Seção 3.1:

8) $\frac{df}{dt} = 1{,}4 \cdot 5t^4 - 2{,}5 \cdot 2t + 0 = 7t^4 - 5t.$

11) $\frac{dy}{dt} = -\frac{2}{5} x^{-7/5}.$

22) $\frac{dy}{dx} = \frac{x^2 \cdot \left(\frac{1}{2\sqrt{x}} + 1\right) - 2x(\sqrt{x} + x)}{x^4} = \frac{\frac{x^2}{2\sqrt{x}} - 2x\sqrt{x} - x^2}{x^4} =$

$= \frac{1}{2x^2\sqrt{x}} - \frac{2\sqrt{x}}{x^3} - \frac{1}{x^2} = \frac{\sqrt{x}}{2x^3} - \frac{4\sqrt{x}}{2x^3} - \frac{1}{x^2} = -\frac{3\sqrt{x}}{2x^3} - \frac{1}{x^2}.$

24) $\frac{dG}{dt} = \sqrt{5} \cdot \frac{1}{2\sqrt{t}} - \frac{\sqrt{7}}{t^2}.$

32) $\frac{dy}{dx} = \frac{d}{dx}(e^x \cdot e) = e^x \cdot 0 + e^x \cdot e = e^{x+1}.$

33) $\frac{dy}{dx} = 6x^2 - 2x \Rightarrow \left.\frac{dy}{dx}\right|_{x=1} = 6 - 2 = 4 \Rightarrow$

$\Rightarrow t: y = 4x + b \; ; \; (1,3) \in t \Rightarrow 3 = 4 + b \Rightarrow$

$\Rightarrow b = -1 \;\therefore\; t = 4x - 1.$

49) a) $v(t) = \frac{ds}{dt} = 3t^2 - 3 \; ; \; a(t) = \frac{dv}{dt} = 6t.$

b) $a(2) = 6.2 = 12\ m/s^2$.

$t > 0$

c) $v(t) = 0 \Leftrightarrow 3t^2 - 3 = 0 \Leftrightarrow t^2 = 1 \Leftrightarrow t = 1$

$a(1) = 6.1 = 6\ m/s^2$.

55 $\frac{dy}{dx} = 6x^2 + 6x - 12 = 0 \Leftrightarrow x^2 + x - 2 = (x-1)(x+2) = 0$

$\Leftrightarrow x = 1$ ou $x = -2$

$y(1) = -6$

$y(-2) = -2.8 + 3.4 + 12.2 + 1 = 21$

Logo, há tangente horizontal em $(1, -6)$ e $(-2, 21)$.

58 $y = 32x - 15 \Rightarrow m = 32 \Rightarrow$

$\Rightarrow \frac{dy}{dx} = 4x^3 = 32 \Rightarrow x^3 = 8 \Rightarrow x = 2 \Rightarrow y = 2^4 + 1 = 17$

$t: y = 32x + b$; $(2, 17) \in y \Rightarrow 17 = 64 + b \Rightarrow b = -47$

$\therefore t: y = 32x - 47$.

68 $y = Ax^2 + Bx + C \Rightarrow y' = 2Ax + B \Rightarrow y'' = 2A \Rightarrow$

$\Rightarrow y'' + y' - 2y = 2A + 2Ax + B - 2Ax^2 - 2Bx - 2C =$

$= -2Ax^2 + (2A - 2B)x + (2A + B - 2C) \equiv x^2 \Leftrightarrow$

polinômios idênticos

$$\Leftrightarrow \begin{cases} -2A = 1 \\ 2A - 2B = 0 \\ 2A + B - 2C = 0 \end{cases} \Leftrightarrow \begin{cases} A = -\frac{1}{2} \\ B = -\frac{1}{2} \\ C = -\frac{3}{4} \end{cases}$$

(69) $\frac{dy}{dx} = 3ax^2 + 2bx + c = 0$ para $x = 2$ e $x = -2$

$$\Rightarrow \begin{cases} 12a + 4b + c = 0 \\ 12a - 4b + c = 0 \end{cases} \Rightarrow \begin{matrix} b = 0 \\ c = -12a \end{matrix}$$

$y = ax^3 - 12ax + d$

$y(-2) = -8a + 24a + d = 16a + d = 6$

$y(2) = 8a - 24a + d = -16a + d = 0$ $\Rightarrow d = 16a \Rightarrow$

$\Rightarrow 32a = 6 \Rightarrow a = \frac{3}{16} \Rightarrow d = 3$

$\therefore y = \frac{3}{16}x^3 - \frac{9}{4}x + 3$.

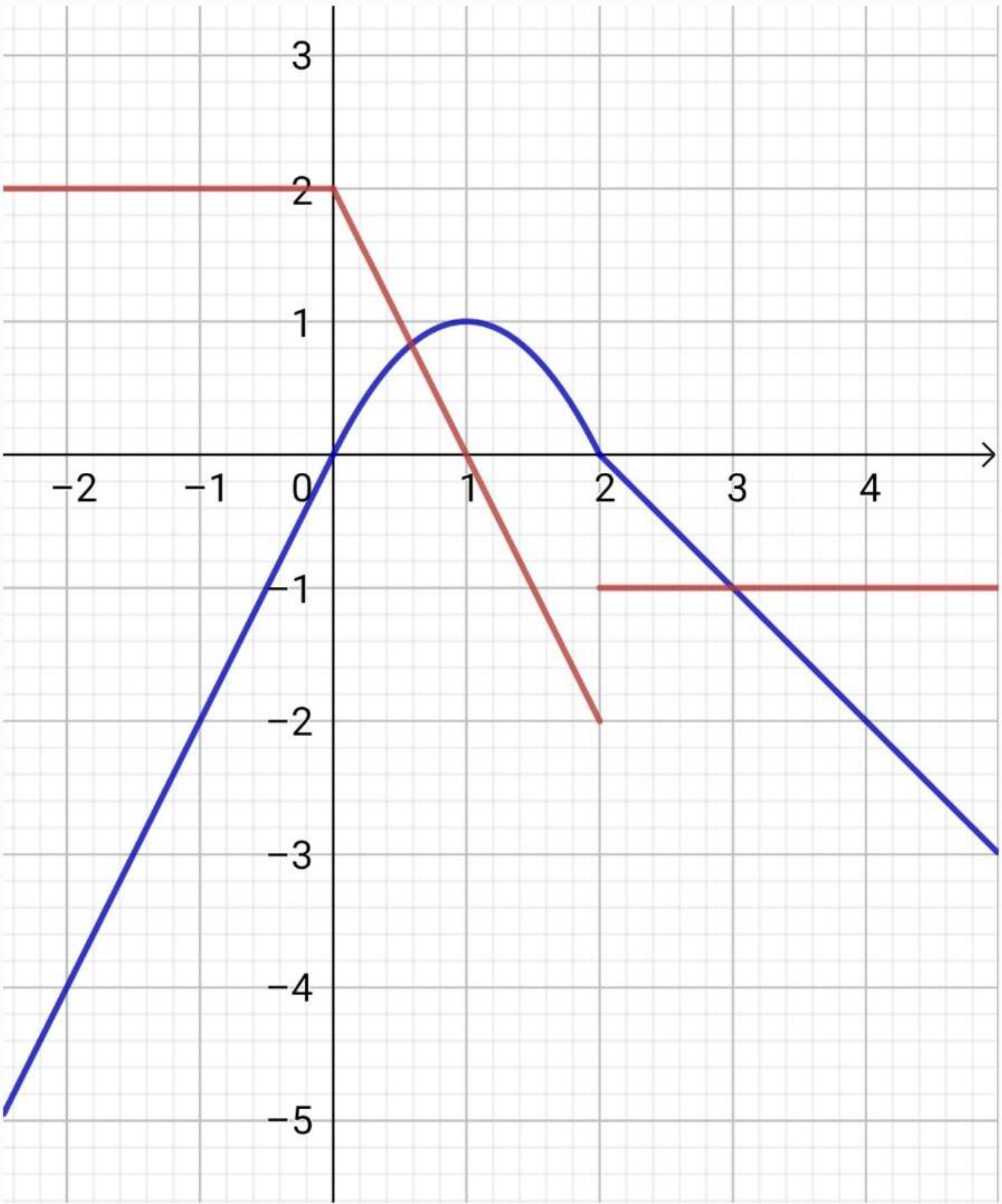
(72) Como $y = 2x$, $y = 2x - x^2$ e $y = 2 - x$ são deriváveis, vamos analisar as derivadas laterais em 0 e 2:

$$\left.\begin{matrix} g'_+(0) = \left[2 - 2x\right]_{x=0} = 2; \\ g'_-(0) = \left[2\right]_{x=0} = 2; \end{matrix}\right\} g'_+(0) = g'_-(0) \Rightarrow g \text{ é derivável em } 0.$$

$$\left.\begin{array}{l} g'_+(2) = [-1]_{x=2} = -1; \\ g'_-(2) = [2-2x]_{x=2} = -2; \end{array}\right\} g'_+(-1) \neq g'_-(-1) \Rightarrow g \text{ não é derivável em } 2.$$

Portanto g é derivável em $\mathbb{R}\setminus\{2\}$.

$$g'(x) = \begin{cases} 2 & \text{se } x \leq 0 \\ 2-2x & \text{se } 0 < x < 2 \\ -1 & \text{se } x > 2 \end{cases}$$



74

$$h(x) = \begin{cases} -2x-1 & \text{se} \quad x < -2 \\ 3 & \text{se} \quad -2 \leq x < 1 \\ 2x+1 & \text{se} \quad x \geq 1 \end{cases}$$

