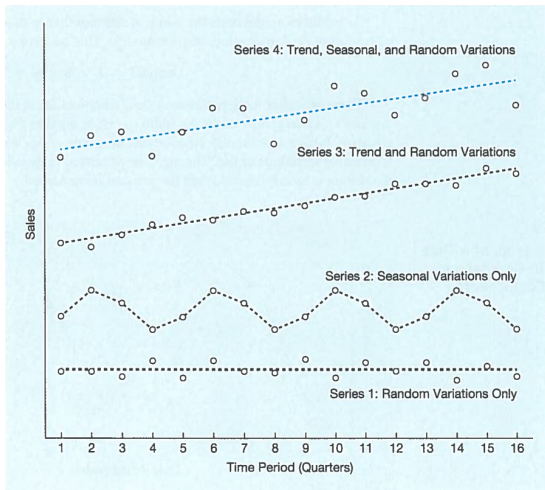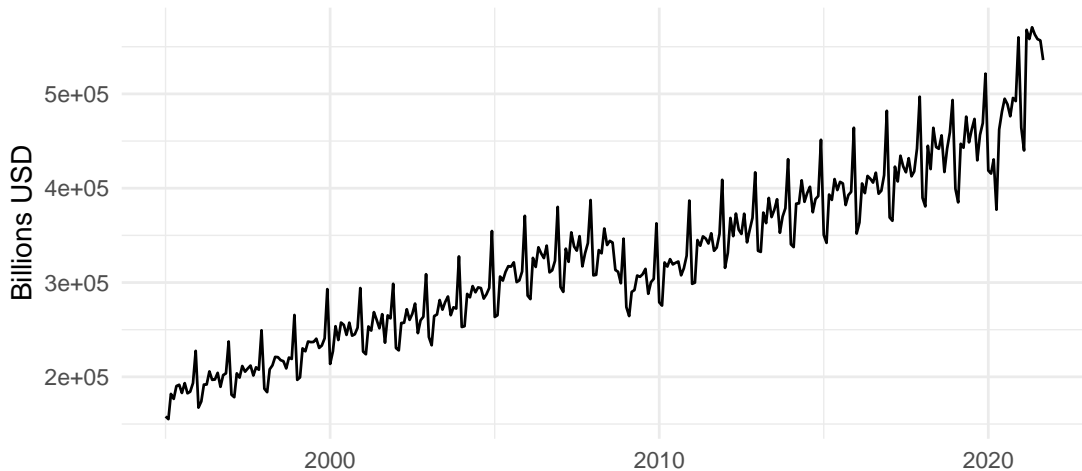# Today's Agenda

1. Review the components of time series data

2. Evaluate four forecasting tools:

   - Naïve

   - Moving Average

   - Weighted Moving Average
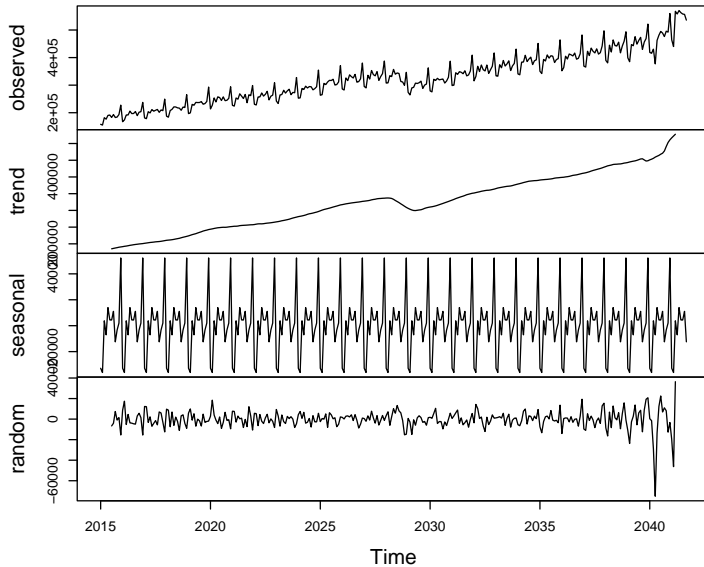
   - Exponential Smoothing

