Vibrations, Structural & Computational Dynamics

October 20th 2020

$$\omega^2\ \mathrm{M}x = \mathrm{K}\ x \Leftrightarrow (K - \lambda M)x = 0$$

ISAE/SUPAERO TOULOUSE
SA403 - ADVANCED COMPUTATIONAL STRUCTURAL MECHANICS

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Vibrations, Structural & Computational Dynamics

Scope/Contents

Let's have fun & consider the next topics in more details in the next slides :

- Reminders
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- Industry Flavours of Dynamics Analysis
- Computational Approaches of Dynamics Analysis
- Reduction of Structures for Dynamics Analysis

Vibrations, Structural & Computational Dynamics

Who am I ?

My name is Julien LE FANIC.

My Final Year Engineer Thesis was about hypervelocity Impact*.

I am an AIRBUS Structural Engineer since 2009†.

Nowadays I spend most of my daily working hours fighting with FEA solver to understand why my dynamic analyses crash and/or/either do not release the expected figures...

* in Paris

† in Toulouse

Reminder

Dynamic Definition

The solution of D'ALEMBERT principle (presented hereafter in the slides... keep calm) for a continuum or a assembly rigid bodies is the purpose of a dynamic study. One is interested in the mechanical quantities such as u, $\dot{u}$, $\ddot{u}$,... up to σ, ε, $\dot{\varepsilon}$ for a system excited with a time dependent phenomena/excitation.

The tasks to be lead depend upon the phenomena and the latter will guide the mathematical approach (solver in a computational mechanics framewrok) to be used.

For the beauty of the formula one reminds that the fundamtal equation of dynamics $F = m \cdot \gamma$ stems from LAGRANGE's equation of motion :

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial P}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = 0 \quad , \quad i \in 1, \ldots, n \qquad (1)$$

with q_i generalized parameters, T the kinetic energy, P the potential energy and D as a dissipative function of the system. Have a look to Tallec [2000], Wells [1967], Komzsik [2005], Roseau [1984], Géradin and Rixen [1996] for more insights. For the latter equaion (1) one can reach for an undamped mass/spring system the the classical equation :

$$m\ddot{x} + kx = 0 \qquad (2)$$

which governs the free vibrations os a single dof system.

Reminder

Dynamic Definition

Approach	Keep it simple. . . but not simpler	. . . for fun. . .	. . . Reality
About dof	s-dof	n-dof	continuum
User's Face	🤫	😉	🤔

Table 1: Classification of Dynamic Classical Models vs. Difficulty. In terms of frequency in the range $10^3 - 10^4$Hz the problem is in the high frequency range, below 10^2Hz the problem is in the low frequency range. Unfortunately many problems may fall in the in-between.

Nota : The art to switch from a dynamic analysis to a *s*-dof analysis doest exist. For example Miles'equation.

Reminder

Single dof System

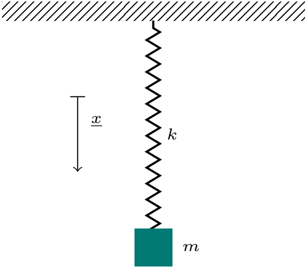

Figure 1: Single dof System without Damping device/sketch.

One has from LAGRANGE or D'ALEMBERT principle :

$$m\ddot{x} + c\dot{x} + kx = F \quad (3)$$

$$\ddot{x} + 2\xi\omega_0\dot{x} + {\omega_0}^2 x = \frac{F}{m} \quad (4)$$

with the damping factor

$$\xi = \frac{c}{2m\omega_0} \quad (5)$$

And :

$$\boxed{\omega_0 = \sqrt{\frac{k}{m}}} \quad (6)$$

Pulsation
is a very important quantity in Dynamics of Structures.

Reminder
Single dof System

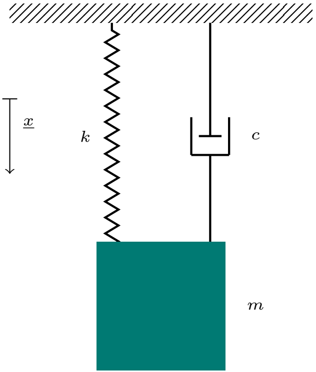

Figure 2: Single dof System with Damping device/sketch.

One has from LAGRANGE or D'ALEMBERT principle :

$$m\ddot{x} + c\dot{x} + kx = F \quad (7)$$

$$\ddot{x} + 2\xi\omega_0\dot{x} + {\omega_0}^2 x = \frac{F}{m} \quad (8)$$

with the damping factor

$$\xi = \frac{c}{2m\omega_0} \quad (9)$$

And :

$$\boxed{\omega_0 = \sqrt{\frac{k}{m}}} \quad (10)$$

Pulsation
is a very important quantity in Dynamics of Structures.

Reminder

Single dof System

The frequency f is defined as :

$$\boxed{f = \frac{\omega}{2\pi}} \tag{11}$$

Thus e.g. for the simple sping :

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{12}$$

Reminder
n-dof System

The n-dof is manageable with simple $i-$indices to generalize up to a certaine points if the parameters are well defined. Otherwise it is much moore practical to fall in a continuum formalism as the FEM.

