to help estimate the probability of a future heart attack (American Heart Association [2016]). Herein, the questions depict "features" that support the underlying risk assessment. Given the goals of machine learning, it is natural that machine-learning methods may be applicable to detecting economic regimes for use in financial planning models.

Identifying economic regimes can be carried out in a variety of ways. A fundamental approach is to search for changes in the relationship between factors and the associated returns of assets via structural breaks, e.g. Timmermann [2001]. Dating turning points between business cycles is an interesting and promising topic in economic research. In the United States, the National Bureau of Economic Research (NBER) calculates these turning points from historical data, which is widely used in business cycle analysis.

For our purposes, we focus on transition dates at which the economy alternates between a "growth" period and a "contraction" period. Several methods are applicable to real-time monitoring of the transition. Bry and Boschan [1971] established a turning point program, while Hamilton [1989] proposed a Markov switching model. Layton [1996] compared the two models. Harding and Pagan [2002] added a Kalman filter approximation. Chauvet and Piger [2008] evaluated a dynamic factor time series model with the Harding and Pagan approach. Alternatively, a hidden Markov model (HMM) can help distinguish growth periods and crash periods in a probabilistic framework, e.g. Cai [1994], Liew and Siu [2010], and Song [2013].

In our study, we aim to identify regimes of the financial markets in a data-driven and model-free context. The algorithm starts with the S&P 500 historical total return data. The basic idea is as follows. We treat S&P 500 wealth path as a nonparametric function of time. If we naively plot this function, it will be a rough curve with a lot of noise. Therefore, we apply smoothing to the nonparametric function, which will "filter out" the noise of the historical data. The resulting function has a clear trend over a persistent period of time, which can help pinpoint regimes of the financial market. Compared to other approaches, this method is purely data-driven and does not rely on the assumption of a certain underlying probability model of the financial returns. Liu and Zhao [2016] and Tibshirani [2014] supply details about the approach and methods for addressing robustness from the machine learning literature.

The trend-filtering algorithm identifies regimes by smoothing a piecewise linear function, labeling growth periods with 1 and contraction periods with -1, with a regularization (penalty) term to dampen the number of changes in regime across a historical period: 1990-2015 in the experiments. The algorithm discovers a reasonable compromise between stability in the curve and errors in the fit, as is common in the machine learning domain (Hastie et al. [2009], Tibshirani [1996, 2014], and Zou and Hastie [2005]). The trend-filtering method deploys an optimization algorithm to directly determine the "status" of each historical time period. Exhibit 1 shows one of the main results. It displays a 2-regime plot of the S&P 500 Index over the 1990-2015 period, where green periods indicate growth and red periods indicate contraction/crash.

Given the classification of the S&P500 time series, Exhibit 2 displays the equilibrium transition matrix across time periods (quarters). The probabilities of movement from one state (growth or contraction) in period t to the state (growth or contraction) in the next period t+1 can be projected and estimated. Thus, if the current quarter is in growth, there is a 90% probability that the next quarter will be selected as growth, and a 10% chance that the next quarter will be contraction. Conversely, if the current quarter is contraction, there is 60% probability of contraction in the following quarter. The transition probabilities form the basis of the switching procedure for the Monte Carlo simulations ahead. The equilibrium values are most appropriate for long-term planning purposes, rather than short-term forecasting.

The next step in developing a long-term planning model is to select the asset categories. Exhibit 3 provides eight common assets and a typical allocation for large U. S. university endowments and foundations. Many of these institutions have followed the shift to alternative categories, as advocated by David Swensen [2009]. Liebowitz et al. [2010] discuss the endowment model. Mulvey and Holen [2016] survey current asset categories for the largest university endowments in the United States; the categories and associated allocation in Exhibit 3 depict a representative portfolio of the larger endowments.

