Oral Thesis Prospectus

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Abstract

The power method, or source iteration, has been the staple of Monte Carlo particle transport eigenvalue calculations for more than fifty years. This method is easy to implement and provides the dominant eigenvalue and eigenfunction, however, this method is slow. The rate of convergence is proportional to the dominance ratio, the ratio of the second largest eigenvalue to the largest eigenvalue.

For this thesis, I propose using an alternative to the power method, Arnoldi's Method of Minimized Iterations. Arnoldi's method is an improvement over the power method; it uses information from all previous iterations—instead of only the last iteration—to calculate the eigenfunction.

Preliminary results indicate that Arnoldi's method is less noisy than the Power method and the eigenvalue and the eigenfunction are converged more quickly. However, the RMS error of the eigenfunction—compared to a deterministic SN calculation—calculated with Arnoldi's method is larger than the RMS error from a power method calculation for the same number of Monte Carlo histories. Other results show that Arnoldi's method can calculate higher eigenmodes which the power method is unable to do without additional calculations.

Several questions must be explored before the use Arnoldi's method in Monte Carlo particle transport can be accepted. Most importantly, how can the uncertainty be calculated? Without a measure of the statistical uncertainty, the answer is meaningless. A simple solution is to restart the method after a few iterations and find the mean and variance of the eigenvalues after each restart. I believe there is a better and more efficient way; this is one aspect of my thesis research.

Arnoldi's method requires the calculation of an inner product between two functions. In Monte Carlo particle transport, these functions are fission sources. In initial studies, these fission sources were spatially binned; the height of these bins are the elements of a vector. Discretizing a fission source in this manner introduces bias to the calculation. The inner product of two vectors is a simple dot product. Is there a way to calculate the inner product of two fission sources without resorting to binning the sources? Can we use basis functions other than histogram step functions to expand the fission source? These are additional questions I plan on addressing in my thesis.

All calculations so far have been performed in 1-D homogeneous slab geometries. What will happen in heterogeneous and multi-dimensional geometries? Is Arnoldi's method still valid and does it still offer the same benefits? Although unknown for certain at this time, I propose the answer is yes. The reduced noise when using Arnoldi's method may lead to dramatic improvements in calculating eigenfunctions of geometries where highly absorbing materials are separated by highly scattering materials (e.g. the eigenvalue of the world type problems). No mathematical obstacles—other than lack of experience of the author—immediately present themselves as problems to using Arnoldi's method in higher dimensions.