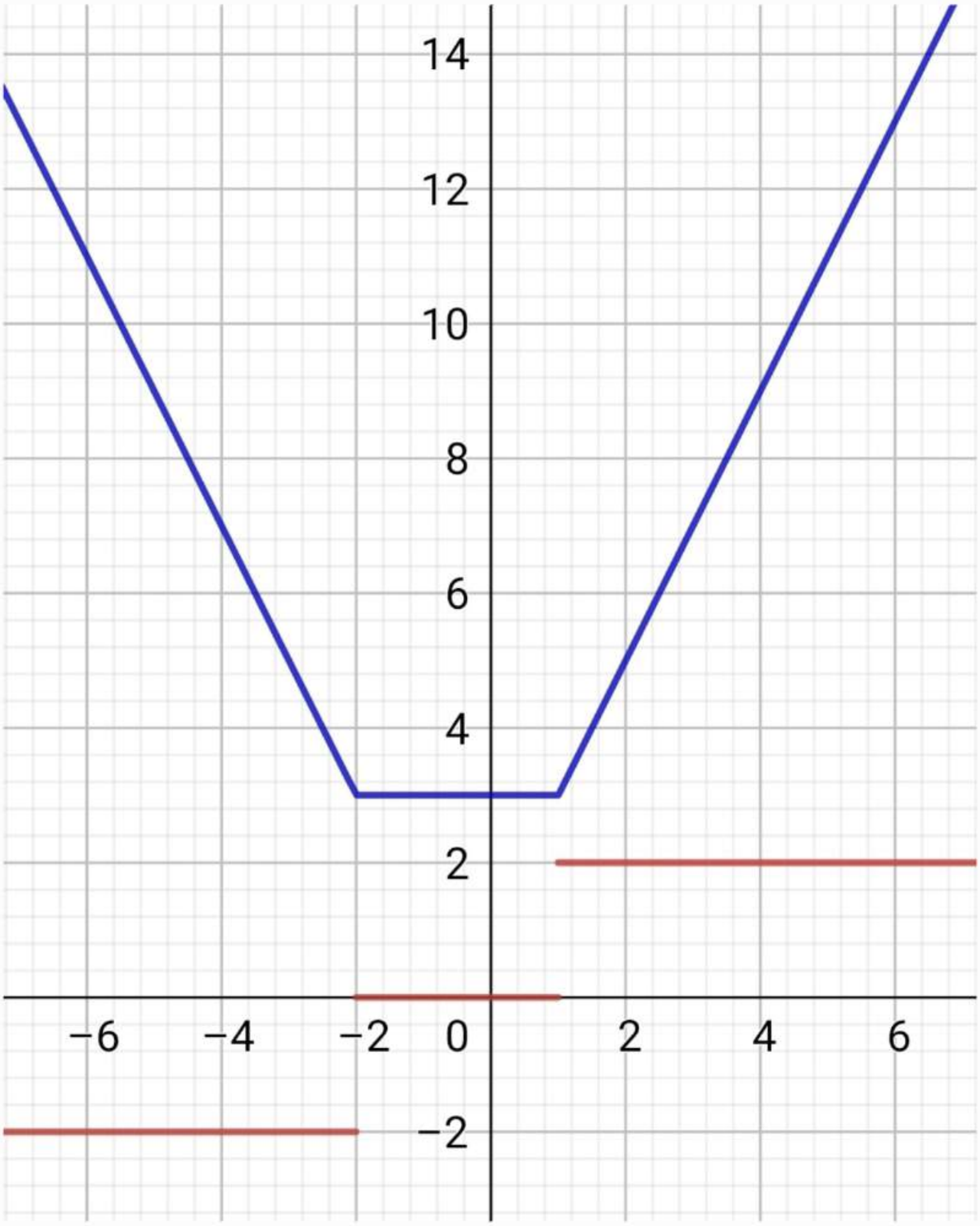
Como $y = -2x-1$, $y = 3$ e $y = 2x+1$ são deriváveis, vamos analisar as derivadas laterais em -2 e 1:

$$\left.\begin{array}{l} h'_+(-2) = [0]_{x=-2} = 0 \,; \\ h'_-(-2) = [-2]_{x=-2} = -2 \,; \end{array}\right\} h'_+(-2) \neq h'_-(-2) \Rightarrow h \text{ não é derivável em } -2.$$

$$\left.\begin{array}{l} h'_+(1) = [2]_{x=1} = 2 \,; \\ h'_-(1) = [0]_{x=1} = 0 \,; \end{array}\right\} h'_+(1) \neq h'_-(1) \Rightarrow h \text{ não é derivável em } 1$$

Portanto h é derivável em $\mathbb{R} \setminus \{-2, 1\}$.

$$h'(x) = \begin{cases} -2 & \text{se} \quad x < -2 \\ 0 & \text{se} \quad -2 < x < 1 \\ 2 & \text{se} \quad x > 1 \end{cases}$$



75) $\frac{dy}{dx} = 2ax + b \Rightarrow \left.\frac{dy}{dx}\right|_{x=1} = 2a + b = 3$;

$(1,1) \in$ parábola $\Rightarrow 1 = a + b$

$$\Rightarrow \begin{cases} 2a + b = 3 \\ a + b = 1 \end{cases} \Rightarrow \begin{matrix} a = 2 \\ b = -1 \end{matrix} \quad \therefore \quad y = 2x^2 - x.$$

78) $\left.\frac{dy}{dx}\right|_{x=x_*} = \left[\frac{c}{2\sqrt{x}}\right]_{x=x_*} = \frac{c}{2\sqrt{x_*}} = \frac{3}{2} \Rightarrow \sqrt{x_*} = \frac{c}{3} \geq 0 \Rightarrow$

$c \geq 0$ e $x_t = \dfrac{c^2}{9} \Rightarrow y_t = c\sqrt{x_t} = \dfrac{c^2}{3}$; $(x_t, y_t) \in$ tangente

$$\Rightarrow \frac{c^2}{3} = \frac{3}{2}\cdot\frac{c^2}{9} + 6 \Rightarrow \frac{c^2}{3} = \frac{c^2}{6} + 6 \Rightarrow \frac{c^2}{6} = 6 \Rightarrow$$

$$\Rightarrow c^2 = 36 \Rightarrow c = \pm 6 \underset{c \geq 0}{\Longrightarrow} c = 6.$$

82 a) $y = \dfrac{c}{x} \Rightarrow \dfrac{dy}{dx} = -\dfrac{c}{x^2} \Rightarrow \left.\dfrac{dy}{dx}\right|_{x=x_p} = -\dfrac{c}{x_p^2}$

$$t: y = -\frac{c}{x_p^2}\cdot x + b \Rightarrow y_p = -\frac{c}{x_p} + b = \frac{c}{x_p} \Rightarrow b = \frac{2c}{x_p}$$

$$t: y = -\frac{c}{x_p^2}\cdot x + \frac{2c}{x_p} \Rightarrow \frac{\left(0, \frac{2c}{x_p}\right) + (2x_p, 0)}{2} = \left(x_p, \frac{c}{x_p}\right) =$$

$$= (x_p, y_p) = P.$$

b) $t: y = -\dfrac{c}{x_p^2}\cdot x + \dfrac{2c}{x_p}$. Vértices do triângulo:

$$\left(0, \frac{2c}{x_p}\right); (2x_p, 0); (0,0) \Rightarrow [\Delta] = \frac{1}{2}\left\| \begin{matrix} 0 & \frac{2c}{x_p} & 1 \\ 2x_p & 0 & 1 \\ 0 & 0 & 1 \end{matrix} \right\| =$$

$= \dfrac{1}{2}\left|2x_p \cdot \dfrac{2c}{x_p}\right| = \dfrac{1}{2}|4c| = 2|c|$, que não depende de P.

83 $$\lim_{x \to 1} \frac{x^{1000} - 1}{x - 1} = \lim_{x\to 1} \frac{(x-1)(x^{999} + x^{998} + \dots + x^2 + x + 1)}{x-1} =$$

$$= \lim_{x \to 1} (x^{999} + x^{998} + \dots + x^2 + x + 1) = 1 + 1 + \dots + 1 = 1000.$$

$$\text{Ou}: \lim_{x \to 1} \frac{x^{1000} - 1^{1000}}{x - 1} = \frac{d}{dx}(x^{1000}) \Big|_{x=1} = \left[1000x^{999}\right]_{x=1} = 1000.$$

Seção 3.2:

1) $f'(x) = 4x(x - x^2) + (1 + 2x^2)(1 - 2x) = 4x^2 - 4x^3 + 1 - 2x + 2x^2 - 4x^3 = -8x^3 + 6x^2 - 2x + 1;$

$f(x) = x - x^2 + 2x^3 - 2x^4 \Rightarrow f'(x) = -8x^3 + 6x^2 - 2x + 1.$

7) $$g'(x) = \frac{(2x+1) \cdot 3 - (3x-1) \cdot 2}{(2x+1)^2} = \frac{5}{(2x+1)^2}.$$

15) $$y' = \frac{(t^2 - 4t + 3) \cdot (3t^2 + 3) - (t^3 + 3t) \cdot (2t - 4)}{(t^2 - 4t + 3)^2} =$$

$$= \frac{3t^4 + 3t^2 - 12t^3 - \cancel{12t} + 9t^2 + 9 - 2t^4 + 4t^3 - 6t^2 + \cancel{12t}}{(t^2 - 4t + 3)^2} =$$

$$= \frac{t^4 - 8t^3 + 6t^2 + 9}{(t^2 - 4t + 3)^2}$$

27) $f'(x) = 3x^2 \cdot e^x + (x^3 + 1) \cdot e^x = (x^3 + 3x^2 + 1)e^x$

$f''(x) = (3x^2 + 6x)e^x + (x^3 + 3x^2 + 1)e^x = (x^3 + 6x^2 + 6x + 1)e^x.$

31) $$y' = \frac{(x^2+x+1).2x - (x^2-1).(2x+1)}{(x^2+x+1)^2} \Rightarrow$$

$$\Rightarrow y'(1) = \frac{3.2-0}{3^2} = \frac{2}{3} \Rightarrow t: y = \frac{2}{3}x + b \;;$$

$$(1,0) \in t \Rightarrow 0 = \frac{2}{3} + b \Rightarrow b = -\frac{2}{3}.$$

$$\therefore\; y = \frac{2}{3}.x - \frac{2}{3}.$$

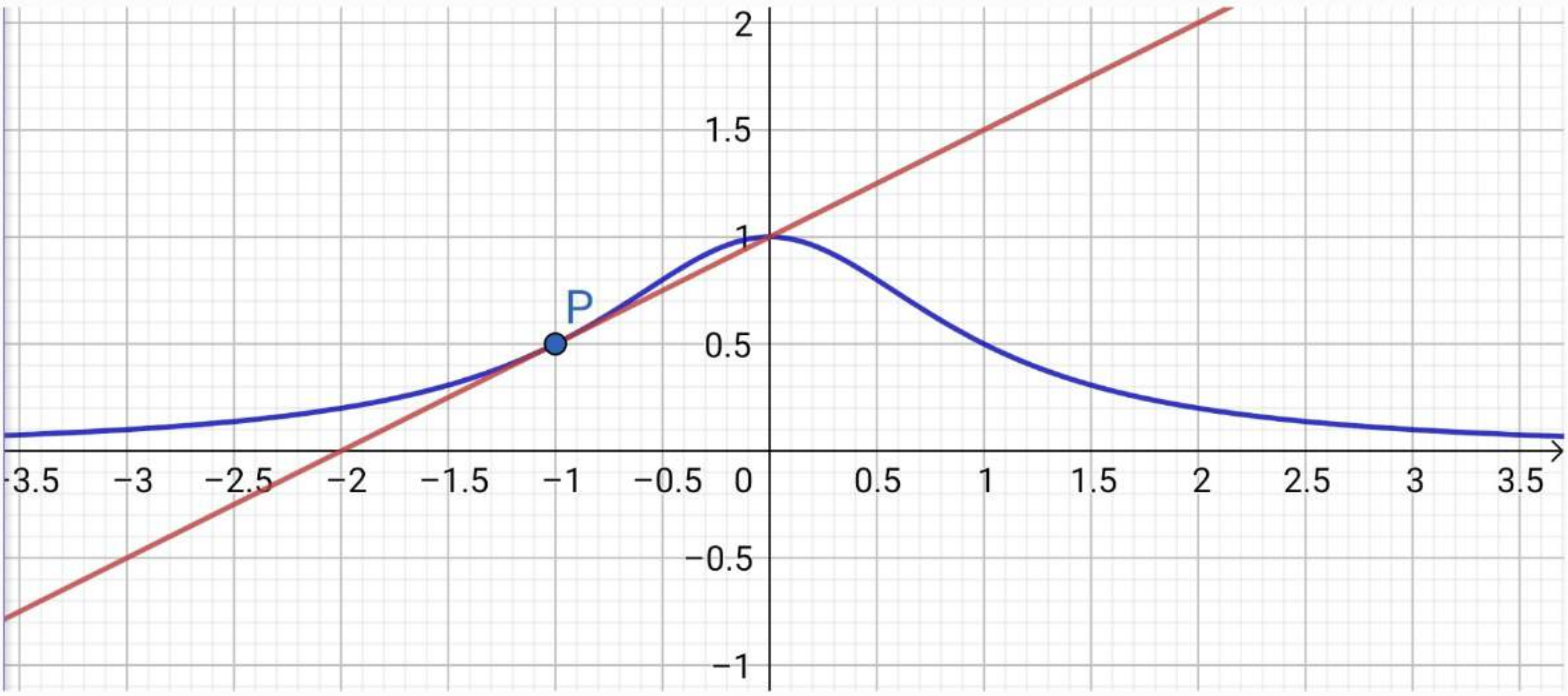
35) a) $$y' = \frac{(1+x^2).0 - 1.2x}{(1+x^2)^2} = \frac{-2x}{(1+x^2)^2} \Rightarrow$$

$$\Rightarrow y'(-1) = \frac{(-2)(-1)}{2^2} = \frac{2}{4} = \frac{1}{2} \Rightarrow t: y = \frac{x}{2} + b \;;$$

$$\left(-1, \frac{1}{2}\right) \in t \Rightarrow \frac{1}{2} = -\frac{1}{2} + b \Rightarrow b = 1$$

$$\therefore\; y = \frac{x}{2} + 1$$

b)



41) $f'(x) = \dfrac{(1+x)\cdot 2x - x^2\cdot 1}{(1+x)^2} = \dfrac{2x+x^2}{(x+1)^2} \Rightarrow f'(1) = \dfrac{3}{4}$.