The frequencies f_i are defined as :

$$\boxed{f_i = \frac{\omega_i}{2\pi}} \quad \ldots \quad (13)$$

Nota : circular frequency (pulsation) ω, frequency, eigenvalues, eigenvectors can be released by a `SOL 103` in NASTRAN. With ANSYS or ABAQUS also of course.

Reminder

Natural vs. Forced

A dynamic system is either in :

1. free vibration
   - No external excitations $F(t) = 0$
   - **Natural** Response/Vibration of the Structure

2. forced vibration
   - External excitations $F(t) \neq 0$
     - Either $F(t)$ is periodic ($\sim$ long timescales : many periods)
     - Or $F(t)$ is transicent ($\sim$ short timescales)
   - **Time** Response in Magnitude and Phase dependent upon the excitation (can be referred as the excitation Spectra)

Reminder

Time domain vs. frequency domain

The response known as frequency analysis is a methode used to compute the answer to oscillations as the ones due to rotations of turbomachinary. It is taken benefit that the excitation is explicitly defined is the frequency.

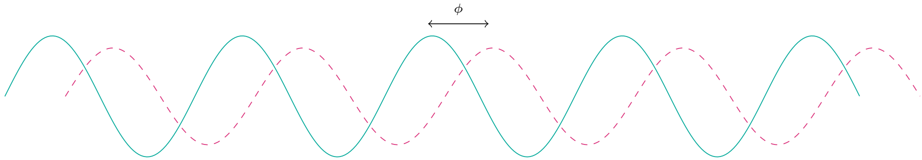

Figure 3: Phase shift.

The outputs of a frequency analysis are computed as complex numbers. Two different methods can be used in frequency response analyis. The direct method solves the coupled equations of motions in terms of forcing frequency. The modal method uses the mode shapes of the structure to reduce and uncouple the equations of motion : the solution is the sum of the modal responses.

Reminder

Rigid body vs. Elastic body

One can have fun understandings gyroscopic effect and devices that take benefit of the gyroscpopic effects...

The main business of complex dynamics phenomena is to bring the modelizaiton towards elastic (at least) solids.

Some solvers can mix rigid & flexible bodies.

Single dof System

Forced Harmonic State - General

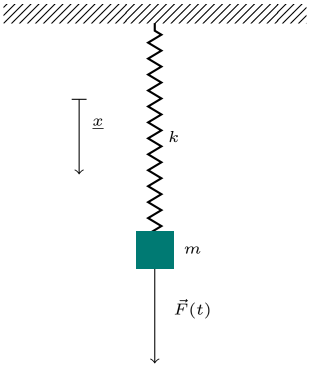

Figure 4: Single dof System without Damping device/sketch.

One has from LAGRANGE or D'ALEMBERT principle :

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t) \quad (14)$$

and

$$F(t) = F_0 \cos \Omega t = F_0 \mathcal{R}\left[\exp i\Omega t\right] \quad (15)$$

The response is the sum of a transicent response $x_t(t)$ and a constant response $x_p(t)$:

- $x_t(t)$ is the natural response
- $x_p(t)$ is the response after the transcient reponse has vanished : with a pulsation of same Ω than the excitation and with a magnitude and a phase to be derived

Single dof System

Forced Harmonic State - Undamped

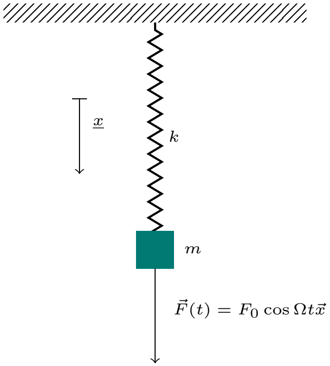

Figure 5: Single dof System.

One has from LAGRANGE or D'ALEMBERT principle :

$$m\ddot{x}(t) + kx(t) = F_0 \cos \Omega t \quad (16)$$

One has :

$$x_t(t) = X_t \cos(\omega_0 t - \varphi_t) \quad (17)$$

and :

$$x_p(t) = X_p \cos \Omega t \quad (18)$$

inserted in Equation 16 one has the magnitude

$$X_p = \frac{F_0}{k - m\Omega^2} = \frac{1}{\omega_0^2 - \Omega_0^2} \cdot \frac{F_0}{m} \quad (19)$$

and of course full solution

$$x(t) = x_t(t) + x_p(t) \quad (20)$$

Nota : Without damping the excitation and the time response are in phase.

Single dof System

Forced Harmonic State - Damped

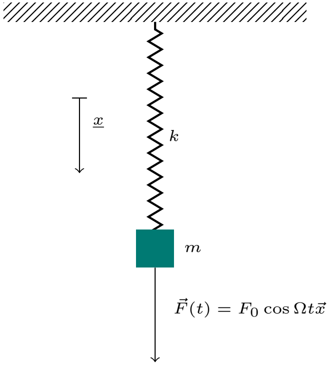

Figure 6: Single dof System.

