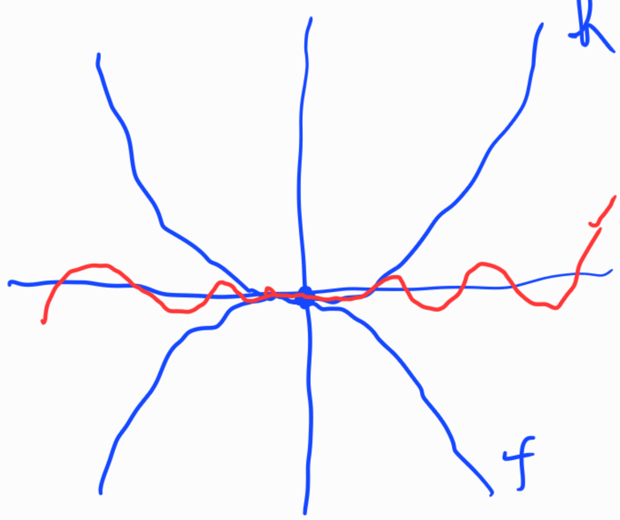
$\boxed{2.3}$ 35) $\lim_{x\to 0}(x^2\cos 20\pi x) = 0$

$-1 \le \cos 20\pi x \le 1 \Rightarrow \underbrace{-x^2}_{f(x)} \le \underbrace{x^2\cos 20\pi x}_{g(x)} \le \underbrace{x^2}_{h(x)}$ e

$\lim_{x\to 0}(-x^2) = \lim_{x\to 0} x^2 = 0 \Rightarrow \lim_{x\to 0} g(x) = 0$




49) $g(x) = \dfrac{x^2+x-6}{|x-2|}$

a) (i) $\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} \dfrac{x^2+x-6}{|x-2|} = \lim_{x\to 2^+} \dfrac{x^2+x-6}{x-2} =$

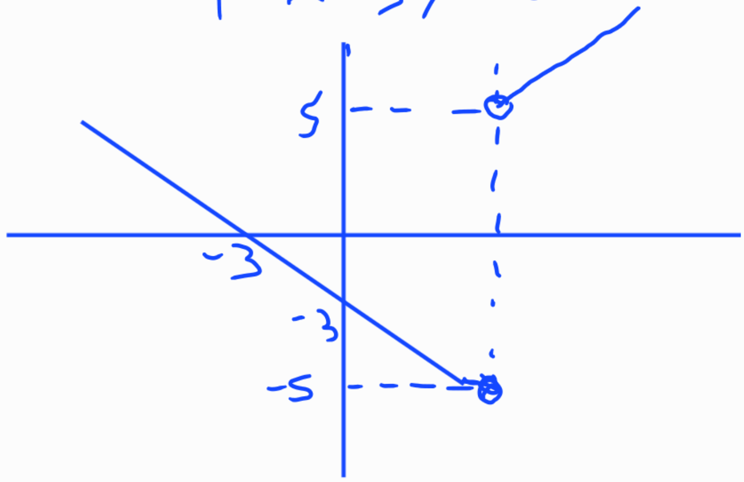
$= \lim_{x\to 2^+} \dfrac{(x-2)(x+3)}{(x-2)} = \lim_{x\to 2^+}(x+3) = 2+3 = 5.$

(ii) $\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} \dfrac{x^2+x-6}{-(x-2)} = \lim_{x\to 2^-} \dfrac{(x-2)(x+3)}{-(x-2)} =$

$= \lim_{x\to 2^-} -(x+3) = -(2+3) = -5.$

b) $\lim_{x \to 2} g(x)$ não existe pq $\lim_{x \to 2^-} g(x) \neq \lim_{x \to 2^+} g(x)$.

c) $g(x) = \begin{cases} x+3, & \text{se } x > 2 \\ -x-3, & \text{se } x < 2 \end{cases}$




5) $\lim_{x \to 1} \frac{f(x)-8}{x-1} = 10 \Rightarrow \lim_{x \to 1} (f(x)-8) =$

$= \lim_{x \to 1} \left[ \frac{f(x)-8}{x-1} \cdot x-1 \right] = \lim_{x \to 1} \left( \frac{f(x)-8}{x-1} \right) \cdot \lim_{x \to 1} (x-1) =$

$= 10 \cdot 0 = 0 \Rightarrow \lim_{x \to 1} f(x) - \lim_{x \to 1} 8 = 0 \Rightarrow \lim_{x \to 1} f(x) = 8.$