To build a forward-looking planning model, we must estimate future performance over the long planning horizon. To this end, there are a host of methods for estimating capital market assumptions and calibrating stochastic forecasting models. As illustrations, see

Bogle and Nolan [2015], Fabozzi et al. [2010], and Ilmanen [2011]. For our case example, we surveyed multiple institutional sources and derived consensus projections in Exhibit 4 (adjusted for inflation). These values are relatively conservative as compared with historical performance due to the current ultra-low interest rates and earnings yield, and modest growth of cyclically adjusted earnings (Pedersen [2015]). For simplicity, the volatility values reference historical performance.

To estimate the expected values under the two regimes, the relative performance of the historical values over the two regimes is preserved for the forward regimes. The goal is to generate similar "average" compound returns between the single-regime and the multi-regime scenarios for the mean of all of the generated scenarios, while maintaining the relative performance across the regimes. The expected values in Exhibits 5 and 6 are calculated so that this goal is achieved (shown in the simulation results).

Several items are worth mentioning. First, although the trend-filtering algorithm employs only the U.S. equity market in its regime identification, the returns of the other asset categories under the two regimes are consistent with *a priori* beliefs. For example, the international equity markets give performance similar to the U.S. Likewise, U.S. government bond returns perform much better during contraction than in growth due to the flight to quality pattern during crash periods. The other assets display anticipated relationships as well. Due to the fundamental causes of contagion during contraction periods, we might expect similar patterns to occur in the future; we build on this assumption in our long term planning model.

Applying Multi-Regime Simulations to Reduce Downside Risks

In this section, we implement Monte Carlo simulations for long-term investors. The case involves an institution that supports a substantial portion of its operating budget from endowment capital. There are two main financial objectives, as indicated in the endowment's policy statement: 1) provide steady income to maintain and improve the organization's activities; and 2) protect the endowment capital over long time periods. At times, the goals will conflict with each other; the trustees in conjunction with the endowment staff must set procedures to balance these goals.

As background, prominent U.S. universities have established spending rules for reconciling these two goals. A typical rule might be to spend 3 to 4% of the endowment capital each year for the operating budget and other needs, where the capital amount is averaged over the past several years. The 3 to 4% range is deemed sufficiently conservative to achieve the second goal - protect capital over long time periods, whereas the averaging process reduces the volatility of spending so that large swings are avoided. In this section, we evaluate spending rules in conjunction with a typical asset allocation by means of multi-year Monte Carlo simulation models. The experiments will compare a traditional single-regime model with a multi-regime version. In the later case, we will employ two regimes to illustrate the methodology. It is a simple matter to extend the approach to a larger number of regimes (Appendix).

The initial baseline simulations assume eight asset categories, an asset mix that is similar to many leading universities, and a single-regime structure (Exhibits 3 and 4). In addition, the asset mix is set to the target allocation each period (quarter). Two extensions to the baseline are proposed. First, we implement the two-regime switching model, with a focus on the downside risks (goal #2). Next, the analysis considers conditional variations on the spending rule, modifying spending when shocks occur. Herein, spending will decrease for a fixed period during and after a protracted drawdown episode.

The baseline experiments employ a single set of portfolio parameters. In particular, a fixed covariance matrix Σ of asset returns is derived from historical performance from 1990-2015, as well as an expected return vector = μ (Exhibit 4). The expected returns are forward looking, based on a consensus forecast from several sources. Next, the returns are sampled from a multi-normal distribution $\mathbb{N}(\mu, \Sigma)$. Given these assumptions, we can readily generate random future paths for the endowment capital, along with associated spending values over the modeling horizon. Each path is designated as a scenario. The Monte Carlo simulation will evaluate a relatively large number of scenarios. For each random scenario s, the generation formula is:

$$r_s = u + V * \varepsilon_s \qquad (1)$$

where μ is the expected return vector, V is a Cholesky factor of Σ (i.e. $V V^t = \Sigma$), and ε_s is a random normal vector with zero mean and unit volatility = $\mathbb{N}(0, I)$.