One has from LAGRANGE or D'ALEMBERT principle :

$$m\ddot{x}(t) + c\dot{x}(t)kx(t) = F_0 \cos \Omega t \quad (21)$$

One has :

$$x_t(t) = X_t \exp(-\xi\omega_0 t)\cos(\omega_0 t - \varphi_t) \quad (22)$$

and :

$$x_p(t) = X_p \cos(\Omega t - \varphi_p) \quad (23)$$

inserted in Equation 16 one has the magnitude

$$X_p = \frac{F_0}{m} \cdot \frac{1}{\sqrt{(\omega_0^2 - \Omega^2)^2 + 4(\xi\Omega\omega_0)^2}} \quad (24)$$

$$\varphi_p = \tan^{-1} \frac{2\xi\omega_0\Omega}{\omega_0^2 - \Omega^2} \quad (25)$$

and of course full solution

$$x(t) = x_t(t) + x_p(t) \quad (26)$$

Nota : amplification of the applied force is maximum when $\Omega \to \omega_0$.

Single dof System

Forced Harmonic State - Damped

- As you maybe aware of... if time force $F(t)$ is periodic thus one can write if as a FOURIER Serie of harmonic functions $F_n(t)$
- if T is the period of $F(t)$ and $\Omega = \dfrac{2\pi}{T}$ and the harmonic own a pulsation $n\Omega$
- each harmonic functions have its own time response $x_n(t)$ and finally the linear superposition principle gives the total time response

$$x(t) = \sum_{n=0}^{\infty} x_n(t) = \sum_{n=0}^{\infty} X_n \cos(n\Omega t - \psi_n - \varphi_n) \qquad (27)$$

- LAPLACE transform can be very fruitful to solve classical dynamic problems for some sdof configurations

Some classical real world examples

Tapeï 101 Tower

Picture from peellden [2005]

Some classical real world examples

Tapeï 101 Tower

Picture from du Plessis [2010]

Some classical real world examples

Tapeï 101 Tower

Situé à Taïwan, la tour Taipei 101 mesure plus de 500 mètres de haut pour une masse totale de près de 700 000 tonnes. Construite en 2003, elle est restée le plus haut gratte-ciel du monde jusqu'en 2007 avec l'inauguration du Burj Dubaï (828 m).

Son TMD est constitué d'une boule d'acier de 660 tonnes pour un rayon de 2,7 m suspendue entre le 92e et le 87e étage. Elle pendule grâce à 4 câbles d'acier de 11,5 m et est amortie par 8 vérins hydrauliques. Sous l'action des typhons, le déplacement des étages les plus hauts peut être de 3 m.

Ainsi, il est prédit par les constructeurs que les oscillations de la tour peuvent être atténuées de 30 % à 40 %. Le système est étudié pour résister à un tremblement de terre de magnitude 7 sur l'échelle de Richter. Son efficacité a été vérifiée lors du séisme Sichuan qui a frappé Taiwan en 2008. De plus, le TMD ne représente que 0.2 % du coût total de construction du bâtiment.

wikipedia – 2020.

Nota : Further materials. . . about 2008 Tuned Mass Damper V&V Oberkampf and Roy [2010].

Created by P Thanga Vignesh from Noun Project

Created by P Thanga Vignesh from Noun Project

Some classical real world examples

Tapeï 101 Tower

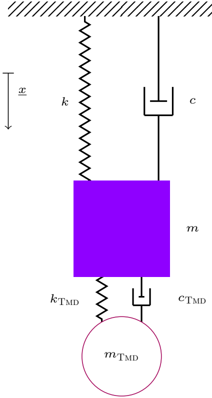

Figure 7: Single dof System with Damping & Tuned Mass Damper devices. The tuned mass damper basically permits the system of mass m to dissipate energy.

Some classical real world examples

Chinook Helicopter Ground Test

An Helicopter Ground Test that went particularly well.

Created by P Thanga Vignesh
from Noun Project

Some classical real world examples

A/C Take-off

Main loads-drivers for Engine Attachments under a take-off loadcase are :

- Engine Thrust & Torque
- Air Intake F_y & F_z aero resultants @A1

- Inertial nz ~ 1.2
- Overall Aeroloads acting on Nacelle : F_y, F_z and M_x, M_y and M_z
- No sensitivity to A/C precession rate ($\dot{\alpha} \cdot \sum I \cdot \omega$ low means no gyroscopic effect)

© AIRBUS 2017 - photo by H. GOUSSÉ / master films

Some classical real world examples

A/C Take-off

The F/T A/C parameters allow to find easily the date t for which $\max \mathrm{nz}$ is reached. FT11 nz=1.2 is reached for take-off. Numerical values for illustration purposes.

