Homework 1

Julian Lehrer

Question 1 Suppose A is both unitary and upper-triangular, that is, $A^*A = AA^* = UU^{-1} = I$, Therefore, $a_{ij} = 0$ for $i > j$, that is, A is upper triangular. Then we have that A^*, the conjugate transpose, is a lower triangular matrix and that $a^*_{ij} = 0$ for $j < i$. Then for the ith row, $A^*A_{i,} = \sum_{j=1}^m A_{i,j}A^*_{i,j} = A_{i,i}A^*_{i,i} + 0 + ... + 0 = AA^*1,$. So, $A_{i,j} = 0$ for $j \neq i$, so A is diagonal.
Question 2.

- Let x be such that $Ax = \lambda x$. Then

  $$\begin{aligned} A^{-1}Ax &= A^{-1}(\lambda x) \\ \implies x &= A^{-1}(\lambda x) \\ \implies x &= A^{-1}\lambda x \\ \implies A^{-1}x &= 1/\lambda x \end{aligned}$$

  Therefore, $/\lambda$ is an eigenvalue of A^{-1}.

- Suppose $AB = \lambda x$. Then $BABx = B\lambda x$. Since linear maps are associative, we have that $(BA)Bx = \lambda(Bx)$, that is, the eigenvalue of BA is the same as AB with a different eigenvector. Therefore, the eigenvalues of AB and BA are the same

- x