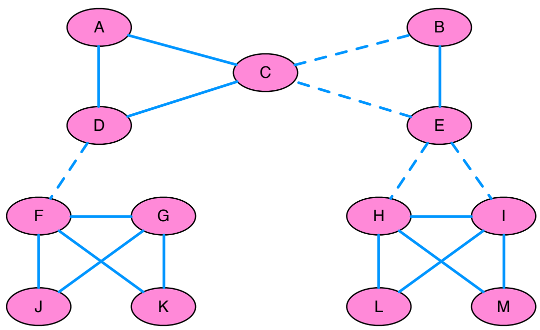
Strong and Weak Ties

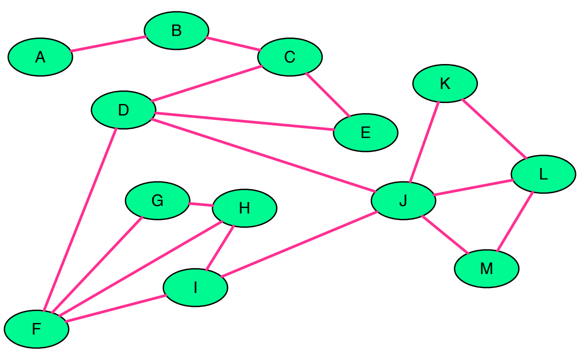
Due: January 21, 2015

1. In the graph below, which missing edge(s) are the most likely ones to be formed in the future? Motivate your answer in terms of the notions from the book that we have discussed in class.



Edges J-K and L-M are most likely to be formed in the future. The existence of these edges satisfy the Strong Triadic Closure properly.

2. Consider the graph below. Which edge(s) are bridges? Which edge(s) are local bridges with finite span? For any local bridges, give the span.

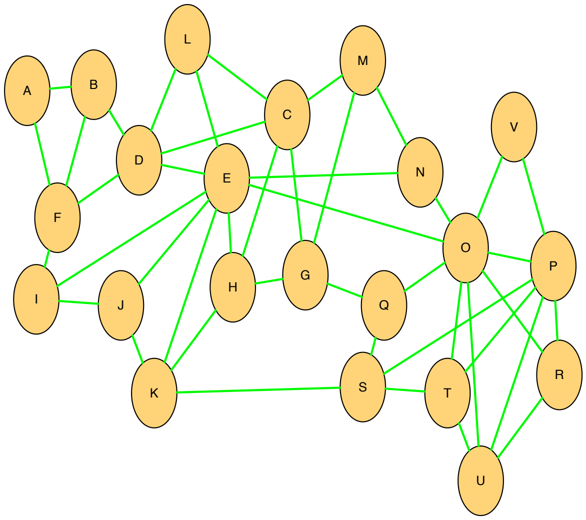


Bridges: A-B and B-C. If either A-B is removed from the graph, the graph will be split into two components. Therefore A-B is a bridge. The same scenario applies to edge B-C. If B-C is removed from the graph, the graph will be split into two components. Therefore B-C is also a bridge.

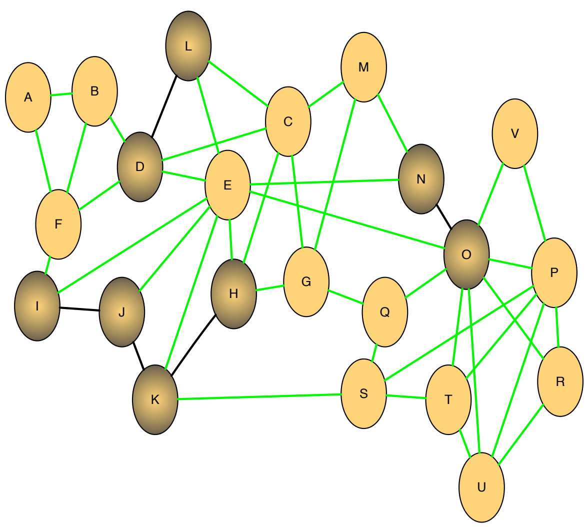
The minimum span of edge D-F is minimum number of the edges that takes use from D to F if edge D-F is removed from the graph. In this example it would be D-J, J-I, and I-F. Therefore the minimum span number of edge D-F is 3. The same scenario applies for D-J and I-J.

Local Bridge (Finite Span)	*Span*
D-F	*3*
D-J	*3*
I-J	*3*

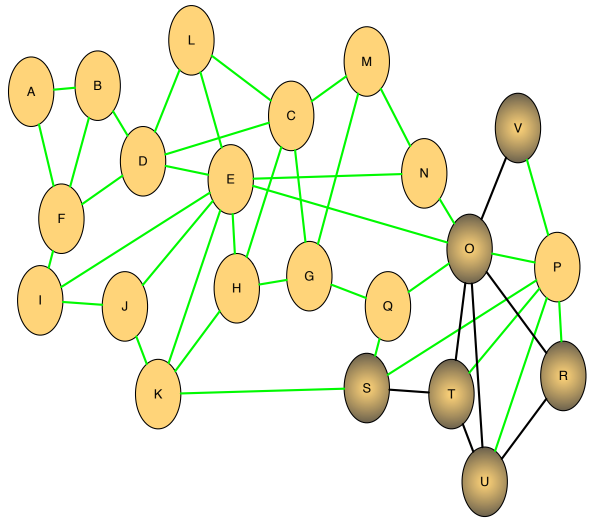
3. Use the following graph to answer the questions below.