For simplicity, the return generating process assumes temporal independence. Thus, at each quarter t and under each scenario $s \epsilon \mathcal{S}$, we generate a random vector of returns for the eight assets, based only on equation 1. The simulation takes place over a long time horizon, equal to fifty years in our case. The experiments employ 10,000 scenarios. Each scenario is a consistent path of random returns over the 2016 to 2066 period for the eight assets. This type of forward simulation is relatively common for large institutional investors such as pension plans.

The baseline spending-rule goes as follows. The target is to spend 4% of endowment capital each year: .5% from annual giving and other sources, and 3.5% from the endowment capital, where the capital is averaged over the past four years. For the first four years, the target is implemented. After year four, the target is compared with a specified range 3.5% to 4.75% of current capital. This range is enforced whenever the target falls outside, by setting spending at the top or bottom of the range.

The results of the baseline simulation appear in Exhibit 7. Wealth paths for several quantiles across the 10,000 scenarios are displayed in Exhibit 8. It is evident that the majority of scenarios indicate growth of the endowment capital. To analyze the worst cases, we fix three risk measures: 1) the probability that drawdown, in inflation adjusted terms, will exceed 25% over the next five years, 2) the probability of 25% or worse drawdown over ten years, and 3) the probability of a 50% or worse drawdown over fifty years.

The baseline risk numbers (Exhibit 7) appear reasonable, given that the institution generates positive cash flows at the 4% target to fund its operations – its primary goal. The 25% drawdown event probabilities over five and ten-year horizons, approximately 10% and 20%, respectively, while significant, are to be expected in an uncertain world fraught with economic and political risks. And the 4.9% probability of a 50% or worse drawdown event over the next fifty years is again within a reasonable range. Similarly, the mean values of the endowment continue to increase (starting at 1): 1.07 at five years, 1.15 after ten years, and 1.58 after fifty years. The baseline spending rule and the asset allocation are relatively consistent with each other, and the trustees' responsibilities seem to fall within a prudent person criterion – at least to the authors' judgment. However, the simulation results depend upon the single regime framework.

In this second set of experiments, we separate each quarter into one of two states: growth or contraction. The simulations begin at 2016 first quarter in the growth state. The state for the next period (quarter) will depend upon the equilibrium Markov transition matrix (Exhibit 2). There is a 90% chance that the next period will be growth, and if so, the random returns will be drawn from the multi-normal distribution under growth. Alternatively, there is a 10% chance that the next quarter will be contraction, and in this case, the random returns for the eight assets will be drawn from the multi-normal distribution under contraction. And so it goes. A full path under the two-regime model will be a mixture that is driven by this transition matrix, with returns coming from the two multi-normal distributions, sometimes growth and sometimes contraction. On average, the number of quarters in growth will be about 80%, whereas the number quarters in contraction will be about 20%, which is consistent with the historical record. The identical asset allocation (Exhibit 3) is implemented over 10,000 scenarios, drawn with the same random number seeds.

Exhibit 9 depicts an example of the future wealth path of the S&P 500 Index for a single scenario over the 50 year planning horizon. Growth and contraction periods are indicated. The simulation generates 10,000 such paths.

Turning to the two-regime results (Exhibit 7), the size of the mean endowment over the 10,000 scenarios is comparable between the single and multi-regime simulations at each future time juncture (5, 10, 20, and 50 years). Also, on average, the endowment continues to grow under two-regimes. However, the probabilities of worst-case outcomes are much higher: such as the 19.9% probability of 50% drawdown over fifty years under two-regimes, as compared with 4.9% previously. A similar relationship occurs for the other two risk measures. Clearly, modeling contagion in the simulation causes the worst-case outcomes to be higher. Exhibit 10 provides another view on the left tails by plotting the respective cumulative distributions: the downside risks are much lower for the single-regime simulation, as compared with the two-regime model. Exhibit 11 displays the ranges across scenarios for the two-regime simulation, again showing the wider ranges at the lower extremes for the two-regime model (compare Exhibits 8 and 11).

