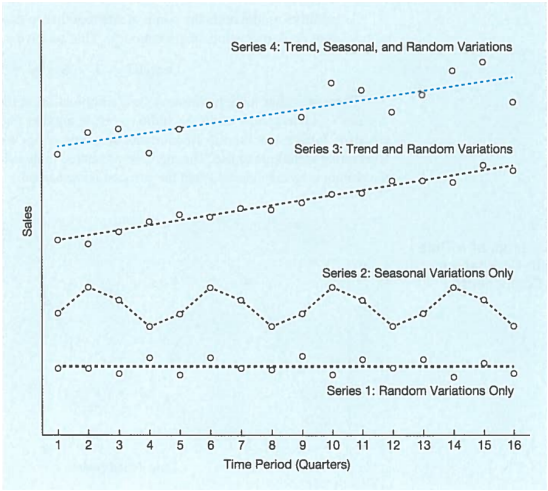
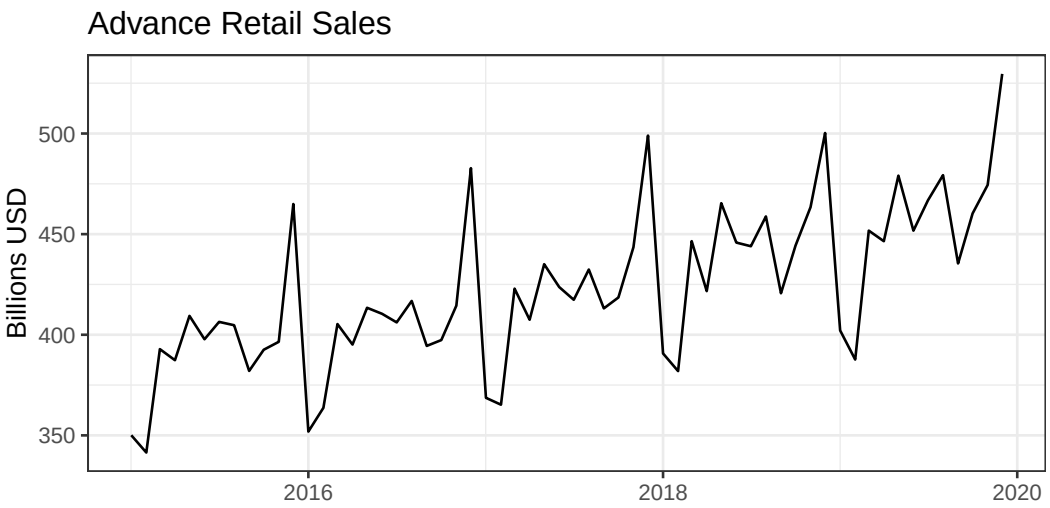
Today's Agenda

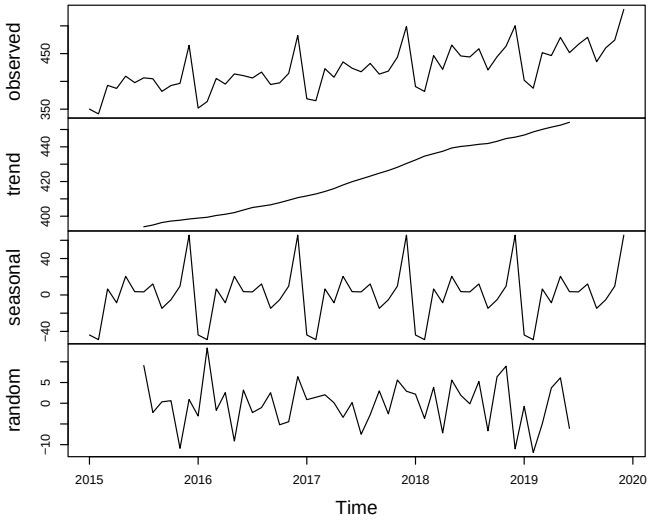
1. Review the components of time series data
2. Evaluate four forecasting tools:
   - Naïve
   - Moving Average
   - Weighted Moving Average
   - Exponential Smoothing

