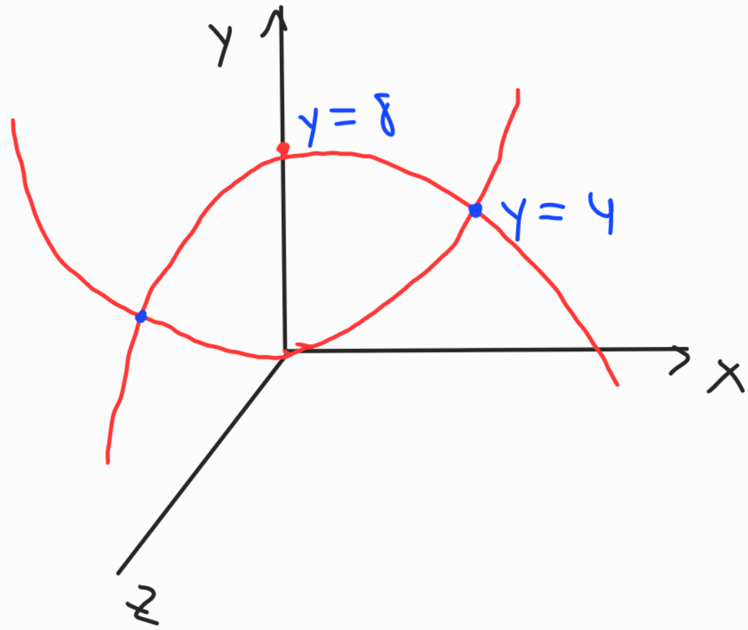
15.7 20

$y = x^2 + z^2$

$y = 8 - x^2 - z^2$

$x = r\cos\theta$

$z = r\,\mathrm{sen}\,\theta$,



$\theta \in [0, 2\pi]$

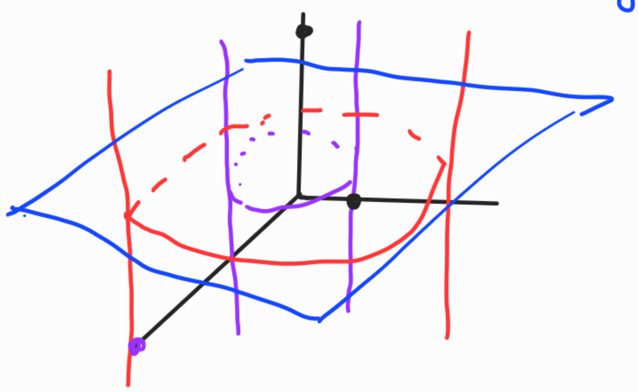
$r \in [0, 2]$

$r^2 \leq y \leq 8 - r^2$

$$V = \int_0^{2\pi}\int_0^2\int_{r^2}^{8-r^2} r \cdot dy\,dr\,d\theta = \int_0^{2\pi} d\theta \int_0^2\int_{r^2}^{8-r^2} r\,dy\,dr =$$

$$= 2\pi \int_0^2 r(8-2r^2)\,dr = 2\pi\left[4r^2 - \frac{r^4}{2}\right]_0^2 = 16\pi.$$

15.8 20



$$E: \begin{cases} z = 0 \\ z = x + y + 5 \\ x^2 + y^2 = 4 \\ x^2 + y^2 = 9 \end{cases}$$

$0 \leq z \leq x + y + 5$

$\begin{cases} x = r\cos\theta \\ y = r\,\mathrm{sen}\,\theta \end{cases}$, $r \in [2,3]$

$\theta \in [0, 2\pi]$

$$\iiint_E x\, dV = \int_0^{2\pi} \int_2^3 \int_0^{r\cos\theta + r\,\mathrm{sen}\,\theta + 5} r^2 \cos\theta \, dz \, dr \, d\theta =$$

$$= \int_0^{2\pi} \int_2^3 r^2 \cos\theta \left(r\cos\theta + r\,\mathrm{sen}\,\theta + 5 \right) dr \, d\theta =$$

$$= \int_0^{2\pi} \int_2^3 \left(r^3 \cos^2\theta + r^3 \,\mathrm{sen}\,\theta \cos\theta + 5r^2 \cos\theta \right) dr \, d\theta =$$

$$= \int_0^{2\pi} \left[\frac{r^4}{4} \cos^2\theta + \frac{r^4}{4} \,\mathrm{sen}\,\theta \cos\theta + \frac{5r^3}{3} \cos\theta \right]_2^3 d\theta =$$

$$= \int_0^{2\pi} \left(\frac{65}{4} \left(\cos^2\theta + \mathrm{sen}\,\theta \cos\theta \right) + \frac{95}{3} \cos\theta \right) d\theta =$$

$$= \frac{65}{4} \int_0^{2\pi} \left(\frac{1 + \cos 2\theta}{2} \right) d\theta + \frac{65}{4} \int_0^{2\pi} \frac{\mathrm{sen}\, 2\theta}{2} \cdot d\theta =$$

$$= \frac{65}{4} \left[\frac{\theta}{2} + \frac{\mathrm{sen}\, 2\theta}{4} \right]_0^{2\pi} + \frac{65}{4} \left[-\frac{\cos 2\theta}{4} \right]_0^{2\pi} =$$