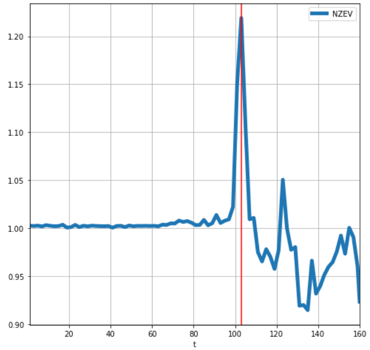

Figure 8: A330neo FT11 - nz vs. t.

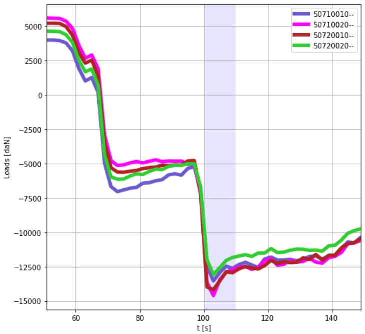

Figure 9: A330neo FT11 - E/M FM [daN] vs. t.

Some classical real world examples

A/C Engine Failure Loads

Dynamics phenomna involved e.g. within :

- Noise / Pax
- Engine
- Rotorburst
- Imbalance
- Fluid/Structure interacton e.g. Fuel
- and of course many more...

For an A/C an infamous dynamic event is loss of part of an engine as turbomachinery may lose parts to due their wear or whatever... The structure is regulated by two Technical Papers

- CS25.362 (Engine Failure Loads) from CS-25 Easa [2018]
- AMC 25-24 (Sustained Engine Imbalance)

Some classical real world examples

A/C Engine Failure Loads

CS 25.362 Engine failure loads

ED Decision 2009/017/R

(a) For engine mounts, pylons and adjacent supporting airframe structure, an ultimate loading condition must be considered that combines 1g flight loads with the most critical transient dynamic loads and vibrations, as determined by dynamic analysis, resulting from failure of a blade, shaft, bearing or bearing support, or bird strike event. Any permanent deformation from these ultimate load conditions should not prevent continued safe flight and landing.

(b) The ultimate loads developed from the conditions specified in paragraph (a) are to be:

(1) multiplied by a factor of 1.0 when applied to engine mounts and pylons; and

(2) multiplied by a factor of 1.25 when applied to adjacent supporting airframe structure.

[Amdt 25/8]

Figure 10: CS25.362 Flavour.

Some classical real world examples

A/C Engine Failure Loads

4. BACKGROUND.

a. Requirements. CS 25.362 (“Engine failure loads”) requires that the engine mounts, pylons, and adjacent supporting airframe structure be designed to withstand 1g flight loads combined with the transient dynamic loads resulting from each engine structural failure condition. The aim being to ensure that the aeroplane is capable of continued safe flight and landing after sudden engine stoppage or engine structural failure, including ensuing damage to other parts of the engine.

b. Engine failure loads. Turbine engines have experienced failure conditions that have resulted in sudden engine deceleration and, in some cases, seizures. These failure conditions are usually caused by internal structural failures or ingestion of foreign objects, such as birds or ice. Whatever the source, these conditions may produce significant structural loads on the engine, engine mounts, pylon, and adjacent supporting airframe structure. With the development of larger high-bypass ratio turbine engines, it became apparent that engine seizure torque loads alone did not adequately define the full loading imposed on the engine mounts, pylons, and adjacent supporting airframe structure. The progression to high-bypass ratio turbine engines of larger diameter and fewer blades with larger chords has increased the magnitude of the transient loads that can be produced during and following engine failures. Consequently, it is considered necessary that the applicant performs a dynamic analysis to ensure that representative loads are determined during and immediately following an engine failure event.

A dynamic model of the aircraft and engine configuration should be sufficiently detailed to characterise the transient loads for the engine mounts, pylons, and adjacent supporting airframe structure during the failure event and subsequent run down.

Figure 11: CS25.362 AMC Flavour - Background.

Some classical real world examples

A/C Engine Failure Loads

5. EVALUATION OF TRANSIENT FAILURE CONDITIONS

a. Evaluation. The applicant's evaluation should show that, from the moment of engine structural failure and during spool-down to the time of windmilling engine rotational speed, the engine-induced loads and vibrations will not cause failure of the engine mounts, pylon, and adjacent supporting airframe structure. (Note: The effects of continued rotation (windmilling) are described in AMC 25-24).

Major engine structural failure events are considered as ultimate load conditions, since they occur at a sufficiently infrequent rate. For design of the engine mounts and pylon, the ultimate loads may be taken without any additional multiplying factors. At the same time, protection of the basic airframe is assured by using a multiplying factor of 1.25 on those ultimate loads for the design of the adjacent supporting airframe structure.

b. Blade loss condition. The loads on the engine mounts, pylon, and adjacent supporting airframe structure should be determined by dynamic analysis. The analysis should take into account all significant structural degrees of freedom. The transient engine loads should be determined for the blade failure condition and rotor speed approved per CS-E, and over the full range of blade release angles to allow determination of the critical loads for all affected components.

The loads to be applied to the pylon and airframe are normally determined by the applicant based on the integrated model, which includes the validated engine model supplied by the engine manufacturer.

