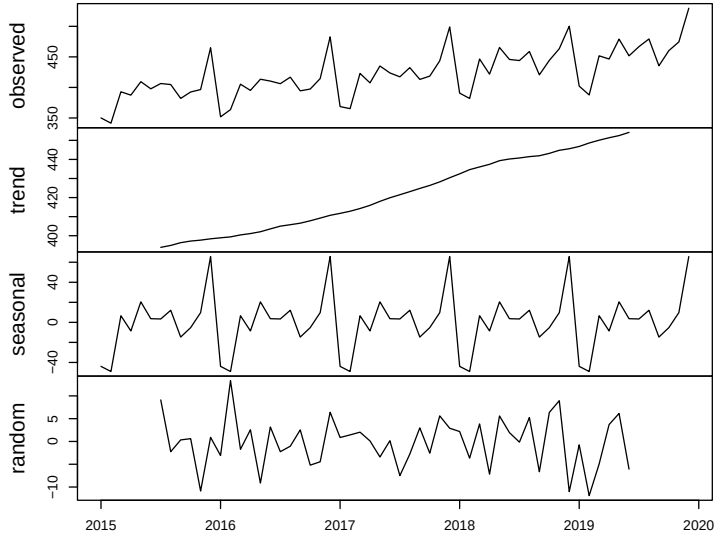
Today's Agenda

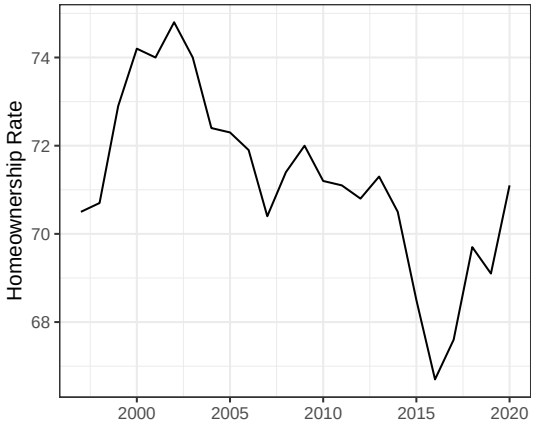
Comparing random variation models to linear trend models of time series data

Justin Leinaweaver (Spring 2022)

