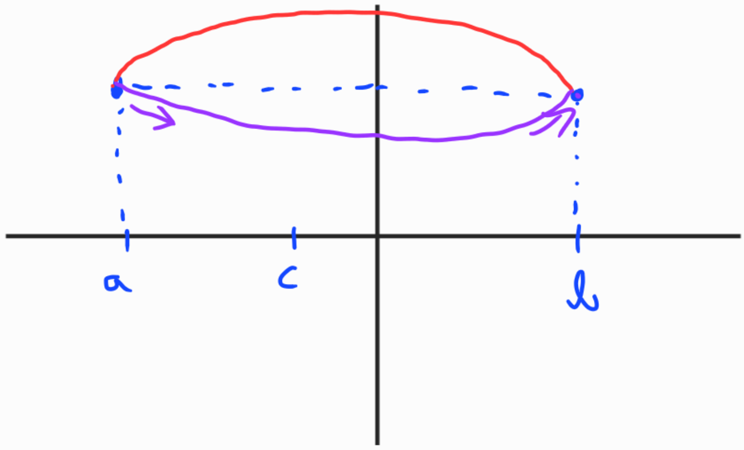
**Teorema de Rolle**: $f$ é contínua em $[a, b]$, derivável em $(a, b)$, $f(a) = f(b) \Rightarrow \exists c \in (a, b)$ tq. $f'(c) = 0$




**Ex. 2** $x^3 + x - 1 = 0$ tem exatamente 1 raiz real.

$$\left.\begin{array}{l} f(0) = -1 < 0 \\ f(1) = 1 > 0 \end{array}\right\} \text{TVI} : \exists c \in (0,1) \text{ tq. } f(c) = 0$$

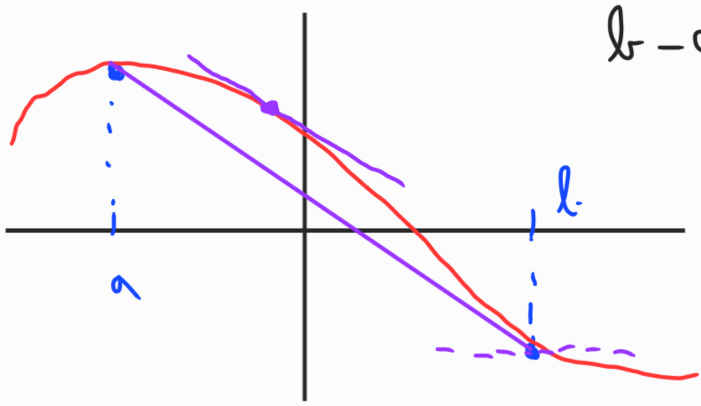
Supõe que $\exists\, a, b$ raízes $\Rightarrow f(a) = f(b) = 0$

$f'(x) = 3x^2 + 1 \geq 3 \cdot 0^2 + 1 = 1, \ \forall x \in \mathbb{R}$

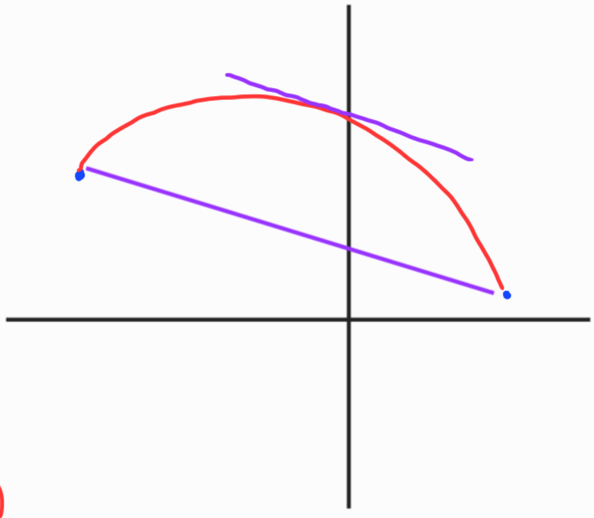
É impossível que $f'(c) = 0$, para algum $c \in (a, b) \Rightarrow$