Do the new higher risks exceed the reasonableness threshold? The guardians of the institution, typically the trustees, have the primary responsibility to address this question. A more appropriate question for the reader is: What can be done to reduce the downside risks? While there are a number of options, an obvious area to explore involves institutional spending. One approach is to manage the drawdown periods by adjustments to spending under adverse circumstances. And the smoothing of the endowment capital over a multi-year back period is a small step in this direction. Another approach is to reduce spending in a dynamic fashion when large drawdown thresholds are breached. This dynamic spending rule is analogous to peak pricing strategies in energy applications.

To illustrate the impact of conditional spending on the worst-case outcomes, the next Monte Carlo simulations lower spending during and after a significant drawdown. Spending drops by 20% for three years, when drawdown exceeds 20%, as measured by the initial capital level. Exhibit 7 shows the impact of this dynamic spending rule. As expected, the reduction has little impact over the five and ten year horizons, due to the rarity of entering the so-called adverse periods. On the other hand, dynamic spending has a noticeable improvement in the worst-case risks over the fifty-year period, for instance dropping from 19.9% to 13.1% under the two-regime case. Note, also, that the total time in adverse events drops as well. Reducing spending at one period helps protect against adverse events in later periods.

Importantly, changes to the base spending (4% inflation adjusted per year) can be part of a sensitivity exercise. To wit, Exhibits 12 and 13 depict the impacts. Increasing the spend rate to a 4.5% target per year has a noticeable impact on the worst-case outcomes. Not surprisingly, decreasing the spending rate to 3.5% per year reduces the downside risks. A conservative strategy is to spend at the 3.5% target (.5% giving and 3% from endowment capital) per year coupled with the dynamic spending rule – resulting in the lowest, 6.3%, probability of the 50% drawdown over the next fifty years. Again, the institution must select an acceptable tradeoff among its opportunities, goals and risks.

Future Directions

The multi-regime framework gives a more realistic depiction of the left tail events than the traditional static (single-regime) approach and thus provides better information to "simulate" the institution's cash needs and viability in the future. In this way, the regime model helps the institution achieve its dual long-term goals to provide a steady source of income, and to protect the endowment capital. To this end, the advantages of dynamic spending rules are quantified.

A primary purpose of this paper has been to illustrate the importance of modeling contagion on the worst-case outcomes for long-term investors. Accordingly, we kept the modeling details sparse. Several extensions of the multi-regime approach stand out. The Monte Carlo simulations can include significant considerations such as limitations on illiquid asset transactions, market impact costs, and other realistic constraints. A dynamic asset allocation strategy might be applied in a modest way, given the difficulty of moving across the illiquid alternative asset categories.

Likewise, the asset allocation decisions and the spending rules can be linked in an integrated fashion via an "Enterprise Risk Management" system for a non-profit institution. Herein, the portfolio is expanded to include the endowment and the sponsoring organization. The reduced spending rule discussed in this report is only one version among others. Undoubtedly, there are better procedures for balancing the needs of the institution to deploy its endowment resources to achieve current and intermediate goals, while at the same time, protecting its capital over long time horizons. As an example, in conjunction with the reduced spending, a non-profit institution should look to implement operational strategies that can be reversed relatively easily during severe drawdown periods. For example, a recently proposed increase in sabbatical leaves for faculty at Princeton University, costing about $10 to $15 million per year (rough estimate by Mulvey), might be suspended during a crash. Other reductions might be possible: temporarily freezing wages, postponing the replacement of non-essential employees who have retired or resigned, and lagging non-critical capital improvements. Similarly, the institution may have some control over donations and contribution campaigns. The long-term benefits of return-contingent spending rates revealed by the two-regime simulation support the importance to the institution of contingent planning for such events, despite their rare occurrences.

ENDNOTES

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The case study involves a major university endowment, with inflation measured by the Higher Education Price Index (HEPI), which is somewhat higher than the CPI.

Other tail risk measures such as Conditional Value at Risk can be evaluated readily within the multi-regime framework.

Reduced spending rules are pertinent for individuals during their retirement years. There are similar advantages for applying contingent spending rules to protect the retirement capital during severe drawdown periods.

