Vibrations, Structural & Computational Dynamics

October 20^{th} 2020

$$\omega^2 \, \mathrm{M}x = \mathrm{K}\, x \Leftrightarrow (K - \lambda M)x = 0$$

ISAE/SUPAERO TOULOUSE
SA403 - ADVANCED COMPUTATIONAL STRUCTURAL MECHANICS

Julien LE FANIC —AIRBUS Engineer

Table of contents

Vibrations, Structural & Computational Dynamics

Scope/Contents

Let's have fun & consider the next topics in more details in the next slides :

- Reminders
- s-dof system
- Industry Flavours of Dynamics Analysis
- Computational Approaches of Dynamics Analysis
- Reduction of Structures for Dynamics Analysis

Vibrations, Structural & Computational Dynamics

Who am I ?

My name is Julien LE FANIC.

My Final Year Engineer Thesis was about hypervelocity Impact*.

I am an AIRBUS Structural Engineer since 2009†.

Nowadays I spend most of my daily working hours fighting with FEA solver to understand why my dynamic analyses crash and/or/either do not release the expected figures...

* in Paris

† in Toulouse

Reminder

Dynamic Definition

The solution of D'ALEMBERT principle (presented hereafter in the slides... keep calm) for a continuum or a assembly rigid bodies is the purpose of a dynamic study. One is interested in the mechanical quantities such as u, $\dot{u}$, $\ddot{u}$,... up to σ, ε, $\dot{\varepsilon}$ for a system excited with a time dependent phenomena/excitation.

The tasks to be lead depend upon the phenomena and the latter will guide the mathematical approach (solver in a computational mechanics framewrok) to be used.

For the beauty of the formula one reminds that the fundamtal equation of dynamics $F = m \cdot \gamma$ stems from LAGRANGE's equation of motion :

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial P}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = 0 \quad , \quad i \in 1, \ldots, n \tag{1}$$

with q_i generalized parameters, T the kinetic energy, P the potential energy and D as a dissipative function of the system. Have a look to Tallec [2000], Wells [1967], Komzsik [2005], Roseau [1984], Géradin and Rixen [1996] for more insights. For the latter equaion (1) one can reach for an undamped mass/spring system the the classical equation :

$$m\ddot{x} + kx = 0 \tag{2}$$

which governs the free vibrations os a single dof system.

Reminder

Dynamic Definition

Approach	Keep it simple... but not simpler	...for fun...	...Reality
About dof	s-dof	n-dof	continuum
User's Face	🤫	😉	🤔

Table 1: Classification of Dynamic Classical Models vs. Difficulty.

Nota : The art to switch from a dynamic analysis to a s-dof analysis doest exist. For example Miles'equation.

Reminder

Single dof System

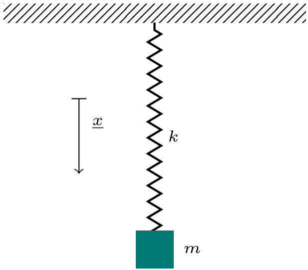

Figure 1: Single dof System without Damping device/sketch.

One has from LAGRANGE or D'ALEMBERT principle :

$$m\ddot{x} + c\dot{x} + kx = F \quad (3)$$

$$\ddot{x} + 2\xi\omega_0\dot{x} + {\omega_0}^2 x = \frac{F}{m} \quad (4)$$

with the damping factor

$$\xi = \frac{c}{2m\omega_0} \quad (5)$$

And :

$$\boxed{\omega_0 = \sqrt{\frac{k}{m}}} \quad (6)$$

Pulsation
is a very important quantity in Dynamics of Structures.

Reminder

Single dof System

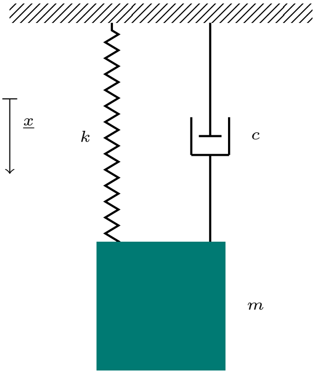

Figure 2: Single dof System with Damping device/sketch.