46) $\dfrac{d}{dx}\left(\dfrac{h(x)}{x}\right) = \dfrac{x\cdot h'(x) - h(x)\cdot 1}{x^2} \Rightarrow \dfrac{d}{dx}\left(\dfrac{h(x)}{x}\right)\Bigg|_{x=2} =$

$= \dfrac{2\cdot h'(2) - h(2)}{2^2} = \dfrac{2\cdot(-3)-4}{4} = -\dfrac{10}{4} = -\dfrac{5}{2}$.

49) a) $u'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x) \Rightarrow u'(1) = f'(1)\cdot g(1) +$
$+ f(1)\cdot g'(1) = 2\cdot 1 + 2\cdot(-1) = 0$.

b) $v'(x) = \dfrac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{[g(x)]^2} \Rightarrow$

$$\Rightarrow v'(5) = \frac{g(5)\cdot f'(5) - f(5)\cdot g'(5)}{[g(5)]^2} = \frac{2\cdot\left(-\frac{1}{3}\right) - 3\cdot\frac{2}{3}}{4} =$$

$$= \frac{-\frac{8}{3}}{4} = -\frac{2}{3}.$$

51 a) $y' = 1\cdot g(x) + x\cdot g'(x) \Rightarrow y' = g(x) + x\cdot g'(x).$

b) $y' = \frac{g(x)\cdot 1 - x\cdot g'(x)}{[g(x)]^2} \Rightarrow y' = \frac{g(x) - x\cdot g'(x)}{[g(x)]^2}.$

c) $y' = \frac{x\cdot g'(x) - g(x)\cdot 1}{x^2} \Rightarrow y' = \frac{x\cdot g'(x) - g(x)}{x^2}.$

52 a) $y' = 2x\cdot f(x) + x^2\cdot f'(x).$

b) $y' = \frac{x^2\cdot f'(x) - 2x f(x)}{x^4}.$

c) $y' = \frac{f(x)\cdot 2x - x^2 f'(x)}{[f(x)]^2}.$

d) $$y' = \frac{\sqrt{x}\left(1\cdot f(x) + x\cdot f'(x)\right) - \left(1 + x f(x)\right)\cdot\frac{1}{2\sqrt{x}}}{x} =$$

$$= \frac{\sqrt{x} f(x) + x\sqrt{x} f'(x) - \frac{1}{2\sqrt{x}} - \frac{\sqrt{x} f(x)}{2}}{x} =$$

$$= \frac{\sqrt{x}\, f(x)}{2x} + \sqrt{x}\, f'(x) - \frac{\sqrt{x}}{2x^2}.$$

53) $$\frac{dy}{dx} = \frac{(x+1)\cdot 1 - x\cdot 1}{(x+1)^2} = \frac{1}{(x+1)^2} \Rightarrow \left.\frac{dy}{dx}\right|_{x=x_p} = \frac{1}{(x_p+1)^2} \Rightarrow$$

$$\Rightarrow t: y = \frac{1}{(x_p+1)^2}\cdot x + b \;;\; (1,2) \in t \Rightarrow$$

$$\Rightarrow 2 = \frac{1}{(x_p+1)^2} + b \Rightarrow b = 2 - \frac{1}{(x_p+1)^2} \Rightarrow$$

$$\Rightarrow t: y = \frac{x-1}{(x_p+1)^2} + 2 \;;\; \left(x_p, \frac{x_p}{x_p+1}\right) \in t \Rightarrow$$

$$\Rightarrow \frac{x_p}{x_p+1} = \frac{x_p - 1}{(x_p+1)^2} + 2 \Rightarrow x_p(x_p+1) = x_p - 1 + 2(x_p+1)^2$$

$$\Rightarrow x_p^2 + x_p = x_p - 1 + 2x_p^2 + 4x_p + 2 \Rightarrow x^2 + 4x_p + 1 = 0 \Rightarrow$$

$$\Rightarrow x_p = \frac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$$ $\Rightarrow$ há 2 pontos x_p possíveis

$\Rightarrow$ há 2 retas tangentes passando por $(1,2)$, tocando a curva em $\left(\sqrt{3}-2, \frac{\sqrt{3}-2}{\sqrt{3}-1}\right)$ e $\left(-\sqrt{3}-2, \frac{-\sqrt{3}-2}{-\sqrt{3}-1}\right)$.

57) $P(1999) \simeq 961400$ pes. ; $R(1999) = 30593$ \$/pes.

$\Delta P(1999) \simeq 9200$ pes. ; $\Delta R(1999) = 1400$ \$/pes. ;

$$\Delta(P\cdot R) \cong \Delta P \cdot R + P \cdot \Delta R = 9200 \cdot 30593\$ + 961400 \cdot 1400\$ =$$

$= 1627415600\$$ ao ano.

62 a) $F' = (f \cdot g)' = f' \cdot g + f \cdot g'$

$\Rightarrow F'' = (f' \cdot g + f \cdot g')' = (f' \cdot g)' + (f \cdot g')' =$

$= f'' \cdot g + f' \cdot g' + f' \cdot g' + f \cdot g'' = f'' \cdot g + 2 f' \cdot g' + f \cdot g''.$

b) $F''' = (f'' \cdot g + 2 f' \cdot g' + f \cdot g'')' = (f'' \cdot g)' + 2 (f' \cdot g')' +$

$+ (f \cdot g'')' = f''' \cdot g + f'' \cdot g' + 2 f'' \cdot g' + 2 f' \cdot g'' + f' \cdot g'' + f \cdot g''' =$

$= f''' \cdot g + 3 f'' \cdot g' + 3 f' \cdot g'' + f \cdot g'''.$

$F^{(4)} = (f''' \cdot g + 3 f'' \cdot g' + 3 f' \cdot g'' + f \cdot g''')' =$

$= f^{(4)} \cdot g + f''' \cdot g' + 3 f''' \cdot g' + 3 f'' \cdot g'' + 3 f'' \cdot g'' +$

$+ 3 f' \cdot g''' + f' \cdot g''' + f \cdot g^{(4)} =$

$= f^{(4)} \cdot g + 4 f''' \cdot g' + 6 f'' \cdot g'' + 4 f' \cdot g''' + f \cdot g^{(4)}$

c) $F^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(k)} \cdot g^{(n-k)}.$

Seção 3.3:

5 $g'(t) = 3t^2 \cdot \cos t + t^3 \cdot (-\operatorname{sen} t) = 3t^2 \cos t - t^3 \operatorname{sen} t.$

13 $\frac{dy}{dt} = \frac{(1+t) \cdot (1 \cdot \operatorname{sen} t + t \cdot \cos t) - t \operatorname{sen} t \cdot 1}{(1+t)^2} =$

$$= \frac{\text{sen}\, t + t\cos t + t\,\text{sen}\, t + t^2\cos t - t\,\text{sen}\, t}{(1+t)^2} =$$

$$= \frac{\text{sen}\, t + t\cos t + t^2\cos t}{(1+t)^2}.$$

22 $\frac{dy}{dx} = e^x \cos x + e^x . (-\text{sen}\, x) = e^x(\cos x - \text{sen}\, x)$

$\left.\frac{dy}{dx}\right|_{x=0} = 1.1 = 1 \Rightarrow t: y = x + b$;

$(0,1) \in t \Rightarrow 1 = 0 + b \Rightarrow b = 1$

$\therefore t: y = x + 1$.

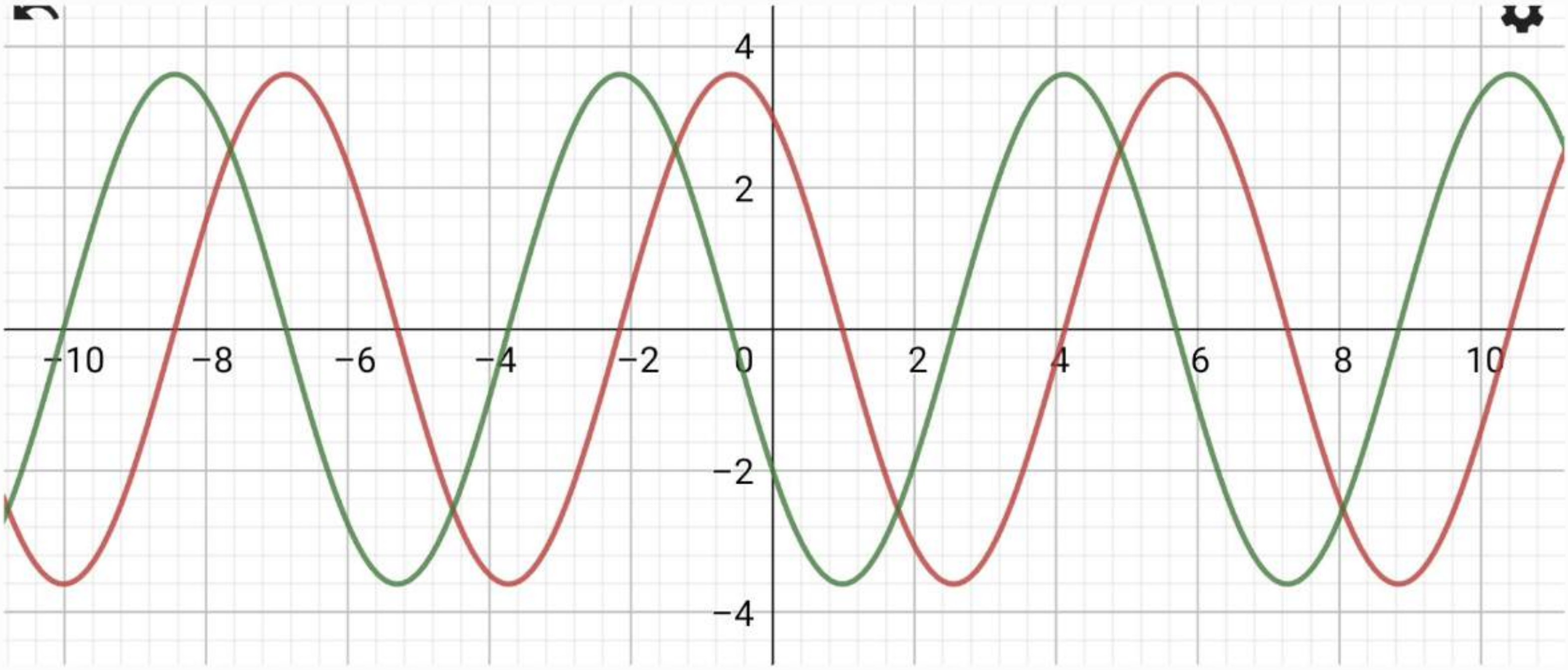
33 $f'(x) = 1 + 2\cos x = 0 \Leftrightarrow 2\cos x = -1 \Leftrightarrow$

$\Leftrightarrow \cos x = -\frac{1}{2} \Leftrightarrow x = \pm\frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$.

