Mathematics IA

Using the Lotka-Volterra Differential Equations to Explore and Model the Predator-Prey Population Dynamics between Wolves and Moose.

Math Analysis and Approaches HL

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Table of Contents:

Introduction:

Rationale:

After studying different aspects of biology throughout high school, one of the topics that I found particularly interesting was ecology, specifically predator-prey dynamics. As Gregor Fussmann, a professor in McGill's Department of Biology puts it, predators face a dilemma, where if they hunt and overexploit the prey, they are killing what keeps them alive. However, when they are less efficient, then prey populations can "bounce back" (Gombay.) With this theory, predators and prey can undergo an endless cycle, and I found it interesting how predators and prey can co-exist over long periods.

Delving more into this field of study, I learned about the Lotka-Volterra Differential Equations. They are a pair of first-order nonlinear differential equations created by Alfred J. Lotka and Vito Volterra that are used to describe the population dynamics of biological systems. This ties directly into predator-prey interactions, leading me to investigate it for my mathematical IA. With my knowledge from HL Math AA and aspects of programming, this is an opportunity for growth and new understanding.

Aim:

Altogether, I decided to center my investigation on modeling Wolves (Predator) and Moose (Prey) using Lotka-Volterra Equations. Specifically, I will be using data from a study which tracks the population of Wolves and Moose in Isle Royale, an island in Lake Superior. Going more in-depth with the Lotka-Volterra equations, they are two differential equations that I will manipulate to create a graph.

Next, this graph will later be compared to real-life empirical data, which I can formulate a response to how accurate the model is today.

Exploring the Lotka-Volterra Differential Equations:

Independent Variable:	Time, t
Dependant Variable:	Prey (Moose) population, M Predator (Wolves) population, W

Table 1: Table introducing the Independent and Dependent Variables.

Constant Variables	Variable
Growth/Birth rate of prey	α
The rate at which predator kill prey	β
Natural Death rate of predators	γ
The rate at which predators increase by consuming prey	δ

Table 2: Table introducing the constant variables.

Prey Differential Equation:

$$\frac{dM}{dt} = \alpha M - \beta MW$$

Equation (1)

First, $\frac{dM}{dt}$ represents the change in prey population (M) in respect to time.

The prey population would increase at a rate of $dM = \alpha M \times dt$. However, it would also decrease by a rate of $dM = \beta MW \times dt$. Hence, a relationship between predators and prey exists. I.e the decrease in prey population depends on the population of the predators.

Predator Differential Equation:

$$\frac{dW}{dt} = \delta MW - \gamma W$$

Equation (2)

First, $\frac{dW}{dt}$ represents the change in the predator population (W) in respect to time.

The predator population will decrease by a rate of $dM = \gamma W \times dt$. It will also increase by a rate of $dW = \delta MW \times dt$. This is because the increase in population of predators depends on the population of the prey.

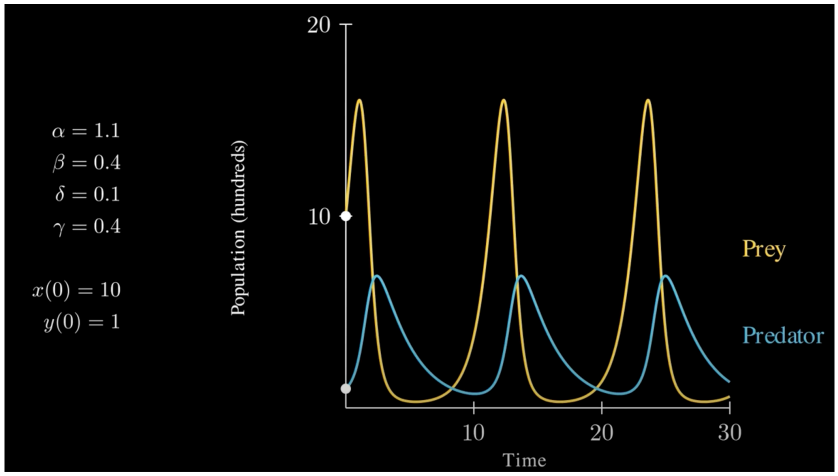

Figure 1: Graphical Example of the Lotka-Volterra Equations with sample values.

