Teste 1

1 a) $x_n = \left(\frac{-1}{a}\right)^n \Rightarrow |x_n| = \frac{1}{a^n}$; $a \geq 1,\ a \in \mathbb{N}$

$a^n \geq n \Rightarrow a^n \to \infty \Rightarrow |x_n| \to 0 \Rightarrow \lim\limits_{n \to \infty} x_n = 0$.

x_n converge.

b) $x_n = a + \cos(2\pi n) = a + \cos 0 = a + 1 \Rightarrow$

$\Rightarrow \lim\limits_{n \to \infty} x_n = a + 1$. x_n converge.

c) $x_n = \frac{1}{a} x_{n-1}$, $x_0 = 100$

$x_0 = 100$, $x_1 = \frac{100}{a}$, $x_2 = \frac{100}{a^2} \Rightarrow x_n = \frac{100}{a^n}$

$$\left.\begin{array}{l} x_n = \frac{x_{n-1}}{a} \\ x_{n-1} = \frac{x_{n-2}}{a} \\ \vdots \\ x_2 = \frac{x_1}{a} \\ x_1 = \frac{x_0}{a} \end{array}\right\} \quad x_n \cdot \cancel{x_{n-1}} \cdots \cancel{x_2} \cdot \cancel{x_1} = \frac{\cancel{x_{n-1}}\,\cancel{x_{n-2}} \cdots \cancel{x_1}\, x_0}{a^n}$$

$$x_n = \frac{x_0}{a^n} = \frac{100}{a^n}$$

$x_0 = 100 = \frac{100}{a^0} = 100$ (V)

$$x_{n-1} = \frac{100}{2^{n-1}} \Rightarrow x_n = \frac{x_{n-1}}{2} = \frac{1}{2} \cdot \frac{100}{2^{n-1}} = \frac{100}{2^n} \quad (✓)$$

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty} \frac{100}{2^n} = 0 . \quad x_n \text{ converge}$$

d) $x_n = 2 + 1{,}1 \cdot x_{n-1}$; $x_0 = 0$

$$x_n = 2 + \frac{11}{10} \cdot x_{n-1} \Rightarrow \left(\frac{10}{11}\right)^n x_n = 2\left(\frac{10}{11}\right)^n + \left(\frac{10}{11}\right)^n \cdot \left(\frac{11}{10}\right) \cdot x_{n-1}$$

$$\Rightarrow \underbrace{\left(\frac{10}{11}\right)^n x_n}_{y_n} = 2 \cdot \left(\frac{10}{11}\right)^n + \underbrace{\left(\frac{10}{11}\right)^{n-1} x_{n-1}}_{y_{n-1}} \Rightarrow y_n = 2\left(\frac{10}{11}\right)^n + y_{n-1}$$

$$\left.\begin{aligned}
y_n &= 2\left(\frac{10}{11}\right)^n + \cancel{y_{n-1}} \\
\cancel{y_{n-1}} &= 2\left(\frac{10}{11}\right)^{n-1} + \cancel{y_{n-2}} \\
&\;\vdots \\
\cancel{y_2} &= 2\left(\frac{10}{11}\right)^2 + \cancel{y_1} \\
\cancel{y_1} &= 2\left(\frac{10}{11}\right)^1 + y_0
\end{aligned}\right\} y_n = 2\left[\left(\frac{10}{11}\right)^n + \left(\frac{10}{11}\right)^{n-1} + \dots + \left(\frac{10}{11}\right)^2 + \frac{10}{11}\right]$$

$$y_n = \left(\frac{10}{11}\right)^n x_n = 2 \cdot \sum_{k=1}^{n} \left(\frac{10}{11}\right)^k \Rightarrow x_n = \frac{2\sum_{k=1}^{n} \left(\frac{10}{11}\right)^k}{\left(\frac{10}{11}\right)^n}$$

$$\Rightarrow x_n = 2 + 2\left(\frac{11}{10}\right) + 2\left(\frac{11}{10}\right)^2 + \dots + 2\left(\frac{11}{10}\right)^{n-1} \quad (a = 2)$$

$$x_n = a\left[1 + \frac{11}{10} + \left(\frac{11}{10}\right)^2 + \dots + \left(\frac{11}{10}\right)^{n-1}\right]$$

$$\lim_{n\to\infty} x_n = a \cdot \lim_{n\to\infty}\left[1 + \frac{11}{10} + \left(\frac{11}{10}\right)^2 + \dots + \left(\frac{11}{10}\right)^{n-1}\right] =$$

