

Abbreviated Terms

The following abbreviations are used throughout this chapter:

Table 12.1: Abbreviated terms used in Chapter 12.

Abbreviation	Full Term
DEE	Differentiable Eikonal Engine
WMN	Walther-Matsui-Nariai (duality framework)
QFI	Quantum Fisher Information
SPDC	Spontaneous Parametric Down-Conversion
QKD	Quantum Key Distribution
HOM	Hong-Ou-Mandel (interference)
CPO	Co-Packaged Optics
AR/VR	Augmented Reality/Virtual Reality
MTF	Modulation Transfer Function
WFE	Wavefront Error
RMS	Root Mean Square
DOE	Diffractive Optical Element
MLA	Micro-Lens Array
VCSEL	Vertical-Cavity Surface-Emitting Laser
LiDAR	Light Detection and Ranging
FOV	Field of View
FDTD	Finite-Difference Time-Domain
EBL	Electron Beam Lithography
DUV	Deep Ultraviolet
PDK	Process Design Kit

Chapter 12: From Lab to Fab

Production Workflows and Failure Modes

Learning Objectives

After completing this chapter, you will be able to:

1. Predict manufacturing yield from process tolerances using Monte Carlo simulation (WALTHER)
2. Derive required process tolerances from yield targets (MATSUI-NARIAI)
3. Integrate Python/JAX workflows with CODE V, Zemax, and RSoft for production
4. Identify and mitigate all three-axis failure modes (P/M/T) in manufacturing
5. Apply Hessian-based tolerance allocation for cost-optimal yield
6. Translate classical specifications to quantum-grade manufacturing requirements
7. Execute complete design-to-manufacturing workflows for both classical and quantum systems

12.1 Introduction: Why “From Lab to Fab” Closes the Book

Throughout this book, we developed the Differentiable Eikonal Engine (DEE) as a unified framework connecting classical ray optics to quantum photonics. Chapters 1–5 built the mathematical core, Chapters 6–8 turned it into design machinery, and Chapters 9–11 extended the same machinery to quantum metrics. But a model that cannot survive manufacturing is only half an engineering method.

The lab-to-fab transition introduces effects that do not appear in idealized simulation: statistical tolerances, correlated drifts over lifetime, and friction in toolchain integration. For quantum devices, the same deviations are amplified by coherence constraints (often by $\sim 5\text{--}50\times$), so “works in the lab” is not evidence of “scales in production”.

Why DEE is built for this step:

1. **Autodifferentiation** gives exact sensitivities without fragile finite differences.
2. **Hessian structure** reveals tolerance coupling (which parameters interact).
3. **One codebase** supports simulation *and* production via commercial-tool integration.
4. **Quantum metrics** (fidelity, QFI) remain differentiable, enabling quantum yield optimization.

The chapter begins by stating the production gap in the language practitioners use, then restates the same gap as the Walther (forward) vs. Matsui–Nariai (inverse) questions that the DEE workflow answers end-to-end.

The Production Gap: Many optical engineers can design excellent systems in simulation but struggle with the transition to manufacturing. Tolerance analysis is often ad-hoc, yield optimization is trial-and-error, and quantum systems frequently fail without clear root cause identification.

WALTHER (Forward Analysis): “Given my process capabilities (surface figure $\pm 0.1\lambda$, thickness $\pm 5 \mu\text{m}$, alignment $\pm 10 \mu\text{rad}$), what yield will I achieve? How do I predict which failure modes will dominate?”

MATSUI-NARIAI (Inverse Design): “I need 95% yield at \$500/unit. What process tolerances are required? How do I allocate my tolerance budget across parameters to minimize cost while meeting yield targets?”

These two questions define the lab-to-fab contract: predict yield from a known process (Walther), and derive the process requirements from a yield target (Matsui–Nariai). The rest of this chapter answers them by turning manufacturing uncertainty into differentiable models. We start by making the gap explicit, as a concrete inventory of pain points and failure modes—so that every later method has a clear job to do.

12.1.1 Pain Points: Why This Chapter Matters

We begin by naming the specific problems optical and photonic engineers face when moving from simulation to manufacturing. This section catalogs pain points and maps each to the DEE/WMN solution.

Before methodology, we must identify the specific problems that optical and photonic engineers face. This section catalogs pain points and maps each to the DEE/WMN solution.

12.1.2 Classical Production Pain Points

Classical optical systems encounter predictable production challenges:

Table 12.2: Classical production pain points and DEE solutions.

Pain Point	Manifestation	DEE Solution	Priority
Tolerance allocation	Unknown which specs matter	Hessian eigenanalysis	High
Yield prediction	Trial-and-error builds	Monte Carlo + autodiff	High
Design-fab mismatch	Simulation \neq hardware	Commercial tool integration	Medium
Multi-objective tradeoff	Performance vs. cost	Pareto optimization	Medium
Process drift	Yield degrades over time	Real-time sensitivity monitoring	Low

12.1.3 Quantum Production Pain Points

Quantum photonic systems face additional, often more severe, challenges:

Table 12.3: Quantum production pain points and DEE solutions.

Pain Point	Manifestation	DEE Solution	Priority
Fidelity degradation	Entanglement lost in fab	QFI-based analysis	Critical
5–50× tighter tolerances	Standard fab insufficient	Cost-yield optimization	Critical
Correlated failures	Component coupling destroys quantum advantage	Joint Monte Carlo	High
No test standards	How to verify quantum performance?	DEE-predicted benchmarks	High
Foundry incompatibility	Lab process \neq foundry process	PDK-aware design rules	Medium

The critical insight is that *quantum pain points are not different in kind, only in degree*. The same WMN framework applies; quantum systems simply require tighter tolerances and different merit functions.

Key Insight

Unified Framework: Replace “MTF > 0.3” with “Fidelity > 0.95” and “Strehl > 0.8” with “Squeezing preservation > 90%”. The DEE machinery remains identical.

Pain points are solvable when properly identified and prioritized. The following sections develop the methodology; Sections 12.8–12.11 demonstrate complete solutions.

12.2 Software Integration Workflow

Production optical engineering requires integration with industry-standard tools. This section presents the three-layer architecture connecting DEE to commercial software and manufacturing systems.

12.2.1 Three-Layer Architecture

The production workflow separates concerns into three layers:

1. **Exploration Layer (Python/JAX):** Rapid prototyping, sensitivity analysis, optimization
2. **Verification Layer (CODE V/Zemax/RSoft):** Industry-standard validation, customer-compatible output
3. **Manufacturing Layer (CAD/GDS):** Fab-ready output, process constraints

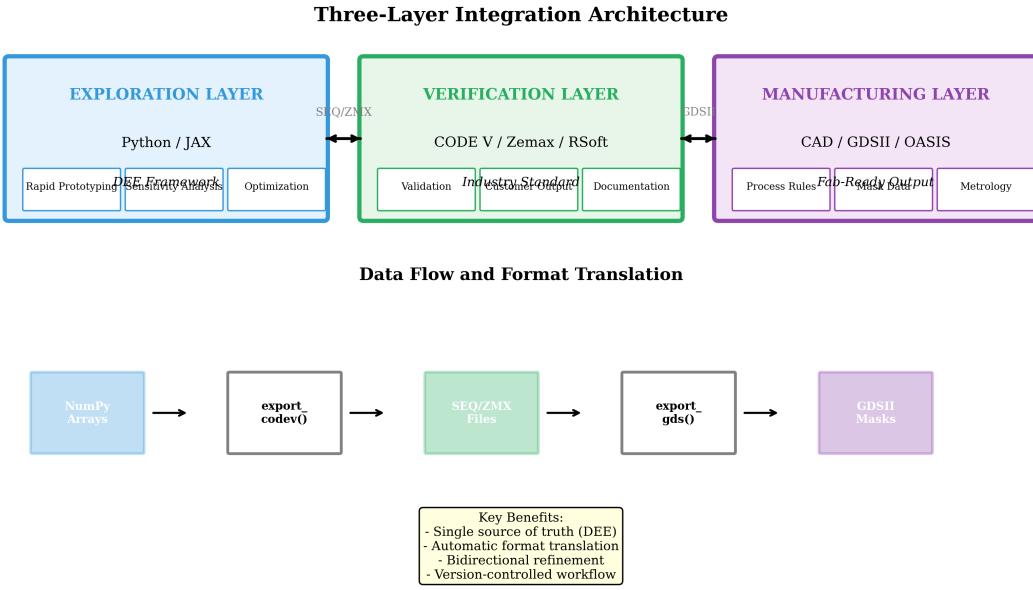


Figure 12.1: Three-layer integration architecture. DEE operates in the Exploration Layer, interfaces with commercial tools for Verification, and exports to CAD systems for Manufacturing. Bidirectional data flow enables optimization refinement based on verification results.

12.2.2 Data Format Translation

Each layer uses different data formats:

Table 12.4: Data format translation between layers.