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Appendix

The appendix describes the results of constructing a three-regime breakdown based on the trend-filtering algorithm. Exhibit 14 depicts the three regimes over the 1990-2015 historical period. Liu and Zhang [2016] and Tibshirani [2014] provide details about the approach and establishing robustness of the recommended solution.

The trend-filtering algorithm is detailed as follows:

The trend filtering estimate is defined as the minimizer of a penalized least squares criterion, in which the penalty term sums the absolute k-th order discrete derivatives over the input points. We apply this method to the S&P500 index. Let $\boldsymbol{Y} = (Y_1, \ldots, Y_n)^T \in \mathbb{R}^n$ denote the return of the S&P500 index over n periods. For a given integer k, the trend filtering estimate $\widehat{\boldsymbol{\beta}} = (\widehat{\beta}_1, \ldots, \widehat{\beta}_n) \in \mathbb{R}^n$ is solved by the following penalized optimization problem:

$$\widehat{\boldsymbol{\beta}} = \operatorname*{argmin}_{\boldsymbol{\beta} \in \mathbb{R}^n} \frac{1}{2} \|\boldsymbol{Y} - \boldsymbol{\beta}\|_2^2 + \frac{n^k}{k!} \lambda \|D^{(k+1)} \boldsymbol{\beta}\|_1, \tag{2.1}$$

where $\lambda \geq 0$ is a regularization parameter, and $D^{(k+1)} \in \mathbb{R}^{(n-k-1)\times n}$ is the discrete difference operator of the order $k+1$. For example, when $k = 0$,

$$D^{(1)} = \begin{pmatrix} 1 & -1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & -1 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 1 & -1 \end{pmatrix} \in \mathbb{R}^{(n-1)\times n}$$

so that $\|D^{(1)}\boldsymbol{\beta}\|_1 = \sum_{i=1}^{n-1} |\beta_i - \beta_{i+1}|$.

Eq. (2.1) is in a form of L_2-loss function plus a L_1 penalty. The minimizer of such forms is known to have sparse structure, meaning that many entries vanish in the entire vector. A well-known estimator is the LASSO (Least Absolute Shrinkage and Selection Operator) for high-dimensional linear regression and variable selection. In our case, this implies that the solution $\widehat{\boldsymbol{\beta}}$ satisfy $\widehat{\beta}_i - \widehat{\beta}_{i+1} = 0$ for many i's. Therefore, the fitted function $\widehat{\boldsymbol{\beta}}$ has a piecewise constant structure. Those i that satisfy $\widehat{\beta}_i \neq \widehat{\beta}_{i+1}$ are called knot points. The fitted function reflects the mean return over some periods of time. We categorize those periods with high positive mean returns as normal periods, and those with high negative mean returns as crash periods. When mean returns are close to 0, we categorize them as transition periods.

An alternative method is to apply trend filtering with $k = 1$ to the price data, instead of return data. In this case, the solution $\widehat{\boldsymbol{\beta}}$ will have a piecewise linear structure, representing the trends of the S&P index price. The regimes can be identified using the slope of the trend data.

We apply trend filtering with $k = 0$ to the S&P weekly return data over the period of 9.18.1989 - 06.12.2015. The algorithm can be practically implemented by the R package "genlasso" for generalized lasso methods. For the tuning parameter, we choose $\lambda = 0.05$. For each $1 \leq i \leq n$, we categorize i as in the normal regime if $\widehat{\beta}_i \geq 0.02/52$, crash regime if $\widehat{\beta}_i \leq -0.03/52$, and transition regime if $-0.03/52 < \widehat{\beta}_i < 0.02/52$. The experiment result is shown in Figure 3.1, along with the S&P 500 index. In particular, the green areas represent the normal periods, red areas the crash periods, and grey areas the transition periods.

The result shows that the regimes identified are quite reliable by inspection. The method is further applied to some sector indices as shown in Figures 3.2 - 3.4.