$= \infty \Rightarrow x_n$ diverge.

$$x_n > a\cdot(n-1)$$

② a) $\displaystyle\lim_{x\to a} \frac{3x - 8x^2 - 5}{x^2 - 3x}$

$\to a \neq 3 \Rightarrow \displaystyle\lim_{x\to a} \frac{3x - 8x^2 - 5}{x^2 - 3x} \Rightarrow \frac{3a - 8a^2 - 5}{a^2 - 3a}$

$\to a = 3 \Rightarrow \displaystyle\lim_{x\to 3} \frac{3x - 8x^2 - 5}{x^2 - 3x} = \lim_{x\to 3} \frac{-78}{x(x-3)}$

$$\lim_{x\to 3}(3x - 8x^2 - 5) = 3\cdot 3 - 8\cdot 3^2 - 5 = -78$$

$\displaystyle\lim_{x\to 3^+} \frac{-78}{x(x-3)} = -\infty$; $\displaystyle\lim_{x\to 3^-} \frac{-78}{x(x-3)} = +\infty$

2) $\lim\limits_{x \to 3} \dfrac{3x - 8x^2 - 5}{x^2 - 3x}$

b) $\lim\limits_{x \to a} \dfrac{\sqrt{2x} - \sqrt{x+a}}{x - a} \cdot \dfrac{\sqrt{2x} + \sqrt{x+a}}{\sqrt{2x} + \sqrt{x+a}} = \dfrac{2x - (x+a)}{(x-a)(\sqrt{2x} + \sqrt{x+a})} =$

$= \lim\limits_{x \to a} \dfrac{\cancel{x-a}}{\cancel{(x-a)}(\sqrt{2x} + \sqrt{x+a})} = \lim\limits_{x \to a} \dfrac{1}{\sqrt{2x} + \sqrt{x+a}} =$

$= \dfrac{1}{\sqrt{2a} + \sqrt{2a}} = \dfrac{1}{2\sqrt{2a}}.$

c) $\lim\limits_{x \to -1} \dfrac{x+a}{\sqrt{2 + (x^2-1)\cos\frac{1}{x+1}}} = \dfrac{a-1}{\sqrt{2}}$

$-1 \le \cos\dfrac{1}{x+1} \le 1 \Rightarrow \left|(x^2-1)\cos\left(\dfrac{1}{x+1}\right)\right| \le |x^2-1|$

$\lim\limits_{x \to -1} \left|(x^2-1)\cos\dfrac{1}{x+1}\right| \le \lim\limits_{x \to -1} |x^2-1| = 0 \Rightarrow$

$\Rightarrow \lim\limits_{x \to -1} (x^2-1)\cos\dfrac{1}{x+1} = 0$

3) $f(x) = \begin{cases} |x+a| & \text{ie } x < -2 \\ \dfrac{(kx+x^2)(x+2)}{x^2+5x+6} & \text{ie } x > -2 \end{cases}$

$$\lim_{x \to -2^+} f(x) = \lim_{x\to -2^+} \frac{(Kx+x^2)(x+2)}{x^2+5x+6} = \lim_{x\to -2^+} \frac{(Kx+x^2)\cancel{(x+2)}}{\cancel{(x+2)}(x+3)}$$

$$= \lim_{x\to -2^+} \frac{Kx+x^2}{x+3} = \frac{-2K+4}{1} = 4-2K$$

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} |x+a| = |a-2|$$

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x) = f(-2)$$

$$4-2K = |a-2| = f(-2) \Rightarrow 2K = 4-|a-2| \Rightarrow$$

$$\Rightarrow K = 2 - \frac{|a-2|}{2} \;;\; f(-2) = |a-2|$$

$a>2$: $a-2>0 \Rightarrow |a-2| = a-2 \Rightarrow K = 2 - \frac{a-2}{2} =$

$$= \frac{4-a+2}{2} = \frac{6-a}{2} = \boxed{3-\frac{a}{2} = K} \;;\; \boxed{f(-2) = a-2}.$$

$a=2$: $|a-2|=0 \Rightarrow \boxed{K=2}$; $\boxed{f(-2)=0}$.

