7.1 Integração por partes

$$\frac{d}{dx}[f(x)g(x)] = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

$$f(x)\cdot g(x) = \int [f(x)\cdot g'(x) + g(x)\cdot f'(x)]\,dx$$

$$f(x)\cdot g(x) = \int f(x)g'(x)\,dx + \int g(x)\cdot f'(x)\,dx$$

$$\int f(x)g'(x)\,dx = f(x)\cdot g(x) - \int g(x)f'(x)\,dx$$

$$\int u\,dv = uv - \int v\,du$$

$u = f(x)$ $\quad du = f'(x)\,dx$

$v = g(x)$ $\quad dv = g'(x)\,dx$

$$\int \ln x\,dx = x\ln x - \int x\frac{dx}{x} = x\ln x - \int dx = \boxed{x\ln x - x + C}$$

$u = \ln x$ $\quad du = \frac{dx}{x}$

$dv = dx$ $\quad v = x$

$$\int_a^b f(x)g'(x)\,dx = \underbrace{\big[f(x)g(x)\big]_a^b}_{f(b)g(b) - f(a)g(a)} - \int_a^b g(x)f'(x)\,dx$$

Como escolher u?

- → L : logarítmica
- → I : inversa trigonométrica
- → A : algébrica
- → T : trigonométrica
- → E : exponencial

$$\int e^x \operatorname{sen} x \, dx = e^x \operatorname{sen} x - \int e^x \cos x \, dx$$

$u = \operatorname{sen} x \qquad du = \cos x \, dx$

$dv = e^x dx \qquad v = e^x$

$$\int e^x \cos x \, dx = e^x \cos x + \int e^x \operatorname{sen} x \, dx$$

$u = \cos x \qquad du = -\operatorname{sen} x \, dx$

$dv = e^x dx \qquad v = e^x$

$$\int e^x \operatorname{sen} x \, dx = e^x \operatorname{sen} x - \left[e^x \cos x + \int e^x \operatorname{sen} x \, dx \right]$$

$$2 \int e^x \operatorname{sen} x \, dx = e^x \operatorname{sen} x - e^x \cos x$$

$$\int e^x \,\mathrm{sen}\, x \, dx = \frac{1}{2} e^x \left[\mathrm{sen}\, x - \cos x \right]$$

$$\int t^2 e^t \, dt = t^2 e^t - 2 \underbrace{\int t e^t \, dt}$$

$u = t^2$ $\qquad$ $du = 2t\, dt$

$dv = e^t dt$ $\qquad$ $v = e^t$

$$\int t e^t \, dt = t e^t - \int e^t \, dt = t e^t - e^t + C_1$$

$u = t$ $\qquad$ $du = dt$

$dv = e^t dt$ $\qquad$ $v = e^t$

$$\int t^2 e^t \, dt = t^2 e^t - 2 \left[t e^t - e^t + C_1 \right] =$$

$$= t^2 e^t - 2 t e^t + 2 e^t + C.$$ $\qquad$ $-2C_1 = C$

$\boxed{7.2}$ Integrais Trigonométricas

$\mathrm{sen}^2 x + \cos^2 x = 1$

$\mathrm{tg}\, x = \dfrac{\mathrm{sen}\, x}{\cos x}$, $\mathrm{cotg}\, x = \dfrac{\cos x}{\mathrm{sen}\, x}$, $\sec x = \dfrac{1}{\cos x}$, $\mathrm{cosec}\, x = \dfrac{1}{\mathrm{sen}\, x}$

$$\frac{\text{sen}^2 x}{\cos^2 x} + 1 = \frac{1}{\cos^2 x} \Rightarrow \text{tg}^2 x + 1 = \sec^2 x$$

$$1 + \frac{\cos^2 x}{\text{sen}^2 x} = \frac{1}{\text{sen}^2 x} \Rightarrow 1 + \text{cotg}^2 x = \text{cossec}^2 x$$

$$\text{sen}(a+b) = \text{sen}\, a \cos b + \text{sen}\, b \cos a$$

$$\text{sen}(a-b) = \text{sen}\, a \cos b - \text{sen}\, b \cos a$$

$$\text{sen}\, a \cos b = \frac{\text{sen}(a+b) + \text{sen}(a-b)}{2}$$

$$\cos(a+b) = \cos a \cos b - \text{sen}\, a\, \text{sen}\, b$$

$$\cos(a-b) = \cos a \cos b + \text{sen}\, a\, \text{sen}\, b$$

$$\text{sen}\, 2x = 2\, \text{sen}\, x \cos x$$

$$\cos 2x = \cos^2 x - \text{sen}^2 x = 1 - 2\,\text{sen}^2 x = 2\cos^2 x - 1$$ (arco duplo)