Looking at figure 1, the graph suggests that as the prey population increases, the population of the predator also increases. This is because as the prey population increases, there is abundance of prey for the predators, hence their population also increases. Also, as the prey population decreases, the predator population follows. This is because there is a high population of predators, hence they require a high amount of prey. However, the supply of prey will decrease, and predators begin to have no food. As a result, they naturally starve to death without different methods of nutrition.

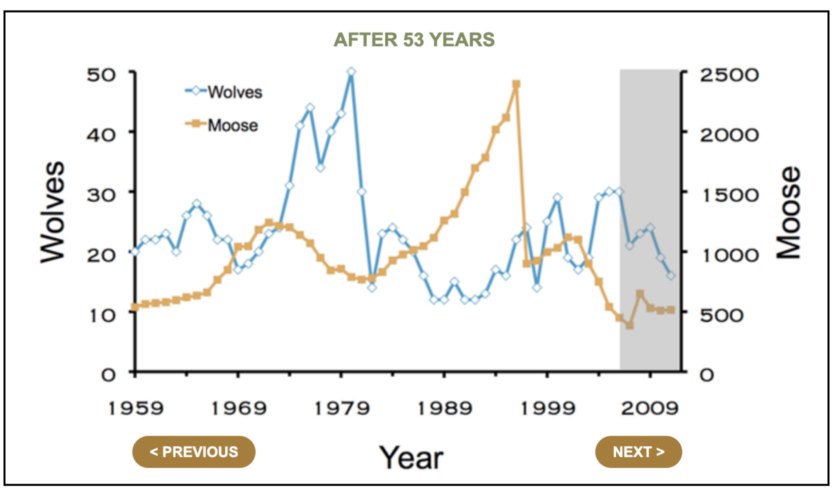

Figure 2: Wolves and Moose Population Graph from 1959 to 2009.

Looking at figure 2, this theory somewhat still follows. Looking at real-life data, *Wolves & Moose at Isle Royale* is a project that studies population dynamics of Wolves and Moose at Isle Royale, a remote island in Lake Superior. Nevertheless, as the population of moose was low, the population of wolves began to decrease. This is because there was a low supply of food, and some wolves would starve. As the population of wolves were low, the population of moose were able to increase due to the lack of predators, and the population of wolves could then increase again. Then the cycle repeats. Although there are some fluctuations in the graph, this can be explained by factors that have been explained in the study. For example, a major drop in the population of wolves was due to Canine parvovirus, a disease caused by humans (isleroyalewolf.)

Parameters and Assumptions:

Furthermore, there are certain assumptions that need to be made when developing the Lotka-Volterra model of the populations of Moose and Wolves.

Assumptions:

1. Assumption 1:

Only two species exist in this Model. Although predators feed off of more than one type of prey, this model assumes that predators (Wolves) will only feed off of one prey (Moose).

2. Assumption 2:

There is no shortage of food for the prey population. As a result, without predators, the prey population can only increase.

3. Assumption 3:

There is no limit on the appetite of predators (Wolves) — i.e they can consume an infinite amount of prey. However, they can only feed off the specific prey (Moose). Therefore, without prey, they will starve and the population of predators will decrease.

4. Assumption 4:

The rate of change of the population is proportional to its size. Using the Malthusian principle of population growth, this theory suggests that a population will grow exponentially. Under the previously mentioned assumption that there is no shortage of food for the prey population, the Malthusian principle of population growth can be also assumed to be true.

5. Assumption 5:

The environment is constant, and factors such as disease, mutualism and competition are negligible.

Parameters:

Returning to the constant variables, α, β, γ and δ must be defined in order to be substituted into the equation and hence formulate the final equation.