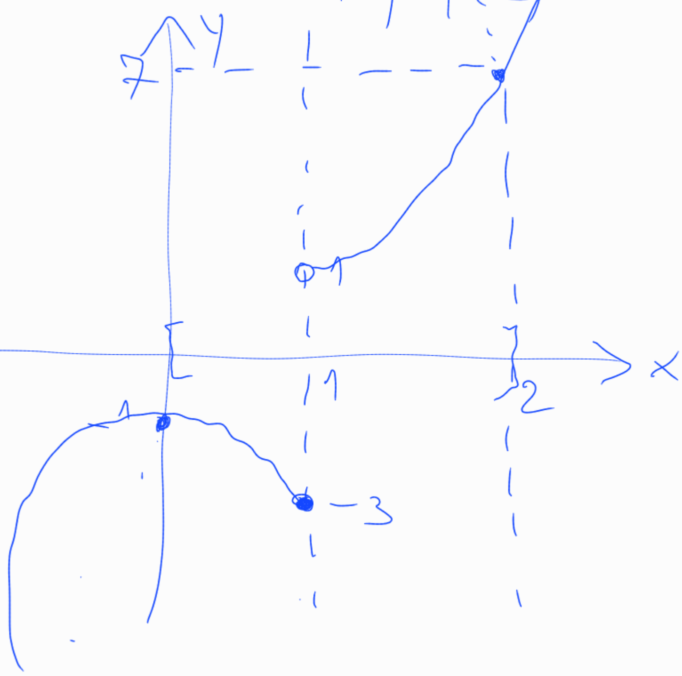
$a<2$: $a-2<0 \Rightarrow |a-2| = 2-a \Rightarrow K = 2 - \frac{2-a}{2}$

$\Rightarrow \boxed{K = \dfrac{2+n}{2}}$; $\boxed{f(-2) = 2-n}$,

(9) $f(x) = \begin{cases} n x^2 - 1, & \text{se } x > 1 \\ -n x^2 - 1, & \text{se } x \leq 1 \end{cases}$

a) $f(0) = -n \cdot 0^2 - 1 = -1$, $f(2) = n \cdot 2^2 - 1 = 4n - 1$.

$\boxed{n = 2}$



b) $a > 1$, $f(a) = 0 \Rightarrow n a^2 - 1 = 0 \Rightarrow n a^2 = 1 \Rightarrow$

$\Rightarrow a^2 = \dfrac{1}{n} \leq 1 \Rightarrow -1 \leq a \leq 1$, absurdo!

$n \geq 1 \Rightarrow \dfrac{1}{n} \leq 1$

$a = 1 \Rightarrow -n - 1 = 0 \Rightarrow n = -1$, absurdo!

$a < 1$, $f(a) = 0 \Rightarrow -n a^2 - 1 = 0 \Rightarrow n a^2 = -1$, absurdo, pois $a^2 \geq 0$ e $a \geq 1$,

$\therefore \nexists\, a \in \mathbb{R}$ tq $f(a) = 0$. $\blacksquare$

c) Não pode, pois o TVI assume como hipótese que f é contínua no intervalo fechado analisado.

Como f não é contínua num intervalo que contém o 1, não podemos aplicar o TVI ao intervalo $[0,2]$.

(5) $f(x) = \dfrac{2}{x}$

$$m = \lim_{h \to 0} \frac{f(h-1) - f(-1)}{h} = \lim_{h \to 0} \frac{\dfrac{2}{h-1} - (-2)}{h} =$$

$$= \lim_{h \to 0} \frac{2 - (-2)(h-1)}{h(h-1)} = \lim_{h \to 0} \frac{\cancel{2} - (\cancel{2} - h2)}{h(h-1)} =$$

$$= \lim_{h \to 0} \frac{h2}{h(h-1)} = \lim_{h \to 0} \frac{2}{h-1} = \frac{2}{-1} = -2 \Rightarrow \boxed{m = -2.}$$