The Components of Time Series Data
(Render, Stair, Jr., Hanna and Hale 2018)
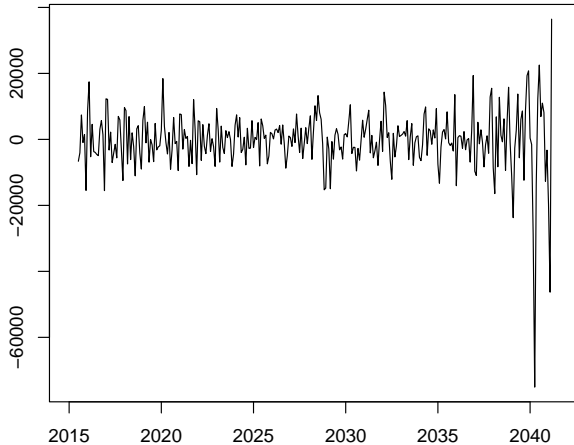
Advance Retail Sales

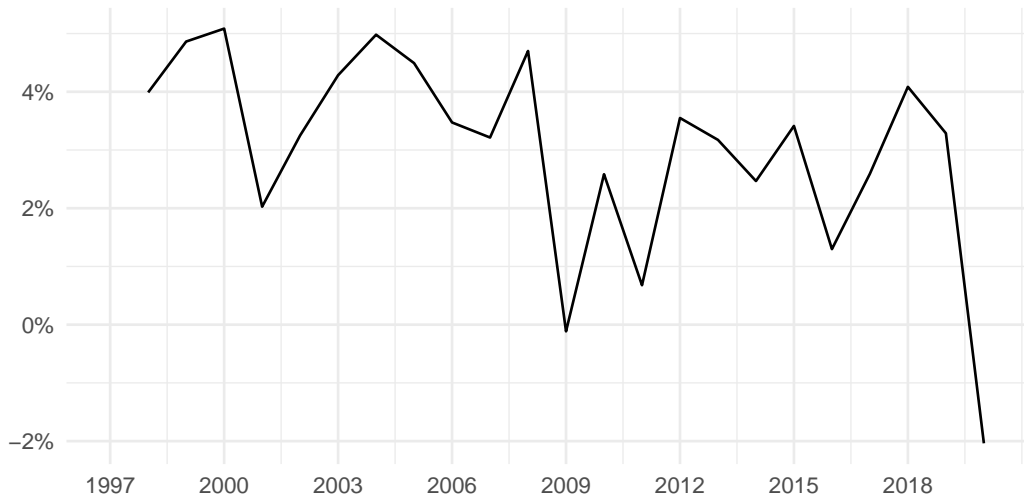Source: U.S. Census Bureau

Decomposition of additive time series

Random Variation

Random Variation Models
1. Naïve Forecast
2. Moving Average
3. Weighted Moving Average
4. Exponential Smoothing

Rate of Change in GDP (Missouri)

Forecast 1: Naive Forecast

Set forecast to the last observation

- $\text{Forecast}_{time+1} = \text{Actual}_{time}$

Forecast 1: Naive Forecast

	A	B	C
1	**Time**	**Actual**	**Forecast**
2	1998	X_{1998}	---
3	1999	X_{1999}	= B2
4	2000	X_{2000}	= B3

Calculate the forecast, extend to 2019 and visualize it.

Rate of Change in GDP (Missouri)

Forecast: −2%

GDP_Rate
Naive
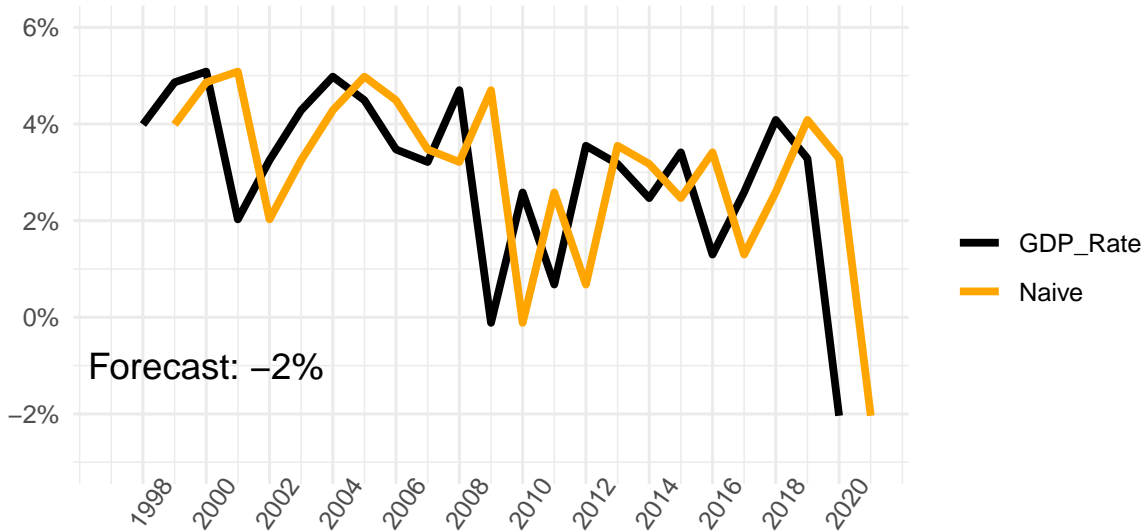
Forecast Accuracy: Mean Squared Error (MSE)