1. Growth/Birth Rate of Prey (Represented by α)

The first constant α, which represents the growth/birth rate of the prey (Moose) is valued at 0.84. This value was determined by Charles C. Scwartz's from his research studying the reproductive biology of Moose in his study titled "Reproductive Biology of North American Moose."

$\alpha = 0.84$

2. The Rate that Predators Kill Prey (Represented by β)

The next constant B, which represents the rate that predators kill prey is determined to be 0.019 moose/wolf/day. (Or 1.9 Moose killed per wolf every 100 days.) This value was determined by Bryce C. Lake, Mark R. Bertram, Nikki Guldager, and Jason R. Caikoski when they studied Moose and Wolves from the Yukon Flats National Wildlife Report from 2009. The empirical data that the formulated equations will be compared to the Isle Royale Wolf Moose Database. However, this value still holds merit because both of these locations are remote and isolated (Lake.)

$\beta = 0.019$

3. Natural Death Rate of Predators (Represented by γ)

The natural death rate of predators or the predator mortality rate can be determined through two different methods — using empirical data or an online study. From a study of Wolves from 2004 to 2019, Roman Teo Oliynyk discovered that the average annual natural mortality rate for Wolves went from 11.6% to 7.6%. This study was conducted in Northern Minnesota.
A more accurate value for this rate can rather be formulated by using empirical data.

The empirical data that will be used and studied in this investigation is displayed in table three.

Year	Number of Wolves	Year	Number of Moose
1980	50	1980	664
1981	30	1981	650
1982	14	1982	700
1983	23	1983	900
1984	24	1984	811
1985	22	1985	1062
1986	20	1986	1025
1987	16	1987	1380
1988	12	1988	1653
1989	11	1989	1397
1990	15	1990	1216
1991	12	1991	1313
1992	12	1992	1600
1993	13	1993	1880
1994	15	1994	1800
1995	16	1995	2400
1996	22	1996	1200
1997	24	1997	500
1998	14	1998	700
1999	25	1999	750
2000	29	2000	850
2001	19	2001	900
2002	17	2002	1000
2003	19	2003	900
2004	29	2004	750
2005	30	2005	540
2006	30	2006	385
2007	21	2007	450
2008	23	2008	650
2009	24	2009	530
2010	19	2010	510
2011	16	2011	515
2012	9	2012	750
2013	8	2013	975
2014	9	2014	1050
2015	3	2015	1250
2016	2	2016	1300
2017	2	2017	1600
2018	2	2018	1500
2019	14	2019	2060

Table 3: Moose and Wolf Population over the period 1980 to 2019.

The death rate can also be viewed as the change in population over a period of time. From the data table, the lowest population for Moose is 500 in 1998 from 1200 from 1996. At the same time period, the Wolf population changed from 24 to 14. Using the slope equation, the gradient represents the change in the population over the change in time, which provides a rough death rate.

$$death\,rate \;=\; gradient \;=\; \frac{y2 - y1}{x2 - x1}$$
$$= \frac{24-14}{1}$$
$$= 10$$
$$\gamma = 10$$

4. <u>The rate at which predators increase by consuming prey (Represented by δ)</u>

To determine the rate at which predators increase by consuming prey, empirical data will be used. Specifically, the value ‘δ’ is the ratio between the increase of population of Wolves compared to its relative decrease in the population of the Moose.

Using Excel, this ratio will be determined by finding the percent change of population of Predator and Prey. Which then will be divided by the average number of years.

