3.5 51 $y = \operatorname{sen}^{-1}(2x+1) = \operatorname{arcsen}(2x+1)$

$\operatorname{sen} y = 2x+1 \Rightarrow \frac{d}{dx}(\operatorname{sen} y) = \frac{d}{dx}(2x+1) \Rightarrow$

$\Rightarrow \cos y \cdot \frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{2}{\cos y} = \frac{2}{\cos(\operatorname{arcsen}(2x+1))}$

58 $y = \cos^{-1}(\operatorname{sen}^{-1} t) = \arccos(\operatorname{arcsen} t)$

$\cos y = \operatorname{arcsen} t \Rightarrow \operatorname{sen}(\cos y) = t \Rightarrow$

$\Rightarrow \frac{d}{dt} \operatorname{sen}(\cos y) = \frac{d}{dt}(t) \Rightarrow$

$\Rightarrow \cos(\cos y) \cdot (-\operatorname{sen} y) \cdot \frac{dy}{dt} = 1 \Rightarrow$

$\Rightarrow -\cos(\cos y) \cdot \operatorname{sen} y \cdot \frac{dy}{dt} = 1$

$\Rightarrow \frac{dy}{dt} = \frac{-1}{\operatorname{sen} y \cdot \cos(\cos y)}$.

60 $y = \operatorname{arctg} \sqrt{\frac{1-x}{1+x}} \Rightarrow \operatorname{tg} y = \sqrt{\frac{1-x}{1+x}} \Rightarrow$

$\Rightarrow \frac{d}{dx}(\operatorname{tg} y) = \frac{d}{dx} \sqrt{\frac{1-x}{1+x}} \Rightarrow$

$$\Rightarrow \sec^2 y \cdot \frac{dy}{dx} = \frac{1}{2\sqrt{\frac{1-x}{1+x}}} \cdot \frac{(1+x)\cdot(-1) - (1-x)\cdot 1}{(1+x)^2} \Rightarrow$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{\cancel{2}} \sqrt{\frac{1+x}{1-x}} \cdot \frac{(-\cancel{2})}{(1+x)^2}}{\sec^2 y} \Rightarrow$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+x)\sqrt{(1-x)(1+x)}\,\sec^2 y} = \frac{-1}{2\sqrt{1-x^2}}.$$

$$\operatorname{sen}^2 y + \cos^2 y = 1 \qquad \frac{1-x}{1+x} + 1 = \sec^2 y$$

$$tg^2 y + 1 = \sec^2 y \qquad \sec^2 y = \frac{2}{1+x}$$

77 a) f injetora, derivável

f^{-1} derivável

$$(f^{-1})'(x) = \frac{1}{\underbrace{f'(f^{-1}(x))}_{\neq 0}},$$

$$f(f^{-1}(x)) = x \Rightarrow \frac{d}{dx}\left[f(f^{-1}(x))\right] = \frac{d}{dx}(x) \Rightarrow$$

$$\Rightarrow f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1 \Rightarrow$$

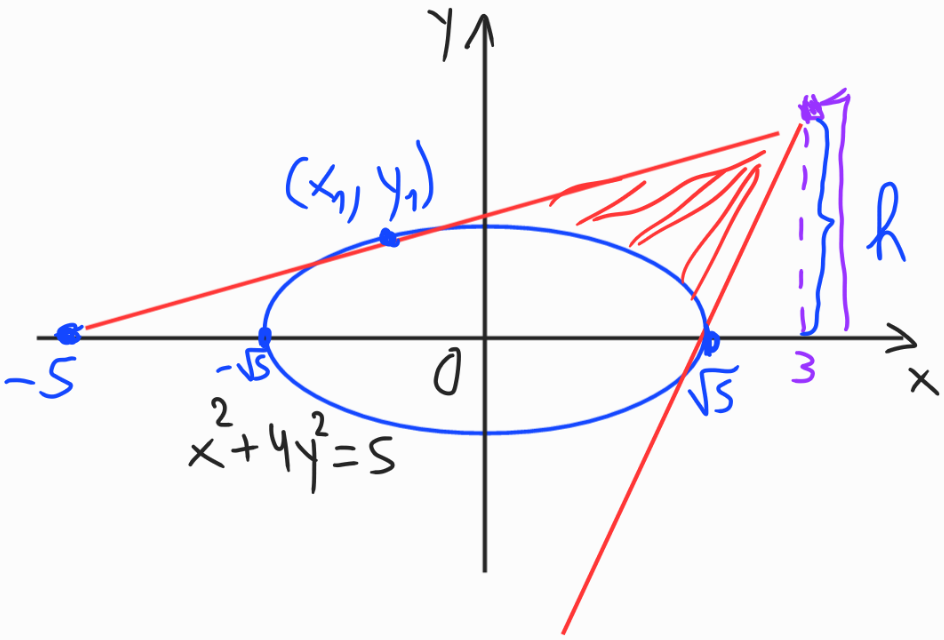
$$\Rightarrow (f^{-1})'(x) = \frac{1}{\underbrace{f'(f^{-1}(x))}_{\neq 0}},$$