One has from LAGRANGE or D'ALEMBERT principle :

$$m\ddot{x} + c\dot{x} + kx = F \quad (7)$$

$$\ddot{x} + 2\xi\omega_0\dot{x} + {\omega_0}^2 x = \frac{F}{m} \quad (8)$$

with the damping factor

$$\xi = \frac{c}{2m\omega_0} \quad (9)$$

And :

$$\boxed{\omega_0 = \sqrt{\frac{k}{m}}} \quad (10)$$

Pulsation
is a very important quantity in Dynamics of Structures.

Reminder
n-dof System

The n-dof

Reminder

Natural vs. Forced

A dynamic system is either in :

1. free vibration
   - No external excitations $F(t) = 0$
   - **Natural** Response/Vibration of the Structure
2. forced vibration
   - External excitations $F(t) \neq 0$
     - Either $F(t)$ is periodic ($\sim$ long timescales : many periods)
     - Or $F(t)$ is transicent ($\sim$ short timescales)
   - **Time** Response in Magnitude and Phase dependent upon the excitation (can be referred as the excitation Spectra)

Reminder

Damping definition

Planned for 2021.

Reminder

Modal basis definition

Planned for 2021.

Reminder

Time domain vs. frequency domain

Planned for 2021.

Reminder

Rigid body vs. elastic body

One can have fun understandings gyroscopic effect and devices that take benefit of the gyroscpopic effects...

The main business of complex dynamics phenomena is to bring the modelizaiton towards elastic (at least) solids.

Single dof System

Forced Harmonic State - General

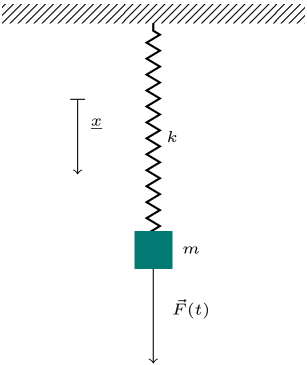

Figure 3: Single dof System without Damping device/sketch.

One has from LAGRANGE or D'ALEMBERT principle :

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t) \quad \ldots \ldots \quad (11)$$

and

$$F(t) = F_0 \cos \Omega t = F_0 \mathcal{R} [\exp i\Omega t] \quad \ldots \ldots \quad (12)$$

The response is the sum of a transcient response $x_t(t)$ and a constant response $x_p(t)$:

- $x_t(t)$ is the natural response
- $x_p(t)$ is the response after the transcient reponse has vanished : with a pulsation of same Ω than the excitation and with a magnitude and a phase to be derived

Single dof System

Forced Harmonic State - Undamped

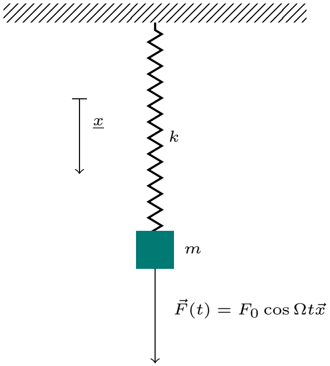

Figure 4: Single dof System.

One has from LAGRANGE or D'ALEMBERT principle :

$$m\ddot{x}(t) + kx(t) = F_0 \cos \Omega t \qquad (13)$$

One has :

$$x_t(t) = X_t \cos(\omega_0 t - \varphi_t) \qquad (14)$$

and :

$$x_p(t) = X_p \cos \Omega t \qquad (15)$$

inserted in Equation 13 one has the magnitude

$$X_p = \frac{F_0}{k - m\Omega^2} = \frac{1}{\omega_0^2 - \Omega_0^2} \cdot \frac{F_0}{m} \qquad (16)$$

and of course full solution

$$x(t) = x_t(t) + x_p(t) \qquad (17)$$

Nota : Without damping the excitation and the time response are in phase.

