(1) $m(a+bX) = a + b \times m(X)$

$y_i = a + bx_i$

$m(a+bX) = m(Y) = \frac{1}{N}\sum_{i=1}^{N} y_i = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i) = \frac{1}{N}\left(\sum_{i=1}^{N} a\right) + \frac{b}{N}\sum_{i=1}^{N} x_i = a + bm(X)$

(2) $cov(X,X) = s^2$

$cov(X,X) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(x_i - m(X)) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))^2 = s^2$

(3) $cov(X, a+bY) = b\,cov(X,Y)$

$Z = a + bY$ so $z_i = a + by_i$

$m(Z) = a + bm(Y)$

$cov(X,Z) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(z_i - m(Z)) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))((a+by_i) - a + bm(Y)))$

$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X)) \cdot b(y_i - m(Y)) = b \cdot \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y)) = b\,cov(X,Y)$

(4) $cov(a+bX, a+bY) = b^2 cov(X,Y)$

$U = a+bX$

$V = a+bY$

$u_i - m(U) = b(x_i - m(X))$

$v_i - m(V) = b(y_i - m(Y))$

$cov(U,V) = \frac{1}{N}\sum_{i=1}^{N}(u_i - m(U))(v_i - m(V)) = \frac{1}{N}\sum_{i=1}^{N}(b(x_i - m(X)))(b(y_i - m(Y)))$

$= b^2 \cdot \frac{1}{N}\sum_{i=1}^{N}(x_i - m(X))(y_i - m(Y)) = b^2 cov(X,Y)$

$Y = X$

$cov(bX, bX) = b^2 cov(X,X) = b^2 s^2$

(5) Yes, for $b>0$ because the transformation $x \mapsto a+bx$ is strictly increasing so it preserves order of the data

Median is $Q_{0.5}$ so $a + b\,med(X)$

$IQR(a+bX) = (a + bQ_{.75}(X)) - (a + bQ_{.25}(X)) = b(Q_{.75}(X) - Q_{.25}(X)) = b\,IQR(X)$

(6) $m(X^2) \neq (m(X))^2$ and $M(\sqrt{X}) \neq \sqrt{m(X)}$

$X = \{0,1\}$ then $m(X) = \frac{0+1}{2} = \frac{1}{2}$ $\quad \sqrt{m(X)} = \sqrt{\frac{1}{2}} \approx 0.707$

$(m(X))^2 = (\frac{1}{2})^2 = \frac{1}{4}$ $\quad$ so $m(\sqrt{X}) \neq \sqrt{m(X)}$