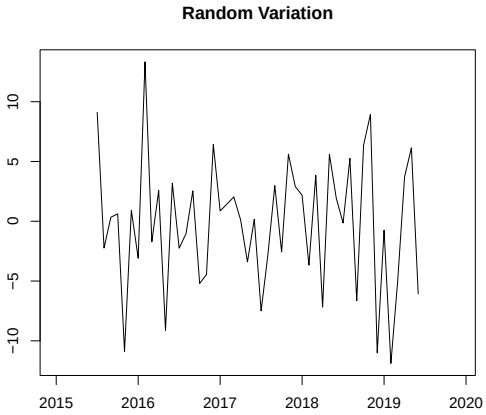


The Components of Time Series Data
(Render, Stair, Jr., Hanna and Hale 2018)



Source: U.S. Census Bureau


Decomposition of additive time series
observed
trend
seasonal
random
350
450
400
420
440
−40
0
40
−10
0
5
2015
2016
2017
2018
2019
2020
Time

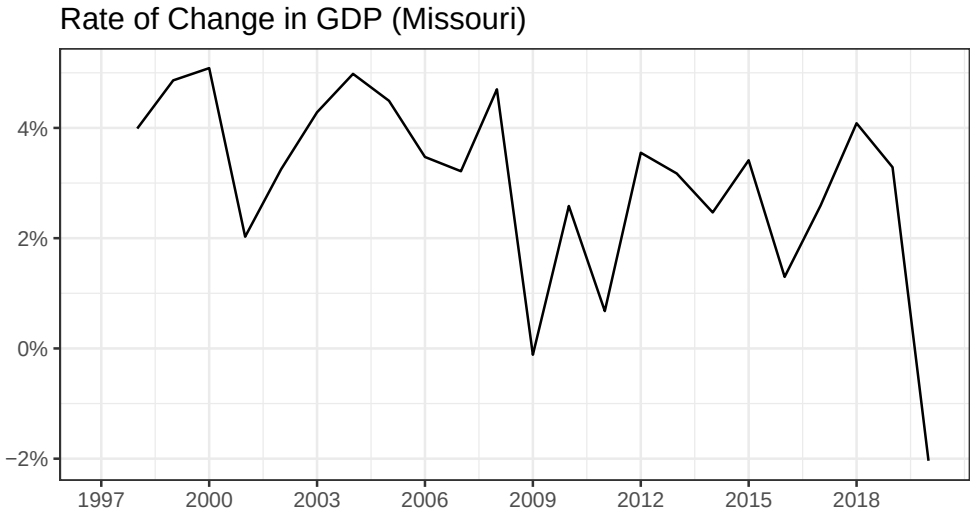


Random Variation Models

1. Naïve Forecast
2. Moving Average
3. Weighted Moving Average
4. Exponential Smoothing

Let's Build a Forecast

1. Grab Dataset1-All_Years.xlsx from Moodle
2. Copy gdp_rate for Missouri to a new sheet
3. Insert a line plot


Rate of Change in GDP (Missouri)
4%
2%
0%
−2%
1997
2000
2003
2006
2009
2012
2015
2018

Forecast 1: Naive Forecast

Set forecast to the last observation

- $\text{Forecast}_{time+1} = \text{Actual}_{time}$

Forecast 1: Naive Forecast

	A	B	C
1	**Time**	**Actual**	**Forecast**
2	1998	X_{1998}	---
3	1999	X_{1999}	= B2
4	2000	X_{2000}	= B3

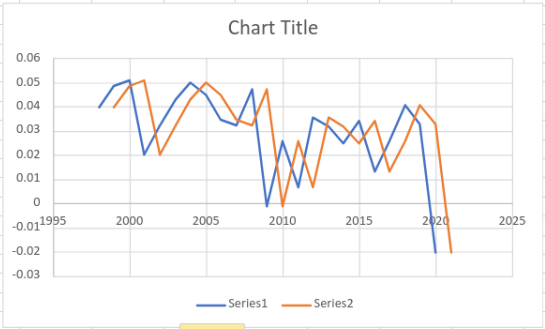
Forecast 1: Naive Forecast

	A	B	C
1	Time	Actual	Forecast
2	1998	X_{1998}	---
3	1999	X_{1999}	= B2
4	2000	X_{2000}	= B3