Decomposition of additive time series



Assignment for Today

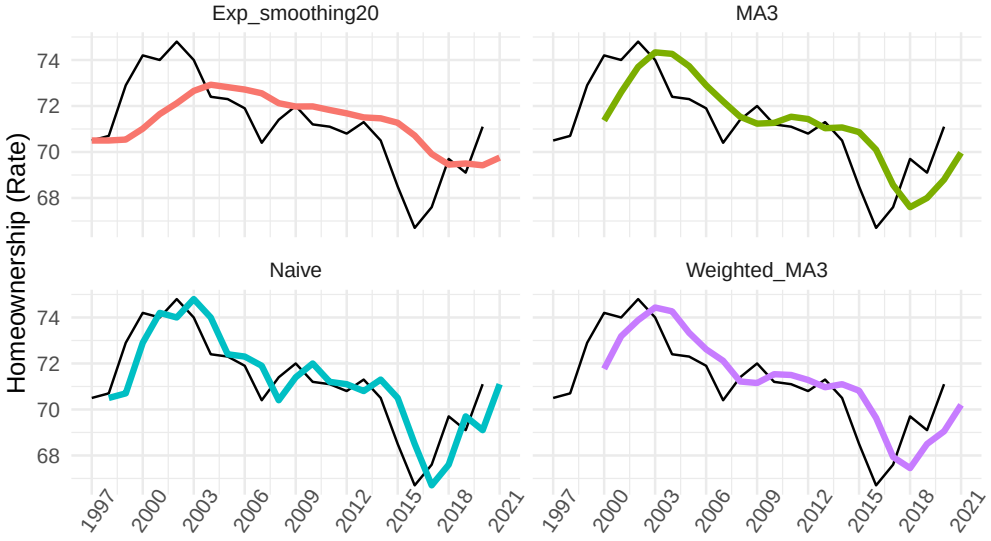


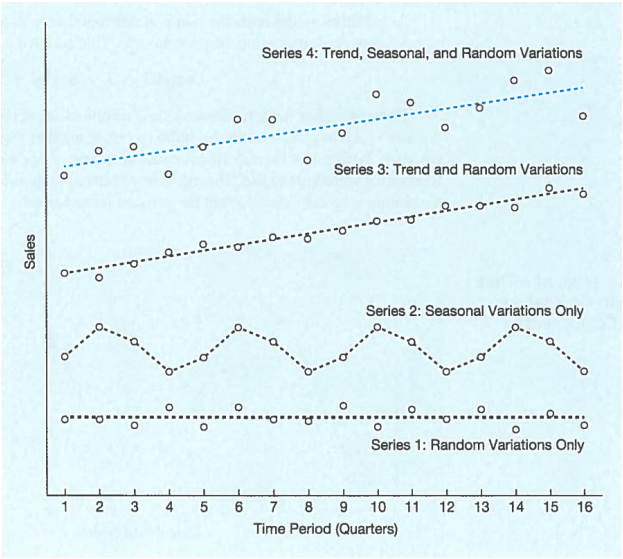
What is the best forecast model of homeownership rates in MO?

- Naïve
- MA (3)
- Weighted MA (3), or
- Exponential Smoothing (0.2)

Predict 2021 and calculate the MSE!

Forecast	MSE	Prediction
Naive	1.32	71.1
MA-3	2.46	70
WMA-3	1.96	70.2
Exp Smooth (.2)	3.15	69.8

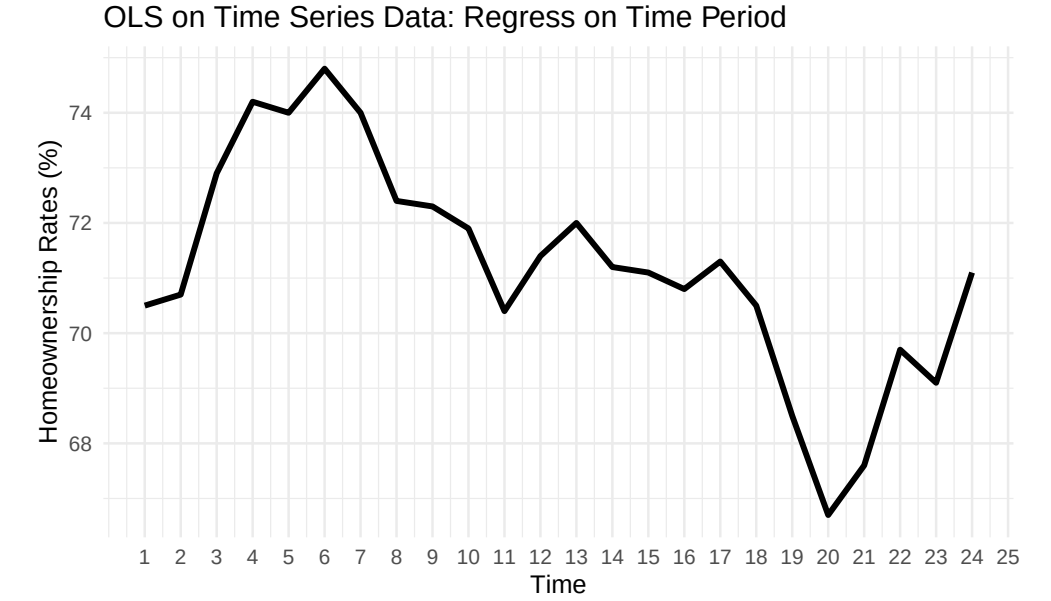

Exp_smoothing20
MA3
Naive
Weighted_MA3
Homeownership (Rate)
74
72
70
68
1997
2000
2003
2006
2009
2012
2015
2018
2021