The calculation of transient dynamic loads should consider:

– the effects of the engine mounting station on the aeroplane (i.e., right side, left side, inboard position, etc.); and

– the most critical aeroplane mass distribution (i.e., fuel loading for wing-mounted engines and payload distribution for fuselage-mounted engines).

For calculation of the combined ultimate airframe loads, the 1g component should be associated with typical flight conditions.

Figure 12: CS25.362 AMC Flavour - Transcient Loads.

Some classical real world examples

A/C Engine Failure Loads

Scope of the analysis. The analysis of the aircraft and engine configuration should be sufficiently detailed to determine the transient and steady-state loads for the engine mounts, pylon, and adjacent supporting airframe structure during the engine failure event and subsequent run-down.

CS-25.362 AMC §6 Analysis Methodology – 2018.

Some classical real world examples

A/C Engine Failure Loads

7. MATHEMATICAL MODELLING AND VALIDATION

a. Components of the integrated dynamics model. The applicant should calculate airframe dynamic responses with an integrated model of the engine, engine mounts, pylon, and adjacent supporting airframe structure. The model should provide representative connections at the engine-to-pylon interfaces, as well as all interfaces between components (e.g., inlet-to-engine and engine-to-thrust reverser). The integrated dynamic model used for engine structural failure analyses should be representative of the aeroplane to the highest frequency needed to accurately represent the transient response. The integrated dynamic model consists of the following components that must be validated:

– Airframe structural model.

– Propulsion structural model (including the engine model representing the engine type-design).

Figure 13: CS25.362 Amc Flavour - Mathematical Modelling.

Some classical real world examples

A/C Engine Failure Loads

Picture from Rolls-Royce [2014]

Created by P Thanga Vignesh
from Noun Project

Some classical real world examples

A/C Engine Failure Loads

Picture from Rolls-Royce [2020]

Some classical real world examples

A/C Engine Failure Loads

Picture from Rolls-Royce [2020]

Some classical real world examples

A/C Engine Failure Loads

Created by P Thanga Vignesh
from Noun Project

Some classical (future) real world examples

Maveric

Picture from Airbus.

Some classical (future) real world examples

∆ Zerœ

Picture from Airbus.

Some classical (future) real world examples

Zerœ A/C w/ Propellers

Picture from AIRBUS.

The propeller considered as a minimal discrete dof system clamped at wing can be cast into :

$$\begin{bmatrix} m_1+m_2 & 0 & 0 \\ 0 & m_1 l_1^2 + m_2 l_2^2 & 0 \\ 0 & 0 & I_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} \ddddot{x}_1 \\ \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{28}$$

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Some classical (future) real world examples

A/C Engine Failure Loads

The propeller dynamics ...

Planned for 2021.

Some classical real world examples

A/C Engine Failure Loads

Engines can lead to e.g. two phenomena linked with flow instabilities :

- Surge
- Stall

Have a look to Dixon and Hall [2013] for more insights.

Picture from Airbus.

Some classical real world examples

A/C Engine Failure Loads

If one engine is shut down the Aircraft has to withstand Windmilling Loads.

Picture from Airbus (Same credits as Slide 39/66).

Some classical real world examples

Rotordynamics

References : Lalanne and Ferraris [1998], Muszynska [2005].

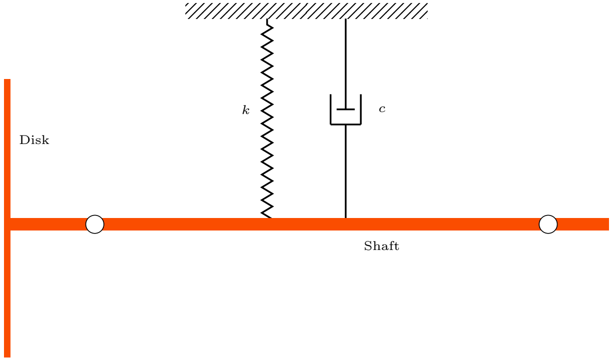

Figure 14: A Rotordynamics Simple model. Rotors can be characterized by their structural Modes vs. their rotational speeds ω (CAMPBELL diagram). The bearings $\circ$ can be idealized with the same behaviour of the s-dof system in cylindrical coordinate system. One has to consider the sum of the kinetic energy of the disk, the shaft,. . . for simple sysem one can reach closase form solution.

Some classical real world examples

Aeroelasticity

Planned for 2021.

Some classical real world examples

Flight Dynamics

Planned for 2021.

Fluid Structure Interacton

A flavour of Fluid Structure Interaction (e.g. de Langre [2001]).

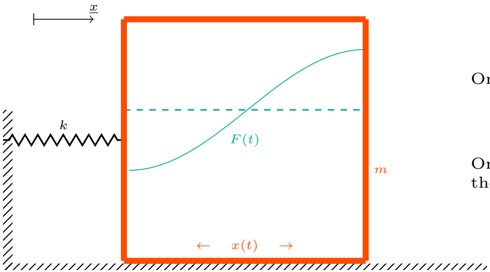

Figure 15: A Fluid Structure Interaction Simple Configuration. One can easily think e.g. to a bio-fuel tank embedded in a flying machine. Under manoeuvers the bio-fuel is swung.