Predators (Wolves)		Prey (Moose)	
Number of Years	Percent Change in population (%)	Number of Years	Percent Change in population
3	72	2	2.10843
6	54.16667	2	9.88888
2	20	3	26.43678
2	40.66667	3	79.16667
3	34.48276	4	61.5
14	93.33333	13	-357.7778
Average Number of Years	Average Percent Change (%)	Average Number of Years	Average Percent Change (%)
5	52.441572	4.5	-29.77951

Table 4: Percentage change in population from a minimum population to a maximum population

Sample Calculation:

$$Percent\ Change\ (\%) = \frac{initial - final}{initial} \times 100$$

$$Percent\ Change\ (\%) = \frac{50 - 14}{50} \times 100 = 72\%$$

To calculate δ, the following calculations occur:

$$\frac{Average\ percent\ change\ of\ Wolves}{Average\ percent\ change\ of\ Moose} = \left|\frac{52.441572}{-29.77951}\right| = 1.760995313 \approx 1.761$$

$$\frac{Average\ number\ of\ years\ (predators) + Average\ number\ of\ years\ (prey}{2} = \frac{5 + 4.5}{2} = 4.75$$

$$\frac{1.760995313}{4.75} = 0.3707358554 = 0.37$$

$\delta = 0.37$

5. Initial Population of Prey [Moose] (Represented by M_0)

The initial population represents the amount of prey (Moose) when time is zero. From the database/empirical data, the first datapoint available is in 1980, where the population of prey (Moose) is 664.

$M_0 = 664$

6. Initial Population of Predators [Wolves] (Represented by W_0)

The initial population represents the amount of predators (Wolves) when time is zero. From the database/empirical data, the first datapoint available is in 1980, where the population of predators (Wolves) is 50.

$W_0 = 50$

Modelling Wolves Population:

$$\frac{dW}{dt} = \delta MW - \gamma W$$

$$\frac{dW}{dt} = W(\delta M - \gamma)$$

$$[dt] \times [\frac{1}{W}] \times \frac{dW}{dt} = W(\delta M - \gamma) \times [\frac{1}{W}] \times [dt]$$

$$\int \frac{1}{W} dy = (\delta M - \gamma) \times \int 1\, dt$$

$$ln|W| + C = (\delta M - \gamma) \times t + C$$

$$e^{ln|W|} = e^{(\delta M - \gamma)t + C}$$

$$W = e^{\delta Mt} \times e^{-\gamma t} \times W_0$$

Equation (4)

When modeling the predator population looking at assumption 1 states that the predators will only feed off of prey. If there are no prey in the environment, the predators have no food, and will die. This is represented by c, and the rate of change of the population over time will decrease. This changes in the presence of prey. When predators and prey meet (demonstrated by $M \times W$. The birth rate of the predators can be represented by 'δ', which is the rate at which predators increase by consuming prey. Putting these variables together, the amount that the population of Wolves increases is δMW.

Once again, this equation also has an unaccounted variable, e^C. In this situation, the constant of integration will be considered the initial population of the predators ('W_0') Therefore, e^C can rather be considered as 'W_0'.

<u>Modeling Moose Population:</u>

$$\frac{dM}{dt} = \alpha M - \beta MW$$
$$\frac{dM}{dt} = M(\alpha - \beta W)$$
$$[dt] \times [\frac{1}{M}] \times \frac{dM}{dt} = M(\alpha - \beta W) \times [\frac{1}{M}] \times [dt]$$
$$\int \frac{1}{M} dx = (\alpha - \beta W) \times \int 1\, dt$$
$$ln|M| + C = (\alpha - \beta W) \times t + C$$
$$e^{ln|M|} = e^{(\alpha - \beta W)t + C}$$
$$M = e^{\alpha t} \times e^{-\beta W t} \times e^{C}$$
$$M = e^{\alpha t} \times e^{-\beta W t} \times M_0$$

Equation (3)

Starting from equation (1), following assumption 2 — that there is no shortage of food, there is no term required related to food in this equation. Next, assumption 4 states that the rate of change of the population is proportional to its size. Returning to the Malthusian population theory, this assumption means the population of prey increases exponentially. This holds true because of assumption (2), saying there are no shortages of food.

Finally, following assumption 1, since there are only two species (Moose & Wolves), this scope means that when the predator and prey meet, the predators will kill the prey for food. Finally, after solving for x, there is one final unaccounted variable, e^{C}. This was created because when integrating, there is a constant of integration. In this context, this value will be considered as the initial population of the prey (‘M_0’). Therefore, e^{C} can rather be considered as ‘M_0’.