The Components of Time Series Data

(Render, Stair, Jr., Hanna and Hale 2018)

Homeownership in Missouri
Homeownership Rates (%)
74
72
70
68
1997
2000
2003
2006
2009
2012
2015
2018
2021


OLS on Time Series Data: Regress on Time Period
Homeownership Rates (%)
74
72
70
68
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
Time

Using OLS on Time Series Data

Regress homeownership rate on the time period

- Visualize the model as a line plot
- Calculate the MSE

Create Time Period Variable

	A	B	C	D
1	state	year	homeowner_rate	time
2	Missouri	1997	70.5	1
3	Missouri	1998	70.7	2
4	Missouri	1999	72.9	3
5	Missouri	2000	74.2	4
6	Missouri	2001	74	5
7	Missouri	2002	74.8	6
8	Missouri	2003	74	7
9	Missouri	2004	72.4	8
10	Missouri	2005	72.3	9
11	Missouri	2006	71.9	10
12	Missouri	2007	70.4	11
13	Missouri	2008	71.4	12
14	Missouri	2009	72	13

RESIDUAL OUTPUT			
Observation	*Predicted homeowner_rate*	*Residuals*	*Error2*
1	70.95	-0.45	0.20
2	71.83	-1.13	1.27
3	72.49	0.41	0.17
4	72.97	1.23	1.52
5	73.27	0.73	0.53
6	73.42	1.38	1.91
7	73.42	0.58	0.33
8	73.31	-0.91	0.83
9	73.09	-0.79	0.63
10	72.79	-0.89	0.79
11	72.41	-2.01	4.04
12	71.98	-0.58	0.34

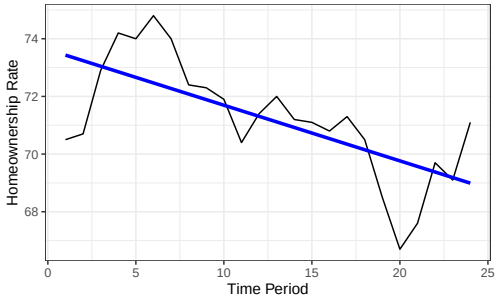
MSE = Average of the Squared Residuals

Using OLS on Time Series Data

Regress homeownership rate on the time period

- Visualize the model as a line plot
- Calculate the MSE

	homeowner_rate
time	-0.19^{*}
	(0.04)
Constant	73.63^{*}
	(0.64)
Observations	24
Adjusted R^2	0.43
Residual Std. Error	1.52 (df = 22)
F Statistic	18.61^{*} (df = 1; 22)
Note:	$^{*}p < 0.05$



Forecast	MSE	Prediction
Exp Smooth (.2)	3.15	69.8
MA-3	2.46	70
OLS	2.11	68.99
WMA-3	1.96	70.2
Naive	1.32	71.1

Make Predictions

	homeowner_rate
time	−0.19*
	(0.04)
Constant	73.63*
	(0.64)
Observations	24
Adjusted R^2	0.43
Residual Std. Error	1.52 (df = 22)
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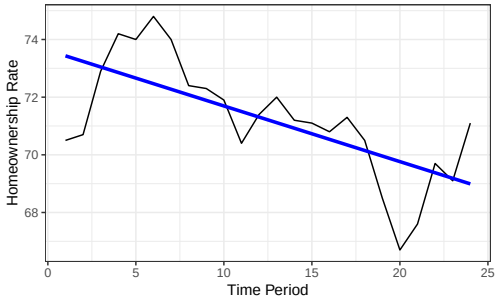
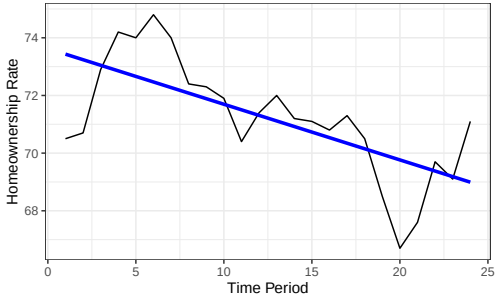


Prediction = 73.63 + -0.19 x Time

Use the model to predict homeownership for the next two years.

