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1. **Introduction**

This report presents a simulated design of a control system in which the goal is to keep an Inertia Wheel Pendulum (IWP) in the upright position through the use, and implementation, of the state-space representation of the mathematical model for the pendulum seen in **Figure 1**. This state-space representation of the pendulum will be implemented with the use of Matlab and a negative feedback control loop, for this system, will be implemented using Simulink in synchrony with the Matlab results. The negative feedback loop will result in greater stability, minimize disturbance signaling, and decrease sensitivity to the parametric variation for this system. The system will be modeled and simulated using CoppeliaSim to test and demonstrate the results of our control system, and algorithm to control the Inertia Wheel Pendulum.

The IWP will consist of a Link, mounted to a rigid surface, with a wheel at the end of that link that will be driven a motor to utilize the mass and geometric properties of the wheel to swing the link into the upright position. As the motor spins the wheel, this will create a reaction in the link to begin swinging, as a pendulum, with a controlled input to the system to provide sufficient angular acceleration and velocity to create enough momentum to swing the pendulum in the upright position.

1. **Mathematical Modeling**

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| **Nomenclature** | **Description** |
| *θl* | The pendulum (link) angular position. (*rad/s*) |
| *θw* | The wheel angular position. (*rad/s*) |
| *Ml* | Mass of the link. (*kg*) |
| *Mw* | Mass of the wheel. (*kg*) |
| *Tmotor* | Torque applied by the motor, at the wheel. (*Nm*) |
| *d* | Length of the link. (*m*) |
| *Jl* | Inertia of the link (about center of mass). (*kgm2*) |
| *Jw* | Inertia of the wheel and motor rotor. (*kgm2)* |
| *rw* | Radius of the wheel (*m*) |
| *g* | Acceleration due to gravity. *(m/s2*) |
| *R* | Motor armature resistance. (*Ohms*) |
| *L* | Motor armature inductance. (*H*) |
| *kt* | Motor’s torque constant (*Nm/A)* |
| *kb* | Electromotive force constant (*V/m/s)* |

**Figure 1**: Inertia Wheel Pendulum

*θw*

*Tmotor*

*rw*

*Ml, Jl*

*θl*

*Mw, Jw*

*d*

The mathematical model for the IWP will be derived using the *Euler-Lagrange equations* (**Eq. 1&2**) to demonstrate the interacting bodies as a second-order ordinary differential equation.

The “L” term will be determined by an analysis of both the kinetic energies and potential energies from both the wheel and link. The difference of the summed kinetic energies and potential energies from both bodies will give us the *Lagrangian* as shown in **Eq. 2, 3, and 4**.

The kinetic energy seen at the link is based on the geometric properties, and mass, of the link as a function of the change in angle when the system accelerates. This can be viewed as a rectangular extrusion with a given mass *Ml* as well as the energy it takes it to rotate about its center of mass.

The kinetic energy at the wheel can be achieved by a similar process, although, the angle of the wheel will be affected by the angle of the link which can be calculated by the sum of the 2.

The potential energy equations can be formed under the assumption that the angle, *θl,* is in a rested positionandboth potential energies will equate to 0. The pendulum should only gain potential energy as the pendulum begins swinging to a position and certain height, where the kinetic is transferred to potential energy after, and just before, the pendulum swings back down when the acceleration and velocity is at 0.

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Thus, given the *Euler-Lagrange equations* stated in the beginning, the *Lagrangian* can be viewed and constructed as:

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After plugging in the “L” term to **Eq. 1 & 2,** and a bit of rearranging, we can view the equation in matrix form.

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To verify that the inverse of the matrix exists (which is necessary to find the solution of linear equations), it is necessary to take the determinant of the D matrix and confirm that the solution is NOT equal to 0.