1. Calculate the forecast error
   - Forecast Error = Actual Value - Forecast Value

2. Square each forecast error

3. Calculate the mean of the squared errors

$$\text{MSE} = \frac{\Sigma(Error)^2}{n}$$

$$\text{MSE} = \frac{\Sigma(Error)^2}{n}$$

	A	B	C	D
1	**Time**	**Actual**	**Forecast**	**Error^2**
2	1998	X_{1998}	---	---
3	1999	X_{1999}	= B2	= (B3 - C3)^2
4	2000	X_{2000}	= B3	= (B4 - C4)^2

$$\text{MSE} = \text{AVERAGE(D3:D4)}$$

Rate of Change in GDP (Missouri)

Forecast: −2%
MSE: 0.00045

GDP_Rate
Naive
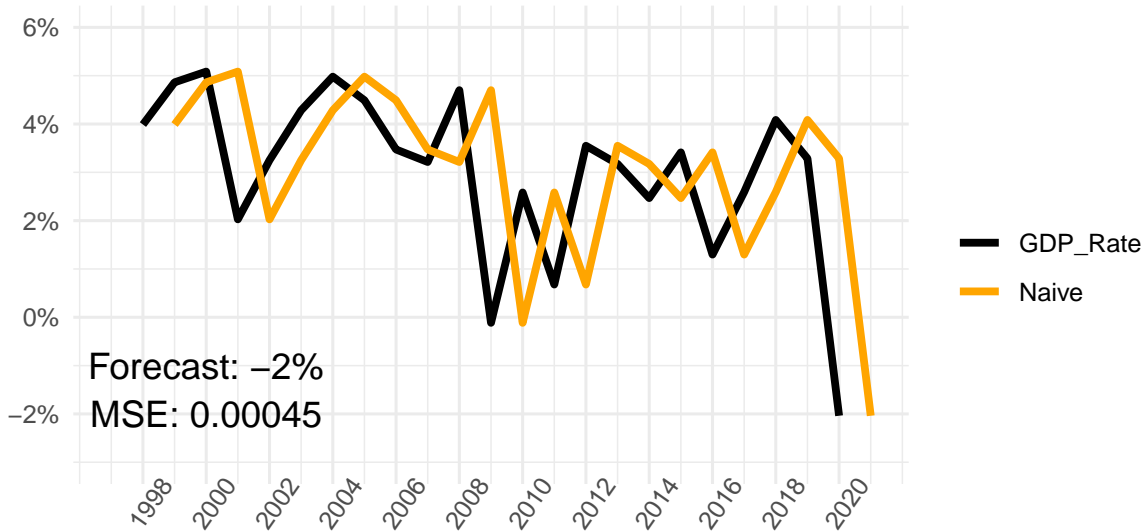
Forecast 2: Moving Average Forecast

$$F_{t+1} = \frac{Y_t + Y_{t-1} + \cdots + Y_{t-n+1}}{n}$$

F_{t+1} = forecast for time period $t + 1$

Y_t = actual value in time period t

n = number of periods to average

Forecast 2: Moving Average Forecast (3)

	A	B	C
1	**Time**	**Actual**	**Forecast**
2	1998	X_{1998}	---
3	1999	X_{1999}	---
4	2000	X_{2000}	---
5	2001	X_{2001}	= (B2 + B3 + B4) / 3
6	2002	X_{2002}	= (B3 + B4 + B5) / 3
7	2003	X_{2003}	= (B4 + B5 + B6) / 3