Single dof System

Forced Harmonic State - Damped

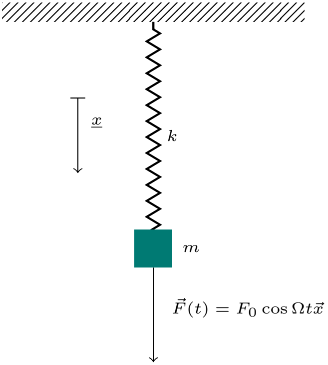

Figure 5: Single dof System.

One has from LAGRANGE or D'ALEMBERT principle :

$$m\ddot{x}(t) + c\dot{x}(t)kx(t) = F_0 \cos \Omega t \quad (18)$$

One has :

$$x_t(t) = X_t \exp(-\xi\omega_0 t)\cos(\omega_0 t - \varphi_t) \quad (19)$$

and :

$$x_p(t) = X_p \cos(\Omega t - \varphi_p) \quad (20)$$

inserted in Equation 13 one has the magnitude

$$X_p = \frac{F_0}{m} \cdot \frac{1}{\sqrt{(\omega_0^2 - \Omega^2)^2 + 4(\xi\Omega\omega_0)^2}} \quad (21)$$

$$\varphi_p = \tan^{-1}\frac{2\xi\omega_0\Omega}{\omega_0^2 - \Omega^2} \quad (22)$$

and of course full solution

$$x(t) = x_t(t) + x_p(t) \quad (23)$$

Nota : amplification of the applied force is maximum when $\Omega \to \omega_0$.

Single dof System

Forced Harmonic State - Damped

- As you maybe aware of... if time force $F(t)$ is periodic thus one can write if as a Fourier Serie of harmonic functions $F_n(t)$
- if T is the period of $F(t)$ and $\Omega = \frac{2\pi}{T}$ and the harmonic own a pulsation $n\Omega$
- each harmonic functions have its own time response $x_n(t)$ and finally the linear superposition principle gives the total time response

$$x(t) = \sum_{n=0}^{\infty} x_n(t) = \sum_{n=0}^{\infty} X_n \cos(n\Omega t - \psi_n - \varphi_n) \quad (24)$$

- Laplace transform can be very fruitful to solve classical dynamic problems for some sdof configurations

Some classical real world examples

Tapeï 101 Tower

Picture from peellden [2005]

Some classical real world examples

Tapeï 101 Tower

Picture from du Plessis [2010]

Some classical real world examples

Tapeï 101 Tower

Situé à Taïwan, la tour Taipei 101 mesure plus de 500 mètres de haut pour une masse totale de près de 700 000 tonnes. Construite en 2003, elle est restée le plus haut gratte-ciel du monde jusqu'en 2007 avec l'inauguration du Burj Dubaï (828 m).

Son TMD est constitué d'une boule d'acier de 660 tonnes pour un rayon de 2,7 m suspendue entre le 92e et le 87e étage. Elle pendule grâce à 4 câbles d'acier de 11,5 m et est amortie par 8 vérins hydrauliques. Sous l'action des typhons, le déplacement des étages les plus hauts peut être de 3 m.

Ainsi, il est prédit par les constructeurs que les oscillations de la tour peuvent être atténuées de 30 % à 40 %. Le système est étudié pour résister à un tremblement de terre de magnitude 7 sur l'échelle de Richter. Son efficacité a été vérifiée lors du séisme Sichuan qui a frappé Taiwan en 2008. De plus, le TMD ne représente que 0.2 % du coût total de construction du bâtiment.

wikipedia – 2020.

Nota : Further materials... about 2008 Tuned Mass Damper V&V Oberkampf and Roy [2010].

