$$x_{t_{i+1}} = x_{t_i} + \int_{t_i}^{t_{i+1}} f(x_s)\, ds$$

$$x_{t_{i+1}} = \phi\left(x_{t_i},\ t_{i+1} - t_i,\ x_{t_{i-1}},\ t_i - t_{i-1}, \dots \right)$$

$$x_{t_{i+1}} = \phi\left(x_{t_i},\ t_{i+1} - t_i\right)$$

$$\begin{aligned} \phi(x, t+s) &= \phi\left(\phi(x,t),\ s\right) \\ &= \phi\left(\phi(x,s),\ t\right) \end{aligned}$$

$$\begin{cases} \phi(x, t+s) = \phi(z, s) \\ z = \phi(x, t) \end{cases} \qquad \forall \underbrace{x, t, s}$$

$$\Downarrow$$

$$x_i, t_i, s_i$$

$$\begin{aligned} \min_{z,\phi}\quad & \|\phi\|_K \\ \text{s.t.}\quad & \phi(x, t) = z \\ & \phi(z, s) = \phi(x, t+s) \end{aligned}$$

$$\min_{z,\phi} \quad \|\phi\|_K$$

$$\text{s.t.} \quad \phi(x,t) = z$$

$$\phi(z,s) = \phi(x,t+s)$$

$$\min_{z} \quad \begin{pmatrix} z \\ 0 \end{pmatrix}^T K(\psi_z, \psi_z)^{-1} \begin{pmatrix} z \\ 0 \end{pmatrix}$$

Gauss-Newton

$$\left|\begin{array}{l} \phi((x,t)) = z \\ \phi(z,t) - \phi(x,t+s) = 0 \end{array}\right.$$

$$\left|\begin{array}{l} [\overbrace{\delta_{(x,t)}}^{\psi_1}, \phi] = z \\ [\underbrace{\delta_{(z,t)} - \delta_{(x,t+s)}}_{\psi_2(z)}, \phi] = 0 \end{array}\right.$$

$$\min_z \left| \begin{array}{l} \min \|\phi\|_K \\ [\psi_1, \phi] = z \\ [\psi_2, \phi] = 0 \end{array} \right.$$

$$K(\psi, \psi) = \begin{pmatrix} K(\psi_1, \psi_1) & K(\psi_1, \psi_2) \\ K(\psi_2, \psi_1) & K(\psi_2, \psi_2) \end{pmatrix}$$

$$K(\psi_i, \psi_j) = \int \psi_i(x') K(x', y') \psi_j(y') \, dx' dy'$$

$$\min_z \quad (z, 0) \, K(\psi, \psi)^{-1} \begin{pmatrix} z \\ 0 \end{pmatrix}$$

$$\min_{\substack{z_j \\ \theta}} \; \|\phi\|_{K(\theta)}$$

$$\text{s.t.} \quad x_{t_{i+1}} = \phi(x_{t_i}, t_{i+1} - t_i)$$

$$\phi(x'_j, t'_j) = z_j$$

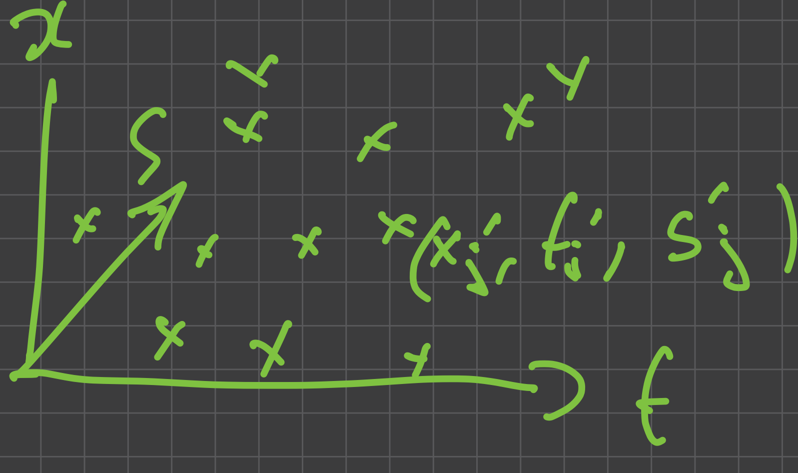
$$\phi(z_j, s'_j) = \phi(x'_j, t'_j + s'_j)$$

$$\min_{\substack{z_j \\ \theta}} \; \|\phi\|_{K(\theta)}$$

$$\text{s.t.} \quad [\overbrace{\delta_{(x_{t_i}, t_{i+1} - t_i)}}^{\psi_i^1}, \phi] = \overbrace{x_{t_{i+1}}}^{X_i}$$

$$\psi_j^2 \quad [\underbrace{\delta_{(x'_j, t'_j)}}, \phi] = \overbrace{z_j}^{Z}$$

$$[\underbrace{\delta_{(z_j, s'_j)} - \delta_{(x'_j, t'_j + s'_j)}}_{\psi_j^3}, \phi] = 0$$



$$\min_{z,\theta} \begin{pmatrix} X \\ z \\ 0 \end{pmatrix}^T K(\Psi, \Psi)^{-1} \begin{pmatrix} X \\ z \\ 0 \end{pmatrix}$$

$$K(\Psi, \Psi)$$

$$\Psi = \begin{pmatrix} \psi^1 \\ \psi^2 \\ \psi^3 \end{pmatrix} = \begin{pmatrix} \psi^1_1 \\ \vdots \\ \psi^1_i \\ \vdots \\ \psi^2_j \\ \vdots \\ \psi^3_? \end{pmatrix}$$

$$\boxed{\phi = K_\theta(\cdot, \Psi) K_\theta(\Psi, \Psi)^{-1} \begin{pmatrix} X \\ z \\ 0 \end{pmatrix}}$$

$$\downarrow$$

$$\phi(x, t)$$