a)

```
void f1(int n)
{
    int i=2;
    while(i<n){
        /* do something that takes O(1) time */
        i = i*i;
    }
}
```

$i=2$
$i=4$
$i=8$
$i=16$
$2^{2^i} = n$

$\log 2^{2^i} = \log n$
$2^i = \log n$
$\log 2^i = \log\log n$
$i = \log\log n$

$$\sum_{i=2}^{\log\log n} \theta(1) = \log\log n = \theta(\log\log n)$$

b)

```
void f2(int n)
{
  for(int i=1; i<=n; i++){
    if((i % (int)sqrt(n)) == 0){
      for(int k=0; k<pow(i,3); k++){
        /* do something that takes O(1) time */
      }
    }
  }
}
```

$$T(n) = \sum_{i=1}^{n}\left(O(1) + O\left(\sum_{k=0}^{n^3} O(1)\right)\right) \qquad \sum_{i}^{\sqrt{n}} \cdot \sum_{k=0}^{n^3} \theta(1) = n^{\frac{1}{2}} \cdot n^3 = n^{\frac{1}{2}+\frac{6}{2}} = n^{\frac{7}{2}}$$

$$= \sum_{i=1}^{n} \theta(1) + \theta(n^{7/2})$$

$$= \theta(n) + \theta(n^{7/2})$$

$$= \theta(n^{7/2})$$

c)

```
for (int i=1; i <= n; i++){
    for(int k=1; k<=n; k++){
      if(A[k]==i){
        for(int m=1; m <= n; m=m+m){
          O(1)
        }
      }
    }
}
```

$m^2 = n$

$m = \log n$

$$T(n) = \sum_{i=1}^{n} \left(\sum_{k=1}^{n} + \sum_{m=1}^{\log n} \right)$$

$$= \sum_{i=1}^{n} \sum_{k=1}^{n} \theta(1) + \sum_{i=1}^{n} \sum_{m=1}^{\log n} \theta(1)$$

$$= \theta(n^2)$$

d)

```
int f (int n)
{
  int *a = new int[10]; O(1)
  int size = 10 O(1)
  for(int i=0; i<n; i++){
    if(i==size){
      int newsize = 3*size/2;
      int *b = new int [newsize];
      for (int j=0; j<size; j++){
        b[j] = a[j];
      }
      delete [] a;
      a=b;
      size = newsize;
    }
    a[i] = i*i; O(1)
  }
}
```

$\sum_{j=0}^{size} \theta(1)$

$\sum_{i=0}^{n-1}$

x	newsize
1	10
2	15
3	22.5

$10(1.5)^{x-1}$

$(1.5)^x = n/10$

$\log_{1.5}(1.5)^x = \log_{1.5}(n/10)$

$x = \log_{1.5}(n/10)$

$$\sum_{i=0}^{n} \left(\theta(1) \left(\sum_{i=1}^{\log_{1.5}(n/10)} \theta(1) + \theta(1) + \sum_{j=0}^{(10)(1.5)^{x-1}} \theta(1) + \theta(1) + \theta(1) \right) \right)$$

$$\sum_{i=0}^{n} (\theta(1)) + \sum_{i=0}^{n} \left(\log_{1.5}(n/10) + (10)(1.5)^{x-1} \right)$$

$$\theta(n) + \theta((10)(1.5)^{x-1})$$

$\theta(n)$