Created by P Thanga Vignesh from Noun Project

Created by P Thanga Vignesh from Noun Project

Some classical real world examples

Tapeï 101 Tower

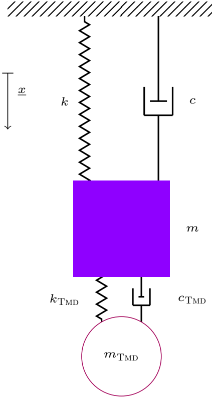

Figure 6: Single dof System with Damping & Tuned Mass Damper devices. The tuned mass damper basically permits the system of mass m to dissipate energy.

Some classical real world examples
Helicopter

A

Some classical real world examples

Rotordynamics

References : Lalanne and Ferraris [1998], Muszynska [2005].

Some classical real world examples

A/C & A/C Engine Failure Loads

Dynamics phenomna involved e.g. within :

- Noise / Pax
- Engine
- Rotorburst
- Imbalance
- Fluid/Structure interacton e.g. Fuel
- And ...

For an A/C an infamous dynamic event is loss of part of an engine as turbomachinery may lose parts to due their wear or whatever.. The structure is regulated by two Technical Papers

- CS25.362 (Engine Failure Loads)
- AMC 25-24 (Sustained Engine Imbalance)

Some classical real world examples

A/C & A/C Engine Failure Loads

Picture from Rolls-Royce [2014]

Some classical real world examples

A/C & A/C Engine Failure Loads

Picture from Rolls-Royce [2020]

Some classical real world examples

A/C & A/C Engine Failure Loads

Picture from Rolls-Royce [2020]

Some classical (future) real world examples

A/C & A/C Engine Failure Loads

Picture from Airbus.

Some classical (future) real world examples

A/C & A/C Engine Failure Loads

Picture from Airbus.

Some classical (future) real world examples

A/C & A/C Engine Failure Loads

Picture from AIRBUS.

Some classical (future) real world examples

Impact

The propeller can be set into equations this way

Some classical real world examples

Impact

A

Some classical real world examples

Whatever !?!

A

Direct Method & Uncertainties Propagation

The NEWMARK family integration scheme

More or less one has to consider

$$M\,\ddot{u} + C\,\dot{u} + Ku = f_{ext} \tag{25}$$

or

$$M\,\ddot{u} + C\,\dot{u} + f_{int} = f_{ext} \tag{26}$$

The NEWMARK method states that one can consider velocity $\dot{u}$ can be derived as

$$\dot{u}_{n+1} = \dot{u}_n + \Delta t\,\ddot{u}_\gamma \tag{27}$$

whith $\ddot{u}_\gamma$ assumed

$$\ddot{u}_\gamma = (1-\gamma)\ddot{u}_n + \gamma\ddot{u}_{n+1} \tag{28}$$

with $0 \leq \gamma \leq 1$ and therefore

$$\dot{u}_{n+1} = \dot{u}_n + (1-\gamma)\Delta t\,\ddot{u}_n + \gamma\Delta t\,\ddot{u}_{n+1} \tag{29}$$

Direct Method & Uncertainties Propagation

The Newmark family integration scheme

and as acceleration is also t-dependent one can carry on and one has

$$u_{n+1} = u_n + \Delta t\, \dot{u}_n + \frac{1}{2}\Delta t^2 \ddot{u}_\beta \tag{30}$$

with

$$\ddot{u}_\beta = (1 - 2\beta)\ddot{u}_n + 2\beta \ddot{u}_{n+1} \tag{31}$$

and $0 \leq 2\beta \leq 1$.

Direct Method & Uncertainties Propagation

The NEWMARK family integration scheme

Thus

$$\dot{u}_{n+1} = \dot{u}_n + (1-\gamma)\Delta t\, \ddot{u}_n + \gamma \Delta t \ddot{u}_{n+1} \quad (32)$$

$$u_{n+1} = u_n + \Delta t\, \dot{u}_n + \frac{\Delta t^2}{2}\left\{(1-2\beta)\ddot{u}_n + 2\beta\ddot{u}_{n+1}\right\} \quad (33)$$

$$M\ddot{u}_{n+1} + C\dot{u}_{n+1} + f_{\text{int}} = f_{\text{ext}\ n+1} \quad (34)$$

A random choice of β and γ is not recommended ! They will define the stability of the t-integration scheme. Stability analysis is generally not presented but give a try.