Graphing the Equations:

Final Equations	
Moose Manipulated Lotka-Volterra Equation:	$\frac{dM}{dt} = \alpha M - \beta MW$ $M = e^{\alpha t} \times e^{-\beta Wt} \times M_0$
With Values:	$\frac{dM}{dt} = (0.84)\, M - (0.019)\, MW$
Wolves Manipulated Lotka-Volterra Equation:	$\frac{dW}{dt} = \delta MW - \gamma W$ $W = e^{\delta Mt} \times e^{-\gamma t} \times W_0$
With Values:	$\frac{dW}{dt} = (0.37)\, MW - (10)\, W$

Table 5: Table showing the derived Lotka-Volterra equations before and after substituting values.

In terms of graphing these equations through online graphing softwares, these equations involves slope fields, which is outside of the scope of the HL Math AA curriculum. Due to the complexity of the graph, a programming algorithm with python will instead be used to develop a graph.

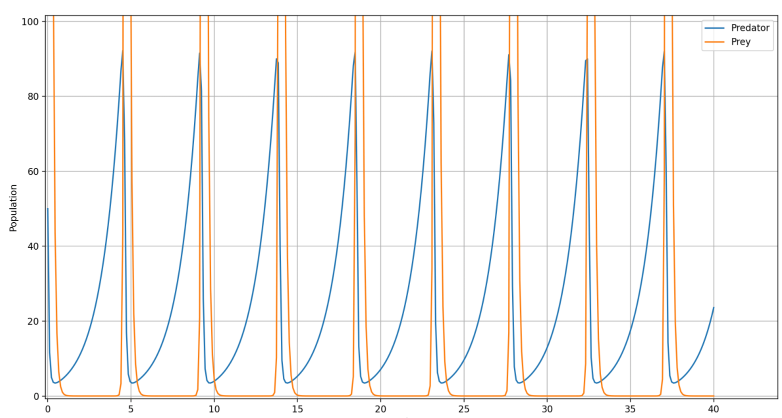

Figure 3: Graph of Lotka-Volterra Equation using student derived parameters. ($y \in 0 \leq y \leq 100$)

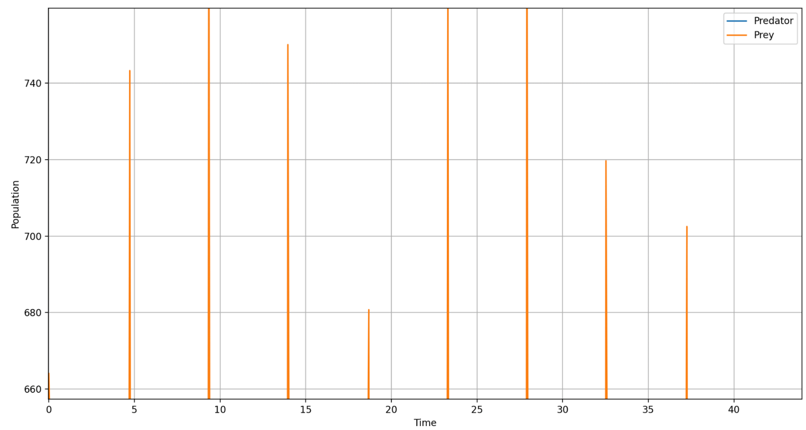

Figure 4: Graph of Lotka-Volterra Equation using student derived parameters. ($y \in 660 \leq y \leq 740$)

Evaluation:

Comparing Equation with Empirical Data:

Returning to table 3, using Excel, a scatter plot of the values can be created, forming the following figures.