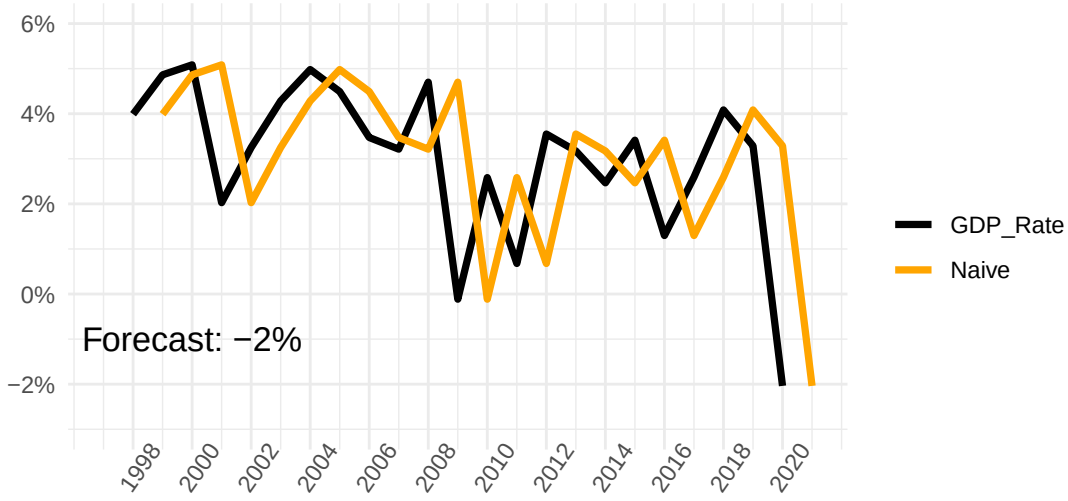
- Forecast up to 2021 and update the line plot

	A	B	C	D
1	State	year	gdp_rate	Naive
2	Missouri	1997		
3	Missouri	1998	0.039886236	
4	Missouri	1999	0.048625647	0.039886236
5	Missouri	2000	0.050845722	0.048625647
6	Missouri	2001	0.02026713	0.050845722
7	Missouri	2002	0.032502356	0.02026713
8	Missouri	2003	0.04283998	0.032502356
9	Missouri	2004	0.049801603	0.04283998
10	Missouri	2005	0.044917396	0.049801603
11	Missouri	2006	0.034729818	0.044917396
12	Missouri	2007	0.032139964	0.034729818
13	Missouri	2008	0.047005328	0.032139964
14	Missouri	2009	-0.001148231	0.047005328
15	Missouri	2010	0.025834259	-0.001148231
16	Missouri	2011	0.006776338	0.025834259
17	Missouri	2012	0.035498084	0.006776338
18	Missouri	2013	0.03173488	0.035498084
19	Missouri	2014	0.024668865	0.03173488
20	Missouri	2015	0.034130297	0.024668865
21	Missouri	2016	0.012976533	0.034130297

	A	B	C	D
1	State	year	gdp_rate	Naive
2	Missouri	1997		
3	Missouri	1998	0.039886236	
4	Missouri	1999	0.048625647	0.039886236
5	Missouri	2000	0.050845722	0.048625647
6	Missouri	2001	0.02026713	0.050845722
7	Missouri	2002	0.032502356	0.02026713
8	Missouri	2003	0.04283998	0.032502356
9	Missouri	2004	0.049801603	0.04283998
10	Missouri	2005	0.044917396	0.049801603
11	Missouri	2006	0.034729818	0.044917396
12	Missouri	2007	0.032139964	0.034729818
13	Missouri	2008	0.047005328	0.032139964
14	Missouri	2009	-0.001148231	0.047005328
15	Missouri	2010	0.025834259	-0.001148231
16	Missouri	2011	0.006776338	0.025834259
17	Missouri	2012	0.035498084	0.006776338
18	Missouri	2013	0.03173488	0.035498084
19	Missouri	2014	0.024668865	0.03173488
20	Missouri	2015	0.034130297	0.024668865
21	Missouri	2016	0.012976533	0.034130297
22	Missouri	2017	0.025923535	0.012976533
23	Missouri	2018	0.040847359	0.025923535
24	Missouri	2019	0.032871614	0.040847359
25	Missouri	2020	-0.020376942	0.032871614
26		2021		-0.020376942
27				



Rate of Change in GDP (Missouri)



Accuracy: Mean Squared Error (MSE)