Calculate the forecast, extend to 2019 and visualize it.
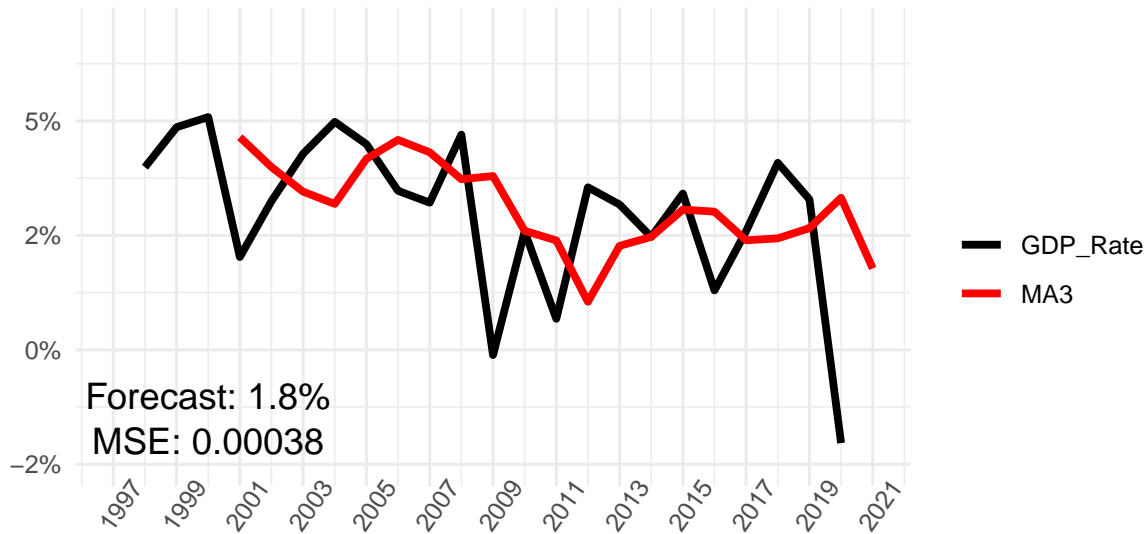
Rate of Change in GDP (Missouri)

Forecast: 1.8%
MSE: 0.00038

Legend:
— GDP_Rate
— MA3

Forecast 3: Weighted Moving Average Forecast

$$F_{t+1} = \frac{\Sigma(\text{Weight in period i})(\text{Actual value in period i})}{\Sigma(\text{Weights})}$$

Forecast 3: Weighted Moving Average Forecast

$$F_{t+1} = \frac{\Sigma(\text{Weight in period i})(\text{Actual value in period i})}{\Sigma(\text{Weights})}$$
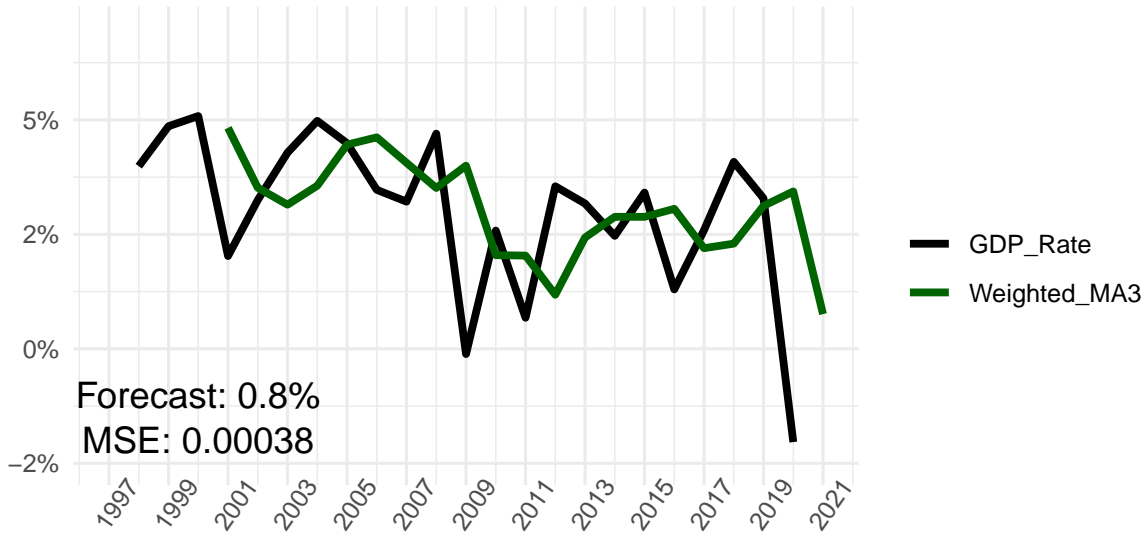
$$\text{Forecast}_t = \frac{(\text{Actual}_{t-1} \times 3 + \text{Actual}_{t-2} \times 2 + \text{Actual}_{t-3} \times 1)}{6}$$