$$\text{sen}\, a \cos b = \frac{\text{sen}(a+b) + \text{sen}(a-b)}{2}$$

$$\text{sen}\, a\, \text{sen}\, b = \frac{\cos(a-b) - \cos(a+b)}{2}$$

$$\begin{cases} \text{sen}^2 x = \dfrac{1 - \cos 2x}{2} \\ \cos^2 x = \dfrac{1 + \cos 2x}{2} \end{cases}$$

$$\cos a \cos b = \frac{\cos(a-b) + \cos(a+b)}{2}$$

$$\cos a\, \text{sen}\, b = \frac{\text{sen}(a+b) - \text{sen}(a-b)}{2}$$

$$\int \operatorname{sen} 4x \cos 5x \, dx = \int \frac{\operatorname{sen} 9x + \operatorname{sen}(-x)}{2} \, dx =$$

$$= \frac{1}{2} \int (-\operatorname{sen} x) \, dx + \frac{1}{2} \int \operatorname{sen} 9x \, dx = \frac{1}{2} \cos x +$$

$$+ \frac{1}{2} \int \operatorname{sen} u \cdot \frac{du}{9} = \boxed{\frac{1}{2} \cos x + \frac{1}{18} (-\cos 9x) + C}$$

$$9x = u \Rightarrow du = 9 \, dx \Rightarrow dx = \frac{du}{9}$$

$$\int tg^3 x \, dx = \int tg\, x \cdot (\sec^2 x - 1) \, dx = \int tg\, x \cdot \sec^2 x \, dx -$$

$$- \int tg\, x \, dx = \int u \cdot du - \ln|\sec x| + C =$$

$$u = tg\, x \Rightarrow du = \sec^2 x \, dx$$

$$= \frac{u^2}{2} - \ln|\sec x| + C = \frac{tg^2 x}{2} - \ln|\sec x| + C.$$

Olhar no livro: $\int \operatorname{sen}^m x \cos^n x \, dx$, $\int tg^m x \sec^n x \, dx$

$$\int \operatorname{sen}^2 x \, dx = \int \frac{1 - \cos 2x}{2} \, dx = \int \frac{dx}{2} - \frac{1}{2} \int \cos 2x \, dx =$$

$$= \frac{x}{2} - \frac{1}{2}\int \cos u \cdot \frac{du}{2} + C = \frac{x}{2} - \frac{1}{4}\,\text{sen}\,2x + C$$

$$u = 2x \Rightarrow du = 2dx \Rightarrow dx = \frac{du}{2}$$

$\boxed{7.3}$ Substituições Trigonométricas

$$9 - x^2 \geq 0 \Rightarrow x^2 \leq 9 \Rightarrow -3 \leq x \leq 3 \Rightarrow$$

$$\Rightarrow x = 3\,\text{sen}\,\theta, \quad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Rightarrow$$

$$\Rightarrow \sqrt{9 - x^2} = \sqrt{9 - 9\,\text{sen}^2\theta} = 3\sqrt{1 - \text{sen}^2\theta} = 3|\cos\theta| = 3\cos\theta$$

$$\int \frac{\sqrt{9 - x^2}}{x^2}\,dx = \int \frac{3\cos\theta}{9\,\text{sen}^2\theta} \cdot 3\cos\theta\,d\theta = \int \frac{\cos^2\theta}{\text{sen}^2\theta}\,d\theta =$$

$$x = 3\,\text{sen}\,\theta \Rightarrow dx = 3\cos\theta\,d\theta$$

$$= \int \text{cotg}^2\theta\,d\theta = -\text{cotg}\,\theta - \theta + C = -\frac{\sqrt{9 - x^2}}{x} - \text{arcsen}\left(\frac{x}{3}\right) + C$$

$$\text{cotg}\,\theta = \frac{\cos\theta}{\text{sen}\,\theta} = \frac{\frac{\sqrt{9 - x^2}}{3}}{\frac{x}{3}} = \frac{\sqrt{9 - x^2}}{x}$$

$$\text{sen}\,\theta = \frac{x}{3} \Rightarrow \theta = \text{arcsen}\left(\frac{x}{3}\right)$$