A1:

1) a) $x_n = \dfrac{1}{(-2)^n} + n$

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} \frac{1}{(-2)^n} + \lim_{n\to\infty} n = 0 + \lim_{n\to\infty} n = \infty$$

$\Rightarrow x_n$ diverge para $+\infty$.

b) $x_n = \dfrac{2n^3+1}{n^3+8n^5}$

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} \frac{\overset{\to 0}{\cancel{\frac{2}{n^2}}} + \overset{\to 0}{\cancel{\frac{1}{n^5}}}}{\underset{\to 0}{\cancel{\frac{1}{n^2}}} + 8} = \lim_{n\to\infty} \frac{0}{8} = 0.$$

c) $x_n = 2 + 0{,}3\,x_{n-1}$, $x_0 = 2$

$$\underbrace{\frac{x_n}{0{,}3^n}}_{f(n)} = \frac{2}{0{,}3^n} + \underbrace{\frac{x_{n-1}}{0{,}3^{n-1}}}_{f(n-1)}$$

$$f(n) = \frac{2}{0{,}3^n} + \cancel{f(n-1)}$$

$$\cancel{f(n-1)} = \frac{2}{0{,}3^{n-1}} + \cancel{f(n-2)}$$

$x_n = L$

$x_{n-1} = L$

$L = 2 + 0{,}3L$

$0{,}7L = 2$

$L = \dfrac{2}{\frac{7}{10}} = \dfrac{20}{7}$

$$\cancel{f(n-2)} = \frac{a}{0{,}3^{n-2}} + \cancel{f(n-3)}$$

$$\vdots$$

$$\cancel{f(1)} = \frac{a}{0{,}3^{1}} + f(0)$$

$$+$$

$$f(n) = \frac{a}{0{,}3^{n}} + \frac{a}{0{,}3^{n-1}} + \dots + \frac{a}{0{,}3^{1}} + f(0) =$$

$$= a\left[\left(\frac{10}{3}\right)^{n} + \left(\frac{10}{3}\right)^{n-1} + \dots + \left(\frac{10}{3}\right)\right] + f(0) =$$

$$= a \cdot \left[\frac{\left(\frac{10}{3}\right)^{n+1} - \frac{10}{3}}{\frac{10}{3} - 1}\right] + 2 =$$

$$= a \cdot \frac{3}{7} \cdot \left[\left(\frac{10}{3}\right)^{n+1} - \frac{10}{3}\right] + 2$$

$$x_n = \frac{\frac{3a}{7}\left[\left(\frac{10}{3}\right)^{n+1} - \frac{10}{3}\right] + 2}{\left(\frac{10}{3}\right)^{n}}$$

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty}\left(\frac{\cancel{3s}}{7}\cdot\frac{10}{\cancel{3}} - \cancel{\frac{\frac{3s}{7}}{\left(\frac{10}{3}\right)^{n-1}}}^{\to 0} + \cancel{\frac{2}{\left(\frac{10}{3}\right)^{n}}}^{0}\right) =$$

$$= \frac{10s}{7}.$$

3) $g(x) = \begin{cases} ax+b, & \text{ze } x<2 \\ sx^2-1, & \text{ze } x\geq 2 \end{cases}$

$$\left.\begin{aligned} \lim_{x\to 2^-} g(x) &= \lim_{x\to 2^-}(ax+b) = 2a+b \\ \lim_{x\to 2^+} g(x) &= \lim_{x\to 2^+}(sx^2-1) = 4s-1 = g(2) \end{aligned}\right\} \boxed{2a+b = 4s-1}$$

$$g'_-(2) = \lim_{h\to 0^-}\frac{g(2+h)-g(2)}{h} = \lim_{h\to 0^-}\frac{a(2+h)+b-4s+1}{h} =$$

$$= \lim_{h\to 0^-}\left(a + \frac{2a+b-4s+1}{h}\right) = \lim_{h\to 0^-} a + \frac{0}{h} = a$$

$$g'_+(2) = \lim_{h\to 0^+}\frac{g(2+h)-g(2)}{h} = \lim_{h\to 0^+}\frac{s\cdot(2+h)^2-1-4s+1}{h} =$$

$$= \lim_{h\to 0^+}\frac{s(h^2+4h+4)-4s}{h} = \lim_{h\to 0^+}(sh+4s) = 4s$$

$$\begin{cases} a = 4n \\ 2a + b = 4n - 1 \end{cases} \Rightarrow a = -b - 1 \Rightarrow b = -a - 1 = -4n - 1$$

$\therefore (a, b) = (4n, -4n - 1)$.

4 a) $f(x) = \dfrac{n}{x} - \dfrac{1}{x^2}$

$D(f) = \mathbb{R} \setminus \{0\} = \mathbb{R}^*$

$y = 0 \Rightarrow f(x) = \dfrac{n}{x} - \dfrac{1}{x^2} = 0 \Rightarrow \dfrac{n}{x} = \dfrac{1}{x^2} \Rightarrow x = \dfrac{1}{n}$

Intercepta o eixo x em $\left(\dfrac{1}{n}, 0\right)$.

c) $f'(x) = \dfrac{d}{dx}\left(\dfrac{n}{x}\right) - \dfrac{d}{dx}\left(\dfrac{1}{x^2}\right) = (-1) \cdot n x^{-2} - (-2) \cdot x^{-3} =$

$= -\dfrac{n}{x^2} + \dfrac{2}{x^3}$; $f'(x) = 0 \Leftrightarrow \dfrac{2}{x^3} = \dfrac{n}{x^2} \Leftrightarrow x = \dfrac{2}{n}$.

d) $f'(x) = -\dfrac{n}{x^2} + \dfrac{2}{x^3} > 0 \Leftrightarrow \dfrac{n}{x^2} < \dfrac{2}{x^3} \Leftrightarrow$

$\Leftrightarrow x < \dfrac{2}{n}$ e $x \neq 0$.

$f'(x) < 0 \Leftrightarrow \dfrac{n}{x^2} > \dfrac{2}{x^3} \Leftrightarrow x > \dfrac{2}{n}$.

6) $f(x) = 5x^2 - 7x + n \quad , \quad P = \left(\frac{2}{5}, n-2\right)$.

$f'(x) = 5 \cdot 2x^1 - 7 \cdot 1 \cdot x^0 = 10x - 7$

$f'\left(\frac{2}{5}\right) = 10 \cdot \frac{2}{5} - 7 = 4 - 7 = -3$

$t : y = -3x + b_1 \; ; \quad P \in t \Rightarrow n - 2 = -3 \cdot \frac{2}{5} + b_1$

$\Rightarrow b_1 = n - 2 + \frac{6}{5} = n - \frac{4}{5}$

$\therefore \; t : y = -3x + \left(n - \frac{4}{5}\right) ;$

$(a, b) \in t \Rightarrow b = -3a + n - \frac{4}{5} \Rightarrow$

$\Rightarrow 3a + b = n - \frac{4}{5} \Rightarrow \boxed{B}$.