0.1 Introduction

This paper is about extending the kernel flows method to answer the following problems:

1) How can we make extrapolations when the time series is irregularly observed?

2) How well does this adaptation work in the context of attractor reconstruction?

There are also ongoing questions about GRU-ODE-Bayes

3) Noisy RNNs (2021): we can try to extend the work to GRU-ODE-Bayes and study whether replacing the ODE with SDE improves the regularisation properties.

4) Probabilistic Neural ODEs

0.2 Kernel flows with dynamic adaptation of the step-size

Given a kernel $K_\theta(x, y)$, the adaptation is done as follows

$$\frac{d\theta}{dt} = -\eta \frac{\partial \rho}{\partial \theta}$$

Let t' be the new time. So

$$\frac{d\theta}{dt'} \frac{dt'}{dt} = -\eta \frac{\partial \rho}{\partial \theta}$$

$$\frac{d\theta}{dt'} = -\eta \frac{dt}{dt'} \frac{\partial \rho}{\partial \theta}$$

Once discretized, this becomes

$$\theta_{i+1} = \theta_i - \eta(t_{i+1} - t_i)\frac{\partial \rho}{\partial \theta}$$

which is the same kernel flows algorithm except that now one has to adapt the step-size in an irregular way due to the irregularity of the measurements.

0.3 Include the irregularity of time in the kernel itself