b) $f(4) = 5 \Rightarrow f^{-1}(5) = 4$

$f'(4) = \frac{2}{3} \Rightarrow (f^{-1})'(5)$

$$(f^{-1})'(5) = \frac{1}{f'(f^{-1}(5))} = \frac{1}{f'(4)} = \frac{3}{2}.$$

80



$$\frac{d}{dx}(x^2+4y^2) = \frac{d}{dx}(5) \Rightarrow 2x + 4 \cdot 2y \cdot \frac{dy}{dx} = 0 \Rightarrow$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x}{8y} = \frac{-x}{4y}$$

$(x_1, y_1) \in$ elipse $\Rightarrow x_1^2 + 4y_1^2 = 5$

$$t_1 : \begin{vmatrix} 3 & k & 1 \\ -5 & 0 & 1 \\ x_1 & y_1 & 1 \end{vmatrix} = kx_1 - 5y_1 - 3y_1 + 5k = 0$$

$$\left.\begin{array}{l} kx_1 = 8y_1 - 5k \\ x_1^2 + 4y_1^2 = 5 \end{array}\right\} \left(\frac{8y_1 - 5k}{k}\right)^2 + 4y_1^2 = 5 \Rightarrow$$

$$\Rightarrow \frac{64y_1^2 - 80ky_1 + 25k^2}{k^2} + 4y_1^2 = 5$$

$$-\frac{x_1}{4y_1} = \frac{k}{8} \Rightarrow x_1 = -\frac{ky_1}{2}$$

$$kx_1 = 8y_1 - 5k = -\frac{k^2 y_1}{2} \Rightarrow \left(8 + \frac{k^2}{2}\right) y_1 = 5k \Rightarrow$$

$$x_1^2 + 4y_1^2 = 5 \Rightarrow \left(-\frac{k}{2} y_1\right)^2 + \frac{4 \cdot 25k^2}{\left(8 + \frac{k^2}{2}\right)^2} = 5$$

$\Rightarrow$

$\boxed{3.6}$ 55 $\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$

$$= \lim_{h\to 0} \frac{\ln(1+h) - \ln 1}{h} = \underbrace{\frac{d}{dy}(\ln y)\Big|_{y=1}}_{f(y) = \ln y} = \frac{1}{y}\Big|_{y=1} = 1.$$

$$\lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \frac{\ln(1+h) - \overbrace{\cancel{\ln 1}}^{f'(1)}}{h} = \frac{\ln(1+h)}{h}$$

0

$$\frac{d}{dx}(g(x)) = \lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$$

56 $y = \left(1 + \frac{x}{n}\right)^n \Rightarrow \sqrt[x]{y} = \left(1 + \frac{x}{n}\right)^{n/x} = \left(1 + \frac{1}{K}\right)^K$, sendo $K =$

$= \frac{n}{x}$. Temos $\lim_{n\to\infty} K = \lim_{n\to\infty}\left(\frac{n}{x}\right) = \infty \Rightarrow \lim_{n\to\infty} \sqrt[x]{y} =$

$= \lim_{K\to\infty}\left(1 + \frac{1}{K}\right)^K = e \Rightarrow \lim_{n\to\infty} y = e^x$ $\therefore$

$\therefore \lim_{n\to\infty}\left(1 + \frac{x}{n}\right)^n = e^x$, $x > 0$.