EXHIBIT 1
Two-Regimes for the S&P 500 Index (1990-2015)

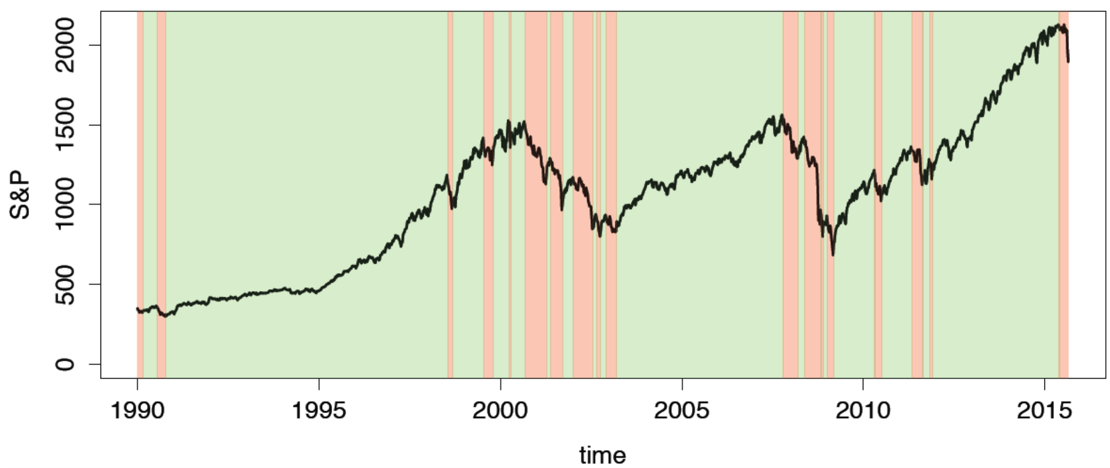

EXHIBIT 2
Equilibrium Transition Matrix
(Probability, period t to t+1)

	Growth Regime at Time t+1	Contraction Regime at Time t+1
Growth Regime at Time t	.9	.1
Contraction Regime at Time t	.4	.6

EXHIBIT 3
Asset Categories and Representative Allocation

	Private Equity	Real Estate	Hedge Funds	Real Assets	U.S. Equities	International Equity - Developed	International Equity - Emerging	U.S. Government Bond
Allocation	**20.0%**	**12.0%**	**18.0%**	**7.0%**	**15.0%**	**12.0%**	**8.0%**	**8.0%**

EXHIBIT 4
Projected Returns for Asset Categories – Single Regime
(Inflation adjusted)

Annualized Rate	Private Equity	Real Estate	Hedge Fund	Real Assets	U.S. Equities	International Equity - Developed	International Equity - Emerging	U.S. Government Bond
Single Regime Returns	6.50%	5.50%	5.00%	4.00%	4.50%	4.20%	4.80%	1.00%
Annual Volatility	14.212%	9.827%	7.681%	6.045%	17.988%	19.503%	26.344%	5.546%

EXHIBIT 5
Historical Returns for Assets under Growth Regime
(Inflation adjusted)

Annualized Rate	Private Equity	Real Estate	Hedge Fund	Real Assets	U.S. Equities	International Equity - Developed	International Equity - Emerging	U.S. Government Bond
Return under growth	15.00%	9.00%	8.50%	4.50%	17.00%	16.50%	18.00%	0.30%
Annual Volatility	12.089%	7.881%	5.887%	6.157%	13.556%	15.731%	23.762%	4.236%

EXHIBIT 6
Historical Returns for Assets under Contraction Regime
(Inflation adjusted)

Annualized Rate	Private Equity	Real Estate	Hedge Fund	Real Assets	U.S. Equities	International Equity - Developed	International Equity - Emerging	U.S. Government Bond
Return under crash	-21.66%	-7.41%	-7.91%	2.02%	-33.50%	-33.31%	-34.80%	3.85%
Annual Volatility	9.455%	12.345%	8.678%	5.561%	12.898%	13.654%	18.716%	7.601%