Make Predictions

	homeowner_rate
time	−0.19*
	(0.04)
Constant	73.63*
	(0.64)
Observations	24
Adjusted R^2	0.43
Residual Std. Error	1.52 (df = 22)
F Statistic	18.61* (df = 1; 22)
Note:	*p < 0.05



$$\text{Prediction} = 73.63 + \text{-}0.19 \text{ x } (\text{Time} = 25) = 68.88$$

$$\text{Prediction} = 73.63 + \text{-}0.19 \text{ x } (\text{Time} = 26) = 68.69$$

Using OLS on Time Series Data

Compare our results to two model transformations:

1. Regress homeownership rate on a quadratic function of time period
2. Regress homeownership rate on a cubic function of time period

	A	B	C	D	E	F
1	state	year	homeowner_rate	time	time2	time3
2	Missouri	1997	70.5	1	1	1
3	Missouri	1998	70.7	2	4	8
4	Missouri	1999	72.9	3	9	27
5	Missouri	2000	74.2	4	16	64
6	Missouri	2001	74	5	25	125
7	Missouri	2002	74.8	6	36	216
8	Missouri	2003	74	7	49	343
9	Missouri	2004	72.4	8	64	512
10	Missouri	2005	72.3	9	81	729
11	Missouri	2006	71.9	10	100	1000
12	Missouri	2007	70.4	11	121	1331
13	Missouri	2008	71.4	12	144	1728
14	Missouri	2009	72	13	169	2197

time2

E2 = D2^2

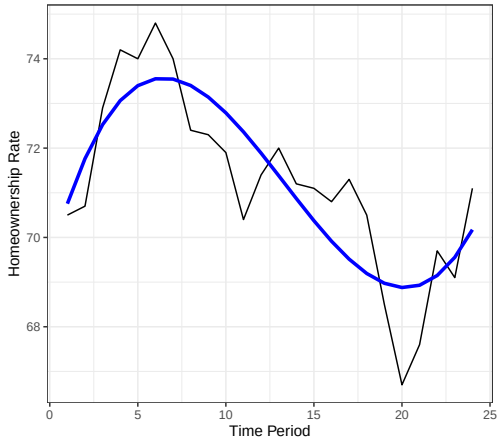
time3

F2 = D2^3

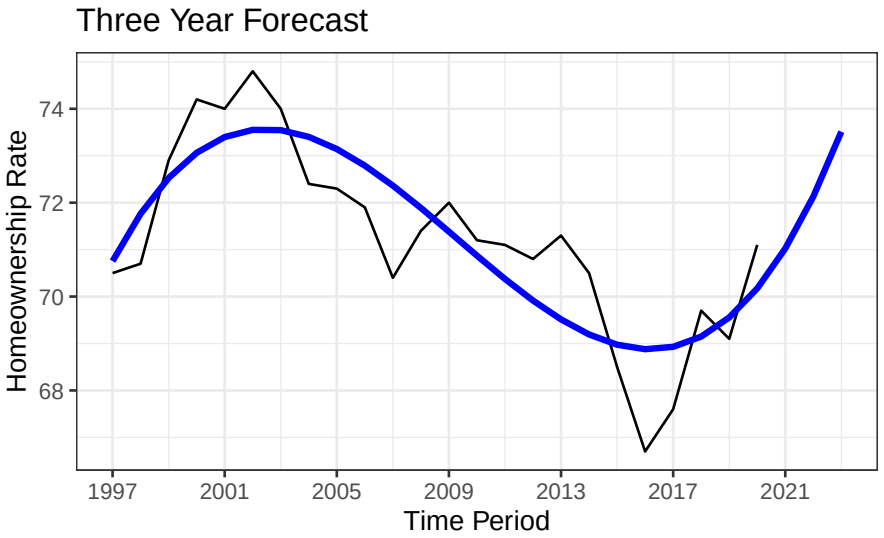
	Homeownership		
	(1)	(2)	(3)
Time	−0.19*	0.03	1.42*
	(0.04)	(0.18)	(0.37)
Squared		−0.01	−0.15*
		(0.01)	(0.03)
Cubed			0.004*
			(0.001)
Constant	73.63*	72.66*	69.48*
	(0.64)	(1.00)	(1.10)
Observations	24	24	24
Adjusted R^2	0.43	0.45	0.68
Residual Std. Error	1.52 (df = 22)	1.50 (df = 21)	1.14 (df = 20)
F Statistic	18.61* (df = 1; 22)	10.30* (df = 2; 21)	17.18* (df = 3; 20)
Note:			*p < 0.05