1. Calculate the forecast error
   - Forecast Error = Actual Value - Forecast Value
2. Square each forecast error
3. Calculate the mean of the squared errors

$$\text{MSE} = \frac{\Sigma(Error)^2}{n}$$

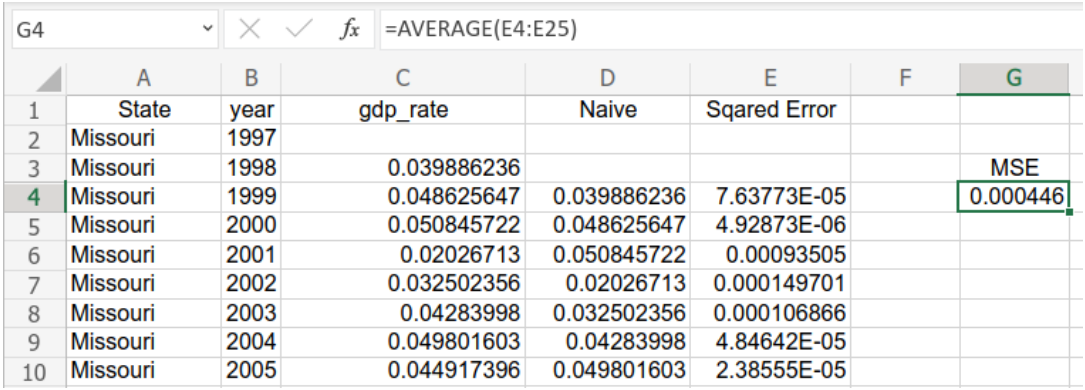
$$\text{MSE} = \frac{\Sigma(Error)^2}{n}$$

	A	B	C	D
1	**Time**	**Actual**	**Forecast**	**Error^2**
2	1998	X_{1998}	---	---
3	1999	X_{1999}	= B2	= (B3 - C3)^2
4	2000	X_{2000}	= B3	= (B4 - C4)^2

$$\text{MSE} = \text{AVERAGE(D3:D4)}$$

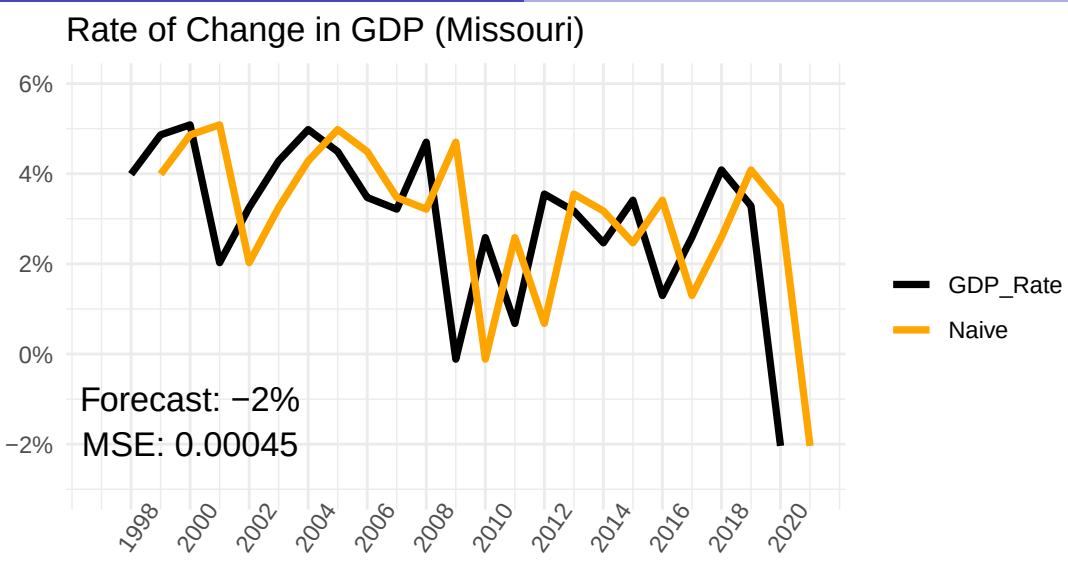
E4 =(C4-D4)^2

	A	B	C	D	E
1	State	year	gdp_rate	Naive	Sqared Error
2	Missouri	1997			
3	Missouri	1998	0.039886236		
4	Missouri	1999	0.048625647	0.039886236	7.63773E-05
5	Missouri	2000	0.050845722	0.048625647	4.92873E-06
6	Missouri	2001	0.02026713	0.050845722	0.00093505
7	Missouri	2002	0.032502356	0.02026713	0.000149701
8	Missouri	2003	0.04283998	0.032502356	0.000106866
9	Missouri	2004	0.049801603	0.04283998	4.84642E-05
10	Missouri	2005	0.044917396	0.049801603	2.38555E-05
11	Missouri	2006	0.034729818	0.044917396	0.000103787
12	Missouri	2007	0.032139964	0.034729818	6.70735E-06