36 a) $v(t) = \frac{d}{dt} s(t) = -2\,\text{sen}\, t + 3\cos t$

$a(t) = \frac{d}{dt} v(t) = -2\cos t - 3\,\text{sen}\, t$

b)



c) $s(t) = 0 \iff 2\cos t + 3 \operatorname{sen} t = 0 \iff$

$\iff 3 \operatorname{sen} t = -2\cos t \iff \dfrac{\operatorname{sen} t}{\cos t} = \dfrac{-2}{3} \iff \operatorname{tg} t = -\dfrac{2}{3}$

Logo, isso ocorre pela primeira vez em $t = \operatorname{arctg}\left(\frac{-2}{3}\right)$.

d) $s(t) = 2\cos t + 3 \operatorname{sen} t = \sqrt{13}\left(\dfrac{2}{\sqrt{13}}\cos t + \dfrac{3}{\sqrt{13}} \operatorname{sen} t\right)$

Seja $\alpha \in \mathbb{R}$ tq. $\operatorname{sen}\alpha = \dfrac{2}{\sqrt{13}}$ e $\cos\alpha = \dfrac{3}{\sqrt{13}} \Rightarrow$

$\Rightarrow s(t) = \sqrt{13}\,(\operatorname{sen}\alpha \cos t + \operatorname{sen} t \cos\alpha) = \sqrt{13} \operatorname{sen}(\alpha + t)$

$\Rightarrow -\sqrt{13} \leq s(t) \leq \sqrt{13} \Rightarrow |s_{MÁX}| = \sqrt{13}$.

e) $v(t) = 3\cos t - 2 \operatorname{sen} t = \sqrt{13}\left(\dfrac{3}{\sqrt{13}}\cos t - \dfrac{2}{\sqrt{13}} \operatorname{sen} t\right)$.

Seja $\theta \in \mathbb{R}$ tq. $\cos\theta = \dfrac{3}{\sqrt{13}}$ e $\operatorname{sen}\theta = \dfrac{2}{\sqrt{13}} \Rightarrow$

$\Rightarrow v(t) = \sqrt{13}\,(\cos\theta\cos t - \operatorname{sen}\theta \operatorname{sen} t) = \sqrt{13}\cos(\theta + t) \Rightarrow$

$$\Rightarrow -\sqrt{13} \leq v(t) \leq \sqrt{13} \Rightarrow |v_{máx}| = \sqrt{13}.$$

39) $$\lim_{x\to 0} \frac{\operatorname{sen} 5x}{3x} = \lim_{x\to 0} \frac{\operatorname{sen} 5x}{5x} \cdot \frac{5}{3} = \frac{5}{3} \lim_{5x\to 0} \frac{\operatorname{sen} 5x}{5x} =$$

$$= \frac{5}{3} \cdot 1 = \frac{5}{3}.$$

41) $$\lim_{t\to 0} \frac{\operatorname{tg} 6t}{\operatorname{sen} 2t} = \lim_{t\to 0} \frac{\frac{\operatorname{tg} 6t}{6t}}{\frac{\operatorname{sen} 2t}{2t}} \cdot 3 = 3 \frac{\lim\limits_{6t\to 0} \frac{\operatorname{tg} 6t}{6t}}{\lim\limits_{2t\to 0} \frac{\operatorname{sen} 2t}{2t}} =$$

$$= 3 \cdot \frac{1}{1} = 3.$$

48) $$\lim_{x\to 0} \frac{\operatorname{sen}(x^2)}{x} = \lim_{x\to 0} \frac{\operatorname{sen}(x^2)}{x^2} \cdot x =$$

$$= \lim_{x^2\to 0} \frac{\operatorname{sen}(x^2)}{x^2} \cdot \lim_{x\to 0} x = 1 \cdot 0 = 0.$$

Seção 3.4:

5) $$\begin{cases} u = \sqrt{x} \\ y = e^u \end{cases} \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot \frac{1}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}.$$

⑦ $\begin{cases} u = 5x^6 + 2x^3 \\ y = u^4 \end{cases} \Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = 4u^3 \cdot (30x^5 + 6x^2) =$

$= 24(5x^6 + 2x^3)^3 \cdot (5x^5 + x^2).$

⑨ $\begin{cases} u = 5x + 1 \\ y = \sqrt{u} \end{cases} \Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = \dfrac{1}{2\sqrt{u}} \cdot 5 = \dfrac{5}{2\sqrt{5x+1}}.$

⑪ $\begin{cases} u = \theta^2 \\ y = \cos u \end{cases} \Rightarrow \dfrac{dy}{d\theta} = \dfrac{dy}{du} \cdot \dfrac{du}{d\theta} = (-\operatorname{sen} u) \cdot 2\theta =$

$= -2\theta \operatorname{sen}(\theta^2).$

㉑ $\begin{cases} u = \dfrac{x}{x+1} \\ y = \sqrt{u} \end{cases} \Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = \dfrac{1}{2\sqrt{u}} \cdot \dfrac{(x+1) \cdot 1 - x \cdot 1}{(x+1)^2} =$

$= \dfrac{1}{2\sqrt{\dfrac{x}{x+1}} \cdot (x+1)^2}.$

㉘ $\begin{cases} u = \dfrac{z}{z-1} \\ y = e^u \end{cases} \Rightarrow \dfrac{dy}{dz} = \dfrac{dy}{du} \cdot \dfrac{du}{dz} = e^u \cdot \dfrac{(z-1) \cdot 1 - z \cdot 1}{(z-1)^2} =$

$= e^u \cdot \dfrac{-1}{(z-1)^2} = \dfrac{-e^{\frac{z}{z-1}}}{(z-1)^2}.$

(35) $\begin{cases} u = 2x \\ v = tg\, u \\ y = sen\, v \end{cases} \Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{dv} \cdot \dfrac{dv}{du} \cdot \dfrac{du}{dx} =$

$= \cos v \cdot sec^2 u \cdot 2 = 2\cos(tg\, 2x) \cdot sec^2(2x).$

(42) $\begin{cases} u = sen\, x \\ v = sen\, u \\ y = sen\, v \end{cases} \Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{dv} \cdot \dfrac{dv}{du} \cdot \dfrac{du}{dx} = \cos v \cdot \cos u \cdot \cos x =$

$= \cos(sen(sen\, x)) \cdot \cos(sen\, x) \cdot \cos x.$

(49) $y' = \dfrac{1}{2\sqrt{1 - sec\, t}} \cdot (-sec\, t \cdot tg\, t) = \dfrac{-sec\, t \cdot tg\, t}{2\sqrt{1-sec\, t}}.$

$$y'' = -\frac{2\sqrt{1-sec\, t} \cdot (sec\, t \cdot tg\, t \cdot tg\, t + sec\, t \cdot sec^2 t) - sec\, t \cdot tg\, t \cdot 2 \cdot \left(\frac{-sec\, t \cdot tg\, t}{2\sqrt{1-sec\, t}}\right)}{\left(2\sqrt{1-sec\, t}\right)^2}$$

$$= -\frac{sec\, t \cdot tg^2 t + sec^3 t}{2\sqrt{1-sec\, t}} - \frac{sec^2 t \cdot tg^2 t}{4\left(\sqrt{1-sec\, t}\right)^3} =$$

$$= \frac{-2(1-sec\, t)(sec\, t \cdot tg^2 t + sec^3 t) - sec^2 t \cdot tg^2 t}{4(1-sec\, t)^{3/2}} =$$

$$= \frac{-2\, sec\, t \cdot tg^2 t - 2\, sec^3 t + 2\, sec^2 t \cdot tg^2 t + 2\, sec^4 t - sec^2 t \cdot tg^2 t}{4(1-sec\, t)^{3/2}} =$$

$$= \frac{tg^2 t\,(\sec^2 t - 2\sec t) - 2\sec^3 t + 2\sec^4 t}{4(1-\sec t)^{3/2}} =$$

$$= \frac{(\sec^2 t - 1)\cdot(\sec^2 t - 2\sec t) - 2\sec^3 t + 2\sec^4 t}{4(1-\sec t)^{3/2}} =$$

$$= \frac{3\sec^4 t - 4\sec^3 t - \sec^2 t + 2\sec t}{4(1-\sec t)^{3/2}}\,.$$

56 a) $y = \dfrac{|x|}{\sqrt{2-x^2}}$; $\left.\dfrac{dy}{dx}\right|_{x=1} = \dfrac{d}{dx}\left(\dfrac{x}{\sqrt{2-x^2}}\right)\Bigg|_{x=1} =$

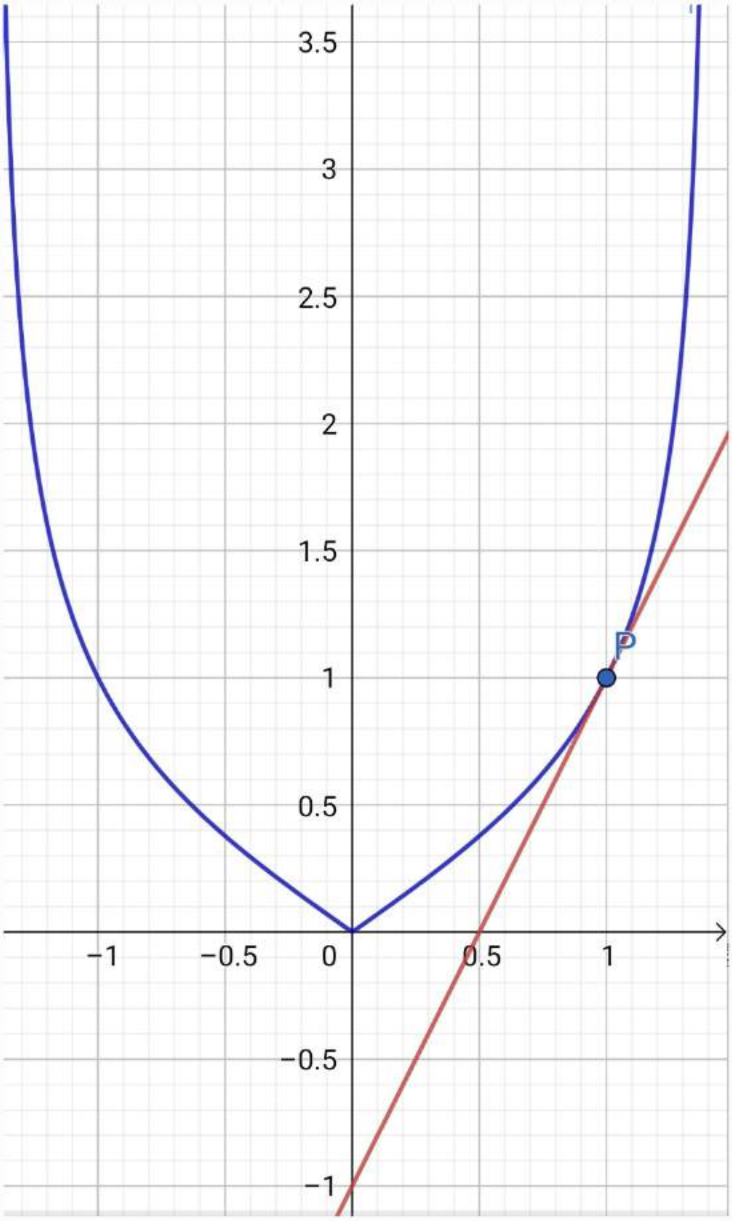
$$= \left.\frac{\sqrt{2-x^2}\cdot 1 - x\cdot\dfrac{1}{2\sqrt{2-x^2}}\cdot(-2x)}{\left(\sqrt{2-x^2}\right)^2}\right|_{x=1} = \left[\frac{1}{\sqrt{2-x^2}} + \frac{x^2}{\left(\sqrt{2-x^2}\right)^3}\right]_{x=1} =$$

$$= \frac{1}{1} + \frac{1}{1} = 2 \Rightarrow t: y = 2x + b;$$

$$(1,1) \in t \Rightarrow 1 = 2.1 + b \Rightarrow b = -1$$

$$\therefore\ t: y = 2x - 1$$

b)