(a) What is the clustering coefficient of vertex E? Vertex P?
Clustering coefficient of node E is number of edges between neighbors of E divided by maximum total number of edges that can exist between neighbors of E.
E has 8 neighbors (as shown with darker color in the graph). Therefore, total number of possible edges that can exist between neighbors of E is $\binom{8}{2}$. As we can see in the graph, there are five black edges that exist between neighbors of E. Therefore the clustering coefficient of node E is $\frac{5}{\binom{8}{2}} = \frac{5}{\frac{(8)\times(7)}{2}} = \frac{10}{56}$

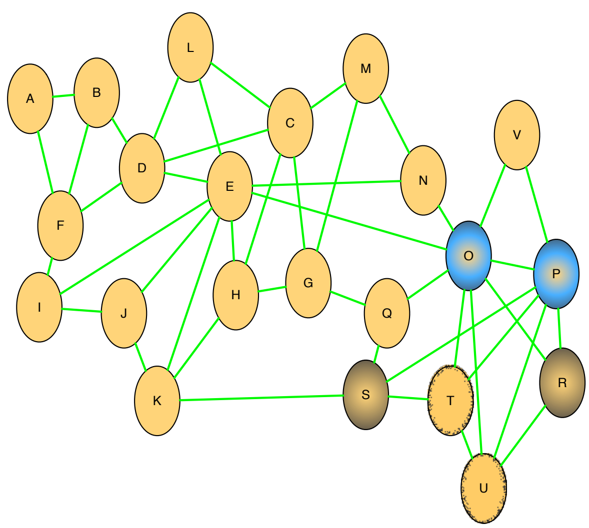


Vertex P:
Clustering coefficient of node P is number of edges between neighbors of P divided by maximum total number of edges that can exist between neighbors of P.
P has 6 neighbors (as shown with darker color in the graph). Therefore, total number of possible edges that can exist between neighbors of E is $\binom{6}{2}$. As we can see in the graph, there are seven black edges that exist between neighbors of E. Therefore the clustering coefficient of node E is $\frac{7}{\binom{6}{2}} = \frac{7}{\frac{(6)\times(5)}{2}} = \frac{7}{15}$



(b) What is the neighborhood overlap of the edge between vertex U and vertex T?

The neighborhood overlap of the edge U-T is the number of the nodes who are neighbors of both U and T divided by number of nodes who are neighbors of U and T. Noted that we do not double count the neighbor nodes that are shared between both U and T. As shown in the graph bellow, two nodes (O and P) are neighbors that are shared between T and U and there are total of 4 nodes that are eight connected to T or U. Therefore the neighborhood overlap of edge U-T is $\frac{2}{4}$.



4. *Networks Crowds and Markets* Section 3.7 Question 1

   According to definition of triadic closure, if two people in a social network has a friend in common, then there is an increased likelihood that they will become friends themselves at some point. The existence of triadic closure tendency in a network increases the tendency of clustering coefficient (i.e., fraction of neighbors who are themselves neighbors).

5. *Networks Crowds and Markets* Section 3.7 Question 2

   Edge B-C has to be a weak edge. If edge B-C were a strong edge, then based on the definition of triadic strong closure, when C-F and C-B are both strong, then there must exist edge F-B that is at least a weak tie. However, we do not see the existence of such an edge in the graph. Therefore, edge B-C cannot be a strong edge. So, it must be a weak edge.

6. *Networks Crowds and Markets* Section 3.7 Question 3

   - *edges A-D and A-B are strong. Therefore, based on Triadic Strong Closure, edge B-D must be at least a weak edge.*

   - *edges D-A and D-E are strong. Therefore, based on Triadic Strong Closure, edge A-E must be at least a weak edge.*

   - *edges B-A and B-C are strong. Therefore, based on Triadic Strong Closure, edge A-C must be at least a weak edge.*

   - *C does not satisfy Strong Triadic Closure because C-B and C-E are strong but there is no edge between B and E. E does not satisfy Strong Triadic Closure because E-D and E-C are strong but there is no edge between C and D. Another way to look at it is that edge C-E should be a weak edge because if edge C-E were to be strong, then there must have existed at least a weak edge between C and E. Given that such an edge does not exist in the graph, we can conclude that edge C-E should not be a strong edge. Therefore it should be a weak edge.*

7. *Networks Crowds and Markets* Section 3.7 Question 4

   Edge B-C is expected to be a weak edge. If edge B-C was a strong edge, then there must have existed edge E-C and B-F in the graph to avoid violation of the Strong Triadic Closure property.

   The two nodes that violate the Strong Triadic Closure property are C and E. C is listed because C-B and C-E are strong, but there is no edge between B and E. Similarly for E where E-D and E-C are strong, but there is no edge between C and D.

8. *Networks Crowds and Markets* Section 3.7 Question 5

   Edges A-B and A-C are both strong. Therefore edge B-C can be either strong or weak. In this example it is assumed to be a strong edge.
   Assuming that edge C-B is labeled correctly and edges C-B and C-E are both strong, according to triadic strong closure, there must exist least a weak edge between B and E which does not. Therefore either C-B or C-E is labeled wrong. And therefore C is the node that does not satisfy the Strong Triadic Closure Property