Direct Method & Uncertainties Propagation

The NEWMARK family integration scheme

Example : For example if one has to solve $\omega^2 Mx = Kx$ and has choosen $\gamma = \frac{1}{2}$ and $\beta = 0$ (explicit central difference scheme) the scheme is stable iff :

$$\omega \Delta t \leq 2 \quad (35)$$

Aeroelasticity

Planned for 2021.

Flight Dynamics

Planned for 2021.

Fluid Structure Interacton

A flavour of Fluid Structure Interacton (e.g. de Langre [2001]).

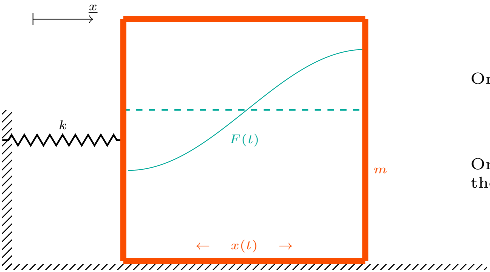

Figure 7: A Fluid Structure Interacton Simple Configuration. One can easily think e.g. to a bio-fuel tank embedded in a flying machine. Under manoeuvers the bio-fuel is swung.

One can assume e.g. :

$$\ddot{x} + x = 0 \qquad (36)$$

One can derive the coupling equations that govern the two parameters (x, F).

$$\begin{aligned} M\ddot{F} + KF &= M_{SF}\ddot{x} \qquad (37) \\ \ddot{x} + x &= -m_{FS}\ddot{F} - m_F\ddot{x} \qquad (38) \end{aligned}$$

Inertial coupling does exist for the two parameters (x, F).

The FEA Formulation

Basics

One solves POISSON's equation :

$$\left\{ \begin{array}{rcl} -\Delta u = f & \text{in} & \Omega \\ u = 0 & \text{on} & \partial\Omega \end{array} \right. \quad (39)$$

It is considered $f \in L^2(\Omega)$. It is assumed $u \in H^2(\Omega)$. Thus by GREEN formula

$$\int_\Omega \nabla u \cdot \nabla v d\Omega = \int_\Omega f v d\Omega + \int_{\partial\Omega} \frac{\partial u}{\partial n} v d\Gamma \quad (40)$$

If it is chosen $v \in H_0^1(\Omega)$ one has

$$\int_\Omega \nabla u \cdot \nabla v d\Omega = \int_\Omega f v d\Omega \qquad , \forall v \in H_0^1(\Omega) \quad (41)$$

The equation (41) is called the variational formulation or the weak form of the differential equation (39).

The FEA Formulation

Basics

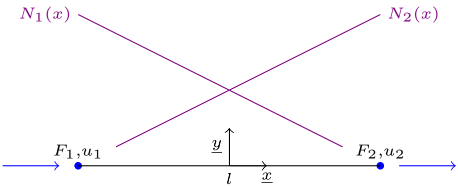

Figure 8: Simple axial element assumption sketch

Linear form function

$$\begin{cases} N_1(x) = \dfrac{1}{l}\left(\dfrac{l}{2} - x\right) \\ N_2(x) = \dfrac{1}{l}\left(x + \dfrac{l}{2}\right) \end{cases} \tag{42}$$

Strain worth

$$\varepsilon = B \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial N_1}{\partial x}, & \dfrac{\partial N_2}{\partial x} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{l}, & \dfrac{1}{l} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{43}$$

The FEA Formulation

Basics

Stiffness matrix is derived as

$$K = \int_{\Omega} B^T E B d\Omega = S \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{E}{l^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} dx \qquad (44)$$

The simple bar element works merely as[‡]

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{ES}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \qquad (45)$$

with stiffness matrix

$$K = \frac{ES}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad (46)$$

Nota : Mass matrix is derived as

$$M = \frac{\rho S l}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad (47)$$

Finite Elements Solver User's user can usually output K, M or any matrix in an file for double check & analysis.