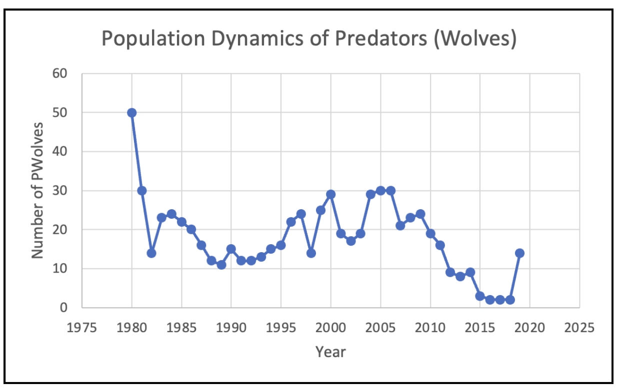

Figure 5: Population of Wolves from 1980 to 2019 using Empirical Data

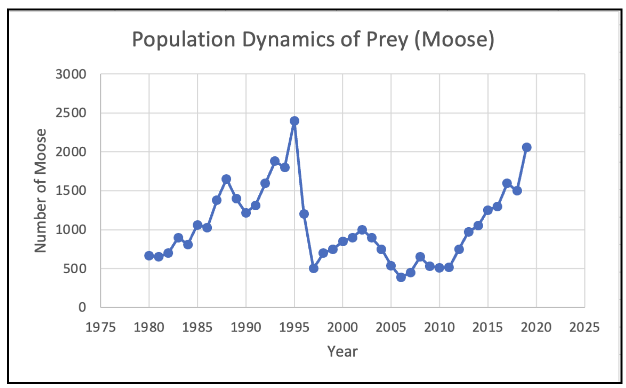

Figure 6: Population of Moose from 1980 to 2019 using Empirical Data

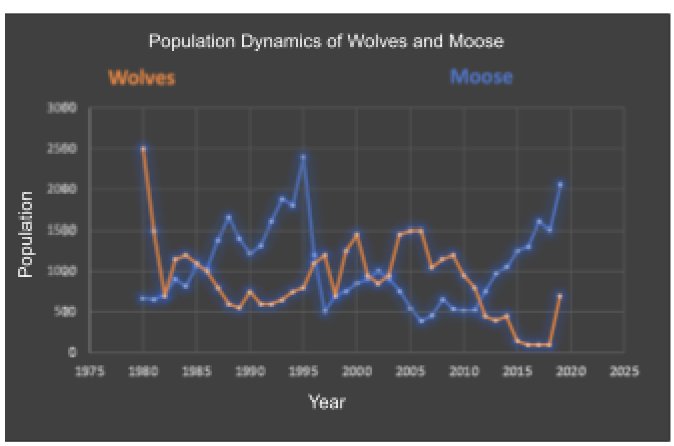

Figure 7: Population Dynamics of both Wolves and Moose using Empirical Data

For visual aid, figure 6 demonstrates the population dynamics of both Wolves and Moose in the time period. This graph is used to purely show the predator-prey dynamic within this empirical data. Due to the differences in populations, the orange graph is manipulated to appear to have higher values.

When evaluating the graphs, figures 3 and 4 are the Lotka-Volterra model whereas figures 5, 6 and 7 are based on empirical data. When comparing figure 4 and figure 7, you can see a similar pattern / relationship that was mentioned when discussing figure 1. Therefore, there are some qualitative similarities between the model and the empirical data.

Next, looking at quantitative data, from figures three and four, the maximums and minimums with their corresponding values will be determined, then recorded in tables six and seven.

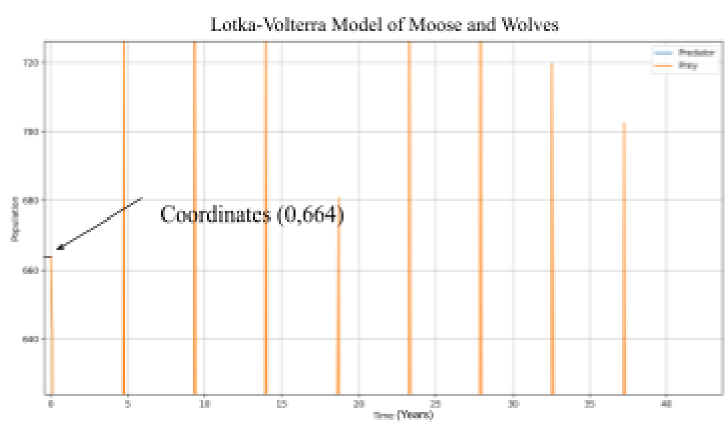

Figure 8: Determining Coordinates of Minimum and Maximums from Lotka-Volterra Model