63 a) $h'(x) = f'(g(x))\cdot g'(x) \Rightarrow h'(1) = f'(g(1))\cdot g'(1) =$

$= f'(2)\cdot 6 = 5\cdot 6 = 30.$

b) $H'(x) = g'(f(x))\cdot f'(x) \Rightarrow H'(1) = g'(f(1))\cdot f'(1) =$

$= g'(3)\cdot 4 = 9\cdot 4 = 36.$

65 a) $u'(1) = f'(g(1))\cdot g'(1) = f'(3)\cdot g'(1) = -\frac{1}{4}\cdot(-3) = \frac{3}{4}.$

b) $v'(1) = g'(f(1))\cdot f'(1) = g'(2)\cdot f'(1) \Rightarrow \nexists\, v'(1)$ pois $\nexists\, g'(2).$

c) $w'(1) = g'(g(1))\cdot g'(1) = g'(3)\cdot g'(1) = \frac{2}{3}\cdot(-3) = -2.$

70 a) $\frac{d}{dx}g(x) = \frac{de^{cx}}{dx} + \frac{d}{dx}f(x) \Rightarrow g'(x) = c\cdot e^{cx} + f'(x)$

$\Rightarrow g''(x) = c^2\cdot e^{cx} + f''(x) \Rightarrow g'(0) = c\cdot e^0 + f'(0) = c+5;$

$g''(0) = c^2\cdot e^0 + f''(0) = c^2 - 2.$

b) $h'(x) = f(x)\cdot \frac{de^{kx}}{dx} + e^{kx}\cdot \frac{d}{dx}f(x) = k\,f(x)e^{kx} +$

$+ f'(x)\cdot e^{kx} \Rightarrow h'(0) = k\cdot f(0)\cdot e^0 + f'(0)\cdot e^0 = 3k+5 \Rightarrow$

$\Rightarrow t: y = (3k+5)x + b;\quad (0, h(0)) \in t \Rightarrow$

$\Rightarrow h(0) = b \Rightarrow e^0\cdot f(0) = b \Rightarrow b = 3$

$\therefore\ t: y = (3k+5)x + 3.$

74 $F'(x) = f'(x\,f(x\,f(x)))\cdot\left[f(x\,f(x)) + x\cdot f'(x\,f(x))\cdot(f(x) + x\,f'(x))\right]$

$\Rightarrow F'(1) = f'(f(f(1)))\cdot\left[f(f(1)) + f'(f(1))\cdot(f(1) + f'(1))\right] =$

$= f'(f(2))\cdot\left[f(2) + f'(2)\cdot(2+4)\right] = f'(3)\cdot(3 + 5.6) =$

$= 33.6 = 198.$

76 $y = e^{rx} \Rightarrow y' = re^{rx} \Rightarrow y'' = r^2e^{rx} \Rightarrow$

$\Rightarrow y'' - 4y' + y = r^2e^{rx} - 4re^{rx} + e^{rx} = \underbrace{e^{rx}}_{>0}(r^2 - 4r + 1) = 0$

$\Rightarrow r^2 - 4r + 1 = 0 \Rightarrow r = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}.$

77) $y = \cos 2x \Rightarrow y^{(1)} = (-\operatorname{sen} 2x) \cdot 2 = -2 \operatorname{sen} 2x \Rightarrow$

$\Rightarrow y^{(2)} = -2 \cdot (\cos 2x) \cdot 2 = -4 \cos 2x \Rightarrow y^{(3)} = -4 \cdot (-\operatorname{sen} 2x) \cdot 2 =$

$= 8 \operatorname{sen} 2x \Rightarrow y^{(4)} = 8 \cdot (\cos 2x) \cdot 2 = 16 \cos 2x = 2^4 \cos 2x \Rightarrow$

$\Rightarrow y^{(48)} = 2^{48} \cos 2x \Rightarrow y^{(49)} = 2^{48} (-\operatorname{sen} 2x) \cdot 2 = -2^{49} \operatorname{sen} 2x \Rightarrow$

$\Rightarrow y^{(50)} = -2^{49} (\cos 2x) \cdot 2 = -2^{50} \cos 2x.$

82) $L'(t) = 2{,}8 \cos\left[\frac{2\pi}{365}(t-80)\right] \cdot \frac{2\pi}{365} = \frac{5{,}6\pi}{365} \cdot \cos\left[\frac{2\pi}{365}(t-80)\right]$

21 de março: $t = 80 \Rightarrow L'(80) = \frac{5{,}6\pi}{365} \cdot \cos 0 = \frac{5{,}6\pi}{365} \cong$

$\cong 0{,}048$;

21 de maio: $t = 141 \Rightarrow L'(141) = \frac{5{,}6\pi}{365} \cos\left[\frac{2\pi}{365} \cdot 61\right] \cong 0{,}048.$

Portanto, nos 2 dias o número de horas de luz do dia aumenta de forma aproximadamente igual.

95) a) $\frac{d}{dx}(\operatorname{sen}^n x \cos nx) = \operatorname{sen}^n x \cdot \frac{d}{dx} \cos nx +$

$+ \cos nx \frac{d}{dx} \operatorname{sen}^n x = \operatorname{sen}^n x \cdot (-\operatorname{sen} nx) \cdot n +$

$+ \cos nx \cdot n \operatorname{sen}^{n-1} x \cdot \cos x = n \operatorname{sen}^{n-1} x (\cos nx \cos x - \operatorname{sen} x \operatorname{sen} nx)$

$= n \operatorname{sen}^{n-1} x (\cos(nx + x)) = n \operatorname{sen}^{n-1} x \cos(n+1)x.$

b) $\frac{d}{dx}(\cos^m x \cos mx) = \cos mx \cdot \frac{d}{dx}\cos^m x +$

$+ \cos^m x \cdot \frac{d}{dx}\cos mx = \cos mx \cdot m\cos^{m-1}x \cdot (-\operatorname{sen} x) +$

$+ \cos^m x \cdot (-\operatorname{sen} mx) \cdot m =$

$= -m\cos^{m-1}x\,(\operatorname{sen} x \cos mx + \cos x \operatorname{sen} mx) =$

$= -m\cos^{m-1}x \cdot \operatorname{sen}(mx + x) =$

$= -m\cos^{m-1}x \cdot \operatorname{sen}(m+1)x$.

97) $\theta = \frac{180}{\pi} \cdot \theta \Rightarrow \frac{d}{d\theta}(\operatorname{sen}\theta) = \frac{d}{d\theta}\operatorname{sen}\left(\frac{180}{\pi} \cdot \theta\right) =$

$= \frac{d}{d\theta}\operatorname{sen}\left(\frac{180\theta}{\pi}\right) = \cos\left(\frac{180\theta}{\pi}\right) \cdot \frac{\pi}{180} \Rightarrow$

$\Rightarrow \frac{d}{d\theta}(\operatorname{sen}\theta) = \frac{\pi}{180}\cos\theta$.

99) $y = f(g(x)) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = f'(g(x)) \cdot g'(x) \Rightarrow$

$\Rightarrow \frac{d^2y}{dy^2} = f'(g(x)) \cdot g''(x) + g'(x) \cdot f''(g(x)) \cdot g'(x) =$

$= \frac{d^2y}{du^2} \cdot \left(\frac{du}{dx}\right)^2 + \frac{dy}{du} \cdot \frac{d^2u}{dx^2}$.

Seção 3.5:

⑤ $\frac{d}{dx}(x^2-4xy+y^2)=\frac{d}{dx}(4) \Rightarrow 2x-4\left[y\cdot\frac{dx}{dx}+x\cdot\frac{dy}{dx}\right]+$

$+2y\cdot\frac{dy}{dx}=0 \Rightarrow 2x-4y-4x\frac{dy}{dx}+2y\frac{dy}{dx}=0 \Rightarrow$

$\Rightarrow \frac{dy}{dx}=\frac{4y-2x}{2y-4x}.$

⑦ $\frac{d}{dx}(x^4+x^2y^2-y^3)=\frac{d}{dx}(5) \Rightarrow 4x^3+\left[2x\cdot y^2+x^2\cdot 2y\frac{dy}{dx}\right]-$

$-3y^2\cdot\frac{dy}{dx}=0 \Rightarrow 4x^3+2xy^2+2x^2y\cdot\frac{dy}{dx}-3y^2\cdot\frac{dy}{dx}=0 \Rightarrow$

$\Rightarrow \frac{dy}{dx}=\frac{4x^3+2xy^2}{3y^2-2x^2y}.$

⑨ $\frac{d}{dx}\left(\frac{x^2}{x+y}\right)=\frac{d}{dx}(y^2+1) \Rightarrow \frac{(x+y)\cdot 2x-x^2\cdot\left(1+\frac{dy}{dx}\right)}{(x+y)^2}=$

$=2y\cdot\frac{dy}{dx} \Rightarrow 2x^2+2xy-x^2-x^2\cdot\frac{dy}{dx}=2y(x+y)^2\cdot\frac{dy}{dx} \Rightarrow$

$\Rightarrow \frac{dy}{dx}=\frac{x^2+2xy}{x^2+2y(x+y)^2}.$

⑪ $\frac{d}{dx}(x^2y^2+x\,\text{sen}\,y)=\frac{d}{dx}(4) \Rightarrow \left[2x\cdot y^2+x^2\cdot 2y\cdot\frac{dy}{dx}\right]+$

$$+\left[1\cdot \operatorname{sen} y + x\cdot \cos y\cdot \frac{dy}{dx}\right] = 0 \Rightarrow \frac{dy}{dx} = \frac{-2xy^2 - \operatorname{sen} y}{2x^2y + x\cos y}.$$

(15) $$\frac{d}{dx}\left(e^{x/y}\right) = \frac{d}{dx}(x-y) \Rightarrow e^{\frac{x}{y}}\cdot\frac{d}{dx}\left(\frac{x}{y}\right) = 1 - \frac{dy}{dx} \Rightarrow$$

$$\Rightarrow e^{\frac{x}{y}}\cdot\frac{y\cdot 1 - x\cdot\frac{dy}{dx}}{y^2} = 1 - \frac{dy}{dx} \Rightarrow e^{\frac{x}{y}}\cdot\left(y - x\cdot\frac{dy}{dx}\right) = y^2\left(1 - \frac{dy}{dx}\right)$$

$$\Rightarrow x e^{\frac{x}{y}}\frac{dy}{dx} - y^2\frac{dy}{dx} = e^{\frac{x}{y}}\cdot y - y^2 \Rightarrow \frac{dy}{dx} = \frac{e^{\frac{x}{y}}\cdot y - y^2}{e^{\frac{x}{y}}\cdot x - y^2}.$$