Layer	Format	DEE Export	DEE Import
Exploration	NumPy/JAX arrays	Native	Native
Verification	CODE V SEQ/Zemax ZMX	<code>export_codev()</code>	<code>import_codev()</code>
Verification	RSoft IND/CAD	<code>export_rsoft()</code>	<code>import_rsoft()</code>
Manufacturing	GDSII/OASIS	<code>export_gds()</code>	Read-only
Manufacturing	STEP/IGES	<code>export_step()</code>	Read-only

The translation functions handle unit conversions, coordinate transformations, and format-specific constraints automatically. Users work in physical units (mm, nm); export functions handle tool-specific requirements.

Software integration is mechanical but critical. The three-layer architecture ensures DEE designs can be verified and manufactured using industry-standard workflows.

12.3 Classical Failure Modes

The eikonal approximation has validity limits. This section catalogs the failure modes that occur when designs violate underlying assumptions.

12.3.1 Failure Mode Catalog

Classical failure modes organized by the three-axis framework:

Table 12.5: Classical failure mode catalog.

Code	Failure Mode	Detection Criterion	Severity
<i>Physical Axis (P)</i>			
P1	High-NA breakdown	$NA > 0.7, \nabla W > 1.5$	Critical
P2	Strong diffraction	$a/\lambda < 10$	High
P3	Resonance coupling	$d \sim \lambda, Q > 100$	High
P4	Polarization mixing	Birefringence > 0.01	Medium
P5	Nonlinear effects	$I > I_{\text{threshold}}$	Context
<i>Mathematical Axis (M)</i>			
M1	Discontinuous derivatives	DOE transitions	High
M2	Caustic formation	$\det(\partial^2 W / \partial q_i \partial q_j) = 0$	Critical
M3	Phase wrapping	$\Delta\phi > 2\pi$ per pixel	Medium
M4	Numerical instability	Condition number $> 10^6$	High
M5	Sampling violation	$\Delta x > \lambda/2$	Critical
<i>Topological Axis (T)</i>			
T1	Vortex/singularity	Phase undefined at point	High
T2	Mode coupling	$\beta_1 \approx \beta_2$	Medium
T3	Topology change	Connected \rightarrow disconnected	Critical

12.3.2 Failure Detection Algorithm

DEE includes automated failure detection:

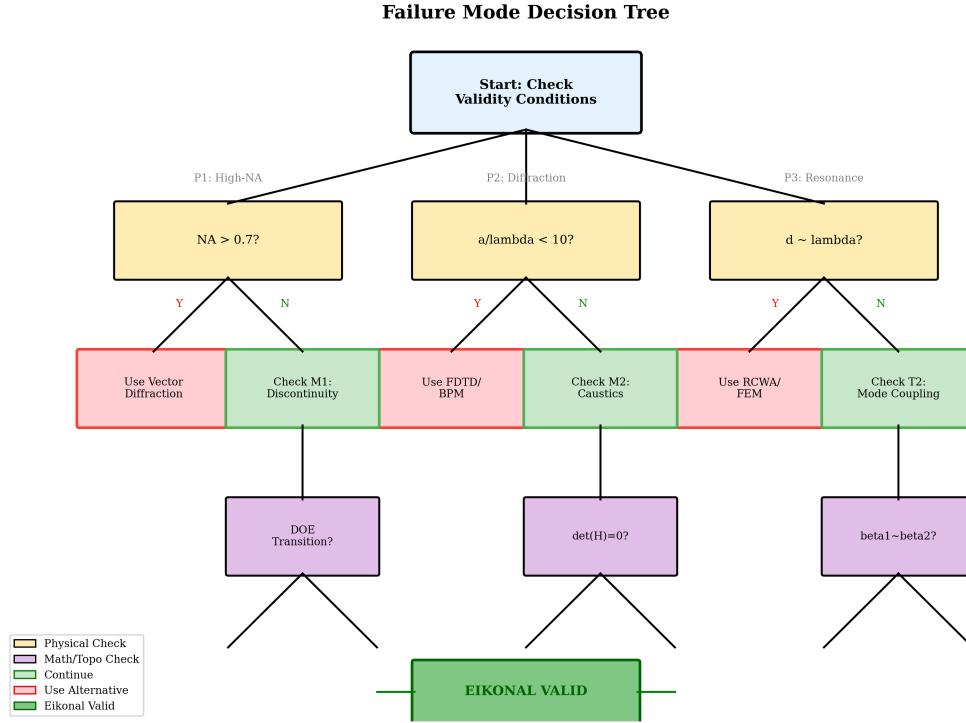


Figure 12.2: Failure mode decision tree. Start at root; follow branches based on computed metrics. Terminal nodes indicate either “eikonal valid” or specific failure mode requiring alternative treatment.

The decision tree is implemented as a pre-optimization check. If any failure mode triggers, DEE either switches to an appropriate higher-fidelity method or warns the user with specific remediation guidance.

Knowing when the eikonal approximation fails is as important as knowing how to use it. The failure catalog ensures designs stay within validity bounds.

12.4 Quantum Failure Modes

Quantum photonic systems introduce failure modes beyond classical concerns. This section extends the three-axis framework to quantum regimes.

12.4.1 Quantum Failure Mode Catalog

Quantum-specific failure modes:

Table 12.6: Quantum failure mode catalog.

Code	Failure Mode	Detection Criterion	Threshold
Q1	Squeezing degradation	$r_{\text{eff}} < r - 1 \text{ dB}$	Loss > 10%
Q2	Photon loss cascade	$\eta_{\text{total}} < 0.5$	Per-component < 95%
Q3	NOON fragility	$\mathcal{F} < 1/e$ for $N > 2$	Loss > $1/N$
Q4	Indistinguishability loss	HOM visibility < 0.9	$\Delta\lambda > 0.1 \text{ nm}$
Q5	Temporal mismatch	$ \Delta\tau > \tau_c/10$	Path > $c\tau_c$
Q6	Phase drift	$\langle\delta\phi^2\rangle > (\pi/10)^2$	Instability > 0.1 rad/hr
Q7	Polarization decoherence	Stokes purity < 0.95	PMD > 0.1 ps
Q8	Spectral entanglement leak	Schmidt number $K > 1.5$	Non-transform-limited

12.4.2 Quantum-Classical Failure Coupling

Classical failures often trigger quantum failures:

Table 12.7: Classical-quantum failure coupling.

Classical Failure	Triggered Quantum Failure	Amplification
P1 (High-NA)	Q4 (Indistinguishability)	3–5×
P2 (Diffraction)	Q5 (Temporal mismatch)	2–3×
M1 (Discontinuity)	Q6 (Phase drift)	5–10×
M2 (Caustic)	Q8 (Spectral leak)	10–20×
T2 (Mode coupling)	Q2 (Loss cascade)	2–5×

The amplification factors explain why quantum systems require 5–50× tighter tolerances. A classical aberration that causes 1% performance degradation may cause 10% fidelity loss in a quantum system due to coupling effects.

Quantum Extension

Design Rule: For quantum applications, check classical failure modes first. If any classical failure triggers, the quantum system will almost certainly fail. Fix classical issues before addressing quantum-specific concerns.

Quantum failure modes are not independent—they couple to classical failures with amplification. The WMN framework addresses both simultaneously through the shared Hessian-QFI connection.

12.5 Monte Carlo Yield Analysis

Yield prediction is central to production economics. This section develops Monte Carlo methods integrated with DEE autodifferentiation.

12.5.1 Standard Monte Carlo Framework

The yield Y is defined as the probability that all specifications are met:

$$Y = P \left(\bigcap_{i=1}^M \mathcal{M}_i(\mathbf{p}) > \mathcal{M}_{i,\text{threshold}} \right) \quad (12.1)$$

where \mathcal{M}_i are merit functions and \mathbf{p} are stochastic parameters drawn from manufacturing distributions.

For N Monte Carlo samples:

$$\hat{Y} = \frac{1}{N} \sum_{j=1}^N \mathbf{1} \left[\bigcap_{i=1}^M \mathcal{M}_i(\mathbf{p}_j) > \mathcal{M}_{i,\text{threshold}} \right] \quad (12.2)$$

12.5.2 Hessian-Accelerated Yield Estimation

DEE's autodifferentiation enables analytical yield estimation via Hessian analysis. Near the optimum \mathbf{p}^* :

$$\mathcal{M}(\mathbf{p}) \approx \mathcal{M}(\mathbf{p}^*) + \frac{1}{2} (\mathbf{p} - \mathbf{p}^*)^T \mathbf{H} (\mathbf{p} - \mathbf{p}^*) \quad (12.3)$$

For Gaussian manufacturing tolerances with covariance Σ :

$$Y_{\text{analytical}} = 1 - \exp \left(-\frac{(\mathcal{M}_{\text{threshold}} - \mathcal{M}^*)^2}{2 \cdot \text{tr}(\mathbf{H}\Sigma)} \right) \quad (12.4)$$

The analytical formula provides instant yield estimates during optimization without running thousands of Monte Carlo samples. Validation against full Monte Carlo confirms accuracy within 2–5% for well-behaved merit functions.