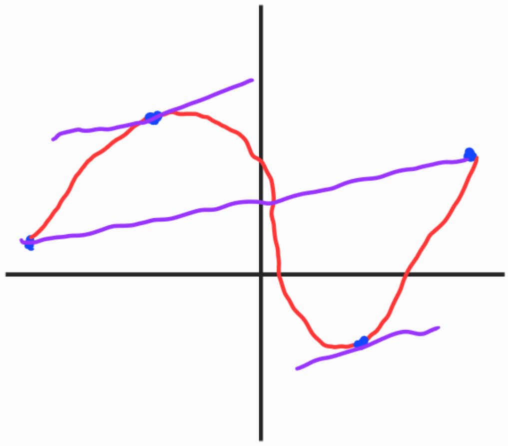
$\Rightarrow f$ possui apenas uma raiz real.

**TVM**: $f$ é contínua em $[a, b]$, derivável em $(a, b)$ $\Rightarrow \exists c \in (a, b)$ tq. $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.




$m = \dfrac{f(b) - f(a)}{b - a}$





21, 29

$\boxed{4.2}$ 9 $f(x) = 1 - x^{2/3} = 1 - \sqrt[3]{x^2}$

$$\left.\begin{aligned} f(-1) &= 1 - \sqrt[3]{(-1)^2} = 1 - \sqrt[3]{1} = 1 - 1 = 0 \\ f(1) &= 1 - \sqrt[3]{1^2} = 1 - 1 = 0 \end{aligned}\right\} f(-1) = f(1)$$

Mostre que $\nexists c \in (-1,1)$ tq. $f'(c) = 0$.

$$f'(x) = -\frac{2}{3} x^{-1/3} = \frac{-2}{3\sqrt[3]{x}}, \quad \begin{cases} \nexists f'(0) \\ f'(x) \neq 0, \ \forall x \neq 0 \end{cases}$$

Porque $f$ ñ é derivável em $(-1,1)$.

27 $x^3 - 15x + c = 0$ tem no máx 1 raiz em $[-2,2]$

$f(x) = x^3 - 15x + c$

$f(-2) = -8 + 30 + c = c + 22$

$f(2) = 8 - 30 + c = c - 22$

Supor que $a,b \in [-2,2]$, $f(a) = f(b) = 0 \Rightarrow \exists c_1 \in (a,b)$ tq $f'(c_1) = 0$

$f'(x) = 3x^2 - 15 \Rightarrow 3c_1^2 - 15 = 0 \Rightarrow c_1 = \pm\sqrt{5} =$

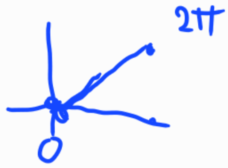
$= \pm 2,\ldots \notin [-2,2] \Rightarrow$ absurdo! $\Rightarrow \exists$ no max 1 raiz em $[-2, 2]$ $\qquad x - \text{sen}\, x > 0 \Rightarrow f(x) > 0$

29) Mostre que $\text{sen}\, x < x$ se $0 < x < 2\pi$ $\qquad 0 < x < 2\pi$

Suponha $\text{sen}\, x \geq x$, $x \in [0, 2\pi]$ $\qquad f(x) = 0$

Defina $f(x) = x - \text{sen}\, x \Rightarrow f(0) = 0$

$f(2\pi) = 2\pi$




$$\left.\begin{array}{r} f'(x) = 1 - \cos x \geq 0 \\ f(0) = 0 \\ f'(0) = 0 \end{array}\right\} \quad f(x) \geq 0, \ \forall x \geq 0$$

$f(0) = 0$, suponha que $\exists\, a \in (0, 2\pi)$ tq. $f(a) = 0$

$\Rightarrow \exists\, c \in (0, a)$ tq. $f'(c) = 0 \Rightarrow 1 - \cos c = 0 \Rightarrow$

$\Rightarrow \cos c = 1$, absurdo! $\Rightarrow f(x) > 0, \ \forall x \in [0, 2\pi]$.