$$= \frac{65}{4}\pi .$$

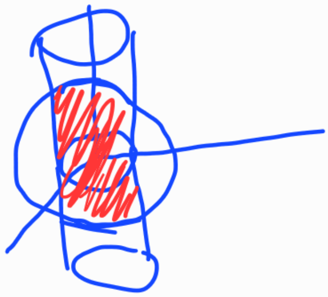
22

$$x^2+y^2=1$$
$$x^2+y^2+z^2=4$$
$$r^2+z^2=4$$
$$z^2=4-r^2$$

$x = r\cos\theta$, $\theta \in [0,2\pi]$

$y = r\sin\theta$ $\quad r \in [0,1]$

$-\sqrt{4-r^2} \leq z \leq \sqrt{4-r^2}$



$r^2 = u$

$2r\,dr = du$

$\sqrt{4-u}\,du$

$$V = \int_0^{2\pi}\int_0^1\int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\cdot dz\,dr\,d\theta = \int_0^{2\pi}\int_0^1 2r\sqrt{4-r^2}\,dr\,d\theta =$$

$$= \int_0^{2\pi}\int_0^1 \sqrt{4-u}\,du\,d\theta = 2\pi\left[-\frac{2}{3}(4-u)^{3/2}\right]_0^1 =$$

$$= -\frac{4\pi}{3}\left(3\sqrt{3}-8\right) = \frac{32\pi}{3} - 4\sqrt{3}\pi = 4\pi\left(\frac{8}{3}-\sqrt{3}\right).$$

$\boxed{15.9}$ 26 $\displaystyle\iiint_E xyz\,dV =$

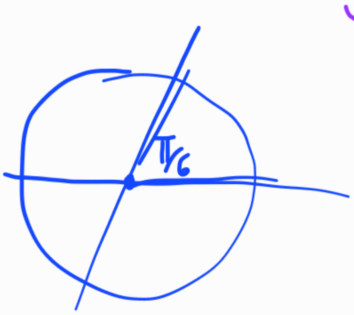
$$E : \begin{cases} \rho = 2 \\ \rho = 4 \\ \varphi = \pi/3 \end{cases} \qquad \begin{array}{l} 2 \le \rho \le 4 \\ 0 \le \varphi \le \pi/3 \\ 0 \le \theta \le 2\pi \end{array}$$

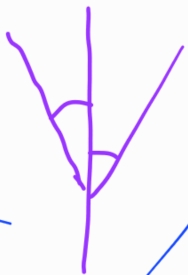
$$= \int_0^{\pi/3} \int_0^{2\pi} \int_2^4 \rho^5 \operatorname{sen}^3\varphi \cos\varphi \operatorname{sen}\theta \cos\theta \, d\rho \, d\theta \, d\varphi =$$

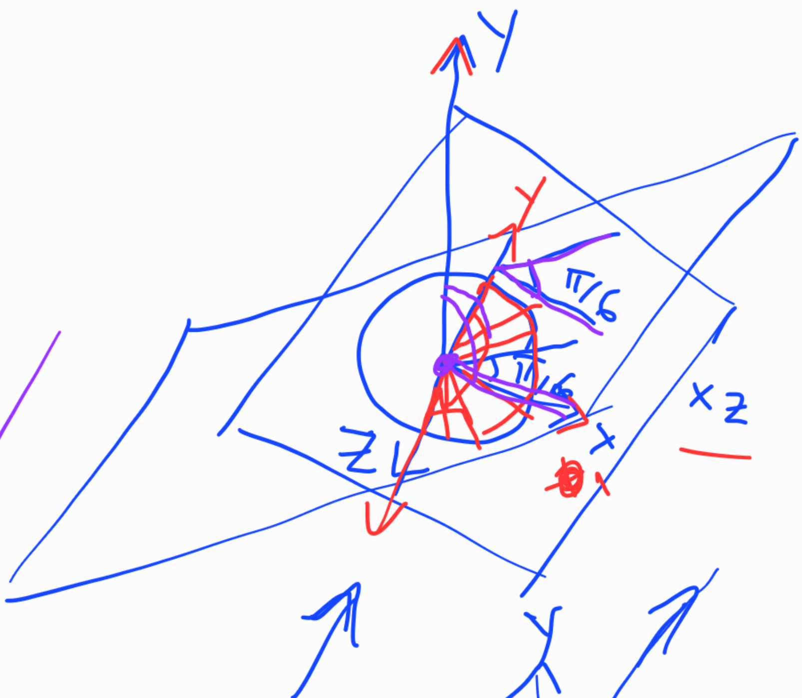
$$= \int_0^{\pi/3} \operatorname{sen}^3\varphi \cos\varphi \, d\varphi \overbrace{\int_0^{2\pi} \frac{\operatorname{sen} 2\theta}{2} \, d\theta}^{0} \int_2^4 \rho^5 \, d\rho = 0.$$

36. $O = (0,0,0)$

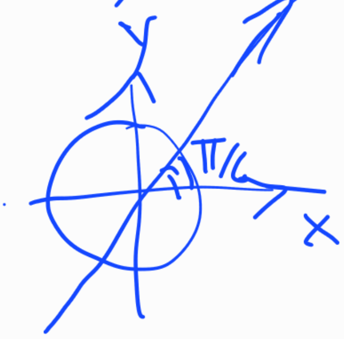
$0 \le$







$$\begin{cases} \rho \in [0, a] \\ \theta \in \left[0, \frac{\pi}{6}\right] \\ \varphi \in [0, \pi] \end{cases}$$



$$V = \int_0^{\pi/6} \int_0^{\pi} \int_0^{a} \rho^2 \operatorname{sen} \varphi \, d\rho \, d\varphi \, d\theta =$$

$$= \frac{\pi}{6} \cdot \int_0^{\pi} \operatorname{sen} \varphi \, d\varphi \cdot \int_0^{a} \rho^2 \, d\rho = \frac{\pi}{6} \cdot \left[-\cos \varphi \right]_0^{\pi} \cdot \frac{a^3}{3} =$$

$$= \frac{\pi a^3}{9} .$$