3.1

72

$$g(x) = \begin{cases} 2x & , \text{ se } x \leq 0 \\ 2x - x^2 & , \text{ se } 0 < x < 2 \\ 2 - x & , \text{ se } x \geq 2 \end{cases}$$

$y = 2x$, $y = 2x - x^2$ e $y = 2 - x$ são deriváveis em todos os seus domínios.

$$\left.\begin{array}{l} g'_-(0) = 2 \cdot x^0 = 2 \\ g'_+(0) = (2 \cdot x^0 - 2 \cdot x^1)_{x=0} = (2-2x)_{x=0} = 2 \end{array}\right\} \text{derivável em } x = 0.$$

$$\left.\begin{array}{l} g'_-(2) = (2-2x)_{x=2} = -2 \\ g'_+(2) = -1 \end{array}\right\} \text{não derivável em } x = 2$$

$\therefore$ g é derivável em $\mathbb{R} / \{2\}$.

3.4

21 $y = \sqrt{\dfrac{x}{x+1}} \Rightarrow u = \dfrac{x}{x+1}$

$$\left.\begin{array}{l} u(x) = \dfrac{x}{x+1} \\ y = \sqrt{u} \end{array}\right\} \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{u}} \cdot \frac{(x+1) \cdot 1 - x \cdot 1}{(x+1)^2} =$$

$$= \frac{1}{2\sqrt{u}\cdot(x+1)^2} = \frac{1}{2(x+1)^2\cdot\sqrt{\frac{x}{x+1}}}.$$

$$y'(x) = \frac{1}{2\sqrt{\frac{x}{1+x}}}\cdot\frac{d}{dx}\left(\frac{x}{x+1}\right).$$

28 $f(z) = e^{\frac{z}{z-1}}$

$$\left.\begin{array}{l} u(z) = \frac{z}{z-1} \\ y(u) = e^{u} \end{array}\right\} \quad f'(z) = y'(u(z))\cdot u'(z) =$$

$$= e^{u}\cdot\frac{(z-1)\cdot 1 - z\cdot 1}{(z-1)^2} =$$

$$= e^{u}\cdot\frac{-1}{(z-1)^2} = -\frac{e^{\frac{z}{z-1}}}{(z-1)^2}.$$

35 $y = \operatorname{sen}(\operatorname{tg} 2x) \qquad \cos(\operatorname{tg} 2x)\cdot\sec^2(2x)\cdot 2$

$v = 2x \qquad \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dx} =$

$u = \operatorname{tg} v$

$$= \cos u\cdot\sec^2 v\cdot 2 =$$

$y = \operatorname{sen} u$

$$= \cos(\operatorname{tg} 2x)\cdot\sec^2(2x)\cdot 2$$

(45) $y = \cos\sqrt{\operatorname{sen}(\operatorname{tg}\pi x)}$

$y = \cos u$

$u = \sqrt{v}$

$v = \operatorname{sen} w$

$w = \operatorname{tg} z$

$z = \pi x$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dw}\cdot\frac{dw}{dz}\cdot\frac{dz}{dx}$$

$$\frac{dy}{dx} = -\operatorname{sen} u\cdot\frac{1}{2\sqrt{v}}\cdot\cos w\cdot\sec^2 z\cdot\pi =$$

$$= -\pi\sec^2(\pi x)\cdot\cos(\operatorname{tg}\pi x)\cdot\frac{1}{2\sqrt{\operatorname{sen}(\operatorname{tg}(\pi x))}}\cdot$$

$$\cdot\operatorname{sen}\sqrt{\operatorname{sen}(\operatorname{tg}\pi x)}\,.$$

$y = \cos\sqrt{\operatorname{sen}(\operatorname{tg}\pi x)}$

$$\frac{dy}{dx} = -\operatorname{sen}\sqrt{\operatorname{sen}(\operatorname{tg}\pi x)}\cdot\frac{1}{2\sqrt{\operatorname{sen}(\operatorname{tg}\pi x)}}\cdot\cos(\operatorname{tg}\pi x)\cdot$$

$$\cdot\sec^2(\pi x)\cdot\pi$$

48) $$y = \frac{1}{(1+tg\,x)^2} = (1+tg\,x)^{-2} \Rightarrow$$

$$\Rightarrow y' = -2 \cdot (1+tg\,x)^{-3} \cdot (0 + sec^2 x) =$$

$$= -2(sec^2 x)(1+tg\,x)^{-3}.$$

$$y'' = -2 \Big[(2 \cdot sec^1 x \cdot sec\,x \cdot tg\,x) \cdot (1+tg\,x)^{-3} +$$

$$+ (sec^2 x) \cdot (-3) \cdot (1+tg\,x)^{-4} \cdot (0 + sec^2 x) \Big] =$$

$$= -2 \left[2 sec^2 x\, tg\,x\, (1+tg\,x)^{-3} - 3 sec^4 x\, (1+tg\,x)^{-4} \right].$$

50) $$y = e^{(e^x)} \Rightarrow y' = e^{e^x} \cdot e^x \Rightarrow y'' =$$

$$= \left(e^{e^x} \cdot e^x \right) \cdot e^x + e^{e^x} \cdot e^x = e^{e^x} \left(e^{2x} + e^x \right).$$