Forecast 3: Weighted MA-3 Forecast

	A	B	C
1	**Time**	**Actual**	**Forecast**
2	1998	X_{1998}	---
3	1999	X_{1999}	---
4	2000	X_{2000}	---
5	2001	X_{2001}	= (B2*1 + B3*2 + B4*3) / 6
6	2002	X_{2002}	= (B3*1 + B4*2 + B5*3) / 6
7	2003	X_{2003}	= (B4*1 + B5*2 + B6*3) / 6

Calculate the forecast, extend to 2019 and visualize it.

Rate of Change in GDP (Missouri)

Forecast: 0.8%
MSE: 0.00038

GDP_Rate
Weighted_MA3

Forecast 4: Exponential Smoothing Forecast

$$F_{t+1} = F_t + \alpha(Y_t - F_t)$$

F_{t+1} = new forecast (for time period $t + 1$)

F_t = previous forecast (for time period t)

α = smoothing constant ($0 \leq \alpha \leq 1$)

Y_t = previous period's actual demand

Forecast 4: Exponential Smoothing (.2)
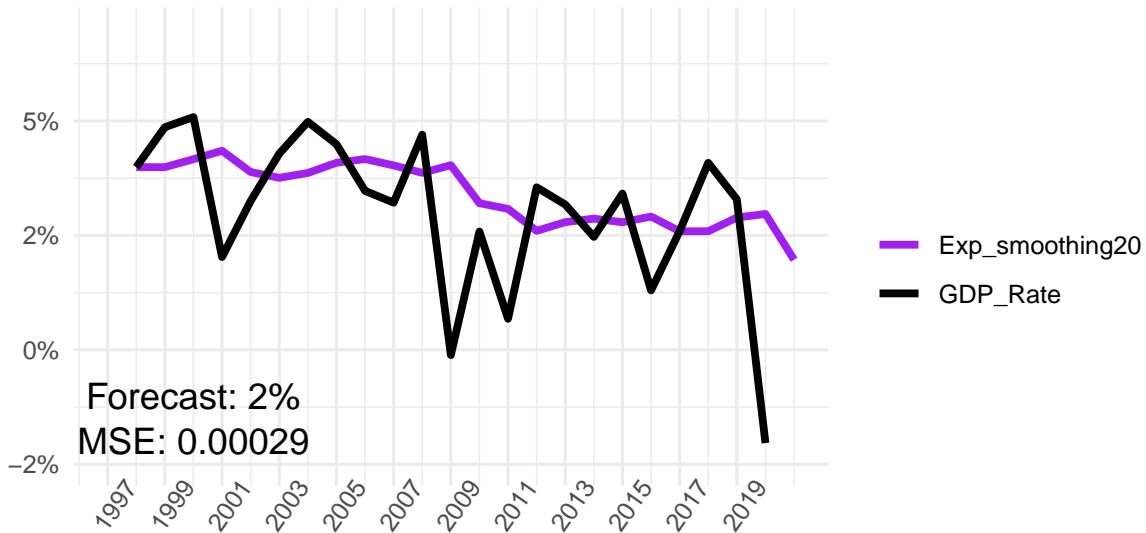
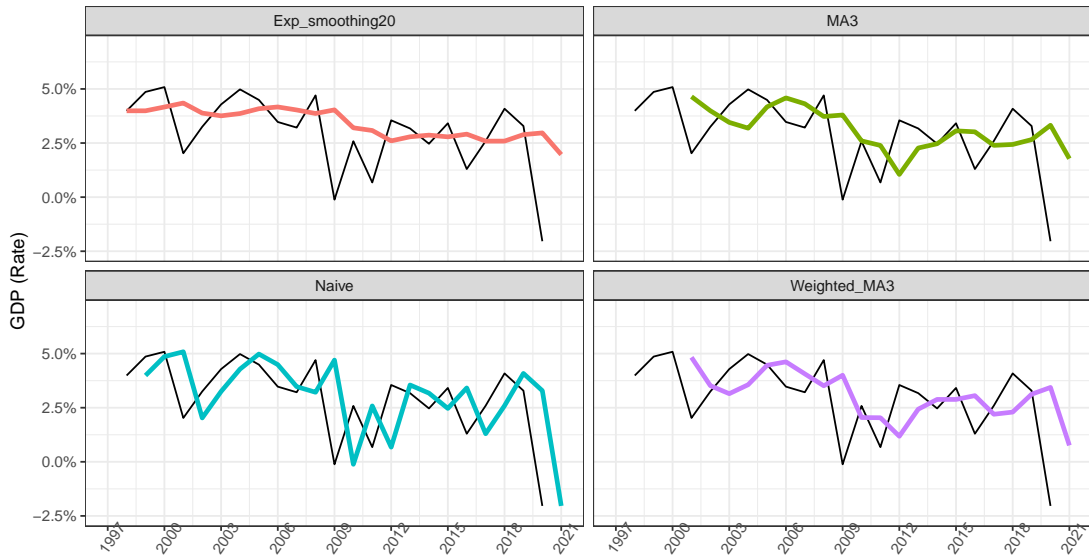|	A	B	C
1	Time	Actual	Forecast
2	1998	X_{1998}	= B2
3	1999	X_{1999}	
4	2000	X_{2000}	
5	2001	X_{2001}	
6	2002	X_{2002}	
7	2003	X_{2003}	