Since the Volterra model is based off of time from an initial population, the x-coordinates represent the number of years after 1980. Thus, an x-coordinate of 0 represents the year 1980. To compare between the empirical and model data, the percentage error will be calculated for each year. The formula to calculate these values is as follows.

$$\%\,Error = \frac{Q_{model} - Q_{Empirical\ Data}}{Q_{model}} \times 100$$

Q_{model} represents the quantity discovered from the model, and $Q_{Empirical\ Data}$ represents the quantity/population from empirical data.

Percentage Error Sample Calculation:

$$\%\ Error\ =\ \frac{Q_{\ model} - Q_{\ Empirical\ Data}}{Q_{\ model}}\ \times\ 100$$

Year	Number of Wolves (Empirical Data)	Number of Wolves (Data from Model)	Percent Error (%)	Year	Number of Moose (Empirical Data)	Number of Moose (Data from Model)	Percent Error (%)
1980	50	50	0	1980	664	664	0
1981	30	5	-500	1981	650	1	-100
1982	14	11	-27.27273	1982	700	0	-100
1983	23	20	-15	1983	900	0	-100
1984	24	26	7.692308	1984	811	0	100
1985	22	4	-450	1985	1062	744	-42.74194
1986	20	5	-300	1986	1025	0	-100
1987	16	15	-6.666667	1987	1380	0	-100
1988	12	26	53.84615	1988	1653	2	-100
1989	11	92	88.04348	1989	1397	794.5	-75.83386
1990	15	4	-275	1990	1216	5	-100
1991	12	10	-20	1991	1313	0	-100
1992	12	20	40	1992	1600	0	-100
1993	13	40	67.5	1993	1880	0	-100
1994	15	20	25	1994	1800	750	-140
1995	16	5	-220	1995	2400	1	-100
1996	22	13	-69.23077	1996	1200	0	-100
1997	24	30	20	1997	500	0	-100
1998	14	91	84.61538	1998	700	0	-100
1999	25	4	-525	1999	750	681	-10.13216
2000	29	7	-314.2857	2000	850	0	-100
2001	19	16	-18.75	2001	900	0	-100
2002	17	39	56.41026	2002	1000	6	-100
2003	19	93	79.56989	2003	900	767.7	-17.23329
2004	29	4	-625	2004	750	0	-100
2005	30	10	-200	2005	540	0	-100
2006	30	21	-42.85714	2006	385	0	-100
2007	21	38	44.73684	2007	450	8	-5525
2008	23	25	8	2008	650	793.5	18.08444
2009	24	5	-380	2009	530	2	-26400
2010	19	13	-46.15385	2010	510	0	-100
2011	16	30	46.66667	2011	515	0	-100
2012	9	80	88.75	2012	750	0	-100
2013	8	17	52.94118	2013	975	719.9	-35.43548
2014	9	30	70	2014	1050	0	-100
2015	3	38	92.10526	2015	1250	0	-100
2016	2	92	97.82609	2016	1300	0	-100
2017	2	4	50	2017	1600	703	-127.596
2018	2	8	75	2018	1500	0	-100
2019	14	23	39.13043	2019	2060	0	-100
Predator Average Percentage Error (%):			-71.18457	**Prey Average Percentage Error (%):**			-876.3972

Table 6: Comparisons between Empirical Data and Model with Predator and Prey.