Regra de L'Hôspital: $f$ e $g$ deriváveis e $g'(x) \neq 0$ para $x \in I$, intervalo aberto que contém $a$ (pode ocorrer $g'(a) = 0$), $\dfrac{f(x)}{g(x)}$ é uma forma indeterminada do tipo $\dfrac{0}{0}$ ou $\dfrac{\infty}{\infty}$ $\Rightarrow \lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to a} \dfrac{f'(x)}{g'(x)}$

quando $\lim\limits_{x \to a} \dfrac{f'(x)}{g'(x)}$ existir ou for $\pm\infty$.

Indeterminações: $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0.\infty$, $\infty-\infty$, $0^0$, $\infty^0$ e $1^\infty$

Exemplos:

① $\lim\limits_{x\to -1} \frac{x^2-1}{x+1} = \lim\limits_{x\to -1} \frac{2x}{1} = 2.(-1) = -2.$

② $\lim\limits_{x\to \frac{\pi}{2}^+} \frac{\cos x}{1-\operatorname{sen} x} = \lim\limits_{x\to \frac{\pi}{2}^+} \frac{-\operatorname{sen} x}{-\cos x} = \lim\limits_{x\to \frac{\pi}{2}^+} \operatorname{tg} x =$

$= -\infty$

③ $\lim\limits_{x\to 0} \frac{\operatorname{sen} 4x}{\operatorname{tg} 5x} = \lim\limits_{x\to 0} \frac{4\cos 4x}{5\sec^2 5x} = \frac{4}{5} . \lim\limits_{x\to 0} (\cos 4x)(\cos^2 5x)$

$= \frac{4}{5} \lim\limits_{x\to 0} \cos 4x \cdot \lim\limits_{x\to 0} \cos^2 5x = \frac{4}{5}.1.1^2 = \frac{4}{5}.$

④ $\lim\limits_{x\to \infty} \frac{\ln x}{\sqrt{x}} = \lim\limits_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim\limits_{x\to\infty} \frac{2}{\sqrt{x}} = 0.$

⑤ $\lim\limits_{t\to 0} \frac{8^t-5^t}{t} = \lim\limits_{t\to 0} \frac{8^t \ln 8 - 5^t \ln 5}{1} = \ln 8 - \ln 5 =$

$= \ln\left(\frac{8}{5}\right).$

$$\boxed{\frac{d}{dx}(b^x) = b^x . \ln b}$$

$$\lim_{x\to\infty} x\cdot \operatorname{sen}\left(\frac{\pi}{x}\right) = \lim_{x\to\infty} \frac{\operatorname{sen}\left(\frac{\pi}{x}\right)}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\cos\left(\frac{\pi}{x}\right)\cdot\left(-\frac{\pi}{x^2}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \pi\cos\left(\frac{\pi}{x}\right) = \pi\cdot 1 = \pi.$$

6 $$\lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{\ln x}\right) = \lim_{x\to 1} \frac{x\ln x - x + 1}{(x-1)\ln x} =$$

$$= \lim_{x\to 1} \frac{\ln x + 1 - 1}{\frac{x-1}{x} + \ln x} = \lim_{x\to 1} \frac{x\ln x}{x-1+x\ln x} =$$

$$= \lim_{x\to 1} \frac{1+\ln x}{1+1+\ln x} = \frac{1}{2}.$$

7 $$\lim_{x\to\infty} \frac{e^x}{x^n} = \lim_{x\to\infty} \frac{e^x}{n x^{n-1}} = \lim_{x\to\infty} \frac{e^x}{n(n-1)x^{n-2}} =$$

$$\cdots = \lim_{x\to\infty} \frac{e^x}{n!} = \infty, \quad n\in\mathbb{Z}_+$$