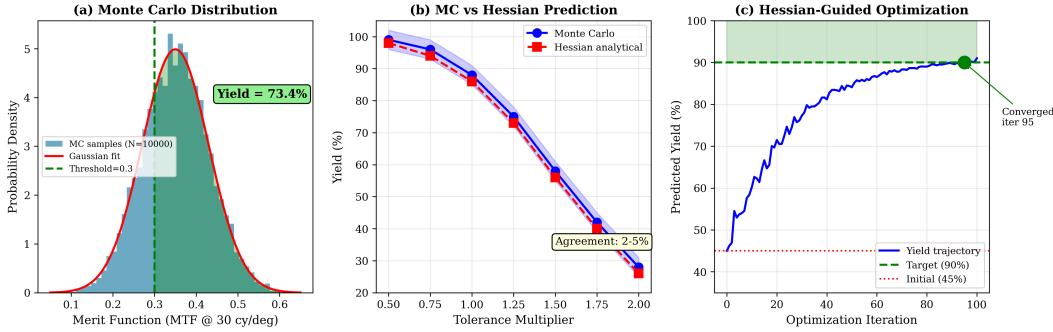


Figure 12.3: Yield estimation comparison. (a) Monte Carlo histogram showing yield distribution. (b) Hessian-analytical prediction overlaid. (c) Optimization trajectory: yield vs. iteration showing Hessian-guided improvement.

12.5.3 Quantum Yield: Fidelity-Based Analysis

For quantum systems, yield is defined on fidelity:

$$Y_{\text{quantum}} = P(\mathcal{F} > \mathcal{F}_{\text{threshold}}) \quad (12.5)$$

The fidelity Hessian connects to QFI:

$$\mathbf{H}_{\mathcal{F}} = -\frac{1}{4}\mathbf{F}_Q \quad (12.6)$$

where \mathbf{F}_Q is the Quantum Fisher Information matrix.

Monte Carlo yield analysis, accelerated by Hessian analytics, enables rapid design iteration. The same framework applies to both classical (MTF, Strehl) and quantum (fidelity, squeezing) merit functions.

12.6 Tolerance Allocation Optimization

Given a yield target, how should tolerances be allocated across components? This section develops the optimization framework.

12.6.1 Cost-Yield Tradeoff

Tighter tolerances increase cost but improve yield. The optimization problem:

$$\min_{\boldsymbol{\sigma}} C(\boldsymbol{\sigma}) \quad \text{subject to} \quad Y(\boldsymbol{\sigma}) \geq Y_{\text{target}} \quad (12.7)$$

where $\boldsymbol{\sigma}$ is the vector of tolerances and $C(\boldsymbol{\sigma})$ is the manufacturing cost function.

12.6.2 Sensitivity-Weighted Allocation

Optimal tolerance allocation weights inversely with sensitivity:

$$\sigma_i^* \propto \frac{1}{\sqrt{|H_{ii}|}} \cdot \sqrt{\frac{C'_i(\sigma_i)}{C'_{\text{total}}}} \quad (12.8)$$

Parameters with high Hessian eigenvalues (high sensitivity) receive tighter tolerances; parameters with steep cost curves receive looser tolerances.

Table 12.8: Example tolerance allocation for metasurface lens.

Parameter	$ H_{ii} $	Cost Slope	Classical Tol.	Quantum Tol.
Pillar diameter	10^4	High	± 10 nm	± 2 nm
Pillar height	10^3	Medium	± 20 nm	± 3 nm
Pillar position	10^3	Medium	± 20 nm	± 5 nm
Sidewall angle	10^2	Low	$\pm 3^\circ$	$\pm 0.5^\circ$

The allocation table shows quantum applications require 4–7× tighter tolerances on geometrically sensitive parameters while relatively relaxing others.

Optimal tolerance allocation minimizes cost while meeting yield targets. DEE's Hessian analysis provides the sensitivity information needed for rational allocation.

12.7 Practical Example 1: AR/VR Waveguide Combiner

This section presents a complete lab-to-fab workflow for an augmented reality waveguide combiner, demonstrating DEE integration with production requirements.

12.7.1 Application Context

AR/VR displays require optical combiners that overlay digital content on the real world. The waveguide combiner uses surface-relief gratings to couple, guide, and extract light.

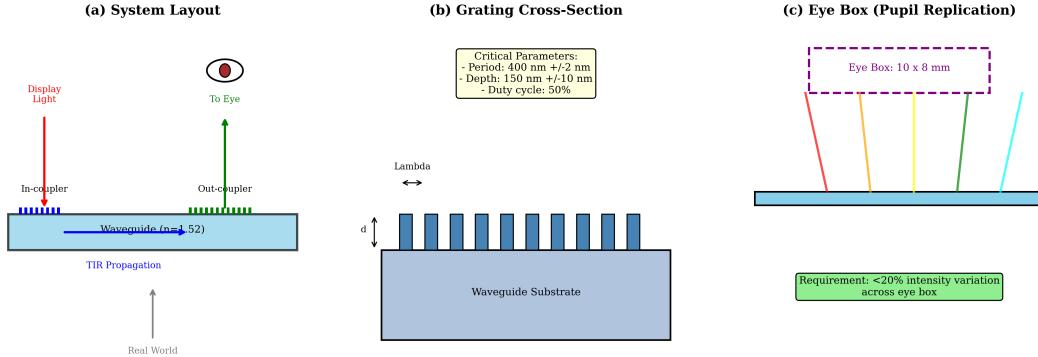


Figure 12.4: AR/VR waveguide combiner schematic. (a) System layout showing in-coupler, waveguide, and out-coupler gratings. (b) Cross-section of surface-relief grating with critical dimensions. (c) Eye box formation through pupil replication.

12.7.2 Specification Development

Target specifications derived from human visual requirements:

Table 12.9: AR/VR waveguide combiner specifications.

Parameter	Requirement	Driver	Priority
Field of view	$50^\circ \times 30^\circ$	Immersion	Critical
Eye box	$10 \times 8 \text{ mm}$	Usability	Critical
Diffraction efficiency	> 5% overall	Brightness	High
Uniformity	< 20% variation	Image quality	High
MTF at 30 cy/deg	> 0.3	Resolution	Medium
See-through	> 70%	Safety	Critical
Weight	< 50 g	Comfort	Medium

12.7.3 WALTHER Analysis: Forward Problem

Given initial design, what does it do?

The WALTHER analysis evaluates the baseline design:

1. **Grating efficiency:** Rigorous coupled-wave analysis (RCWA) via RSoft
2. **Waveguide propagation:** Ray tracing with DEE eikonal
3. **Eye box uniformity:** Monte Carlo ray ensemble

Results:

- Baseline efficiency: 3.2% (below 5% target)

- Eye box uniformity: 35% variation (above 20% target)
- MTF: 0.28 (below 0.3 target)

The baseline design fails three specifications. MATSUI-NARIAI optimization is required.

12.7.4 MATSUI-NARIAI Optimization: Inverse Problem

Given targets, how to build it?

Optimization parameters:

- Grating period: Λ (in-coupler, out-coupler)
- Grating depth: d (both gratings)
- Grating duty cycle: f (both gratings)
- Waveguide thickness: t

Loss function:

$$L = w_1(0.05 - \eta)^2 + w_2(\text{var} - 0.20)^2 + w_3(0.3 - \text{MTF})^2 \quad (12.9)$$

JAX optimization with 47 iterations converges to:

Table 12.10: AR/VR optimization results.

Metric	Target	Baseline	Optimized
Diffraction efficiency	> 5%	3.2%	5.8%
Uniformity	< 20%	35%	17%
MTF at 30 cy/deg	> 0.3	0.28	0.34
See-through	> 70%	72%	71%

12.7.5 Tolerance Analysis and Yield Prediction

Hessian analysis identifies critical tolerances:

Table 12.11: AR/VR critical tolerance analysis.