80 Borda: $x^2 + 4y^2 = 5 \Rightarrow \dfrac{x^2}{5} + \dfrac{y^2}{\frac{5}{4}} = 1$.

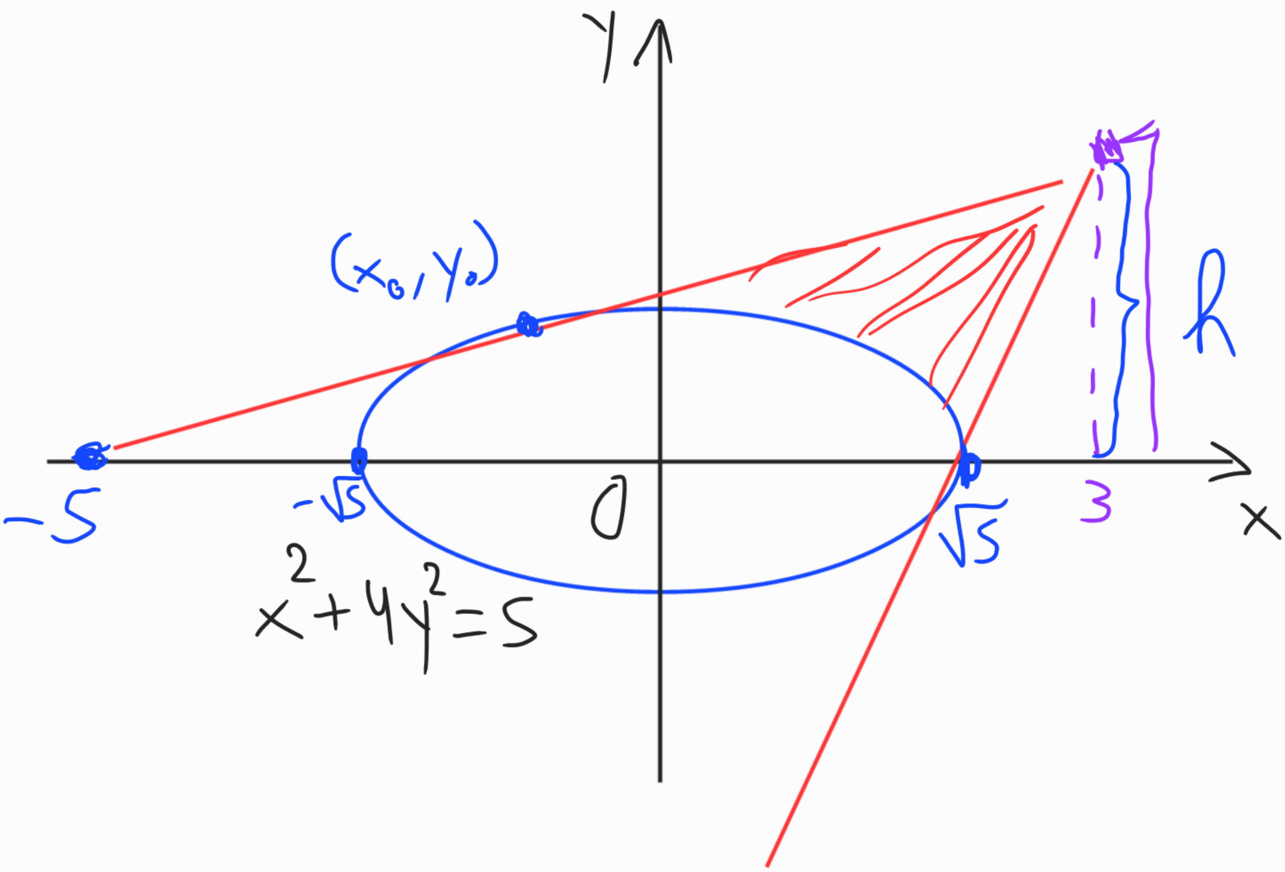
Do exercício 44, a equação da tangente à borda no ponto (x_0, y_0) é $t: \dfrac{xx_0}{5} + \dfrac{yy_0}{\frac{5}{4}} = 1$; $(-5, 0) \in t$

$$\Rightarrow -\frac{5x_0}{5} = 1 \Rightarrow x_0 = -1 \Rightarrow (-1)^2 + 4y_0^2 = 5 \Rightarrow 4y_0^2 = 4$$

$$\Rightarrow y_0^2 = 1 \underset{y_0 > 0}{\Rightarrow} y_0 = 1 \Rightarrow t: -\frac{x}{5} + \frac{4y}{5} = 1;$$

$$(3, L) \in t \Rightarrow -\frac{3}{5} + \frac{4L}{5} = 1 \Rightarrow 4L - 3 = 5 \Rightarrow 4L = 8$$

$\Rightarrow L = 2 \therefore$ A altura da lâmpada é 2.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow T_g$$ no ponto (x_0, y_0) é

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$$

44 $\frac{d}{dx}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=\frac{d}{dx}(1) \Rightarrow \frac{2x}{a^2}+\frac{2y}{b^2}\cdot\frac{dy}{dx}=0 \Rightarrow$

$\Rightarrow \frac{dy}{dx}=-\frac{x}{a^2}\cdot\frac{b^2}{y} \Rightarrow \left.\frac{dy}{dx}\right|_{(x,y)=(x_0,y_0)}=-\frac{x_0}{a^2}\cdot\frac{b^2}{y_0} \Rightarrow$

$\Rightarrow t: y=-\frac{x_0 b^2}{a^2 y_0}\cdot x+K \; ; \; (x_0,y_0)\in t \Rightarrow$

$\Rightarrow y_0=-\frac{x_0^2 b^2}{a^2 y_0}+K \Rightarrow K=\frac{x_0^2 b^2}{a^2 y_0}+y_0 \Rightarrow$

$\Rightarrow t: y=-\frac{x_0 b^2}{a^2 y_0}\cdot x+\frac{x_0^2 b^2}{a^2 y_0}+y_0 \Rightarrow$

$\Rightarrow t: \frac{y y_0}{b^2}=-\frac{x x_0}{a^2}+\frac{x_0^2}{a^2}+\frac{y_0^2}{b^2}=-\frac{x x_0}{a^2}+1 \;\therefore$

$(x_0,y_0)\in$ elipse

$\therefore\; t: \frac{x x_0}{a^2}+\frac{y y_0}{b^2}=1.$