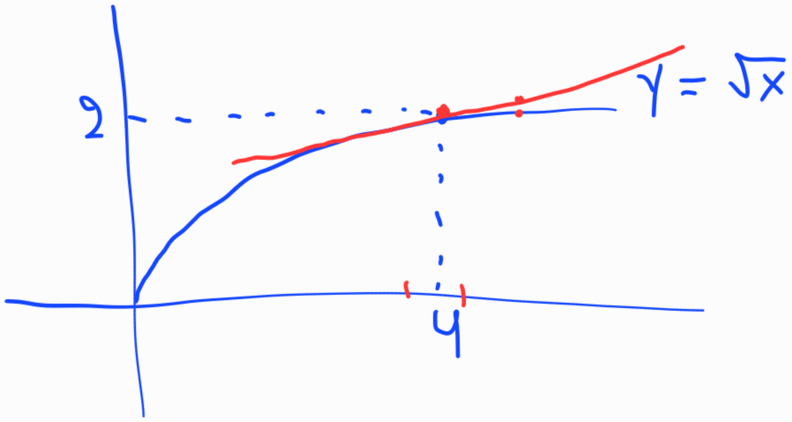

$\sqrt{4{,}01}$

$\boxed{14.2}$

(12) $\displaystyle \lim_{(x,y)\to(1,0)} \frac{xy-x}{(x-1)^2+y^2}$

Dado $\varepsilon > 0$, escolha $\delta > 0$ tal que

$$\xcancel{\|(x,y)-(1,0)\| < \delta \Rightarrow \left\| \frac{xy-x}{(x-1)^2+y^2} - L \right\| < \varepsilon}$$

$$y = 0 \Rightarrow \lim_{x\to 1} \frac{xy-x}{(x-1)^2+y^2} =$$

$$= \lim_{x\to 1} \frac{-x}{(x-1)^2} = -\infty \Rightarrow \nexists \lim.$$

(14) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)(x^2-y^2)}{x^2+y^2} =$

$$= \lim_{(x,y)\to(0,0)} x^2 - y^2 = 0$$

Dado $\varepsilon > 0$, escolha $\delta = \sqrt{\varepsilon} > 0$ tal que

$$\|(x,y) - (0,0)\| < \delta \Rightarrow |x^2 - y^2 - 0| < \varepsilon$$

$$\|(x,y)\| = \sqrt{x^2 + y^2} < \delta \Rightarrow |x^2 - y^2| < \varepsilon$$

$$|x^2 - y^2| = \sqrt{(x^2 - y^2)^2}$$

$$x^2 + y^2 < \delta^2$$

$$\underbrace{x^2 - y^2}_{>0} < x^2 + y^2 < \delta^2$$

$$x^2 - y^2 < \varepsilon \qquad \delta^2 = \varepsilon$$

$$\delta = \sqrt{\varepsilon}$$

$$\sqrt{x^2 + y^2} < \sqrt{\varepsilon} \Rightarrow x^2 + y^2 < \varepsilon \overset{?}{\Rightarrow} |x^2 - y^2| < \varepsilon$$

Caso $x \geq y$: $|x^2 - y^2| = x^2 - y^2 < x^2 + y^2 < \varepsilon$

$\Rightarrow |x^2 - y^2| < \varepsilon$ $(\checkmark)$

Caso $x < y$: $|x^2 - y^2| = y^2 - x^2 < x^2 + y^2 < \varepsilon$ $(\checkmark)$

$$\therefore \lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2+y^2} = 0 .$$

$\boxed{14.4}$ 12 $f(x,y) = x^3 y^4 \quad , \quad (1,1)$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= y^4 \cdot 3x^2 = 3x^2y^4 \\ \frac{\partial f}{\partial y} &= x^3 \cdot 4y^3 = 4x^3y^4 \end{aligned}\right\} \text{ f é de classe } C^1$$

Plano tg:

$f_x(1,1) = 3 \quad , \quad f_y(1,1) = 4$

$z - z_0 = 3(x-1) + 4(y-1)$

$(1,1,1) \in$ plano $\Rightarrow 1 - z_0 = 0 \Rightarrow z_0 = 1$

$z - 1 = 3(x-1) + 4(y-1)$

$L(x,y) = f(1,1) + f_x(1,1).(x-1) + f_y(1,1).(y-1)$

$L(x,y) = 1 + 3(x-1) + 4(y-1)$

$f(1{,}01; 0{,}99)$

$$dz = \frac{\partial z}{\partial x} \cdot dx + \frac{\partial z}{\partial y} \cdot dy = 3\,dx + 4\,dy$$

$\Delta x = 0{,}07 = dx$

$\Delta y = -0{,}02 = dy$

$\Delta z \cong dz = 3\,dx + 4\,dy$