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Programming Report

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Question 1. Here, we write the subroutine to map a square matrix $A \longrightarrow A_H$, where A_H is the Hessenberg form of A. To do so, we write the *hessenberg* subroutine, which uses a modified Householder transform to perform the mapping. This is essentially the same as computing the QR decomposition of A via Householder reflections, except we consider our vector v_j which forms the Householder reflection to have nonzero entries starting at the $j+1$th row index. This way, the subdiagonal of A is nonzero and doesn't get transformed into R. The inputs are A and m, where A is a $m \times m$ matrix. The subroutine changes A to A_H.

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The float-truncated tridiagonal (Hessenberg) form of our symmetric matrix then becomes

$$A_H = \begin{pmatrix} 5 & -4.2426 & 0 & 0 \\ -4.2426 & 6 & 1.1412 & 0 \\ 06 & 1.4142 & 5 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

✓

Question 2. We consider both the QR algorithm with shift, and the QR algorithm without shift. First, we consider solving the problem analytically, which gives us the characteristic equation

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$$-\lambda^3 + 6\lambda^2 - 9\lambda + 2 = 0$$

Therefore,

$$\lambda = 2, 2+\sqrt{3}, 2-\sqrt{3} = 2, \approx 3.7321, \approx 0.26795$$

The QR algorithm without shift reaches an error tolerance of $\epsilon = 1e-6$ in 12 iterations, and recapitulates the eigenvalues $\lambda = 3.7321, 2.0000, 0.26795$ along the diagonal of A, as desired. When writing the QR algorithm with shift, I had to change my QR factorization just slightly, as there are times where the norm of $v_j \longrightarrow 0$, so normalizing v_j is limited by machine precision. After fixing this by stopping the algorithm if $v_j \approx 0$, meaning that the householder reflection is quite close to the identity matrix, we have that the algorithm reveals the same eigenvalues with the same precision, but in only 8 iterations.

hmm...

Question 3. First, we consider a slight modification of the Inverse Iteration algorithm. Instead of considering $y = Bx/\|Bx\|$ where $B = (A-\mu I)^{-1}$, we equivalently solving $(A-\mu I)y = x$ and then $x := x/\|x\|$. This algorithm reveals the eigenvectors with stopping criterion as the norm of the difference of the current and previous eigenvector estimation, with $\|x_{curr} - x_{prev}\| <= \epsilon = 1e-6$.

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report should include the final e-vecs -2