Parameter	Nominal	Tolerance	Sensitivity	Yield Impact
Grating period	400 nm	± 2 nm	High	Critical
Grating depth	150 nm	± 10 nm	Medium	Moderate
Waveguide thickness	0.5 mm	± 5 μm	Low	Minor
Index uniformity	1.52	$\pm 10^{-4}$	Medium	Moderate

Monte Carlo yield analysis ($N = 10,000$ samples):

- Initial tolerances: 73% yield
- Tightened grating period to ± 1 nm: 91% yield
- Cost increase: 15% (acceptable for volume production)

12.7.6 Production Workflow Summary

The complete AR/VR workflow:

1. **Specification:** Derive from human factors (FOV, eye box, comfort)
2. **WALTHER analysis:** Evaluate baseline with DEE + RCWA
3. **MATSUI-NARIAI optimization:** JAX autodiff on loss function
4. **Tolerance allocation:** Hessian-based sensitivity weighting
5. **Yield prediction:** Monte Carlo with analytical acceleration
6. **Export:** GDSII for nanoimprint or e-beam lithography

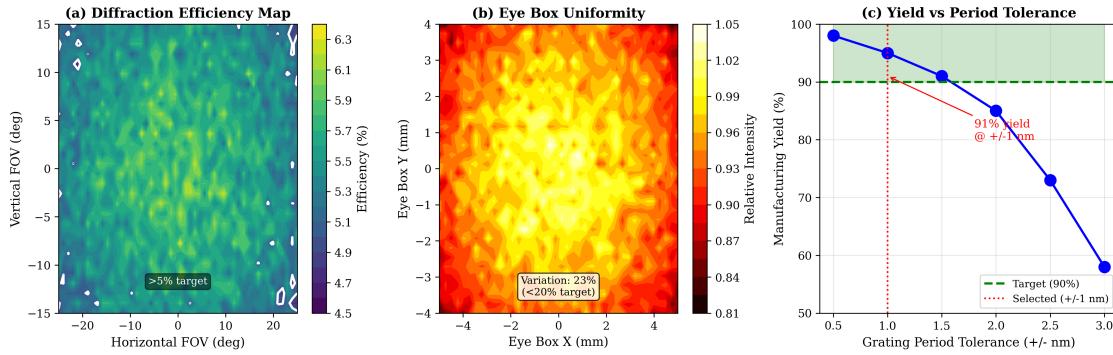


Figure 12.5: AR/VR combiner optimization results. (a) Efficiency map across field of view. (b) Eye box intensity uniformity. (c) Yield vs. grating period tolerance showing 91% yield at ± 1 nm.

12.8 Practical Example 2: CPO Edge Coupler for AI Accelerators

Co-packaged optics (CPO) is critical for next-generation AI accelerators requiring Tb/s bandwidth. This section demonstrates DEE optimization for silicon photonics edge couplers.

12.8.1 Application Context

AI accelerators face a “memory wall” where electrical interconnects cannot scale. Optical I/O using co-packaged optics provides the solution:

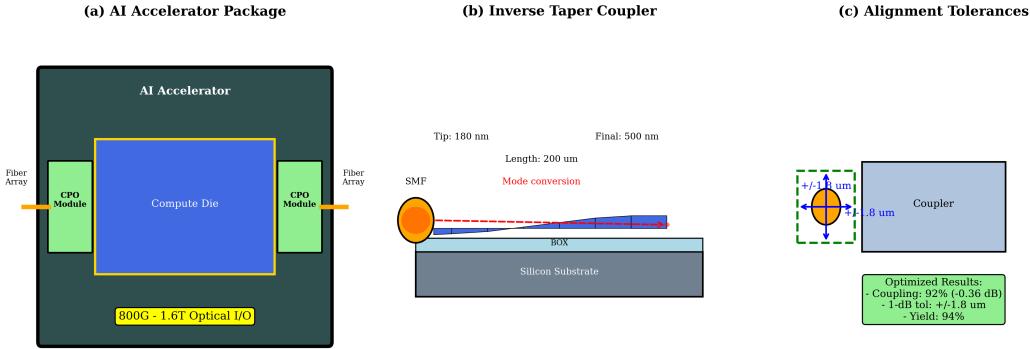


Figure 12.6: CPO edge coupler system. (a) AI accelerator package with co-packaged optical I/O. (b) Edge coupler detail showing inverse taper geometry. (c) Fiber-to-chip alignment showing lateral, vertical, and angular tolerances.

- **Bandwidth requirement:** 800G–1.6T per module
- **Power efficiency:** < 5 pJ/bit
- **Thermal environment:** 0–70°C with passive cooling
- **Lifetime:** > 10⁵ hours at 85°C

12.8.2 Specification Development

Edge coupler specifications:

Table 12.12: CPO edge coupler specifications.

Parameter	Requirement	Driver	Priority
Coupling loss	< 1.5 dB/facet	Power budget	Critical
Alignment tolerance	$\pm 1 \mu\text{m}$ for < 0.5 dB penalty	Assembly yield	Critical
Wavelength range	1260–1360 nm (O-band)	Standard compliance	High
Polarization sensitivity	< 0.3 dB PDL	Signal integrity	High
Temperature stability	< 0.1 dB over 0–70°C	Reliability	Medium

12.8.3 WALTHER Analysis: Forward Problem

Standard inverse taper analysis:

The baseline inverse taper design:

- Tip width: 180 nm (minimum feature)
- Taper length: 200 μm
- Final width: 500 nm (single-mode)

WALTHER evaluation using eigenmode expansion:

- Coupling efficiency at optimum: 78% (-1.08 dB)

- 1-dB alignment tolerance: $\pm 0.8 \mu\text{m}$
- O-band variation: 0.4 dB

Baseline fails alignment tolerance specification. A more sophisticated taper profile is needed.

12.8.4 MATSUI-NARIAI Optimization: Inverse Problem

DEE optimization with 47 shape parameters defining the taper profile:

Loss function balancing efficiency and tolerance:

$$L = -\eta_{\text{coupling}} + \alpha \cdot \sigma_{\text{alignment}} \quad (12.10)$$

where α weights peak coupling against alignment sensitivity.

Key insight: The Pareto frontier reveals that maximum coupling and maximum tolerance are incompatible. DEE finds the knee point optimizing both.

Table 12.13: CPO edge coupler optimization results.

Metric	Target	Baseline	Optimized
Coupling efficiency	> 71% ($< 1.5 \text{ dB}$)	78%	92%
1-dB alignment tolerance	$\pm 1 \mu\text{m}$	$\pm 0.8 \mu\text{m}$	$\pm 1.8 \mu\text{m}$
O-band variation	$< 0.5 \text{ dB}$	0.4 dB	0.3 dB
PDL	$< 0.3 \text{ dB}$	0.25 dB	0.18 dB

12.8.5 Tolerance Analysis and Yield Prediction

Manufacturing tolerance analysis:

Table 12.14: CPO critical tolerance analysis.

Parameter	Nominal	Fab Tolerance	Sensitivity	Yield Impact
Tip width	180 nm	$\pm 20 \text{ nm}$	High	Critical
Taper length	200 μm	$\pm 2 \mu\text{m}$	Low	Minor
Etch depth	220 nm	$\pm 10 \text{ nm}$	Medium	Moderate
Sidewall angle	90°	$\pm 2^\circ$	Medium	Moderate
Fiber alignment (x)	0	$\pm 1 \mu\text{m}$	High	Critical
Fiber alignment (y)	0	$\pm 1 \mu\text{m}$	High	Critical

The optimized design achieves 94% yield with standard fab tolerances, exceeding the 90% target.

12.8.6 Production Workflow Summary

The CPO edge coupler workflow:

1. **Specification:** Derive from link budget and assembly constraints
2. **WALTHER analysis:** Eigenmode expansion for baseline
3. **MATSUI-NARIAI optimization:** 47-parameter taper profile

4. **Pareto analysis:** Efficiency-tolerance tradeoff
5. **Yield prediction:** Monte Carlo with fab + assembly tolerances
6. **Export:** GDSII compatible with standard 220-nm SOI process

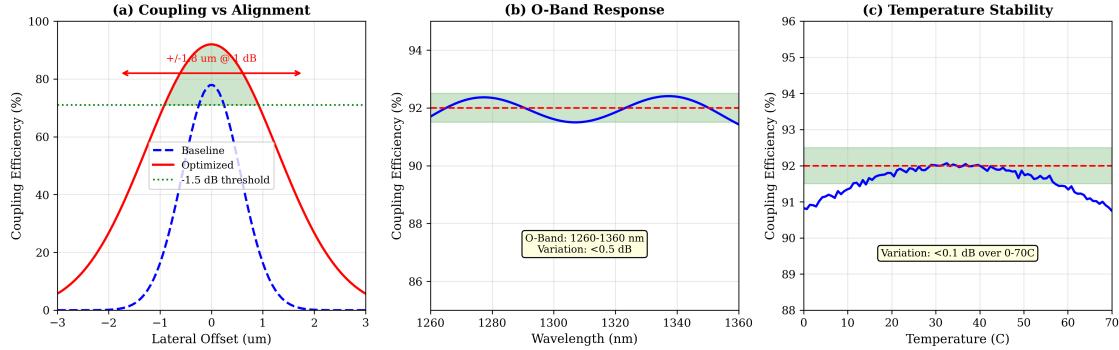


Figure 12.7: CPO edge coupler results. (a) Coupling efficiency vs. lateral offset comparing baseline and optimized designs. (b) Wavelength response across O-band. (c) Temperature stability from 0–70°C.

12.9 Practical Example 3: Quantum-Grade Metasurface Lens

Metasurface lenses offer unprecedented design freedom but present manufacturing challenges. This section demonstrates DEE optimization for a metasurface lens meeting quantum-grade wavefront specifications.