Forecast 4: Exponential Smoothing (.2)

	A	B	C
1	Time	Actual	Forecast
2	1998	X_{1998}	= B2
3	1999	X_{1999}	= C2 + .2 * (B2 - C2)
4	2000	X_{2000}	= C3 + .2 * (B3 - C3)
5	2001	X_{2001}	= C4 + .2 * (B4 - C4)
6	2002	X_{2002}	= C5 + .2 * (B5 - C5)
7	2003	X_{2003}	= C6 + .2 * (B6 - C6)

Rate of Change in GDP (Missouri)

Forecast: 2%
MSE: 0.00029

Exp_smoothing20
GDP_Rate

Forecast Tool	MSE	Prediction
Naive Forecast	0.00045	-2%
MA-3	0.00038	2%
Weighted MA-3	0.00038	1%
Exp Smoothing (.2)	0.00029	2%

State	year	GDP_Rate	Naive	MA3	WMA3	ExpSmth
Missouri	1997	NA	NA	NA	NA	NA
Missouri	1998	0.04	NA	NA	NA	0.04
Missouri	1999	0.05	0.04	NA	NA	0.04
Missouri	2000	0.05	0.05	NA	NA	0.04
Missouri	2001	0.02	0.05	0.05	0.05	0.04
Missouri	2002	0.03	0.02	0.04	0.04	0.04
Missouri	2003	0.04	0.03	0.03	0.03	0.04

To compare forecasts ONLY calculate the MSE on the rows with no missing data!

e.g. starting at row 2001.

Forecast Tool	MSE	Prediction
Naive Forecast	0.00049	-2%
MA-3	0.00038	2%
Weighted MA-3	0.00038	1%
Exp Smoothing (.2)	0.00032	2%

Assignment for Tuesday
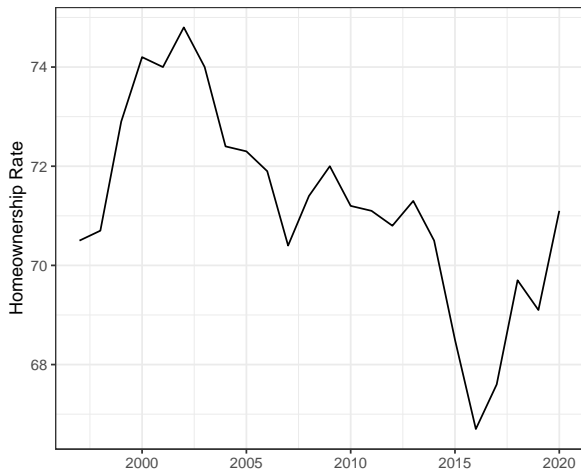
What is the best forecast model of homeownership rates in MO?

- Naïve
- MA (3)
- Weighted MA (3), or
- Exponential Smoothing (0.2)

Predict 2021 and calculate the MSE!