G4 =AVERAGE(E4:E25)

	A	B	C	D	E	F	G
1	State	year	gdp_rate	Naive	Sqared Error		
2	Missouri	1997					
3	Missouri	1998	0.039886236				MSE
4	Missouri	1999	0.048625647	0.039886236	7.63773E-05		0.000446
5	Missouri	2000	0.050845722	0.048625647	4.92873E-06		
6	Missouri	2001	0.02026713	0.050845722	0.00093505		
7	Missouri	2002	0.032502356	0.02026713	0.000149701		
8	Missouri	2003	0.04283998	0.032502356	0.000106866		
9	Missouri	2004	0.049801603	0.04283998	4.84642E-05		
10	Missouri	2005	0.044917396	0.049801603	2.38555E-05		


Rate of Change in GDP (Missouri)
6%
4%
2%
0%
−2%
1998
2000
2002
2004
2006
2008
2010
2012
2014
2016
2018
2020
Forecast: −2%
MSE: 0.00045
GDP_Rate
Naive

Forecast 2: Moving Average Forecast

$$F_{t+1} = \frac{Y_t + Y_{t-1} + \cdots + Y_{t-n+1}}{n}$$

F_{t+1} = forecast for time period $t + 1$

Y_t = actual value in time period t

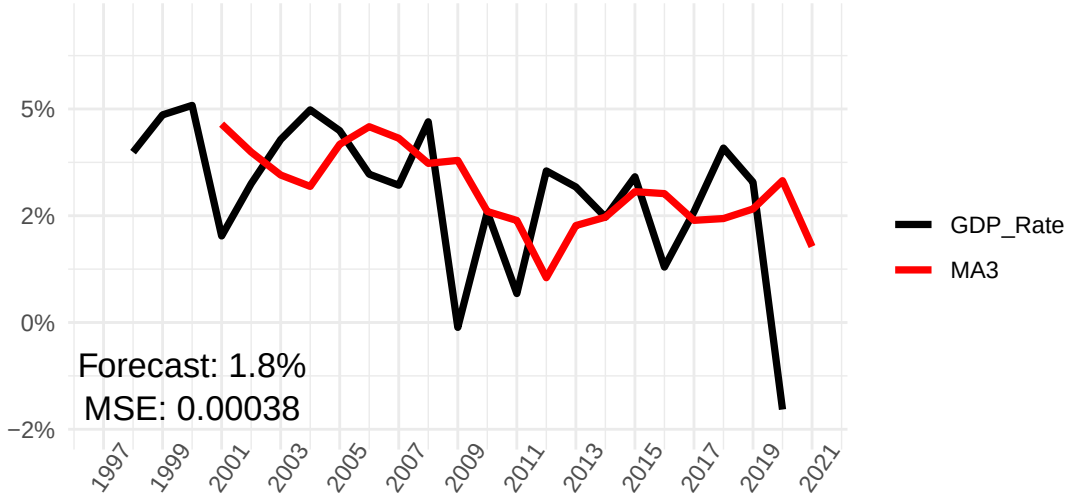
n = number of periods to average

Forecast 2: Moving Average Forecast (3)

	A	B	C
1	Time	Actual	Forecast
2	1998	X_{1998}	---
3	1999	X_{1999}	---
4	2000	X_{2000}	---
5	2001	X_{2001}	= (B2 + B3 + B4) / 3
6	2002	X_{2002}	= (B3 + B4 + B5) / 3
7	2003	X_{2003}	= (B4 + B5 + B6) / 3

- Forecast up to 2021 and update the line plot

Rate of Change in GDP (Missouri)



Forecast 3: Weighted Moving Average Forecast

$$F_{t+1} = \frac{\Sigma(\text{Weight in period i})(\text{Actual value in period i})}{\Sigma(\text{Weights})}$$

Forecast 3: Weighted Moving Average Forecast

$$F_{t+1} = \frac{\Sigma(\text{Weight in period i})(\text{Actual value in period i})}{\Sigma(\text{Weights})}$$

