HW 3: Coding Report

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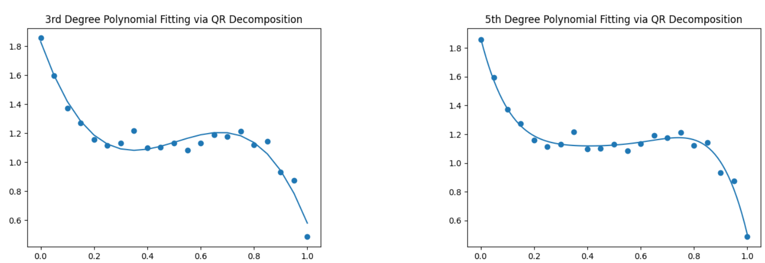
Question 1. The Cholesky decomposition allows us to write a Hermitian, positive-definite matrix A into a lower triangular matrix L and L^*, the congugate transpose. That is, we can write $A = LL^*$. Once we do this, we can use forward substitution to solve $Ly = b$ and backward subsitution to solve $L^*x = y$, which then gives us the solution to a system $Ax = b$. The forward substitution algorithm works by considering the fact that the entry $l_{11}x_1 = b_1$, so we have that $x_1 = b_1/l_{11}$. We can then substitute this *foward* to the next row of our lower triangular matrix, successively calculating x_i for $i = 2, ..., m$. Once we achieve the intermediate solution y by this process, we use backsubstitution, which is essentially the same process but for an upper triangular matrix, so our first solution is given at $l_{mm}x_m = b_m$, and subsituting this solution for x upwards through the rows of L^*.

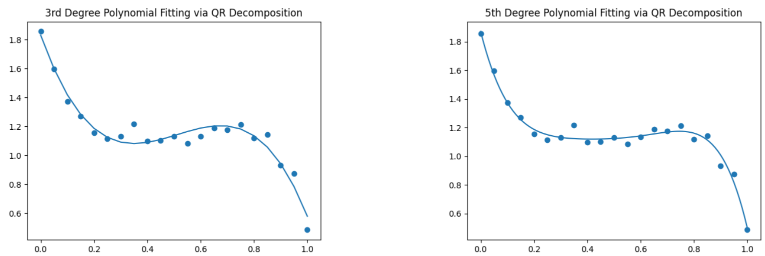
Once we do this process for the Vandermonde equation of the nth degree, we obtain the coefficients a_i of the polynomial fitting equation $\hat{f} = \sum_{i=0}^{n} a_i x^i$ such that the MSE between the data and $\hat{f}$ is minimized. With $n = 3, 5$, we produce the plots below.



(a) 3rd Degree

(b) 5th Degree

Figure 1: 3rd ad 5th degree fitting polynomials, double precision



(a) 3rd Degree (b) 5th Degree

Figure 2: 3rd ad 5th degree fitting polynomials, single precision

The error norm for the 3rd degree polynomial is $E = 0.24457513137092401$, with coefficients (in order of $a_0, a_1, ...$) are $1.8319077733859925 - 5.1704640498915211 1.204369949906708 - 7.2851782508526979$, while the error for the 5th degree polynomial is $E = 0.17274771750962886$ with coefficients $1.8695429787610824 - 7.2643083756061975 28.817794766608223 - 58.761979247240859 61.053318109931141 - 25.212434983094763$ which is lower than the 3rd degree. However, as the degree of the polynomial increases, the absolute magnitude of the coefficients increase as well. Therefore, with $n = 20$, the magnitude of the coefficients is larger than the maximum value for a double precision float, and therefore the polynomial is not well defined, since the coefficients are NaN.

Additionally, the single vs double numerical precision doesn't make a difference for the error of the curves.

Question 2. Householder transformations given by $H = I - 2vv^T$ correspond to reflection through a plane, where the plane is orthogonal to the unit vector v. Now consider applying this matrix to A, that is $H_j A$. WLOG, just consider applying this to a_1 with $j = 1$, we have that

$$(I - 2vv^t)a_1 = a_1 - 2vv^T a_1 = \begin{pmatrix} a_1 \\ a_2 \\ ... \\ a_m \end{pmatrix} - \begin{pmatrix} a_1 - s_j \\ a_2 \\ ... \\ a_m \end{pmatrix} = \begin{pmatrix} -s_j \\ 0 \\ ... \\ 0 \end{pmatrix}$$

Generally, we can see that this zeros out the subdiagonal entries of A, and slowly transforms A into R, and upper triangular matrix, while the sequence of Householder reflections creates Q with is orthogonal since $I - 2vv^T$ is orthogonal, as proved in the theory portion of the homework.