(21) $$\frac{d}{dx}\left[f(x) + x^2\left[f(x)\right]^3\right] = \frac{d}{dx}(10) \Rightarrow$$

$$\Rightarrow f'(x) + 2x\cdot\left[f(x)\right]^3 + x^2\cdot 3\left[f(x)\right]^2\cdot f'(x) = 0 \Rightarrow$$

$$\Rightarrow f'(1) + 2\cdot\left[f(1)\right]^3 + 3\left[f(1)\right]^2\cdot f'(1) = 0 \Rightarrow$$

$$\Rightarrow f'(1) + 2\cdot 2^3 + 3\cdot 2^2\cdot f'(1) = 0 \Rightarrow 13f'(1) = -16 \Rightarrow$$

$$\Rightarrow f'(1) = -\frac{16}{13}.$$

(27) $$\frac{d}{dx}\left(x^2 - xy - y^2\right) = \frac{d}{dx}(1) \Rightarrow 2x - \left(1\cdot y + x\cdot\frac{dy}{dx}\right) -$$

$$- 2y\cdot\frac{dy}{dx} = 0 \Rightarrow 2x - y - x\cdot\frac{dy}{dx} - 2y\cdot\frac{dy}{dx} \Rightarrow$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x-y}{x+2y} \Rightarrow \left.\frac{dy}{dx}\right|_{(x,y)=(2,1)} = \frac{2.2-1}{2+2.1} = \frac{3}{4} \Rightarrow$$

$$\Rightarrow t: y = \frac{3}{4}x + b \;;\; (2,1) \in t \Rightarrow 1 = \frac{3}{4}\cdot 2 + b \Rightarrow$$

$$\Rightarrow b = 1 - \frac{3}{2} = -\frac{1}{2} \quad \therefore \quad t: y = \frac{3}{4}x - \frac{1}{2}.$$

29) $$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}\left[(2x^2+2y^2-x)^2\right] \Rightarrow 2x + 2y\cdot\frac{dy}{dx} =$$

$$= 2(2x^2+2y^2-x)\left(4x + 4y\cdot\frac{dy}{dx} - 2\right) \Rightarrow$$

$$\Rightarrow 2\cdot 0 + 2\cdot\frac{1}{2}\cdot\left.\frac{dy}{dx}\right|_{(x,y)=\left(0,\frac{1}{2}\right)} = 2\cdot\left(2\cdot\frac{1}{4}\right)\cdot\left(2\cdot\left.\frac{dy}{dx}\right|_{(x,y)=\left(0,\frac{1}{2}\right)} - 2\right)$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(x,y)=\left(0,\frac{1}{2}\right)} = 2\left.\frac{dy}{dx}\right|_{(x,y)=\left(0,\frac{1}{2}\right)} - 2 \Rightarrow$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(x,y)=\left(0,\frac{1}{2}\right)} = 2 \Rightarrow t: y = 2x + b \;;\; \left(0,\frac{1}{2}\right) \in t$$

$$\Rightarrow \frac{1}{2} = 2.0 + b \Rightarrow b = \frac{1}{2} \quad \therefore \quad t: y = 2x + \frac{1}{2}.$$

31) $$\frac{d}{dx}\left[2(x^2+y^2)^2\right] = \frac{d}{dx}\left[25(x^2-y^2)\right] \Rightarrow$$

$$\Rightarrow 2\cdot 2\cdot(x^2+y^2)\cdot\left(2x + 2y\cdot\frac{dy}{dx}\right) = 25\left(2x - 2y\cdot\frac{dy}{dx}\right) \Rightarrow$$

$$\Rightarrow 4(9+1)\cdot\left(6+2\cdot 1\cdot \left.\frac{dy}{dx}\right|_{(x,y)=(3,1)}\right) = 25\cdot\left(2\cdot 3 - 2\cdot 1\cdot \left.\frac{dy}{dx}\right|_{(x,y)=(3,1)}\right)$$

$$\Rightarrow 240 + 80\left.\frac{dy}{dx}\right|_{(x,y)=(3,1)} = 150 - 50\left.\frac{dy}{dx}\right|_{(x,y)=(3,1)} \Rightarrow$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{(x,y)=(3,1)} = -\frac{90}{130} = -\frac{9}{13} \Rightarrow t: y = -\frac{9}{13}\cdot x + b$$

$$\Rightarrow 1 = -\frac{9}{13}\cdot 3 + b \Rightarrow b = 1 + \frac{27}{13} = \frac{40}{13} \therefore$$

$$\therefore y = -\frac{9}{13}x + \frac{40}{13}.$$

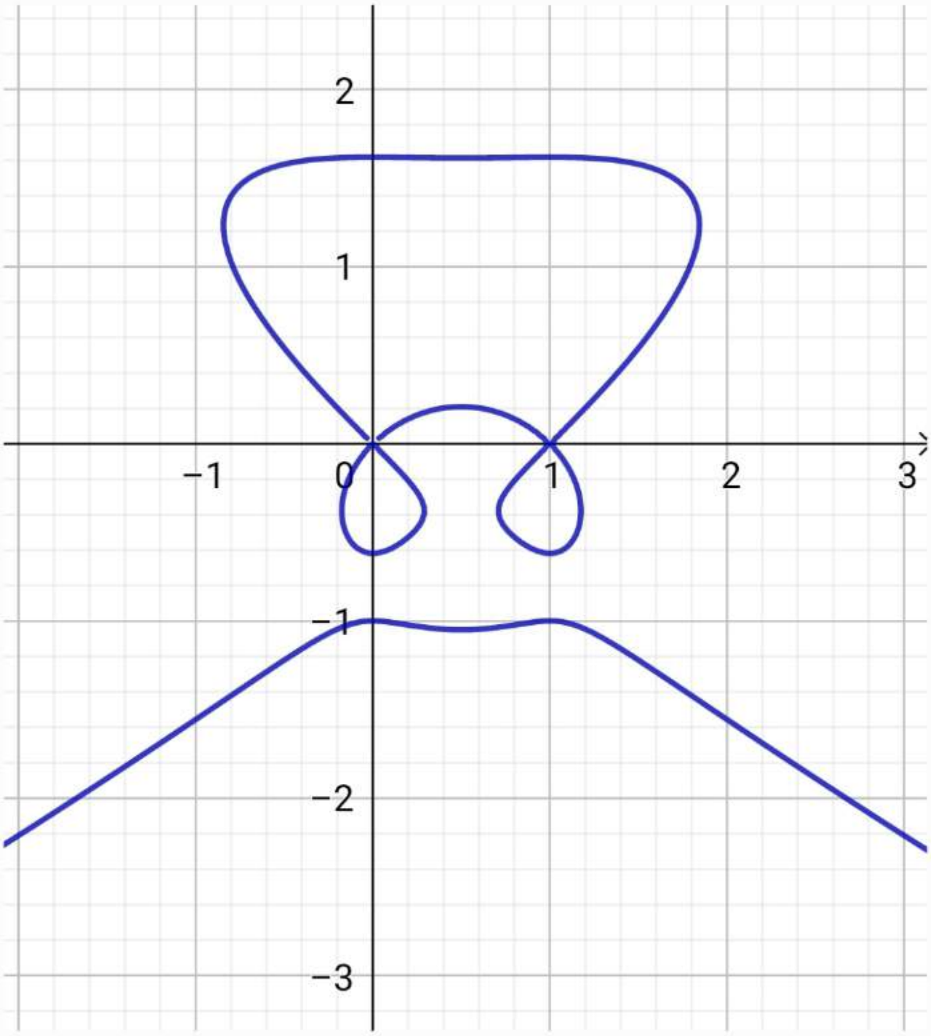
35 $$\frac{d}{dx}(x^2+4y^2) = \frac{d}{dx}(4) \Rightarrow 2x + 4\cdot 2y\cdot\frac{dy}{dx} = 0 \Rightarrow$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{4y} \Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(-\frac{x}{4y}\right) =$$

$$= -\frac{4y\cdot 1 - x\cdot 4\frac{dy}{dx}}{(4y)^2} = \frac{4x\cdot\left(-\frac{x}{4y}\right) - 4y}{16y^2} = \frac{-\frac{x^2}{y} - y}{4y^2} =$$

$$= -\frac{x^2}{4y^3} - \frac{1}{4y} \therefore y'' = -\frac{x^2}{4y^3} - \frac{1}{4y}.$$

42 a)



b) $\frac{d}{dx}\left(2y^3+y^2-y^5\right)=\frac{d}{dx}\left(x^4-2x^3+x^2\right) \Rightarrow$

$\Rightarrow 2\cdot 3y^2\cdot\frac{dy}{dx}+2y\cdot\frac{dy}{dx}-5y^4\cdot\frac{dy}{dx}=4x^3-6x^2+2x \Rightarrow$

$\Rightarrow \frac{dy}{dx}=\frac{4x^3-6x^2+2x}{-5y^4+6y^2+2y}=0 \Leftrightarrow 4x^3-6x^2+2x=0 \Leftrightarrow$

$\Leftrightarrow 4x(x-1)\left(x-\frac{1}{2}\right)=0 \Leftrightarrow x\in\left\{0,\frac{1}{2},1\right\}$ e do gráfico

vemos que há 9 tangentes horizontais para $x\in\left\{0,\frac{1}{2},1\right\}$.

43 Do exercício 31: $4(x^2+y^2)\cdot\left(x+y\frac{dy}{dx}\right)=25\left(x-y\cdot\frac{dy}{dx}\right)$.

$\frac{dy}{dx}=0 \Leftrightarrow 4(x^2+y^2)\cdot x=25x \Leftrightarrow x=0$ ou $x^2+y^2=\frac{25}{4}$.

$$\begin{cases} x^2+y^2=\frac{25}{4} \\ 2(x^2+y^2)^2=25(x^2-y^2) \end{cases} \iff \begin{cases} x^2+y^2=\frac{25}{4} \\ x^2-y^2=\frac{25}{8} \end{cases} \iff \begin{cases} x^2=\frac{75}{16} \\ y^2=\frac{25}{16} \end{cases} \iff$$

$$\iff \begin{cases} x=\pm\frac{5}{4}\sqrt{3} \\ y=\pm\frac{5}{4} \end{cases} \iff (x,y)\in\left\{\left(\frac{5}{4}\sqrt{3},\frac{5}{4}\right),\left(-\frac{5}{4}\sqrt{3},\frac{5}{4}\right),\left(\frac{5}{4}\sqrt{3},-\frac{5}{4}\right),\left(-\frac{5}{4}\sqrt{3},-\frac{5}{4}\right)\right\}.$$

$$\begin{cases} x=0 \\ 2(x^2+y^2)^2=25(x^2-y^2) \end{cases} \iff \begin{cases} x=0 \\ 2y^4=-25y^2 \end{cases} \iff \begin{cases} x=0 \\ y^2\underbrace{(2y^2+25)}_{>0}=0 \end{cases}$$

$$\iff \begin{cases} x=0 \\ y=0 \end{cases} \iff (x,y)\in\{(0,0)\}.$$

$$\therefore \frac{dy}{dx}=0 \iff$$

$$\iff (x,y)\in\left\{(0,0),\left(\frac{5}{4}\sqrt{3},\frac{5}{4}\right),\left(-\frac{5}{4}\sqrt{3},\frac{5}{4}\right),\left(\frac{5}{4}\sqrt{3},-\frac{5}{4}\right),\left(-\frac{5}{4}\sqrt{3},-\frac{5}{4}\right)\right\}.$$