One can assume e.g. :

$$\ddot{x} + x = 0 \qquad (29)$$

One can derive the coupling equations that govern the two parameters (x, F).

$$\begin{aligned} M\ddot{F} + KF &= M_{SF}\ddot{x} && (30) \\ \ddot{x} + x &= -m_{FS}\ddot{F} - m_F\ddot{x} && (31) \end{aligned}$$

Inertial coupling does exist for the two parameters (x, F). One can limit the participation of the fluid to the dynamics of the structure installing anti-swung walls within the tank.

Some classical real world examples

Impact

Planned for 2021.

Transition Slides

So touchy industrial dynamics phenomena could not be solved without massive usage of solvers involving state of the art numerical analysis algorithm.

The FEA Formulation

Basics

One solves POISSON's equation :

$$\begin{cases} -\Delta u = f & \text{in} \quad \Omega \\ u = 0 & \text{on} \quad \partial\Omega \end{cases} \tag{32}$$

It is considered $f \in L^2(\Omega)$. It is assumed $u \in H^2(\Omega)$. Thus by GREEN formula

$$\int_{\Omega} \nabla u \cdot \nabla v d\Omega = \int_{\Omega} f v d\Omega + \int_{\partial\Omega} \frac{\partial u}{\partial n} v d\Gamma \tag{33}$$

If it is chosen $v \in H_0^1(\Omega)$ one has

$$\int_{\Omega} \nabla u \cdot \nabla v d\Omega = \int_{\Omega} f v d\Omega \qquad , \forall v \in H_0^1(\Omega) \tag{34}$$

The equation (34) is called the variational formulation or the weak form of the differential equation (32).

The FEA Formulation

Basics

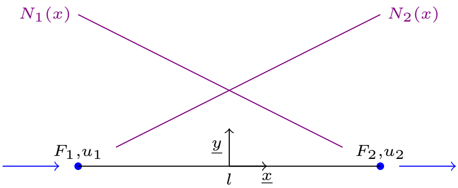

Figure 16: Simple axial element assumption sketch

Linear form function

$$\left\{ \begin{array}{l} N_1\left(x\right) = \dfrac{1}{l}\left(\dfrac{l}{2} - x\right) \\ N_2\left(x\right) = \dfrac{1}{l}\left(x + \dfrac{l}{2}\right) \end{array} \right. \tag{35}$$

Strain worth

$$\varepsilon = B \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial N_1}{\partial x}, & \dfrac{\partial N_2}{\partial x} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{l}, & \dfrac{1}{l} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{36}$$

The FEA Formulation

Basics

Stiffness matrix is derived as

$$K = \int_{\Omega} B^T E B d\Omega = S \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{E}{l^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} dx \quad (37)$$

The simple bar element works merely as‡

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{ES}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad (38)$$

with stiffness matrix

$$K = \frac{ES}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (39)$$

Nota : Mass matrix is derived as

$$M = \frac{\rho S l}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad (40)$$

Finite Elements Solver User's user can usually output K, M or any matrix in an file for double check & analysis.

‡ . . . a $F = k\,u$ spring ☺

The FEA Formulation

Basics

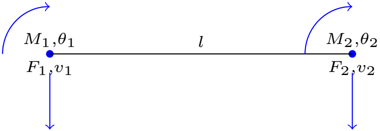

Figure 17: Simple bending element assumption sketch

For a simple bending element one has the relationship

$$\left\{\begin{array}{c} F_1 \\ M_1 \\ F_2 \\ M_2 \end{array}\right\} = \frac{2EI}{l^3}\left[\begin{array}{cccc} 6 & -3l & -6 & -3l \\ -3l & 2l^2 & 3l & l^2 \\ -6 & 3l & 6 & 3l \\ -3l & l^2 & 3l & 2l^2 \end{array}\right]\left\{\begin{array}{c} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{array}\right\} \quad (41)$$

The FEA Formulation

Advanced

Planned for 2021.

The FEA Formulation

Dynamic Reduction

Among the most interesting & funny concept within Dynamics FEA is the Model Reduction. On one hand it can be considred as classical business for Static Analysis with the associated GUYAN approach. On the other hand the approach for dynamic analysis is not numerically exact because upon eigenvalues desired dependent. Nevertheless it remains usual business within large companies dynamics analysis. Mainly to ease the exchange of FEM between e.g. stakeholders of a project. I follow Komzsik [2005] notations.