6) a) $\lim_{x \to 0} \frac{f(x)}{x^2} = 5 \Rightarrow \lim_{x \to 0} f(x) = \lim_{x \to 0} \left( \frac{f(x)}{x^2} \cdot x^2 \right) =$

$= \lim_{x \to 0} \frac{f(x)}{x^2} \cdot \lim_{x \to 0} x^2 = 5 \cdot 0 = 0.$

b) $\lim_{x \to 0} \frac{f(x)}{x} = \lim_{x \to 0} \left( \frac{f(x)}{x^2} \cdot x \right) = \lim_{x \to 0} \frac{f(x)}{x^2} \cdot \lim_{x \to 0} x = 5 \cdot 0 =$

$= 0.$

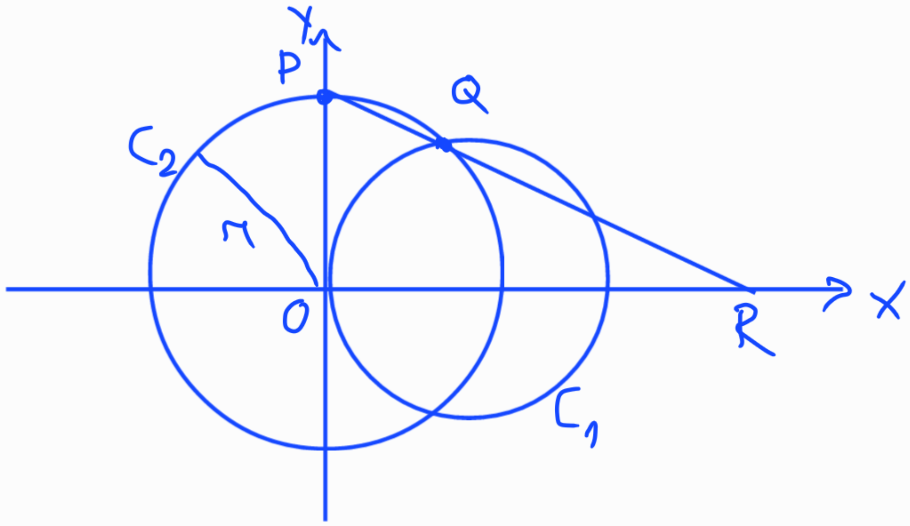
(63) $\lim_{x \to a} [f(x) g(x)]$ existe mas $\lim_{x \to a} f(x)$ e $\lim_{x \to a} g(x)$ não existem.

$$\left.\begin{array}{l} f(x) = \operatorname{sen}\left(\frac{1}{x}\right) \Rightarrow \nexists \lim_{x \to 0} \operatorname{sen}\left(\frac{1}{x}\right) \\ g(x) = \dfrac{1}{\operatorname{sen}\left(\frac{1}{x}\right)} \Rightarrow \nexists \lim_{x \to 0} \dfrac{1}{\operatorname{sen}\left(\frac{1}{x}\right)} \end{array}\right\} \begin{array}{l} f(x) \cdot g(x) = 1 \\ \lim_{x \to 0} 1 = 1 \end{array}$$

(64) $$\lim_{x \to 2} \frac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1} \cdot \frac{\sqrt{3-x} + 1}{\sqrt{3-x} + 1} \cdot \frac{\sqrt{6-x} + 2}{\sqrt{6-x} + 2} =$$

$$= \lim_{x \to 2} \frac{\cancel{(2-x)}(\sqrt{3-x} + 1)}{\cancel{(2-x)}(\sqrt{6-x} + 2)} = \lim_{x \to 2} \frac{(\sqrt{3-x} + 1)}{(\sqrt{6-x} + 2)} = \frac{1+1}{2+2} = \frac{2}{4} = \frac{1}{2}.$$

(66)




$C_1: (x-1)^2 + y^2 = 1$

$C_2: x^2 + y^2 = r^2$

$P = (0, r)$

$$\begin{cases} (x_Q - 1)^2 + y_Q^2 = 1 \\ x_Q^2 + y_Q^2 = r^2 \end{cases} \Rightarrow \begin{cases} x_Q^2 - 2x_Q + 1 + y_Q^2 = 1 \\ x_Q^2 + y_Q^2 = r^2 \end{cases} \Rightarrow$$

$$\Rightarrow r^2 - 2x_Q = 0 \Rightarrow x_Q = \frac{r^2}{2} \Rightarrow y_Q = \sqrt{r^2 - x_Q^2} \Rightarrow$$

$$\Rightarrow y_Q = \sqrt{r^2 - \frac{r^4}{4}} \Rightarrow r\sqrt{1 - \frac{r^2}{4}}$$