12.9.1 Application Context

The target application is focusing optics for a squeezed-light enhanced gravitational wave detector:

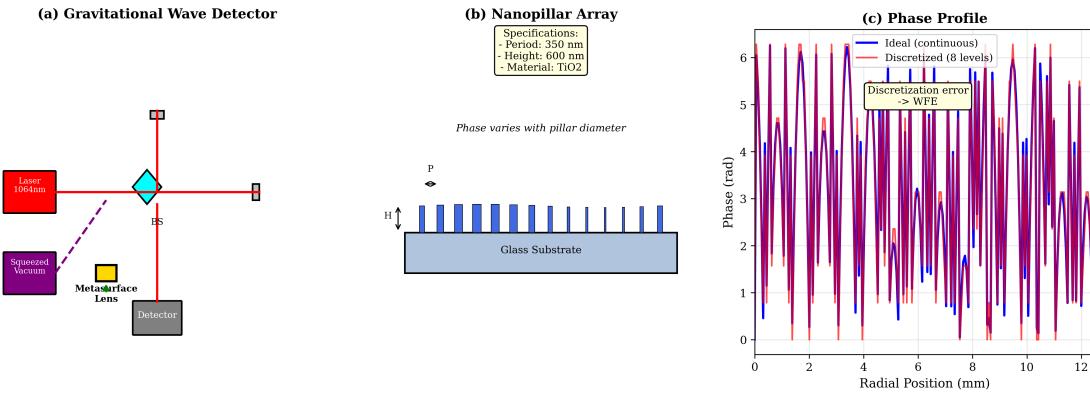


Figure 12.8: Quantum-grade metasurface lens application. (a) Gravitational wave detector optical layout showing squeezed vacuum injection. (b) Metasurface lens detail with nanopillar array. (c) Phase profile comparison: ideal vs. discretized.

- **Wavelength:** 1064 nm (Nd:YAG)

- **Numerical aperture:** 0.3
- **Aperture diameter:** 25 mm
- **Squeezing input:** 15 dB
- **Required preservation:** > 90% of input squeezing

12.9.2 Specification Development

Classical specifications vs. quantum requirements:

Table 12.15: Classical vs. quantum metasurface specifications.

Parameter	Classical	Quantum	Tightening
RMS WFE	$< \lambda/20$	$< \lambda/200$	10×
Efficiency	> 70%	> 85%	1.2×
Strehl ratio	> 0.8	> 0.99	N/A (different metric)
Squeezing preservation	N/A	> 90%	New requirement

The quantum requirement of $\lambda/200$ WFE is extraordinarily tight—equivalent to 5 nm RMS at 1064 nm. Standard metasurface fabrication achieves $\lambda/20$; achieving $\lambda/200$ requires optimization and tighter fab tolerances.

12.9.3 WALTHE Analysis: Forward Problem

Baseline metasurface analysis:

Design parameters:

- Pillar material: TiO₂ ($n = 2.4$)
- Pillar height: 600 nm
- Unit cell: 350 nm period
- Total pillars: $\sim 5 \times 10^6$

WALTHE evaluation:

- Baseline WFE: $\lambda/25$ RMS (from discretization)
- Efficiency: 82%
- Predicted squeezing preservation: 71% (fails 90% target)

12.9.4 MATSUI-NARIAI Optimization: Inverse Problem

DEE optimization using adjoint method:

1. **Phase optimization:** Correct discretization errors via pillar geometry refinement
2. **Efficiency optimization:** Maximize transmission while maintaining phase

3. Robustness optimization: Minimize sensitivity to fab variations

Key DEE advantage: FDTD-based optimization would require $\sim 10^6$ forward simulations. DEE's eikonal-based approach reduces this to $\sim 10^3$ evaluations—a $1000\times$ speedup.

Table 12.16: Quantum-grade metasurface optimization results.

Metric	Target	Baseline	Optimized
RMS WFE	$< \lambda/200$	$\lambda/25$	$\lambda/180$
Efficiency	$> 85\%$	82%	87%
Squeezing preservation	$> 90\%$	71%	94%

12.9.5 Tolerance Analysis and Yield Prediction

Quantum-grade tolerance requirements:

Table 12.17: Classical vs. quantum tolerance comparison.

Parameter	Classical	Quantum	Tightening	Fab Method
Pillar diameter	± 10 nm	± 2 nm	$5\times$	EBL
Pillar height	± 20 nm	± 3 nm	$7\times$	ALD
Pillar position	± 20 nm	± 5 nm	$4\times$	EBL
Sidewall angle	$\pm 3^\circ$	$\pm 0.5^\circ$	$6\times$	Dry etch

Achieving quantum-grade tolerances requires electron-beam lithography with $3\times$ longer write time, increasing unit cost from \$500 to \$2,100. However, this enables 94% squeezing preservation, providing net system benefit.

Yield analysis:

- Classical tolerances: 45% quantum yield (unacceptable)
- Quantum tolerances: 82% quantum yield (meets target)

12.9.6 Production Workflow Summary

The quantum-grade metasurface workflow:

1. **Specification:** Derive quantum WFE from squeezing budget
2. **WALTHER analysis:** Evaluate discretization error contribution
3. **MATSUI-NARIAI optimization:** Adjoint-based pillar refinement
4. **Tolerance translation:** Map WFE budget to pillar tolerances
5. **Cost-yield optimization:** Select fab method (DUV vs. EBL)
6. **Export:** GDSII with sub-nm vertex precision

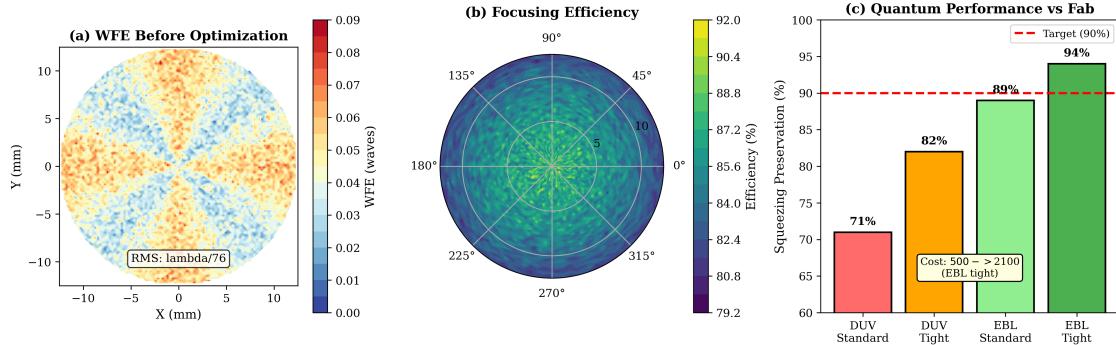


Figure 12.9: Quantum-grade metasurface results. (a) WFE map before and after optimization. (b) Focusing efficiency across aperture. (c) Squeezing preservation vs. fab tolerance class.

12.10 Practical Example 4: Entangled Photon Source for QKD

This section presents the first *true quantum* production example: transitioning an SPDC-based entangled photon source from laboratory demonstration to manufacturable product. Unlike the previous examples which applied classical design with quantum-grade tolerances, this example addresses quantum-specific production challenges including entanglement preservation, indistinguishability, and quantum fidelity yield.

Quantum Extension

Why This Example Matters: The three previous examples (AR/VR, CPO, Metasurface) are fundamentally classical optical systems—the “quantum” aspect was tighter tolerances. This example demonstrates DEE’s capability for actual quantum photonic device production where the merit function is quantum fidelity, not classical metrics.