The FEA Formulation

Dynamic Reduction

For GUYAN Static Reduction the framework is linear elasticity. One solves

$$K_{ff} u_f = F_f \quad (42)$$

with o-set interior degrees of freedom and a-set exterior degrees of freedom one can split the equation (42)

$$\begin{bmatrix} K_{oo} & K_{oa} \\ K^T{}_{oa} & K_{aa} \end{bmatrix} \begin{Bmatrix} u_o \\ u_a \end{Bmatrix} = \begin{Bmatrix} F_o \\ F_a \end{Bmatrix} \quad (43)$$

whose first line of (43) means

$$\begin{aligned} & K_{oo} u_o + K_{oa} u_a = F_o \\ \Leftrightarrow\ & K_{oo}^{-1} \left(K_{oo} u_o + K_{oa} u_a \right) = K_{oo}^{-1} F_o \\ \Leftrightarrow\ & u_o = K_{oo}^{-1} F_o - K_{oo}^{-1} K_{oa} u_a \\ \Leftrightarrow\ & u_o = \underbrace{K_{oo}^{-1} F_o}_{u_o^0} - \underbrace{K_{oo}^{-1} K_{oa}}_{-G_{oa}} u_a \end{aligned} \quad (44)$$

or

$$u_o = u_o^0 + G_{oa} u_a \quad (45)$$

with G_{oa} boundary transformation and u_o^0 fixed boundary displacement.

The FEA Formulation

Dynamic Reduction

Thus the second line of (43) means with the definition (45) of u_o

$$
\begin{aligned}
& {K^T}_{oa} u_o + K_{aa} u_a = F_a \\
\Leftrightarrow\ & {K^T}_{oa} \left(u_o^0 + G_{oa} u_a \right) + K_{aa} u_a = F_a \\
\Leftrightarrow\ & \left(\underbrace{{K^T}_{oa} G_{oa}}_{\overline{K_{aa}}} + K_{aa} \right) u_a = F_a - \underbrace{{K^T}_{oa} u_o^0}_{\overline{F_a}}
\end{aligned} \tag{46}
$$

with $\overline{K_{aa}}$ boundary stiffness and $\overline{F_a}$ boundary load. Typically with a classical NASTRAN run one has $\overline{K_{aa}} \equiv$ `KAAX` and $\overline{F_a} \equiv$ `PAX`. Note that (46) defines the boundary displacement field associated to the `ASET` (NASTRAN language). For Dynamic framework :

The FEA Formulation

Dynamic Reduction

$$\begin{bmatrix} K_{oo} - \lambda M_{oo} & K_{ot} - \lambda M_{ot} \\ K_{ot}^T - \lambda M_{ot}^T & K_{aa}^T - \lambda M_{aa} \end{bmatrix} \begin{Bmatrix} u_o \\ u_t \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (47)$$

One considers first the eigenvalue problem :

$$[K_{oo} - \lambda M_{oo}]\Phi_o = 0 \qquad (48)$$

One is lead to assume a diagonal matrix $\Lambda_{q_o q_o}$ with q_o the eigenvalues and another matrix Φ_{oq_o} with classical :

$$\Phi_{oq_o}^T M_{oo} \Phi_{oq_o} = I_{q_o q_o} \qquad (49)$$

And for the stiffness :

$$\Phi_{oq_o}^T K_{oo} \Phi_{oq_o} = \Lambda_{q_o q_o} \qquad (50)$$

The installed q_o is not today a computational issue for low frequency range of interest for e.g. an Aircraft. It is a gloabl rule of thumb to compute a higher number than in the end the eigenvalues of interest.

The FEA Formulation

Dynamic Reduction

For the boundary

$$[K_{tt} - \lambda M_{tt}]\Phi_t = 0 \quad (51)$$

with the coupled bounday Φ_{tq_o}. The dynamics reduction leads to the introduction of the matrix with $q_t + q_o$ columns and with $t + o$ rows.

$$S = \begin{bmatrix} \Phi_{oq_o} & \\ & \Phi_{tq_o} \end{bmatrix} \quad (52)$$

with $Su_d = u_a$ one has

$$S^T[K_{aa} - \lambda M_{aa}]Su_d = 0 \quad (53)$$

Thus

$$[K_{dd} - \lambda M_{dd}]u_d = 0 \quad (54)$$

Then the problem for the residual structure is :

$$\begin{bmatrix} \Lambda_{q_oq_o} - \lambda I_{q_oq_o} & K_{q_oq_t} - \lambda M_{q_oq_t} \\ K^T_{q_oq_t} - \lambda M^T_{q_oq_t} & \Lambda_{q_tq_t} - \lambda I_{q_tq_t} \end{bmatrix} \left\{ \begin{array}{c} u_{qo} \\ u_{qt} \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ 0 \end{array} \right\} \quad (55)$$

The FEA Formulation

Dynamic Reduction

with the coupling matrices $M_{q_o q_t}$ and $K_{q_o q_t}$ uilt as previously.

$$\Phi_{o q_o} u_{q_o} = u_o \qquad (56)$$

$$\Phi_{t q_t} u_{q_t} = u_t \qquad (57)$$

One ends up with a $q_o + q_t$ problem. All the matrices are note square. The neture of the choice of the q_o leads thus to the introduction of a numerical error. This kind of CRAIG- BAMPTON Reduction is not numerically exact for the modal analysis.