‡ . . . a $F = k\,u$ spring ☺

The FEA Formulation

Basics

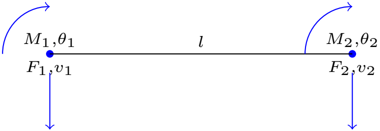

Figure 9: Simple bending element assumption sketch

For a simple bending element one has the relationship

$$\left\{ \begin{array}{c} F_1 \\ M_1 \\ F_2 \\ M_2 \end{array} \right\} = \frac{2EI}{l^3} \begin{bmatrix} 6 & -3l & -6 & -3l \\ -3l & 2l^2 & 3l & l^2 \\ -6 & 3l & 6 & 3l \\ -3l & l^2 & 3l & 2l^2 \end{bmatrix} \left\{ \begin{array}{c} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{array} \right\} \quad (48)$$

The FEA Formulation
Advanced

Planned for 2021.

The Fea Formulation

Dynamic Reduction

Among the most interesting *&* funny concept within Dynamics Fea is the Model Reduction. On one hand it can be consididred as classical business for Static Analysis with the associated Guyan approach. On the other hand the approach for dynamic analysis is not numerically exact because upon eigenvalues desired dependent. Nevertheless it remains usual business within large companies dynamics analysis. Mainly to ease the exchange of Fem between e.g. stakeholders of a project. I follow Komzsik [2005] notations.

The FEA Formulation

Dynamic Reduction

For GUYAN Static Reduction the framework is linear elasticity. One solves

$$K_{ff} u_f = F_f \tag{49}$$

with o-set interior degrees of freedom and a-set exterior degrees of freedom one can split the equation (49)

$$\begin{bmatrix} K_{oo} & K_{oa} \\ K^T{}_{oa} & K_{aa} \end{bmatrix} \begin{Bmatrix} u_o \\ u_a \end{Bmatrix} = \begin{Bmatrix} F_o \\ F_a \end{Bmatrix} \tag{50}$$

whose first line of (50) means

$$\begin{aligned} & K_{oo} u_o + K_{oa} u_a = F_o \\ \Leftrightarrow\ & K_{oo}^{-1} \left(K_{oo} u_o + K_{oa} u_a \right) = K_{oo}^{-1} F_o \\ \Leftrightarrow\ & u_o = K_{oo}^{-1} F_o - K_{oo}^{-1} K_{oa} u_a \\ \Leftrightarrow\ & u_o = \underbrace{K_{oo}^{-1} F_o}_{u_o^0} - \underbrace{K_{oo}^{-1} K_{oa}}_{-G_{oa}} u_a \end{aligned} \tag{51}$$

or

$$u_o = u_o^0 + G_{oa} u_a \tag{52}$$

with G_{oa} boundary transformation and u_o^0 fixed boundary displacement.

The FEA Formulation

Dynamic Reduction

Thus the second line of (50) means with the definition (52) of u_o

$$\begin{aligned} & {K^T}_{oa} u_o + K_{aa} u_a = F_a \\ \Leftrightarrow\ & {K^T}_{oa} \left(u_o^0 + G_{oa} u_a \right) + K_{aa} u_a = F_a \\ \Leftrightarrow\ & \left(\underbrace{{K^T}_{oa} G_{oa} + K_{aa}}_{\overline{K_{aa}}} \right) u_a = F_a - \underbrace{{K^T}_{oa} u_o^0}_{\overline{F_a}} \end{aligned} \quad (53)$$

with $\overline{K_{aa}}$ boundary stiffness and $\overline{F_a}$ boundary load. Typically with a classical NASTRAN run one has $\overline{K_{aa}} \equiv$ `KAAX` and $\overline{F_a} \equiv$ `PAX`. Note that (53) defines the boundary displacement field associated to the `ASET` (NASTRAN language). For Dynamic framework :