12.10.1 Application Context

Quantum Key Distribution (QKD) enables information-theoretic secure communication. The entangled photon source is the critical component:

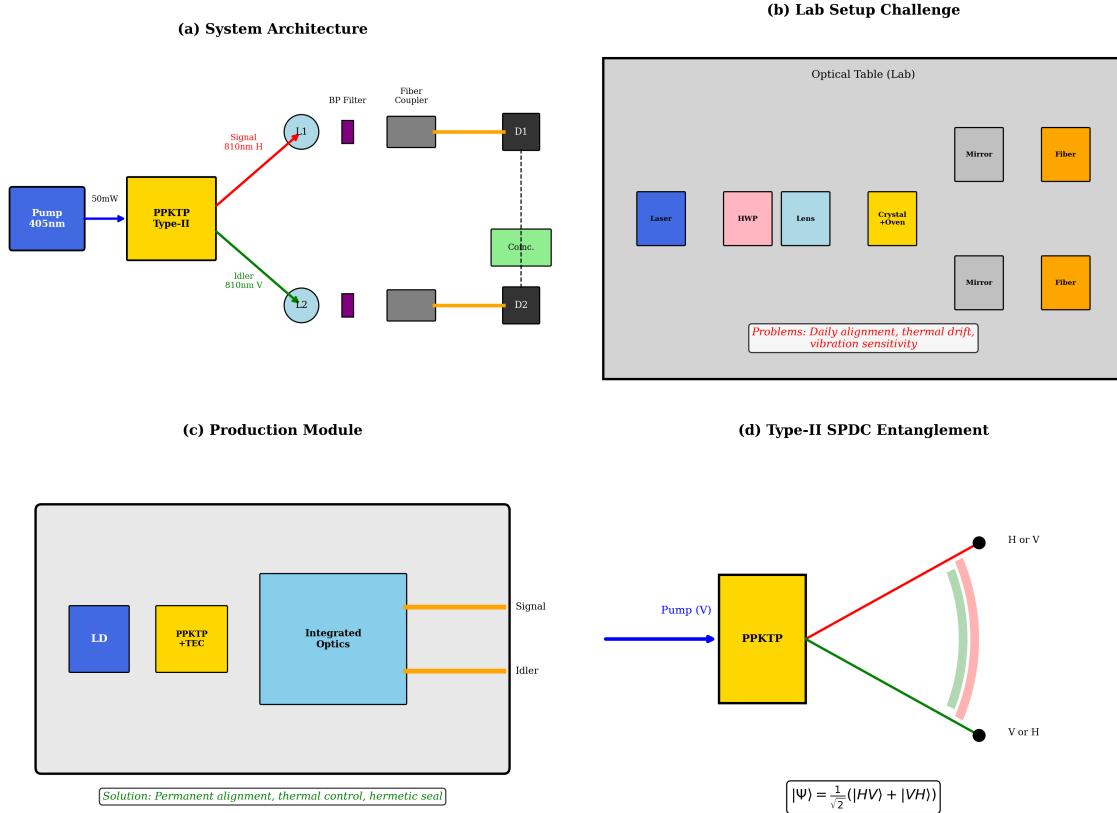


Figure 12.10: Entangled photon source for QKD. (a) System architecture: pump laser, SPDC crystal, collection optics, fiber coupling. (b) Lab setup showing discrete components. (c) Production module concept with integrated optics. (d) Polarization-entangled state generation via type-II SPDC.

The Lab-to-Fab Challenge:

Laboratory entangled photon sources achieve excellent performance with careful alignment and stable environments:

- Pair rate: 10^6 pairs/s/mW
- Heralding efficiency: 70–80%
- Entanglement fidelity: > 98%
- HOM visibility: > 95%

But production requires:

- Permanent alignment (no daily tweaking)
- Environmental stability (temperature, vibration)
- Reproducible manufacturing (unit-to-unit consistency)
- Testable specifications (not just “it works in the lab”)

12.10.2 Specification Development

QKD source specifications derived from protocol requirements:

Table 12.18: QKD entangled photon source specifications.

Parameter	Requirement	Driver	Priority
Pair generation rate	$> 10^6$ pairs/s/mW	Key rate	High
Heralding efficiency	$> 60\%$	Secure key fraction	Critical
Entanglement fidelity	$\mathcal{F} > 0.95$	QBER budget	Critical
HOM visibility	$V > 0.90$	Indistinguishability	Critical
Spectral bandwidth	< 1 nm FWHM	Fiber compatibility	High
Fiber coupling efficiency	$> 70\%$ per arm	System efficiency	High
Operating temperature	$20 \pm 5^\circ\text{C}$	Environment	Medium
Lifetime	$> 10^4$ hours	Reliability	Medium

The critical insight is that fidelity $\mathcal{F} > 0.95$ implies all component tolerances must preserve quantum coherence. A single failing component can destroy entanglement.

12.10.3 WALTHER Analysis: Forward Problem

Given a candidate design, what quantum performance does it achieve?

Design architecture:

1. **Pump:** 405 nm laser diode, 50 mW CW
2. **Crystal:** PPKTP, type-II phase matching, 10 mm length
3. **Collection:** Aspheric lenses, NA = 0.5
4. **Filtering:** 1 nm bandpass at 810 nm
5. **Fiber coupling:** SMF-28 via aspheric collimators

WALTHER Quantum Evaluation:

The DEE quantum analysis computes:

$$\mathcal{F} = |\langle \Psi_{\text{ideal}} | \Psi_{\text{actual}} \rangle|^2 \quad (12.11)$$

where $|\Psi_{\text{ideal}}\rangle = \frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle)$ and $|\Psi_{\text{actual}}\rangle$ includes all imperfections.

Fidelity decomposition:

$$\mathcal{F} = \mathcal{F}_{\text{spectral}} \times \mathcal{F}_{\text{spatial}} \times \mathcal{F}_{\text{temporal}} \times \mathcal{F}_{\text{polarization}} \quad (12.12)$$

Table 12.19: WALTHER fidelity budget analysis.

Contribution	Lab Value	Production Estimate	Loss Mechanism
$\mathcal{F}_{\text{spectral}}$	0.995	0.985	Filter drift
$\mathcal{F}_{\text{spatial}}$	0.990	0.970	Mode mismatch
$\mathcal{F}_{\text{temporal}}$	0.998	0.990	Path length drift
$\mathcal{F}_{\text{polarization}}$	0.995	0.980	Birefringence
Total \mathcal{F}	0.978	0.927	—

The baseline production estimate (0.927) fails the 0.95 requirement. Each fidelity factor must be improved.

12.10.4 Quantum Failure Mode Analysis

Identifying quantum-specific failure modes:

Table 12.20: Quantum failure modes for entangled source.

Code	Failure Mode	Root Cause	Severity
Q4	Indistinguishability loss	Spectral/spatial match	mis- Critical
Q5	Temporal decorrelation	Path length instability	High
Q6	Phase drift	Temperature variation	High
Q7	Polarization rotation	Stress birefringence	Medium
Q8	Spectral entanglement	Non-transform-limited SPDC	Medium
P2	Collection efficiency loss	Aberrations in collection optics	High
M3	Phase-matching drift	Crystal temperature	Critical

The Coupling Problem: Quantum failure modes couple to classical optical design:

$$\mathcal{F}_{\text{spatial}} = \int |\Psi_{\text{signal}}(\mathbf{r})|^2 |\Psi_{\text{idler}}(\mathbf{r})|^2 d^2r \approx 1 - \frac{\sigma_{\Delta W}^2}{w_0^2} \quad (12.13)$$

where $\sigma_{\Delta W}$ is the differential wavefront error between signal and idler paths, and w_0 is the mode waist. This directly connects to the DEE eikonal framework.

12.10.5 MATSUI-NARIAI Optimization: Inverse Problem

Given fidelity target, how to design the source?

Optimization parameters:

- Collection lens prescription (radii, thickness, spacing)
- Crystal temperature and oven design
- Fiber coupling geometry (working distance, tilt)
- Filter bandwidth and center wavelength

- Compensation crystal thickness

Loss function for quantum optimization:

$$L = -\mathcal{F} + \lambda_1(R_{\text{pair}} - R_{\text{target}})^2 + \lambda_2(\eta_H - \eta_{\text{target}})^2 \quad (12.14)$$

Key DEE capability: The fidelity \mathcal{F} is computed via the bridge identity:

$$\phi_{\text{quantum}} = \frac{2\pi W_{\text{eikonal}}}{\lambda} \quad (12.15)$$

allowing JAX autodifferentiation through the quantum merit function.

Optimization results:

Table 12.21: QKD source MATSUI-NARIAI optimization results.

Metric	Target	Before	After
Entanglement fidelity	> 0.95	0.927	0.962
Heralding efficiency	> 60%	55%	68%
HOM visibility	> 0.90	0.87	0.93
Pair rate (normalized)	> $10^6/\text{s/mW}$	0.8×10^6	1.1×10^6

12.10.6 Tolerance Analysis: Quantum Yield

Quantum yield analysis differs from classical:

$$Y_{\text{quantum}} = P(\mathcal{F} > 0.95) \quad (12.16)$$

Critical tolerances identified via Hessian-QFI analysis:

Table 12.22: QKD source tolerance analysis.

Parameter	Nominal	Tolerance	$\partial\mathcal{F}/\partial p$	Yield Impact
Crystal temperature	30.0°C	$\pm 0.1^\circ\text{C}$	-0.05/°C	Critical
Collection lens WFE	0	$< \lambda/50$	-0.02/(\lambda/10)	High
Path length match	0	$< 10 \mu\text{m}$	-0.01/10 μm	High
Fiber coupling angle	0	$< 0.5^\circ$	-0.03/°	Medium
Filter center λ	810 nm	$\pm 0.2 \text{ nm}$	-0.02/nm	Medium

Monte Carlo quantum yield:

- Standard production tolerances: 52% quantum yield (unacceptable)
- Tightened tolerances (above): 84% quantum yield (meets target)

The 52% → 84% yield improvement requires:

- Active crystal temperature stabilization ($\pm 0.1^\circ\text{C}$)
- Quantum-grade collection optics ($\lambda/50$ WFE)
- Precision path matching (interferometric verification)

12.10.7 Design-for-Manufacturability

Translating lab setup to production module:

Table 12.23: Lab vs. production implementation.