EXHIBIT 7
Summary Statistics for Baseline Monte Carlo Simulations

Simulation Results	Without spending-cut rule		Spending Cut by 20%	
	1-Regime	2-Regime	1-Regime	2-Regime
Crash Prob, 5 years	10.3%	18.4%	10.3%	18.4%
Crash Prob, 10 years	20.5%	31.8%	20.0%	31.5%
Crash Prob, 50 years	4.9%	19.9%	2.2%	13.1%
mean-5 years	1.0644	1.0780	1.0644	1.0780
mean-10 years	1.1635	1.1793	1.1692	1.1879
mean-20 years	1.3998	1.4230	1.4173	1.4514
mean-50 years	2.6630	2.6197	2.7152	2.7147
# of simulations	10000	10000	10000	10000
Average % time in "adverse"	2.88%	5.60%	2.22%	4.60%

EXHIBIT 8
Wealth Paths across All Scenarios for Baseline

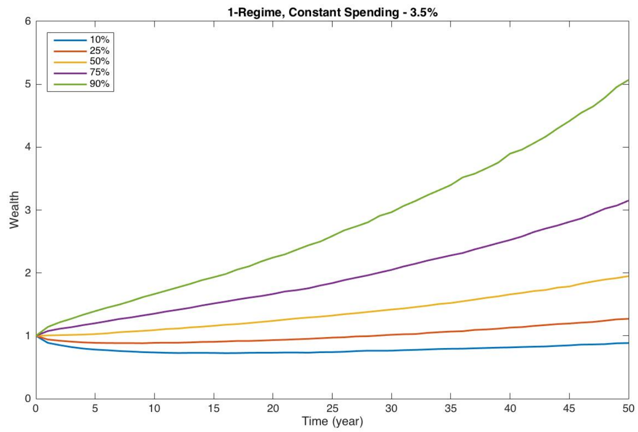

EXHIBIT 9
A Representative Scenario Path over the 50-Year Planning Horizon
(S&P 500 Index)

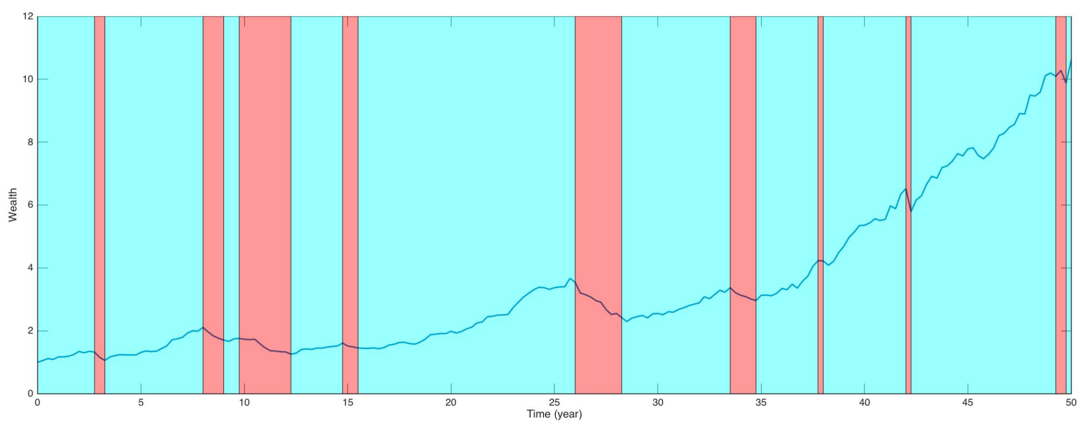

EXHIBIT 10
Cumulative Distribution of Wealth at 50 Year Horizon (inflation adjusted)

