14.2 (13) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0$

Dado $\varepsilon > 0$, escolha $\delta > 0$ tal que

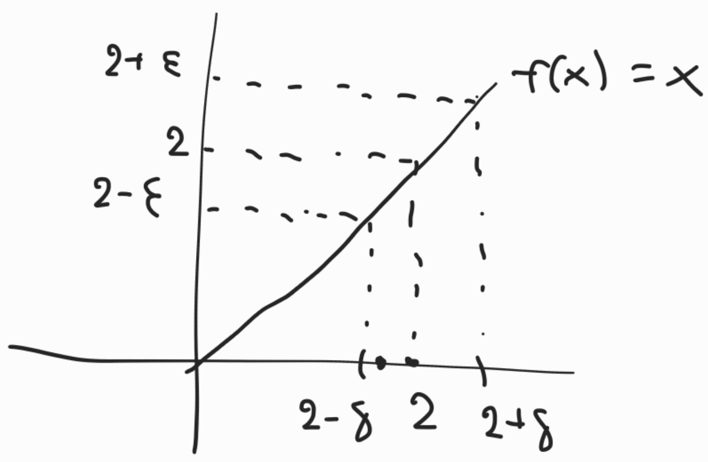
$$\|(x,y)\| < \delta \iff \sqrt{x^2+y^2} < \delta \Rightarrow \left| \frac{xy}{\sqrt{x^2+y^2}} - 0 \right| < \varepsilon$$

$$\iff \frac{|xy|}{\sqrt{x^2+y^2}} < \varepsilon \iff \frac{|x||y|}{\sqrt{x^2+y^2}} < \varepsilon$$

$$\frac{|y|}{\sqrt{y^2+x^2}} < 1 \Rightarrow \frac{|xy|}{\sqrt{x^2+y^2}} < |x| < \sqrt{x^2+y^2} < \delta$$

$|y| = \sqrt{y^2} < \sqrt{y^2+x^2}$

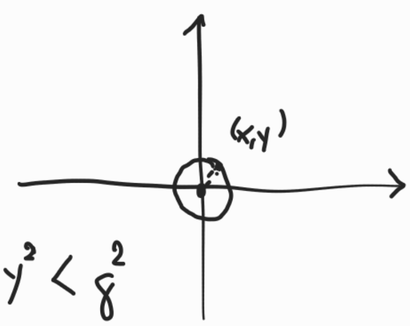
$\therefore\ \delta = \varepsilon$ (qualquer $\delta < \varepsilon$ funciona)



$\displaystyle\lim_{x\to 2} f(x)$

δ

$\mathbb{R}^2 \to \mathbb{R}$



$x^2+y^2 < \delta^2$

$\sqrt{x^2+y^2} < \delta$

$$\left.\begin{array}{c} \|(x,y)-(x_0,y_0)\| < \delta \\ \Downarrow \\ |f(x)-L| < \varepsilon \end{array}\right\} \Longleftrightarrow \lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

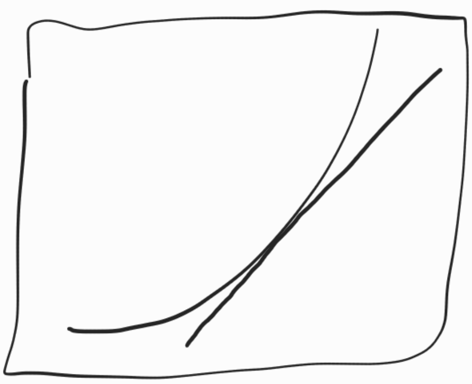
$$\|(x,y)-(0,0)\| < \delta \Rightarrow \left| \frac{xy}{\sqrt{x^2+y^2}} - 0 \right| < \varepsilon$$

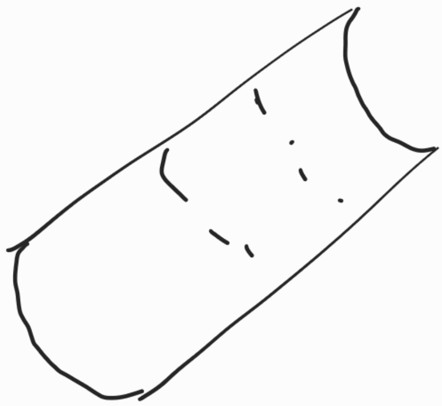
$$f : \mathbb{R}^2 \to \mathbb{R}$$

$$y = ax + b$$

$$z = f(x,y) = ax + by + c$$

$$\frac{dy}{dx}$$





$$\left\{ \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \right\}$$

$$z - z_0 = \underbrace{f_x(x_0,y_0)}\cdot(x-x_0) + \underbrace{f_y(x_0,y_0)}\cdot(y-y_0)$$

$$f_x = \frac{\partial f}{\partial x} \qquad f_y = \frac{\partial f}{\partial y}$$

$f(x,y) = x^2 + y^2$

$g(x,y) = -x^2 - y^2 + xy^3$ $\qquad (x_0, y_0) = (0,0)$

$z_0 = f(x_0,y_0) = g(x_0,y_0) = 0$

$$\frac{\partial f}{\partial x} = 2x + 0 = 2x, \quad \frac{\partial f}{\partial y} = 0 + 2y = 2y$$

$$\frac{\partial g}{\partial x} = -2x + y^3 \cdot 1 = y^3 - 2x, \quad \frac{\partial g}{\partial y} = -2y + x \cdot 3y^2 =$$

$$= 3xy^2 - 2y$$

$f_x(0,0) = 0, \quad f_y(0,0) = 0, \quad g_x(0,0) = 0, \quad g_y(0,0) = 0$

$z = z_0 = 0$ é plano tg a f e g em $(0,0)$

$\therefore$ f e g se tangenciam na origem.

"Extra": $\displaystyle \lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0$.

Dado $\varepsilon > 0$, escolha $\delta = \varepsilon$.

$$\|(x,y) - (0,0)\| = \|(x,y)\| = \sqrt{x^2+y^2} < \delta = \varepsilon \Rightarrow$$

$$\Rightarrow \left| \frac{xy}{\sqrt{x^2+y^2}} - 0 \right| = \left| \frac{xy}{\sqrt{x^2+y^2}} \right| < |x| = \sqrt{x^2} <$$

$$< \sqrt{x^2 + y^2} = \delta = \varepsilon$$

$$\therefore \lim_{(x,y) \to (0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0. \quad \blacksquare$$