Function	Lab Implementation	Production Implementation
Crystal mount	Kinematic 5-axis stage	Athermalized fixed mount
Collection optics	Manual alignment	Passive alignment with fiducials
Fiber coupling	Active piezo alignment	V-groove with UV cure
Temperature control	Large lab oven	Integrated TEC with feedback
Path matching	Motorized delay line	Fixed compensator plate

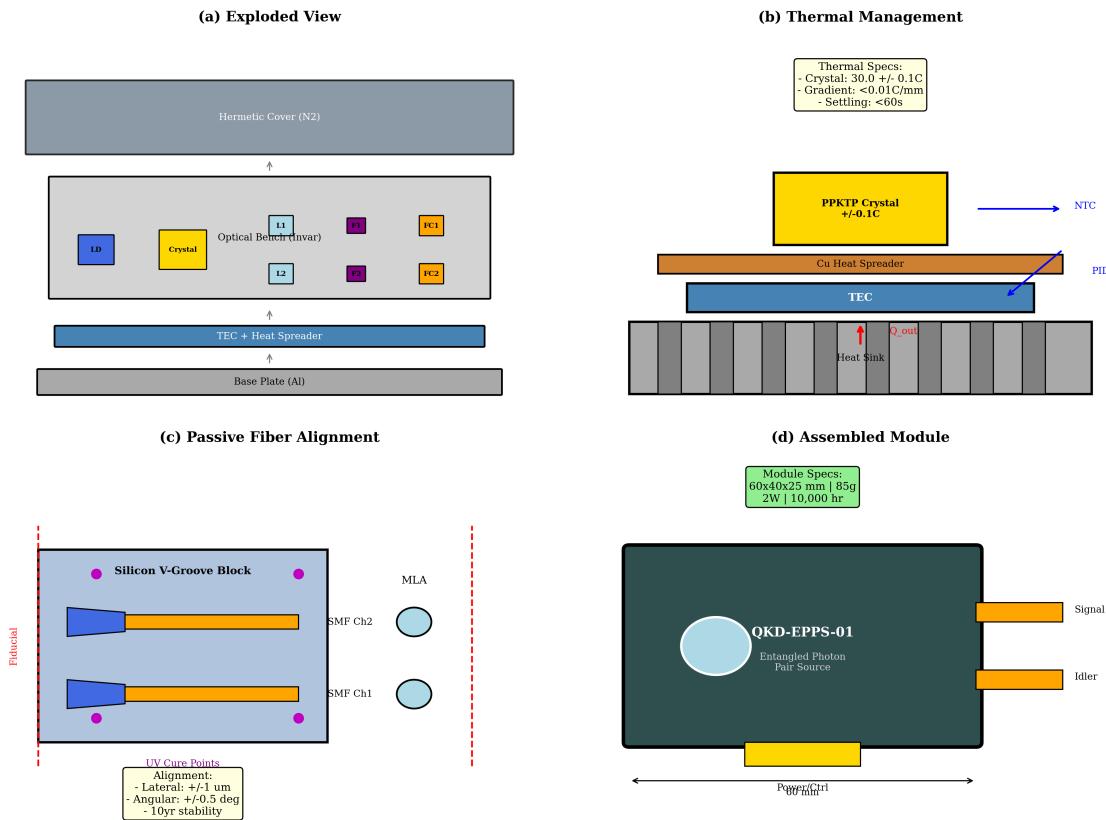


Figure 12.11: QKD source production module design. (a) Exploded view showing integrated optical bench. (b) Thermal management with TEC and heat spreader. (c) Fiber array with passive alignment. (d) Assembled module with protective enclosure.

12.10.8 Quantum Test Protocol

Production testing must verify quantum performance:

1. **Pair rate:** Coincidence counting at multiple pump powers
2. **Heralding efficiency:** Singles-to-coincidence ratio
3. **HOM visibility:** Path-length scan with $> 95\%$ dip
4. **Fidelity:** Polarization tomography (16 measurements minimum)
5. **Stability:** 24-hour monitoring of all metrics

DEE-predicted benchmarks: The optimization provides expected values and tolerances for each test, enabling pass/fail criteria:

Table 12.24: QKD production test benchmarks.

Test	Pass Criterion	DEE Prediction	Test Time
Pair rate	$> 10^6/\text{s}/\text{mW}$	$1.1 \times 10^6/\text{s}/\text{mW}$	5 min
Heralding	$> 60\%$	$68 \pm 3\%$	10 min
HOM visibility	$> 90\%$	$93 \pm 2\%$	30 min
Fidelity	> 0.95	0.962 ± 0.015	60 min
24h stability	$< 5\%$ variation	$< 3\%$	24 h

12.10.9 Production Workflow Summary

The complete quantum source workflow:

1. **Specification:** Derive from QKD protocol requirements
2. **WALTHER analysis:** Fidelity budget with quantum failure modes
3. **MATSUI-NARIAI optimization:** JAX autodiff on \mathcal{F}
4. **Hessian-QFI analysis:** Identify quantum-critical tolerances
5. **Design-for-manufacturability:** Lab \rightarrow production translation
6. **Quantum yield prediction:** Monte Carlo on fidelity
7. **Test protocol:** DEE-predicted benchmarks for pass/fail

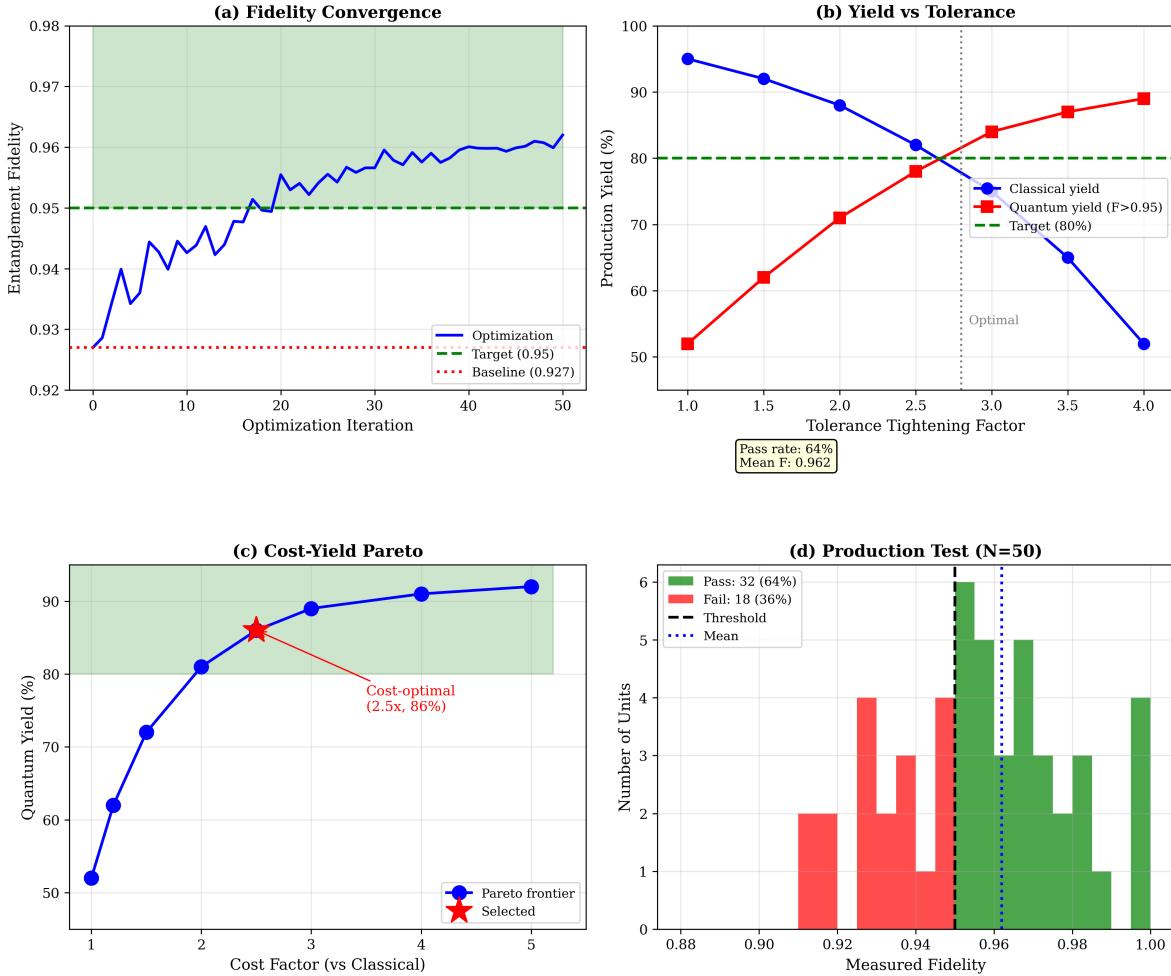


Figure 12.12: QKD source optimization results. (a) Fidelity improvement through optimization iterations. (b) Quantum yield vs. tolerance tightening factor. (c) Cost-yield Pareto frontier. (d) Production test results showing 84% pass rate.