The FEA Formulation

Dynamic Reduction

$$\begin{bmatrix} K_{oo} - \lambda M_{oo} & K_{ot} - \lambda M_{ot} \\ K_{ot}^T - \lambda M_{ot}^T & K_{aa}^T - \lambda M_{aa} \end{bmatrix} \begin{Bmatrix} u_o \\ u_t \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (54)$$

One considers first the eigenvalue problem :

$$[K_{oo} - \lambda M_{oo}]\Phi_o = 0 \qquad (55)$$

One is lead to assume a diagonal matrix $\Lambda_{q_o q_o}$ with q_o the eigenvalues and another matrix Φ_{oq_o} with classical :

$$\Phi_{oq_o}^T M_{oo} \Phi_{oq_o} = I_{q_o q_o} \qquad (56)$$

And for the stiffness :

$$\Phi_{oq_o}^T K_{oo} \Phi_{oq_o} = \Lambda_{q_o q_o} \qquad (57)$$

The installed q_o is not today a computational issue for low frequency range of interest for e.g. an Aircraft. It is a gloabl rule of thumb to compute a higher number than in the end the eigenvalues of interest.

The FEA Formulation

Dynamic Reduction

For the boundary

$$[K_{tt} - \lambda M_{tt}]\Phi_t = 0 \qquad (58)$$

with the coupled bounday Φ_{tq_o}. The dynamics reduction leads to the introduction of the matrix with $q_t + q_o$ columns and with $t + o$ rows.

$$S = \begin{bmatrix} \Phi_{oq_o} & \\ & \Phi_{tq_o} \end{bmatrix} \qquad (59)$$

with $Su_d = u_a$ one has

$$S^T[K_{aa} - \lambda M_{aa}]Su_d = 0 \qquad (60)$$

Thus

$$[K_{dd} - \lambda M_{dd}]u_d = 0 \qquad (61)$$

Then the problem for the residual structure is :

$$\begin{bmatrix} \Lambda_{q_oq_o} - \lambda I_{q_oq_o} & K_{q_oq_t} - \lambda M_{q_oq_t} \\ K^T_{q_oq_t} - \lambda M^T_{q_oq_t} & \Lambda_{q_tq_t} - \lambda I_{q_tq_t} \end{bmatrix} \begin{Bmatrix} u_{qo} \\ u_{qt} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (62)$$

The FEA Formulation

Dynamic Reduction

with the coupling matrices $M_{q_o q_t}$ and $K_{q_o q_t}$ uilt as previously.

$$\Phi_{o q_o} u_{q_o} = u_o \quad (63)$$

$$\Phi_{t q_t} u_{q_t} = u_t \quad (64)$$

One ends up with a $q_o + q_t$ problem. All the matrices are note square. The neture of the choice of the q_o leads thus to the introduction of a numerical error. This kind of CRAIG- BAMPTON Reduction is not numerically exact for the modal analysis.

Outro & Outcomes

Many topics have been left untouched : do not hesitate by discvoring them on your own !

- Dynamics & Instabilities : FLutter, Buffeting, ...
- Component Modal Synthesis
- ...

Optimization under Dynamics Constraints

Two Springs/Three Masses Case Study

One considers the next Two Springs/Three Masses system. One is interested in the frictionless harmonic answer of the system sketched in Figure 10.

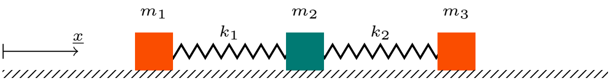

Figure 10: Two Springs/Three Masses system.