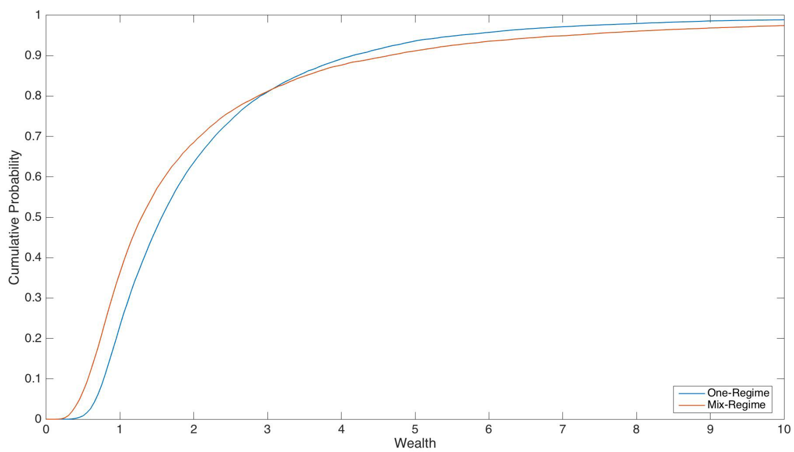

EXHIBIT 11
Wealth Paths across All Scenarios – Two-Regime Mode

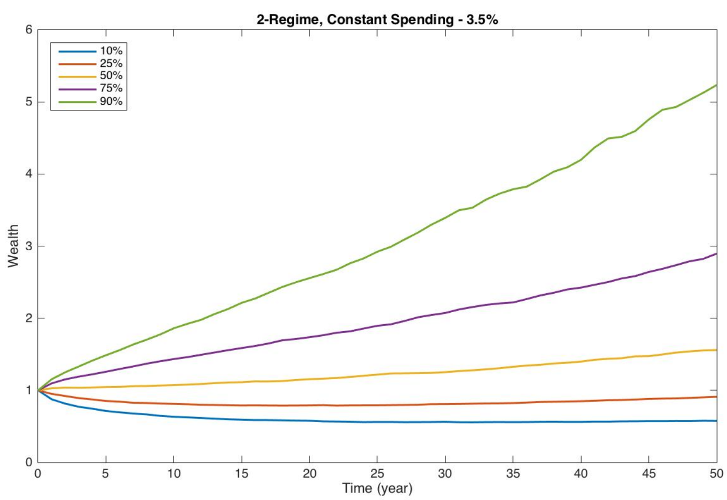

EXHIBIT 12
Performance Statistics for 4.5% Spending Target

	Without spending-cut rule		Spending Cut by 20%	
Simulation Results	1-Regime	2-Regime	1-Regime	2-Regime
Crash Prob, 5 years	12.7%	20.8%	12.7%	20.8%
Crash Prob, 10 years	28.6%	38.2%	27.7%	37.7%
Crash Prob, 50 years	27.9%	43.3%	9.1%	27.3%
mean-5 years	1.0324	1.0462	1.0324	1.0462
mean-10 years	1.0702	1.0913	1.0806	1.1037
mean-20 years	1.1510	1.1897	1.2021	1.2459
mean-50 years	1.5819	1.6233	1.7786	1.8345
# of simulations	10000	10000	10000	10000
Average % time in "adverse"	7.56%	9.51%	5.01%	7.32%

EXHIBIT 13
Performance Statistics for 3.5% Spending Target

	Without spending-cut rule		Spending Cut by 20%	
Simulation Results	1-Regime	2-Regime	1-Regime	2-Regime
Crash Prob, 5 years	8.3%	16.3%	8.3%	16.3%
Crash Prob, 10 years	14.6%	26.6%	14.4%	26.4%
Crash Prob, 50 years	1.2%	8.5%	0.5%	6.3%
mean-5 years	1.0977	1.1112	1.0977	1.1112
mean-10 years	1.2421	1.2609	1.2448	1.2659
mean-20 years	1.5883	1.6267	1.5971	1.6423
mean-50 years	3.6169	3.6499	3.6442	3.7050
# of simulations	10000	10000	10000	10000

EXHIBIT 14
Applying Trend Filtering with Three Regimes (growth, transition, contraction)