# Intro to Graph Theory

1. (a) Is the following graph connected?

   *Yes. There exist paths from every vertex to every other vertex in this graph.*

   (b) How many connected components are there?

   *There is one connected component in this graph.*

   (c) What is the density of the largest component?

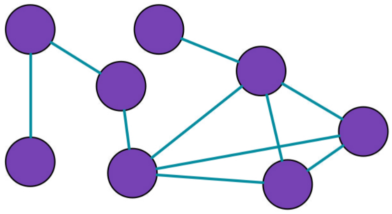
   *The density of a graph $G = (V,E)$ is the ratio of the number of edges in the graph to the maximum possible number of edges. In this example, graph G has 8 vertices and 10 edges.*

   *Therefore, the density of the graph is:*

$$\frac{10}{\binom{8}{2}} = \frac{10}{(8 * 7/2)} = \frac{5}{14}$$

   (d) What is the diameter of the largest component?

   *The diameter of the largest component is 5.*



2. (a) Is the following graph connected?

   *No.*

   (b) How many connected components are there?

   *4 components. Each isolated vertex counts as one component.*

   (c) What is the density of the largest component?

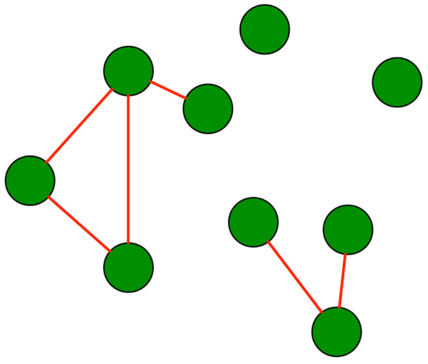
   *The largest component has 4 edges and 4 vertices.*

   *Therefore, the density of the graph is:*

$$\frac{4}{\binom{4}{2}} = \frac{4}{(4 * 3/2)} = \frac{2}{3}$$

   (d) What is the diameter of the largest component?

   *The diameter of the largest component is 2.*

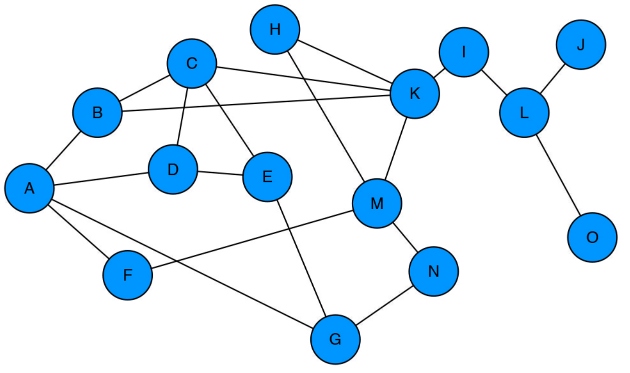


3. (a) Use breadth-first search to compute the distance from $A$ to $J$. Explain the steps of your computation.

*BFS begins at root node A and inspects all the neighboring nodes. Then for each of those neighbor nodes in turn, it inspects their neighbor nodes which were unvisited, and so on. On level one nodes B, F, D, G are discovered. On level two, nodes C,K,E,M,N are discovered. On level three, nodes I, H are discovered. on level four, node L and on level 5, node J is discovered. Therefore, the shortest path between A and J is 5.*

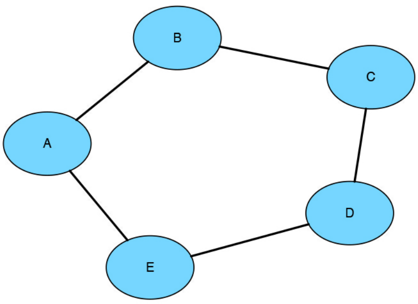
(b) Based on your computation, list all shortest paths from $A$ to $J$.

*There is only one shortest path in this graph from A to J which is A,B,K,I, L, J.*

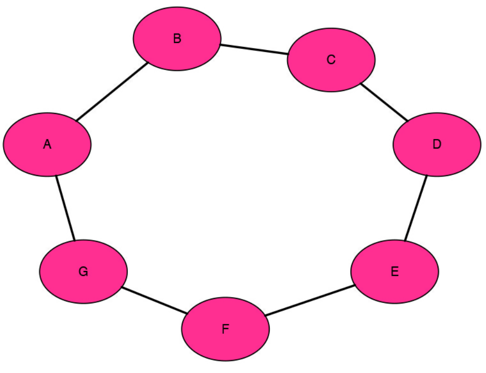



4. *Networks Crowds and Markets* Section 2.5 Question 1

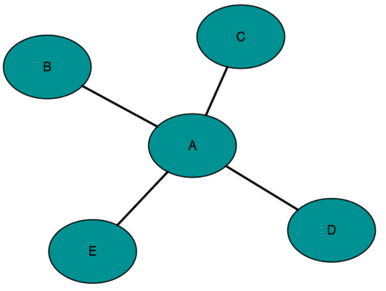
*(a) In the following graph every vertex is pivotal to its two neighbors. Note that there are other examples.*




*(b) In the following graph every vertex is pivotal to at least two different pairs of vertices. Its neighbors and one neighbor and one vertex at distance two in the other direction. For example A is pivotal for (B,G) and (C,G) (and also (B,F)!). Since a cycle is symmetric the same holds for every vertex. Note that there are other examples.*

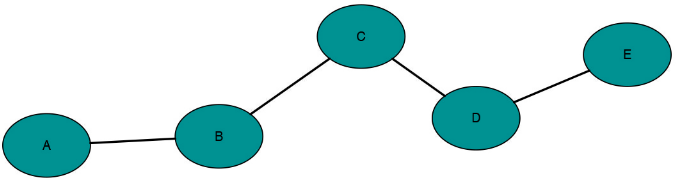


*(c) In the following graph A is pivotal to every pair of vertices. Note that there are other examples.*

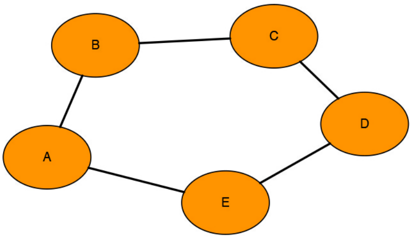



5. *Networks Crowds and Markets* Section 2.5 Question 2

   *(a) In the following graph B, C, and D are gatekeepers. So 3/5 of the vertices are gatekeepers. Note that there are other examples.*

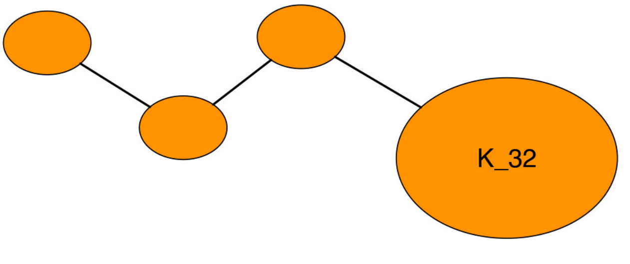



   *(b) In the following graph all vertices are local gatekeepers, but none are local gatekeepers.*




6. *Networks Crowds and Markets* Section 2.5 Question 3

   *(a) The intuition behind this problem is that you want at least one very long shortest path (to make the diameter large) and then a cluster of very interconnected vertices (to make shortest paths small). For example in the following graph, the diameter is 4 and the average distance is 1.318. Note that $K_{32}$ means the complete graph on 32 vertices. (http://en.wikipedia.org/wiki/Complete_graph).*




   *Note that there are many other examples.*

   *(b) We could change how many vertices the complete graph is on and the length of the long tail.*