Transcient Analysis

To solve a time-dependent dynamics probelem the engineer has at least the next 4 approaches available :

1. Direct Approach with Explicit scheme
2. Direct Approach with Implicit scheme
3. Modal Superpostion in time domain
4. Modal Superpostion in frequency domain

Direct Method & Uncertainties Propagation

The Newmark family integration scheme

More or less one has to consider

$$M\,\ddot{u} + C\,\dot{u} + K u = f_{ext} \quad (58)$$

or

$$M\,\ddot{u} + C\,\dot{u} + f_{int} = f_{ext} \quad (59)$$

The Newmark method states that one can consider velocity $\dot{u}$ can be derived as

$$\dot{u}_{n+1} = \dot{u}_n + \Delta t\,\ddot{u}_\gamma \quad (60)$$

whith $\ddot{u}_\gamma$ assumed

$$\ddot{u}_\gamma = (1-\gamma)\ddot{u}_n + \gamma\ddot{u}_{n+1} \quad (61)$$

with $0 \leq \gamma \leq 1$ and therefore

$$\dot{u}_{n+1} = \dot{u}_n + (1-\gamma)\Delta t\,\ddot{u}_n + \gamma\Delta t\,\ddot{u}_{n+1} \quad (62)$$

Direct Method & Uncertainties Propagation

The NEWMARK family integration scheme

and as acceleration is also t-dependent one can carry on and one has

$$u_{n+1} = u_n + \Delta t\, \dot{u}_n + \frac{1}{2}\Delta t^2 \ddot{u}_\beta \quad (63)$$

with

$$\ddot{u}_\beta = (1 - 2\beta)\ddot{u}_n + 2\beta\ddot{u}_{n+1} \quad (64)$$

and $0 \leq 2\beta \leq 1$.

Direct Method & Uncertainties Propagation

The Newmark family integration scheme

Thus

$$\dot{u}_{n+1} = \dot{u}_n + (1-\gamma)\Delta t\,\ddot{u}_n + \gamma \Delta t \ddot{u}_{n+1} \quad (65)$$

$$u_{n+1} = u_n + \Delta t\,\dot{u}_n + \frac{\Delta t^2}{2}\left\{(1-2\beta)\ddot{u}_n + 2\beta\ddot{u}_{n+1}\right\} \quad (66)$$

$$M\ddot{u}_{n+1} + C\dot{u}_{n+1} + f_{\text{int}} = f_{\text{ext}\,n+1} \quad (67)$$

A random choice of β and γ is not recommended ! They will define the stability of the t-integration scheme. Stability analysis is generally not presented but give a try.

Direct Method & Uncertainties Propagation

The NEWMARK family integration scheme

Example : For example if one has to solve $\omega^2 Mx = Kx$ and has choosen $\gamma = \frac{1}{2}$ and $\beta = 0$ (explicit central difference scheme) the scheme is stable iff :

$$\omega \Delta t \leq 2 \quad (68)$$

Optimization under Dynamics Constraints

Two Springs/Three Masses Case Study

One considers the next Two Springs/Three Masses system. One is interested in the frictionless harmonic answer of the system sketched in Figure 18.

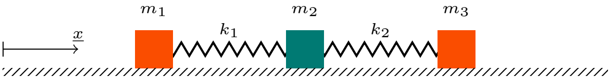

Figure 18: Two Springs/Three Masses system.

Aim of Case study (extracted from Yang [2010]): Choose masses and spring stiffnesses to satisfy next optimization (min-max Danskin [2012]) problem with S and k_0 constants:

$$\text{Problem} \left\{ \begin{array}{c} \min\limits_{p \in \{m_1, m_2, m_3, k_1, k_2\}} \omega = \max \omega_i \\ S = m_1 + m_2 + m_3 \\ m_1 = m_3 \\ k_1 = k_2 \geq k_0 \end{array} \right.$$

Optimization under Dynamics Constraints

Two Springs/Three Masses Case Study

Answer : One can reach the minimum-max ω_i

$$\min \max \omega_i = \left\{ \frac{8k_0}{S} \right\}^{\frac{1}{2}} \qquad (69)$$

with the *ad hoc* mass split (cf. complete demonstration in Yang [2010]).

Outro & Outcomes

Many topics have been left untouched : do not hesitate by discovering them on your own !

- Dynamics & Instabilities : Flutter, Buffeting, . . .
- Component Modal Synthesis
- Dynamics & Nonlinearity
- . . .

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- my nowadays regularly Dynamics Engineers focal points within Engine Manufacturer Companies
- my nowadays Aircraft Dynamics Specialists colleagues @ AIRBUS/Toulouse
- my nowadays Powerplant Integrated FEM Dynamics Specialists colleagues @ AIRBUS/Toulouse

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I did not copy/paste them only to get a *nice* Bibliography. I have been lead to open most of them either as a student or in my professional life. Some are really interesting to dive into more insight Structural *&* Computational Dynamics of Structures and I admit some content chapters, sections §very technical ...To complete the documentation of some finite elements software can be of great help (NASTRAN as Sitton [1997], Herling [1997], ABAQUS, ANSYS, MATLAB,...)

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