44 $$\frac{d}{dx}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=\frac{d}{dx}(1) \Rightarrow \frac{2x}{a^2}+\frac{2y}{b^2}\cdot\frac{dy}{dx}=0 \Rightarrow$$

$$\Rightarrow \frac{dy}{dx}=-\frac{x}{a^2}\cdot\frac{b^2}{y} \Rightarrow \left.\frac{dy}{dx}\right|_{(x,y)=(x_0,y_0)}=-\frac{x_0}{a^2}\cdot\frac{b^2}{y_0} \Rightarrow$$

$$\Rightarrow t: y = -\frac{x_0 b^2}{a^2 y_0} \cdot x + K \; ; \; (x_0, y_0) \in t \Rightarrow$$

$$\Rightarrow y_0 = -\frac{x_0^2 b^2}{a^2 y_0} + K \Rightarrow K = \frac{x_0^2 b^2}{a^2 y_0} + y_0 \Rightarrow$$

$$\Rightarrow t: y = -\frac{x_0 b^2}{a^2 y_0} \cdot x + \frac{x_0^2 b^2}{a^2 y_0} + y_0 \Rightarrow$$

$$\Rightarrow t: \frac{y y_0}{b^2} = -\frac{x x_0}{a^2} + \frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} \underset{\substack{\uparrow \\ (x_0, y_0) \in \text{elipse}}}{=} -\frac{x x_0}{a^2} + 1 \; \therefore$$

$$\therefore \; t: \frac{x x_0}{a^2} + \frac{y y_0}{b^2} = 1 .$$

48 $$\frac{d}{dx}(y^q) = \frac{d}{dx}(x^p) \Rightarrow q y^{q-1} \cdot \frac{dy}{dx} = p x^{p-1} \Rightarrow$$

$$\Rightarrow \frac{dy}{dx} = \frac{p x^{p-1}}{q (x^{p/q})^{q-1}} = \frac{p}{q} \cdot \frac{x^{p-1}}{x^{p - \frac{p}{q}}} = \frac{p}{q} \cdot x^{\frac{p}{q} - 1} .$$

51 $$\text{sen}\, y = 2x + 1 \Rightarrow \frac{d}{dx}(\text{sen}\, y) = \frac{d}{dx}(2x+1) \Rightarrow$$

$$\Rightarrow \cos y \cdot \frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{2}{\cos y} = \frac{2}{\cos[\text{arcsen}\,(2x+1)]} .$$

58) $\cos y = \operatorname{arcsen} t \Rightarrow \frac{d}{dt}(\cos y) = \frac{d}{dt}(\operatorname{arcsen} t) \Rightarrow$

$\Rightarrow -\operatorname{sen} y \cdot \frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}} \Rightarrow \frac{dy}{dt} = \frac{-1}{\sqrt{1-t^2}\cdot\sqrt{1-(\operatorname{arcsen} t)^2}}.$

60) $\operatorname{tg} y = \sqrt{\frac{1-x}{1+x}} \Rightarrow \frac{d}{dx}(\operatorname{tg} y) = \frac{d}{dx}\left(\sqrt{\frac{1-x}{1+x}}\right) \Rightarrow$

$\Rightarrow \sec^2 y \cdot \frac{dy}{dx} = \frac{1}{2}\cdot\sqrt{\frac{1+x}{1-x}}\cdot\frac{(1+x).(-1)-(1-x).1}{(1+x)^2} \Rightarrow$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sec^2\left[\operatorname{arctg}\sqrt{\frac{1-x}{1+x}}\right]}\cdot\frac{1}{\sqrt{1-x^2}}.$

67) $y = cx^2 \Rightarrow \frac{dy}{dx} = 2cx$;

$x^2 + 2y^2 = k \Rightarrow \frac{d}{dx}(x^2+2y^2) = \frac{d}{dx}(k) \Rightarrow$

$\Rightarrow 2x + 4y\cdot\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-x}{2y}$;

Seja (x_0, y_0) ponto de interseção das curvas genéricas:

$$\begin{cases} y_0 = cx_0^2 \Rightarrow \left.\frac{dy}{dx}\right|_{(x,y)=(x_0,y_0)} = 2cx_0 \\ x_0^2 + 2y_0^2 = k \Rightarrow \left.\frac{dy}{dx}\right|_{(x,y)=(x_0,y_0)} = \frac{-x_0}{2y_0} = \frac{-x_0}{2cx_0^2} = -\frac{1}{2cx_0} \end{cases} \Rightarrow$$

$\Rightarrow$ As tangentes são perpendiculares $\Rightarrow$ As curvas são ortogonais.

73) $x^2 - xy + y = 3$; $y = 0 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3} \Rightarrow$

$\Rightarrow$ a elipse toca o eixo x em $(\sqrt{3}, 0)$ e $(-\sqrt{3}, 0)$.

$$\frac{d}{dx}\left(x^2 - xy + y^2\right) = \frac{d}{dx}(3) \Rightarrow 2x - \left(1\cdot y + x\cdot\frac{dy}{dx}\right) + 2y\cdot\frac{dy}{dx} =$$

$$= 0 \Rightarrow (2y - x)\frac{dy}{dx} = y - 2x \Rightarrow \frac{dy}{dx} = \frac{y-2x}{2y-x}.$$

$$\left.\frac{dy}{dx}\right|_{(x,y)=(\sqrt{3},0)} = \frac{-2\sqrt{3}}{-\sqrt{3}} = 2 \Rightarrow t_1 : y = 2x + b_1 ;$$

$(\sqrt{3}, 0) \in t \Rightarrow 2\sqrt{3} + b_1 = 0 \Rightarrow b_1 = -2\sqrt{3} \Rightarrow$

$\Rightarrow t_1 : y = 2x - 2\sqrt{3}$.

$$\left.\frac{dy}{dx}\right|_{(x,y)=(-\sqrt{3},0)} = \frac{-2(-\sqrt{3})}{-(-\sqrt{3})} = 2 \Rightarrow t_2 : y = 2x + b_2 ;$$

$(-\sqrt{3}, 0) \in t_2 \Rightarrow -2\sqrt{3} + b_2 = 0 \Rightarrow b_2 = 2\sqrt{3} \Rightarrow$

$\Rightarrow t_2 : y = 2x + 2\sqrt{3}$.

t_1 e t_2 têm inclinação 2 $\Rightarrow$ são paralelas.

77) a) $f(f^{-1}(x)) = x \Rightarrow \frac{d}{dx} f(f^{-1}(x)) = \frac{d}{dx}(x) \Rightarrow$

$\Rightarrow f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1 \Rightarrow (f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$.

b) $f(4) = 5 \Rightarrow f^{-1}(5) = 4 \Rightarrow (f^{-1})'(5) = \dfrac{1}{f'(f^{-1}(5))} =$

$= \dfrac{1}{f'(4)} = \dfrac{3}{2}$.

80 Borda: $x^2 + 4y^2 = 5 \Rightarrow \dfrac{x^2}{5} + \dfrac{y^2}{\frac{5}{4}} = 1$.

No exercício 44, a equação da tangente à borda no ponto (x_0, y_0) é $t: \dfrac{xx_0}{5} + \dfrac{yy_0}{\frac{5}{4}} = 1$; $(-5, 0) \in t$

$\Rightarrow -\dfrac{5x_0}{5} = 1 \Rightarrow x_0 = -1 \Rightarrow (-1)^2 + 4y_0^2 = 5 \Rightarrow 4y_0^2 = 4$

$\Rightarrow y_0^2 = 1 \underset{y_0 > 0}{\Rightarrow} y_0 = 1 \Rightarrow t: -\dfrac{x}{5} + \dfrac{4y}{5} = 1$;

$(3, L) \in t \Rightarrow -\dfrac{3}{5} + \dfrac{4L}{5} = 1 \Rightarrow 4L - 3 = 5 \Rightarrow 4L = 8$

$\Rightarrow L = 2 \therefore$ A altura da lâmpada é 2.

Seção 3.6:

(3) $f'(x) = \cos(\ln x) \cdot \frac{1}{x} = \frac{\cos(\ln x)}{x}$.

(10) $g'(t) = \frac{1}{2\sqrt{1+\ln t}} \cdot \left(0 + \frac{1}{t}\right) = \frac{1}{2t\sqrt{1+\ln t}}$.

(13) $g'(x) = \frac{1}{x\sqrt{x^2-1}} \cdot \left(1 \cdot \sqrt{x^2-1} + x \cdot \frac{1}{2\sqrt{x^2-1}} \cdot 2x\right) =$

$= \frac{1}{x} + \frac{x}{x^2-1} = \frac{2x^2-1}{x(x^2-1)}$.

(16) $\frac{dy}{dt} = \frac{1}{1+t-t^3} \cdot (0 + 1 - 3t^2) = \frac{1-3t^2}{1+t-t^3}$.

(20) $H'(z) = \frac{1}{\sqrt{\frac{a^2-z^2}{a^2+z^2}}} \cdot \frac{1}{2\sqrt{\frac{a^2-z^2}{a^2+z^2}}} \cdot \frac{(a^2+z^2)\cdot(-2z) - (a^2-z^2)\cdot 2z}{(a^2+z^2)^2}$

$= \frac{1}{2} \cdot \frac{a^2+z^2}{a^2-z^2} \cdot \frac{-4za^2}{(a^2+z^2)^2} = \frac{-2za^2}{(a^2+z^2)(a^2-z^2)}$.

(22) $\frac{dy}{dx} = \frac{1}{x\log_5 x \cdot \ln 2} \cdot \left(1 \cdot \log_5 x + x \cdot \frac{1}{x \ln 5}\right) =$

$= \frac{1}{x \ln 2} + \frac{1}{x \log_5 x \cdot \ln 2 \cdot \ln 5}$.

(25) $\frac{dy}{dx} = \frac{1}{\sec x} \cdot \sec x \cdot \operatorname{tg} x = \operatorname{tg} x \Rightarrow \frac{d^2y}{dx^2} = \sec^2 x$.