$$\begin{vmatrix} 0 & r & 1 \\ x_Q & y_Q & 1 \\ x_R & y_R & 1 \end{vmatrix} = 0 \Rightarrow \cancelto{0}{x_Q y_R} + r\underline{x_R} - \underline{x_R} y_Q - r x_Q = 0$$

$$x_R = \frac{r x_Q}{r - y_Q} = \frac{\frac{r^3}{2}}{r - r\sqrt{1 - \frac{r^2}{4}}} = \frac{\frac{r^2}{2}}{1 - \sqrt{1 - \frac{r^2}{4}}}$$

$$\lim_{r \to 0^+} x_R = \lim_{r \to 0^+} \frac{\frac{r^2}{2}}{1 - \sqrt{1 - \frac{r^2}{4}}} \cdot \frac{1 + \sqrt{1 - \frac{r^2}{4}}}{1 + \sqrt{1 - \frac{r^2}{4}}} =$$

$$= \lim_{r \to 0^+} \frac{\frac{\cancel{r^2}}{2}\left(1 + \sqrt{1 - \frac{r^2}{4}}\right)}{\frac{\cancel{r^2}}{4}} = \lim_{r \to 0^+} 2\left(1 + \sqrt{1 - \frac{r^2}{4}}\right) =$$

$$= 2\left(1 + \sqrt{1}\right) = 2.2 = 4 \Rightarrow R \to (4,0)$$ quando

$r \to 0^+$.

(29) $\lim\limits_{t\to 0}\left(\dfrac{1}{t\sqrt{1+t}}-\dfrac{1}{t}\right)=\lim\limits_{t\to 0}\dfrac{1}{t}\left(\dfrac{1}{\sqrt{1+t}}-1\right)=$

$=\lim\limits_{t\to 0}\dfrac{1}{t}\left(\dfrac{1-\sqrt{1+t}}{\sqrt{1+t}}\right)\cdot\dfrac{(1+\sqrt{1+t})}{1+\sqrt{1+t}}=$

$=\lim\limits_{t\to 0}\dfrac{1}{\cancel{t}}\cdot\dfrac{(-\cancel{t})}{\sqrt{1+t}\,(1+\sqrt{1+t})}=\lim\limits_{t\to 0}\dfrac{-1}{\sqrt{1+t}+1+t}=-\dfrac{1}{2}.$

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$x=\dfrac{6\pm 2}{2}=3\pm 1\ \begin{cases}2\\4\end{cases}$


$(2,4)$

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(4) a) $f(x)=x^5-3x^2+1$

$f(-1)=-3<0$

$f(2)=21>0$

$f(x)=0,\ x\in[-1,2]$

$\boxed{1,3,5 \text{ raíces}}$


$a=-1 \Rightarrow f(a)=-3$

$b=2 \Rightarrow f(b)=21$

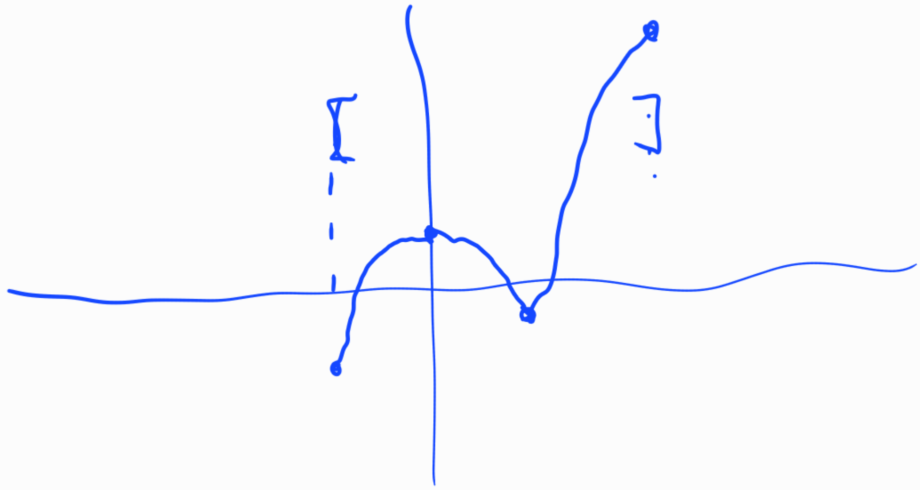
$\exists c\in(a,b)$ tq $f(c)=N$

$\forall N\in[f(a),f(b)]$

b) $f(0)=1$

$f(1)=-1$

$\boxed{1,3,5}$



1, 3, 5

1, 3, 5

1, 3, 5

3, 5