To calculate the overall percent error, the mean error can be calculated. Microsoft Excel was used to complete the table and calculate the percent error.

$$Mean\ Percentage\ Error\ =\ \frac{\Sigma\ Percentage\ Error\ of\ each\ point}{Number\ of\ data\ points}$$

Mean percentage error for Predators (Wolves): $71.18457\ \%\ \approx\ 71.2\%$

Mean percentage error for Prey (Moose): $876.3972\ \%\ \approx\ 876\%$

Overall, the model has a percent error for prey of roughly 71.2% whereas the percent error of prey is roughly 876%. Therefore the model is somewhat accurate for the predators. However, for the latter, the model is extremely inaccurate. Since the overall percent error is very high, this likely means that the model does not accurately represent the population dynamics of Moose and Wolves.

<u>Conclusion:</u>

In conclusion, in this exploration, I explored the Lotka-Volterra equations and manipulated them so that they can be used to model the predator-prey relationship between Moose and Wolves. Investigating this modeling and its characteristics helps provide useful insight into the population dynamics of animals. My chosen species, Moose and Wolves, especially Wolves are at high risk of extinction, as can be seen from the data, where the population of Wolves declined to only two in 2016. Using these models can be used to potentially help researchers or biologists to predict when populations are at risk of extinction and therefore protect these species.

Overall, from this investigation, there is a 71.2% and 876% percent error between empirical data and modeling for predators and prey respectively.

From this investigation, there is limited success in the data, which can be explained by the multiple assumptions behind this model. First, assumption one states that there are only two species that exist in the model. Compared to most natural habitats, this assumption does not hold true because habitats generally have a lot of biodiversity. Therefore, predators would have access to different types of prey, not one single animal. For prey, they would have different types of predators, and not be limited to one predator.

Next, looking back to assumption five, this assumption states that the environment is constant, and factors such as disease, mutualism and competition are negligible. This limitation is more important because, in the empirical data, the wolf population was unfortunately affected by the human-caused Parvovirus (Isle Royale.)

Finally, one unrealistic part of the model is when the moose population goes to zero — which means extinction. This is not possible in a real-life situation, and the next steps in this investigation would be to include further adjustments to the parameters of the data to develop a more accurate model.

The Lotka-Volterra equations are a simple model to model predator-prey population dynamics and give a basic understanding of this topic. Thus, there are multiple possible extensions / next-steps for this investigation.

The first possible next-steps for this investigation are modifications to the parameters to better fit the empirical data. When determining the Natural Death Rate of Predators (γ), this was made through two points from the empirical data. Perhaps a more accurate depiction of γ would yield a more accurate model.

Other possible next-steps for this investigation is adding other variables to the equations that make up this system. For example, considering how a disease can affect the population. This extension would be useful for this specific investigation due to the significant impact of disease on the population of the Wolves. Considering this factor would result in a more accurate representation of the population changes. Furthermore, a more advanced model could be using the Kolmogorov model. The Lotka-Volterra model with its assumptions creates a model with oscillating populations. However, the Kolmogorov model can consider specific interactions and other parameters such as natural disasters, disease and extinction events (Sigmund.) Hence, investigating predator-prey population dynamics through further advanced models that consider more factors would provide more useful insight and implications that can protect or maintain animal species.

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Appendix:

The python algorithm used to create the Lotka-Volterra Model is shown below. This was created based off of Softinery's Calculations in Chemical Engineering Blog titled "Lotka-Volterra model and simulation."

```
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt

def eqs(x,t):
   x1 = x[0]
   x2 = x[1]
   alfa = 0.84
   beta = 0.019
   gamma = 10
   delta = 0.37

   dxdt = [alfa * x1 - beta * x1 * x2,
           delta * x1 * x2 - gamma * x2]
   return dxdt

y0 = [50, 664]

t = np.linspace(0, 40, 365)

sol = odeint(eqs, y0, t)

x1 = sol[:, 0]
x2 = sol[:, 1]

# Plot the solution
plt.plot(t, x1, label='Predator')
plt.plot(t, x2, label='Prey')
plt.xlabel('Time')
plt.ylabel('Population')
plt.legend(loc='upper right')

plt.ylim(0, np.max(x1) + 10)
plt.grid(True)
plt.show()
```