(31) $x \in D(f) \iff x > 0 \ \wedge \ x + \ln x > 0 \iff$

$\iff x > 0 \ \wedge \ \ln x > -x \iff x > x_0$, sendo

$x_0 = e^{-x_0} \cong 0{,}567$.

$$f'(x) = \frac{1}{x+\ln x} \cdot \left(1 + \frac{1}{x}\right) \Rightarrow f'(1) = \frac{1}{1+0} \cdot (1+1) = 2.$$

(39) $\ln y = 5\ln(2x+1) + 6\ln(x^4-3)$

$$\frac{d}{dx}(\ln y) = 5 \cdot \frac{1}{2x+1} \cdot 2 + 6 \cdot \frac{1}{x^4-3} \cdot 4x^3 \Rightarrow$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{10}{2x+1} + \frac{24x^3}{x^4-3} \Rightarrow$$

$$\Rightarrow \frac{dy}{dx} = (2x+1)^5 (x^4-3)^6 \cdot \left(\frac{10}{2x+1} + \frac{24x^3}{x^4-3}\right).$$

(40) $\ln y = \frac{1}{2}\ln x + x^2 + 10\ln(x^2+1)$

$$\frac{y'}{y} = \frac{1}{2} \cdot \frac{1}{x} + 2x + 10 \cdot \frac{1}{x^2+1} \cdot 2x$$

$$y' = \sqrt{x}\, e^{x^2} (x^2+1)^{10} \left(\frac{1}{2x} + 2x + \frac{20x}{x^2+1}\right).$$

(49) $\ln y = \frac{1}{x} \cdot \ln(\operatorname{tg} x)$

$$\frac{y'}{y} = -\frac{1}{x^2} \cdot \ln(\operatorname{tg} x) + \frac{1}{x} \cdot \frac{1}{\operatorname{tg} x} \cdot \sec^2 x$$

$$y' = (\operatorname{tg} x)^{1/x}\left[\frac{\sec^2 x}{x \cdot \operatorname{tg} x} - \frac{\ln(\operatorname{tg} x)}{x^2}\right].$$

53 $f(x) = \ln(x-1) \Rightarrow f^{(1)}(x) = \dfrac{1}{x-1} \Rightarrow f^{(2)}(x) = \dfrac{-1}{(x-1)^2}$

$$\Rightarrow f^{(3)}(x) = \frac{1}{(x-1)^3} \Rightarrow \dots \Rightarrow f^{(n)}(x) = \frac{(-1)^{n+1}}{(x-1)^n} = \frac{-1}{(1-x)^n}.$$

55 $\displaystyle\lim_{x\to 0} \frac{\ln(1+x)}{x} = \lim_{h\to 0} \frac{\ln(1+h) - \ln 1}{h} = \frac{d}{dx}(\ln x)\Big|_{x=1} =$

$$= \left[\frac{1}{x}\right]_{x=1} = \frac{1}{1} = 1.$$

56 $y = \left(1+\dfrac{x}{n}\right)^n \Rightarrow \sqrt[x]{y} = \left(1+\dfrac{x}{n}\right)^{n/x} = \left(1+\dfrac{1}{k}\right)^k$, sendo $k =$

$= \dfrac{n}{x}$. Temos $\displaystyle\lim_{n\to\infty} k = \lim_{n\to\infty}\left(\frac{n}{x}\right) = \infty \Rightarrow \lim_{n\to\infty} \sqrt[x]{y} =$

$$= \lim_{k\to\infty}\left(1+\frac{1}{k}\right)^k = e \Rightarrow \lim_{n\to\infty} y = e^x \therefore$$

$$\therefore \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x,\ x>0.$$

Seção 3.10:

4) $f(x) = 2^x \Rightarrow f'(x) = 2^x \cdot \ln 2$

$L(x) = f(a) + f'(a) \cdot (x-a) \Rightarrow L(x) = f(0) + f'(0) \cdot x \Rightarrow$

$\Rightarrow L(x) = 1 + x \cdot \ln 2.$

11) a) $y = xe^{-4x} \Rightarrow \frac{dy}{dx} = 1 \cdot e^{-4x} + x \cdot (-4)e^{-4x} = e^{-4x}(1-4x)$

$\Rightarrow dy = e^{-4x}(1-4x)\,dx.$

b) $y = \ln\sqrt{1+t^4} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1+t^4}} \cdot \frac{1}{2\sqrt{1+t^4}} \cdot 4t^3 =$

$= \frac{2t^3}{1+t^4} \Rightarrow dy = \frac{2t^3}{1+t^4} \cdot dx.$

18) a) $y = \frac{x+1}{x-1} \Rightarrow \frac{dy}{dx} = \frac{(x-1) \cdot 1 - (x+1) \cdot 1}{(x-1)^2} = \frac{-2}{(x-1)^2} \Rightarrow$

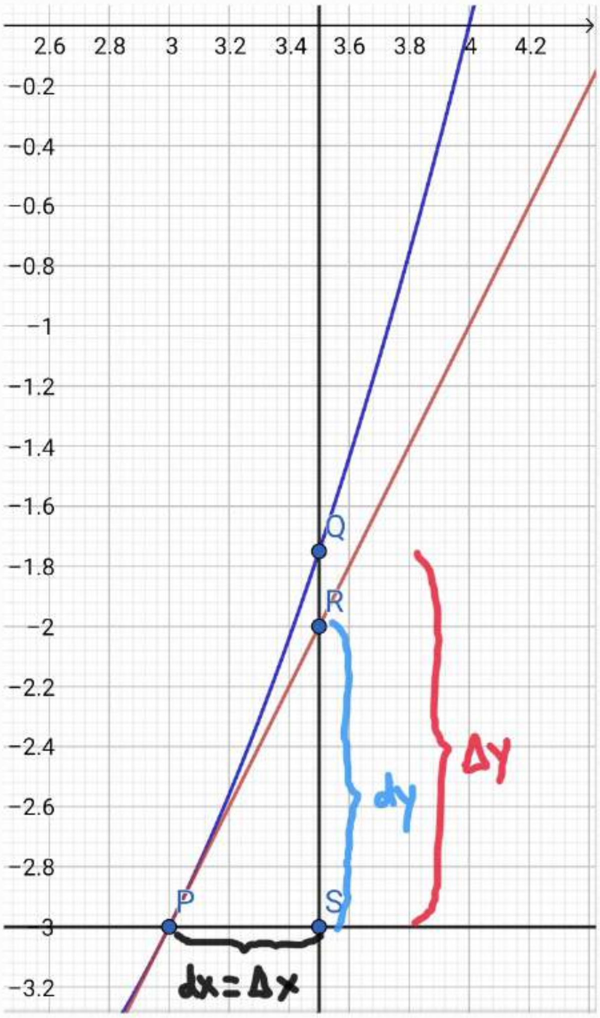
$\Rightarrow dy = \frac{-2}{(x-1)^2}\,dx$

b) $dy = \frac{-2}{(2-1)^2} \cdot 0{,}05 = -2 \cdot 0{,}05 = -0{,}1.$

19) $\Delta y = y(x+\Delta x) - y(x) = y(3{,}5) - y(3) = 3{,}5^2 - 4 \cdot 3{,}5 -$

$- 3^2 + 4 \cdot 3 = 1{,}25;\ \frac{dy}{dx} = 2x - 4 \Rightarrow dy = (2x-4)\,dx =$

$= (2 \cdot 3 - 4) \cdot 0{,}5 = 1.$



(23) $y = x^4 \ , \ x = 2 \ , \ dx = -0{,}001$

$$\frac{dy}{dx} = 4x^3 \Rightarrow dy = 4x^3\,dx = 4 \cdot 2^3 \cdot (-0{,}001) = -0{,}032$$

$$\Rightarrow (1{,}999)^4 \cong 2^4 - 0{,}032 = 15{,}968.$$

(24) $y = \dfrac{1}{x} \ , \ x = 4 \ , \ dx = 0{,}002$

$$\frac{dy}{dx} = -\frac{1}{x^2} \Rightarrow dy = -\frac{dx}{x^2} = -\frac{0{,}002}{16} = -0{,}000125$$

$$\Rightarrow \frac{1}{4{,}002} \cong \frac{1}{4} - 0{,}000125 = 0{,}249875$$

(28) $y = \cos x \ , \ x = \dfrac{\pi}{6} \ , \ dx = -\dfrac{\pi}{180}$

$$\frac{dy}{dx} = -\operatorname{sen} x \Rightarrow dy = -\operatorname{sen} x\,dx = -\frac{1}{2} \cdot \left(\frac{-\pi}{180}\right) = \frac{\pi}{360}$$

$$\Rightarrow \cos 29^\circ \cong \frac{\sqrt{3}}{2} + \frac{\pi}{360} \cong 0{,}8747.$$

(33) a) $V = a^3 \Rightarrow \frac{dV}{da} = 3a^2 \Rightarrow dV = 3a^2\, da = 3 \cdot 30^2 \cdot 0{,}1 =$

$= 270\ cm^3 \Rightarrow \frac{dV}{V} = \frac{3a^2\, da}{a^3} = \frac{3\, da}{a} = \frac{3 \cdot 0{,}1}{30} = 0{,}01 \therefore$

erro máximo $= \pm 270\ cm^3$.
erro relativo $= \pm 0{,}01$.
erro percentual $= \pm 1\%$.

b) $A = 6a^2 \Rightarrow \frac{dA}{da} = 12a \Rightarrow dA = 12a\, da = 12 \cdot 30 \cdot 0{,}1 = 36\ cm^2$

$\Rightarrow \frac{dA}{A} = \frac{12a\, da}{6a^2} = \frac{2\, da}{a} = \frac{2 \cdot 0{,}1}{30} = \frac{0{,}1}{15} \cong 0{,}0066.$

erro máximo $= \pm 36\ cm^2$.
erro relativo $\cong \pm 0{,}0066$.
erro percentual $\cong \pm 0{,}66\%$.

(35) a) $A = \frac{C^2}{\pi} \Rightarrow \frac{dA}{dC} = \frac{2C}{\pi} \Rightarrow dA = \frac{2C}{\pi} \cdot dC = \frac{2 \cdot 84}{\pi} \cdot 0{,}5 =$

$= \frac{84}{\pi} \Rightarrow \frac{dA}{A} = \frac{\frac{2C \cdot dC}{\pi}}{\frac{C^2}{\pi}} = \frac{2}{C} \cdot dC = \frac{2 \cdot 0{,}5}{84} = \frac{1}{84}$

erro máximo $= \pm 26{,}738\ cm^2$.
erro relativo $\cong \pm 0{,}0119$.

b) $V = \frac{1}{6\pi^2} \cdot C \Rightarrow \frac{dV}{dC} = \frac{1}{6\pi^2} \Rightarrow dV = \frac{dC}{6\pi^2} = \frac{0,5}{6\pi^2} = \frac{1}{12\pi^2}.$

$$\frac{dV}{V} = \frac{\frac{dC}{6\pi^2}}{\frac{C}{6\pi^2}} = \frac{dC}{C} = \frac{0,5}{84} = \frac{1}{168}$$

erro máximo $= \pm 0,00844 \text{ cm}^3$.

erro relativo $\cong \pm 0,00595$.

38 a) $h = \frac{20}{\text{sen}\,\theta} \Rightarrow \frac{dh}{d\theta} = \frac{-20}{\text{sen}^2\theta} \cdot \cos\theta \Rightarrow dh = \frac{-20}{\text{sen}^2\theta} \cdot \cos\theta \cdot d\theta$

$$= \frac{-20}{\frac{1}{4}} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\pi}{180} = \frac{-2\pi\sqrt{3}}{9} \cong -1,2092$$

erro máximo $\cong \pm 1,2092$.

b) $$\frac{dh}{h} = \frac{\frac{-20}{\text{sen}^2\theta} \cdot \cos\theta \cdot d\theta}{\frac{20}{\text{sen}\theta}} = -\frac{\cos\theta}{\text{sen}\theta}\, d\theta = -\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \cdot \frac{\pi}{180} = -\frac{\pi\sqrt{3}}{180}$$

erro relativo $\cong \pm 0,0302$.

43 a) $L(x) = f(1) + f'(1) \cdot (x-1) \Rightarrow L(x) = 5 + 2 \cdot (x-1)$

$f(0,9) \cong 5 + 2 \cdot (0,9-1) = 5 - 2 \cdot 0,1 = 4,8.$

$f(1,1) \cong 5 + 2 \cdot (1,1-1) = 5 + 2 \cdot 0,1 = 5,2.$

b) $f'(x)$ decresce com o crescimento de x, logo f tem concavidade para baixo, ou seja, a reta tangente fica acima da curva, nos levando a estimar valores maiores no item a.