Control Systems Design

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MECA 482

Design Report

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1. **Introduction**

This report presents a simulated design of a control system in which the goal is to keep an Inertia Wheel Pendulum (IWP) in the upright position through the use, and implementation, of the state-space representation of the mathematical model for the pendulum seen in **Figure 1**. This state-space representation of the pendulum will be implemented with the use of Matlab and a negative feedback control loop, for this system, will be implemented using Simulink in synchrony with the Matlab results. The negative feedback loop will result in greater stability, minimize disturbance signaling, and decrease sensitivity to the parametric variation for this system. The system will be modeled and simulated using CoppeliaSim to test and demonstrate the results of our control system, and algorithm to control the Inertia Wheel Pendulum.

The IWP will consist of a Link, mounted to a rigid surface, with a wheel at the end of that link that will be driven a motor to utilize the mass and geometric properties of the wheel to swing the link into the upright position. As the motor spins the wheel, this will create a reaction in the link to begin swinging, as a pendulum, with a controlled input to the system to provide sufficient angular acceleration and velocity to create enough momentum to swing the pendulum in the upright position.

1. **Logical and Operational Viewpoint**

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**Figure 1**: Operational Viewpoint of Inertia Wheel Pendulum

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**Figure 2**: Logical Viewpoint of Inertia Wheel Pendulum

**Table 1**: Logical Viewpoint of Inertia Wheel Pendulum

1. **Mathematical Modeling**

|  |  |
| --- | --- |
| **Nomenclature** | **Description** |
| *θl* | The pendulum (link) angular position. (*rad/s*) |
| *θw* | The wheel angular position. (*rad/s*) |
| *Ml* | Mass of the link. (*kg*) |
| *Mw* | Mass of the wheel. (*kg*) |
| *Tmotor* | Torque applied by the motor, at the wheel. (*Nm*) |
| *d* | Length of the link. (*m*) |
| *Jl* | Inertia of the link (about center of mass). (*kgm2*) |
| *Jw* | Inertia of the wheel and motor rotor. (*kgm2)* |
| *rw* | Radius of the wheel (*m*) |
| *g* | Acceleration due to gravity. *(m/s2*) |
| *R* | Motor armature resistance. (*Ohms*) |
| *L* | Motor armature inductance. (*H*) |
| *kt* | Motor’s torque constant (*Nm/A)* |
| *kb* | Electromotive force constant (*V/m/s)* |

*d*

*Ml, Jl*

*θl*

*rw*

*Mw, Jw*

*θw*

*Tmotor*

**Figure 3**: Inertia Wheel Pendulum

The mathematical model for the IWP will be derived using the *Euler-Lagrange equations* (**Eq. 1&2**) to demonstrate the interacting bodies as a second-order ordinary differential equation.

Eq.1

Eq.2

The “L” term will be determined by an analysis of both the kinetic energies and potential energies from both the wheel and link. The difference of the summed kinetic energies and potential energies from both bodies will give us the *Lagrangian* as shown in **Eq. 3, 4, and 5**.

Eq.5

Eq.4

Eq.3

The kinetic energy seen at the link is based on the geometric properties, and mass, of the link as a function of the change in angle when the system accelerates. This can be viewed as a rectangular extrusion with a given mass *Ml* as well as the energy it takes it to rotate about its center of mass.

Eq.6

The kinetic energy at the wheel can be achieved by a similar process, although, the angle of the wheel will be affected by the angle of the link which can be calculated by the sum of the 2.

Eq.7

The potential energy equations can be formed under the assumption that the angle, *θl,* is in a rested positionandboth potential energies will equate to 0. The pendulum should only gain potential energy as the pendulum begins swinging to a position and certain height, where the kinetic is transferred to potential energy after, and just before, the pendulum swings back down when the acceleration and velocity is at 0.

Eq.9

Eq.8

))

))

Thus, given the *Euler-Lagrange equations* stated in the beginning, the *Lagrangian* can be viewed and constructed as:

Eq.10

))

After plugging in the “L” term to **Eq. 1 & 2,** and a bit of rearranging, we can view the equation in matrix form.

Eq.12

Eq.11

To verify that the inverse of the matrix exists (which is needed to find the solution of linear equations), it is necessary to take the determinant of the D matrix and confirm that the solution is NOT equal to 0.