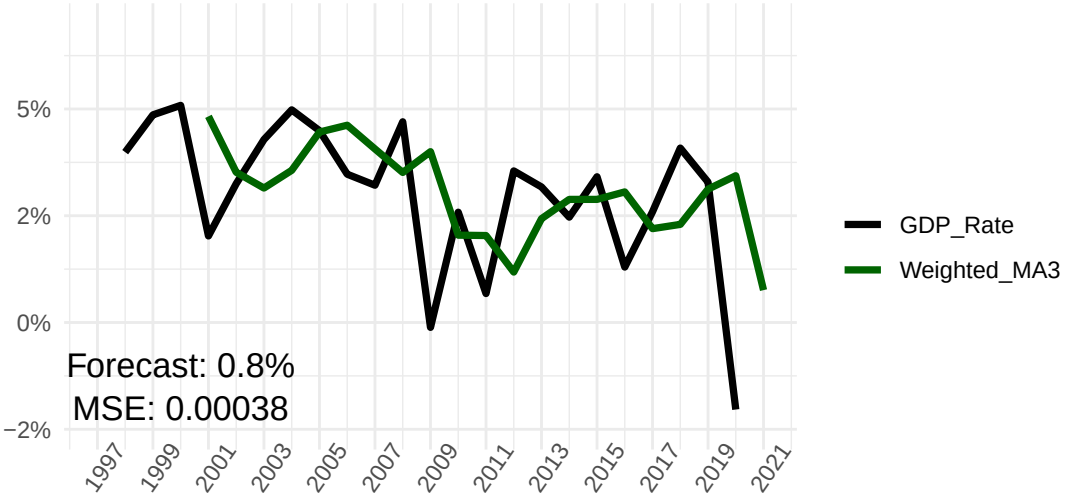
$$\text{Forecast}_t = \frac{(\text{Actual}_{t-1} \text{ x } 3 + \text{Actual}_{t-2} \text{ x } 2 + \text{Actual}_{t-3} \text{ x } 1)}{6}$$

Forecast 3: Weighted MA-3 Forecast

	A	B	C
1	Time	Actual	Forecast
2	1998	X_{1998}	---
3	1999	X_{1999}	---
4	2000	X_{2000}	---
5	2001	X_{2001}	= (B2*1 + B3*2 + B4*3) / 6
6	2002	X_{2002}	= (B3*1 + B4*2 + B5*3) / 6
7	2003	X_{2003}	= (B4*1 + B5*2 + B6*3) / 6

- Forecast up to 2021 and update the line plot

Rate of Change in GDP (Missouri)



Forecast 4: Exponential Smoothing Forecast

$$F_{t+1} = F_t + \alpha(Y_t - F_t)$$

F_{t+1} = new forecast (for time period $t + 1$)

F_t = previous forecast (for time period t)

α = smoothing constant ($0 \leq \alpha \leq 1$)

Y_t = previous period's actual demand

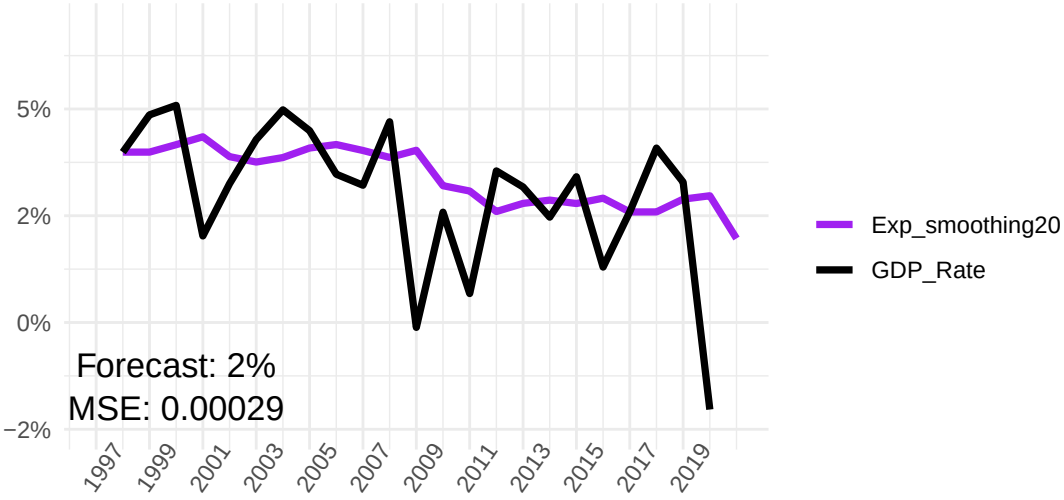
Forecast 4: Exponential Smoothing (.2)