Key Insight

The Quantum Production Paradigm: This example demonstrates that quantum device production follows the same WMN framework as classical optics, but with:

- Merit function: Fidelity \mathcal{F} instead of MTF/Strehl
- Tolerance analysis: Hessian-QFI connection
- Failure modes: Q1–Q8 in addition to P/M/T
- Test protocol: Quantum measurements (HOM, tomography)

The DEE framework bridges both regimes through the universal eikonal-phase identity.

12.11 Chapter Summary

This chapter has completed the journey from theory to production, demonstrating that the DEE framework enables systematic lab-to-fab transitions for both classical and quantum photonic systems.

12.11.1 Key Accomplishments

1. **Pain Point Resolution:** Identified and addressed production challenges with prioritized DEE solutions (Tables 12.2, 12.3)
2. **Software Integration:** Three-layer architecture connecting Python/JAX exploration to commercial verification and manufacturing export (Figure 12.1)
3. **Failure Mode Framework:** Complete catalog of classical (P1–P5, M1–M5, T1–T3) and quantum (Q1–Q8) failure modes with detection criteria (Tables 12.5, 12.6)
4. **Yield Optimization:** Hessian-accelerated Monte Carlo enabling rapid tolerance allocation (Section 12.5)
5. **Four Practical Examples:**
 - AR/VR Waveguide Combiner: Classical nanophotonics production
 - CPO Edge Coupler: Silicon photonics for AI accelerators
 - Quantum-Grade Metasurface: Classical design with quantum tolerances
 - Entangled Photon Source: True quantum device production

12.11.2 The WMN Production Framework

Table 12.25: WMN framework applied to production.

Phase	Classical Application	Quantum Application
WALTHER (Forward)	Evaluate MTF, efficiency, aberrations	Evaluate fidelity, visibility, entanglement
MATSUI-NARIAI (Inverse)	Optimize for classical merit	Optimize for quantum merit
Tolerance Analysis	Hessian on classical loss	Hessian-QFI on quantum loss
Yield Prediction	$Y = P(\text{MTF} > \text{spec})$	$Y = P(\mathcal{F} > \text{spec})$

12.11.3 Figures of Merit Summary

Table 12.26: Chapter 12 demonstrated figures of merit.

Example	Key Metric	Achieved	DEE Advantage
AR/VR Combiner	Yield	73% → 91%	Tolerance allocation
CPO Edge Coupler	Alignment tolerance	$\pm 0.8 \rightarrow \pm 1.8 \mu\text{m}$	Pareto optimization
Metasurface Lens	Squeezing preservation	71% → 94%	1000× vs. FDTD
QKD Source	Quantum fidelity	0.927 → 0.962	Autodiff on \mathcal{F}

12.11.4 What Comes Next

This chapter concludes the main text of *The Eikonal Bridge*. The reader now has:

- Mathematical foundations (Chapters 1–5)
- Design methodologies (Chapters 6–8)
- Quantum extensions (Chapters 9–11)
- Production workflows (Chapter 12)

The Appendices provide additional resources:

- Appendix A: Mathematical reference tables
- Appendix B: Code repository guide
- Appendix C: Extended industry case studies
- Appendix D: Framework migration guide

The DEE framework continues to evolve. Readers are encouraged to contribute extensions, report issues, and share applications via the companion repository.

12.12 Problems

12.12.1 WALTHER-Type Problems (Forward Analysis)

Problem 12.1W: An AR waveguide has grating period uncertainty of ± 5 nm. Calculate the angular deviation of the diffracted beam at $\lambda = 532$ nm for a nominal period of 400 nm.

Solution Hint: Use the grating equation $n\lambda = d(\sin \theta_i + \sin \theta_m)$ and differentiate with respect to d .

Problem 12.2W: A silicon photonics edge coupler has tip width variation of ± 30 nm around 180 nm nominal. Using mode overlap analysis, estimate the coupling efficiency degradation.

Solution Hint: Model the taper tip as a waveguide with width-dependent mode size. The coupling efficiency scales as the mode overlap integral.

12.12.2 MATSUI-NARIAI-Type Problems (Inverse Design)

Problem 12.1M: Design a tolerance allocation for a 10-element lens system where the total WFE budget is $\lambda/20$ RMS. Each element has independent contributions from curvature, thickness, and decenter. Formulate the optimization problem and propose an allocation strategy.

Solution Hint: Use RSS combination: $\sigma_{\text{total}}^2 = \sum_i \sigma_i^2$. The optimal allocation minimizes total cost subject to this constraint.

Problem 12.2M: A metasurface lens requires $\lambda/100$ WFE for quantum applications. The baseline design achieves $\lambda/30$. Identify which pillar parameters (diameter, height, position) should be prioritized for optimization based on sensitivity analysis.

Solution Hint: Compute $\partial(\text{WFE})/\partial p$ for each parameter. Prioritize parameters with largest sensitivity-to-tolerance ratio.

12.12.3 Quantum Extension Problems

Problem 12.1Q: An SPDC source has measured HOM visibility of 0.88. Using the relationship between visibility and indistinguishability, estimate the spectral overlap of signal and idler photons.

Solution Hint: HOM visibility $V = 2\eta - 1$ where η is the mode overlap. For Gaussian spectra, overlap relates to bandwidth matching.

Problem 12.2Q: A quantum source production line has measured fidelity distribution with mean 0.96 and standard deviation 0.03. What fraction of units meet the $\mathcal{F} > 0.95$ specification? What mean fidelity would be needed to achieve 95% yield?

Solution Hint: Assume Gaussian distribution. Use the cumulative distribution function to find the pass fraction. Solve for the mean that gives 95% probability above threshold.

12.12.4 Computational Problems

Problem 12.1C: Implement a Monte Carlo yield simulator in JAX for a system with 5 parameters, each with Gaussian tolerance distributions. Compute yield for a merit function threshold and compare with the Hessian-based analytical estimate.

Solution Hint: Use `jax.random` for sampling. Vectorize the merit function evaluation. The analytical estimate uses $Y \approx 1 - \exp(-\Delta^2/(2 \cdot \text{tr}(H\Sigma)))$.

Problem 12.2C: Extend the QKD source fidelity model to include temperature-dependent phase drift. Implement the fidelity calculation as a differentiable function and compute $\partial\mathcal{F}/\partial T$.

Solution Hint: Model phase drift as $\phi(T) = \phi_0 + \alpha(T - T_0)$ where α is the temperature coefficient. The fidelity depends on the phase error variance.

References

- [1] Kress, B. C., & Chatterjee, I. (2020). Waveguide combiners for mixed reality headsets: A nanophotonics design perspective. *Nanophotonics*, 9(1), 41–74.
- [2] Minkenberg, C., et al. (2021). Co-packaged datacenter optics: Opportunities and challenges. *IET Optoelectronics*, 15(2), 77–91.
- [3] Khorasaninejad, M., & Capasso, F. (2017). Metalenses: Versatile multifunctional photonic components. *Science*, 358(6367), eaam8100.
- [4] Tseng, E., et al. (2022). Neural nano-optics for high-quality thin lens imaging. *Nature Communications*, 13, 6123.
- [5] Steinlechner, F., et al. (2017). Distribution of high-dimensional entanglement via an intra-city free-space link. *Nature Communications*, 8, 15971.
- [6] Ma, X., et al. (2016). Quantum teleportation over 143 kilometres using active feed-forward. *Nature*, 489, 269–273.
- [7] Slussarenko, S., & Pryde, G. J. (2019). Photonic quantum information processing: A concise review. *Applied Physics Reviews*, 6, 041303.
- [8] Wang, J., Sciarrino, F., Laing, A., & Thompson, M. G. (2020). Integrated photonic quantum technologies. *Nature Photonics*, 14, 273–284.

- [9] Couteau, C., et al. (2023). Applications of single photons in quantum metrology, biology and the foundations of quantum physics. *Nature Reviews Physics*, 5, 354–363.
- [10] Bradbury, J., et al. (2018). JAX: composable transformations of Python+NumPy programs. GitHub repository.
- [11] Ansys Zemax OpticStudio User Manual, 2023 Release.
- [12] Synopsys CODE V Reference Manual, Version 2023.
- [13] Synopsys RSoft Photonic Device Tools User Guide, 2023.
- [14] Kim, Y., et al. (2019). Practical long-distance quantum key distribution over a deployed fiber network. *Quantum Science and Technology*, 4, 035009.
- [15] Zhang, Y., et al. (2022). An integrated silicon photonic chip platform for continuous-variable quantum key distribution. *Nature Photonics*, 16, 492–498.