Once confirmed, we can define our state vectors as such:

Eq.13

From ***Eq.12***, we can take the inverse of our D matrix:

Eq.15

Eq.14

After taking the inverse of the D matrix formed from the original *Lagrangian,* in ***Eq.10***, we can represent the state vector model in the form of its derivative as shown:

Eq.16

The state space model can be approximated in the form of a linear representation as:

Eq.18

Eq.17

Where A and B, and C are represented as:

Eq.20

Eq.19

Eq.19

To verify that the system is controllable, we take the determinant of the controllability matrix, C, and verify that the determinant is NOT equal to 0.

1. **Matlab and Simulink Implementation**

The parametric values, shown in ***Table 2*,** were sampled from Automatic Control with Experiments (Silva-Ortigoza & Hernández-Guzmán, 2018).

**Table 2**: Parameters for IWP

|  |  |
| --- | --- |
| **Parameter** | **Value** |
| *Ml* | 0.016 (*kg*) |
| *Mw* | 0.058 (*kg*) |
| *d* | 0.146 (*m*) |
| *Jl* | 0.000048463 (*kgm2*) |
| *Jw* | 0.0000076242 (*kgm2)* |
| *rw* | 0.0189 (*m*) |
| *g* | 9.81 *(m/s2*) |
| *R* | 4.172 (*Ohms*) |
| *kt* | 0.00775 (*Nm/A)* |

The parameters listed in **Table 2** were used to implement the equations and matrices derived from the mathematical model, in state space representation. The code used can be seen in ***Appendix A***.

By using Matlab’s integrated feature, Simulink, this allows us to create a simulated dynamic control system. Utilization of the state-space representation in a negative loop feedback control system, as seen in ***Appendix B***, let us control the IWP in terms of acceleration, angular position, and velocity. This was made possible by reducing the second order system (in the original ***Eq.10***) into a first order differential equation. The control system implemented within Simulink yielded results as seen in ***Appendix C***.

1. **Real World Application**

The IWP can be implemented a number of ways to achieve the desired results. If one is looking for a simplified way to control the IWP, it is necessary to choose a correct and efficient sensing technique that will allow for ease of controller implementation in terms of feedback and as well as algorithmic approach. When it comes to position and motion sensing, using an encoder, and in the case of the IWP: 2 encoders, would be desirable as it can provide feedback on the current position of the link of the pendulum as well as the wheel.

When the pendulum starts in the resting position, we can use it as a reference point or a homing point. This position can generally be called home and can be zeroed. When the wheel receives an initial rotational excitation of the system, by the motor connected to the wheel, it shall bring the pendulum to a new position while the encoder provides information about that said position as it is relative to the home position. With this information, the controller will be able to make a judgement on how much more excitation, and which direction (based on the direction the wheel is already spinning through encoder feedback), is necessary to get that pendulum closer to the position that is 180 degrees from home (completely vertical). The early excitations will have greater angular velocities to provide more momentum to said wheel as more energy is required to reach the vertical position, as the system approaches the desired position, the excitations will become smaller and smaller as the system reaches the steady state in the vertical position. The minimal adjustments made at the end of the process are determined based on the continuous feedback as we know, because we are approaching the final destination, less energy is required to reach that position. When speaking of energy put into the system, this is in reference to the motor’s torque applied to the system which, again, is dependent on the current position of both the link and wheel.

1. **Conclusion**

This project allowed us to explore different physical systems, derive mathematical models for a physical system, and represent that mathematical model in a state-space representation to allow it to be more easily manipulated. This process introduces us to the world of control systems and provides us with the necessary knowledge to apply this same concept to real world situations. The application that we experienced gives you the understanding of how powerful simulation can be. If you are able to take a theoretical physical model of something, derive an equation that represents that physical system, and then simulate it, it greatly helps you predict and further design a successful system as well as improve already existing systems. This process can both aid in design as well as make your design economically efficient by reducing the error in your design process.

# Bibliography

Nise, N. S. (2015). *Control Systems Engineering.* Wiley.

Silva-Ortigoza, R., & Hernández-Guzmán, V. M. (2018). *Automatic Control with Experiments.* Springer.

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Nise, N. S. (2015). *Control Systems Engineering.* Wiley.

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Appendix B.

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Appendix C.

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