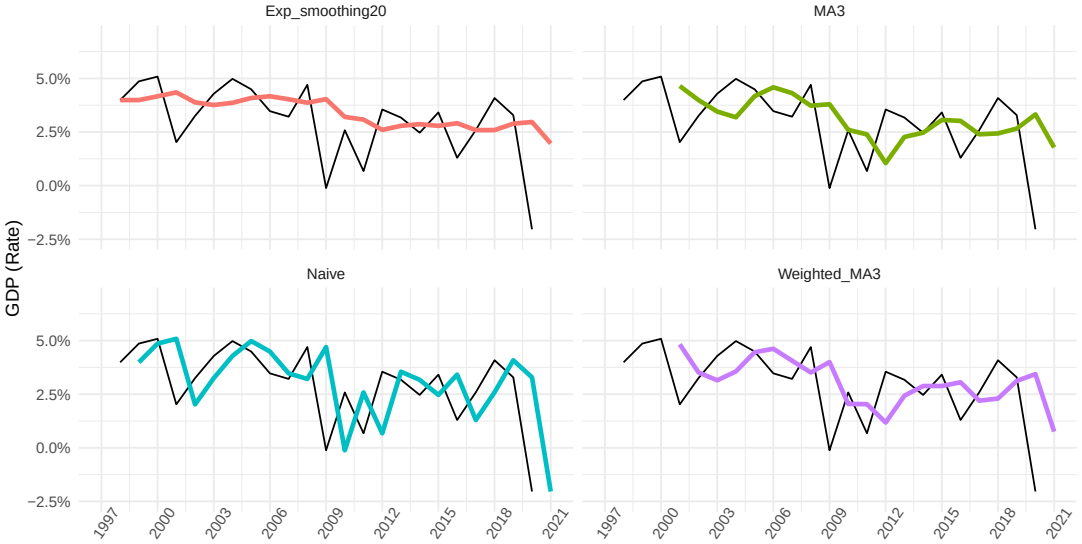
	A	B	C
1	Time	Actual	Forecast
2	1998	X_{1998}	= B2
3	1999	X_{1999}	
4	2000	X_{2000}	
5	2001	X_{2001}	
6	2002	X_{2002}	
7	2003	X_{2003}	

Forecast 4: Exponential Smoothing (.2)

	A	B	C
1	Time	Actual	Forecast
2	1998	X_{1998}	= B2
3	1999	X_{1999}	= C2 + .2 * (B2 - C2)
4	2000	X_{2000}	= C3 + .2 * (B3 - C3)
5	2001	X_{2001}	= C4 + .2 * (B4 - C4)
6	2002	X_{2002}	= C5 + .2 * (B5 - C5)
7	2003	X_{2003}	= C6 + .2 * (B6 - C6)

Rate of Change in GDP (Missouri)




Exp_smoothing20
MA3
Naive
Weighted_MA3
GDP (Rate)
5.0%
2.5%
0.0%
−2.5%
1997
2000
2003
2006
2009
2012
2015
2018
2021

Forecast Tool	MSE	Prediction
Naive Forecast	0.00045	-2%
MA-3	0.00038	2%
Weighted MA-3	0.00038	1%
Exp Smoothing (.2)	0.00029	2%

State	year	GDP_Rate	Naive	MA3	WMA3	ExpSmth
Missouri	1997	NA	NA	NA	NA	NA
Missouri	1998	0.04	NA	NA	NA	0.04
Missouri	1999	0.05	0.04	NA	NA	0.04
Missouri	2000	0.05	0.05	NA	NA	0.04
Missouri	2001	0.02	0.05	0.05	0.05	0.04
Missouri	2002	0.03	0.02	0.04	0.04	0.04
Missouri	2003	0.04	0.03	0.03	0.03	0.04

To compare forecasts ONLY calculate the MSE on the rows with no missing data!

e.g. starting at row 2001.

Forecast Tool	MSE	Prediction
Naive Forecast	0.00049	-2%
MA-3	0.00038	2%
Weighted MA-3	0.00038	1%
Exp Smoothing (.2)	0.00032	2%

Assignment for Tuesday

What is the best forecast model of homeownership rates in MO?

- Naïve
- MA (3)
- Weighted MA (3), or
- Exponential Smoothing (0.2)

Predict 2021 and calculate the MSE!