Aim of Case study (extracted from Yang [2010]): Choose masses and spring stiffnesses to satisfy next optimization (min-max Danskin [2012]) problem with S and k_0 constants:

$$\S E \left\{ \begin{array}{c} \min\limits_{p \in \{m_1, m_2, m_3, k_1, k_2\}} \omega = \max \omega_i \\ S = m_1 + m_2 + m_3 \\ m_1 = m_3 \\ k_1 = k_2 \geq k_0 \end{array} \right.$$

Optimization under Dynamics Constraints

Two Springs/Three Masses Case Study

Answer : One can reach the minimum-max ω_i

$$\min \max \omega_i = \left\{ \frac{8k_0}{S} \right\}^{\frac{1}{2}} \quad \ldots \quad (65)$$

with the *ad hoc* mass split (cf. complete demonstration in **?**).

Acknowledgements

I warmly thank...

- Joseph MORLIER to offer me the possibility to present this lecture
- my ENS - ARTS & MÉTIERS - PARIS VI TACS diploma former colleagues. Mainly those who are Professors of Mechanics Eng. and shared with me their own Slides about *Structural Dynamics*
- my former colleagues & Dynamics Specialists @ DGA/Arcueil
- my nowadays regularly Dynamics Engineers focal points within Engine Manufacturer Companies
- my nowadays Aircraft Dynamics Specialists colleagues @ AIRBUS/Toulouse
- my nowadays Powerplant Integrated FEM Dynamics Specialists @ colleagues @ AIRBUS/Toulouse

References

-

I did not copy/paste them only to get a *nice* Bibliography. I have been lead to open most of them either as a student or in my professional life. Some are really interesting to dive into more insight Structural & Computational Dynamics of Structures and I admit some content chapters, sections §very technical . . . To complete the documentation of some finite elements software can be of great help (NASTRAN as Sitton [1997], Herling [1997], ABAQUS, ANSYS, MATLAB,. . .)

P.L. Tallec. *Introduction à la dynamique structures*. Les Cours de l'École polytechnique. Ellipses Marketing, 2000. ISBN 9782729879945.

D.A. Wells. *Schaum's Outline of Lagrangian Dynamics*. Schaum's outline series. McGraw-Hill Education, 1967. ISBN 9780070692589.

L. Komzsik. *What Every Engineer Should Know About Computational Techniques of Finite Element Analysis*. What every engineer should know. Taylor & Francis, 2005. ISBN 9780849335631.

M. Roseau. *Vibrations des systèmes mécaniques: méthodes analytiques et applications*. Masson, 1984. ISBN 9782225802058.

M. Géradin and D.J. Rixen. *Théorie des vibrations: application à la dynamique des structures*. Collection Physique fondamentale et appliquée. Masson, 1996. ISBN 9782225851735.

peellden. Creative Commons, 2005.

Armand du Plessis. Creative Commons, 2010.

W.L. Oberkampf and C.J. Roy. *Verification and Validation in Scientific Computing*. Cambridge University Press, 2010. ISBN 9781139491761.

M. Lalanne and G. Ferraris. *Rotordynamics Prediction in Engineering*. Wiley, 1998. ISBN 9780471972884.

A. Muszynska. *Rotordynamics*. Mechanical Engineering. CRC Press, 2005. ISBN 9781420027792.

Rolls-Royce. 2014.

Rolls-Royce. 2020.

E. de Langre. *Fluides et solides*. Editions de l'Ecole Polytechnique, 2001. ISBN 9782730208338.

Xin-She Yang. *Engineering Optimization: An Introduction with Metaheuristic Applications*. Wiley Online Library: Books. Wiley, 2010. ISBN 9780470640418.

J.M. Danskin. *The Theory of Max-Min and its Application to Weapons Allocation Problems*. Ökonometrie und Unternehmensforschung Econometrics and Operations Research. Springer Berlin Heidelberg, 2012. ISBN 9783642460920.

G. Sitton. *Basics Dynamic Analysis User's Guide*. MSC/NASTRAN, 1997.

D. N. Herling. *Advanced Dynamic Analysis User's Guide*. MSC/NASTRAN, 1997.