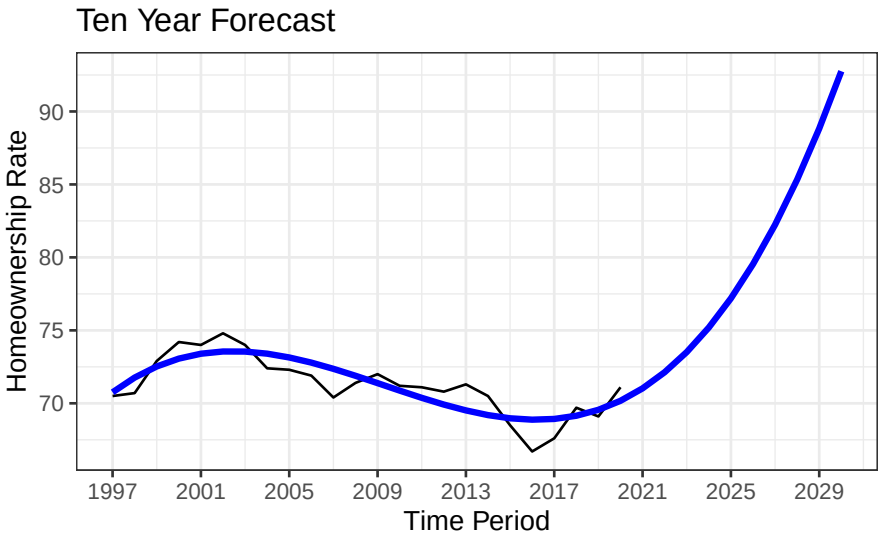
Forecast	MSE	Prediction
Exp Smooth (.2)	3.15	69.8
MA-3	2.46	70
OLS	2.11	68.99
OLS Quadratic	1.97	68.24
WMA-3	1.96	70.2
Naive	1.32	71.1
OLS Cubic	1.09	70.17

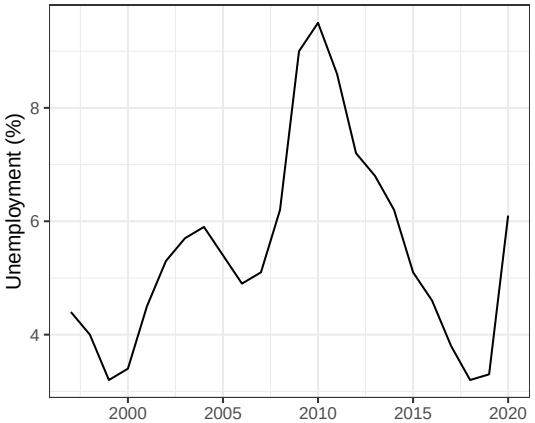
RESIDUAL OUTPUT	
Observation	*Predicted homeowner_rate*
1	70.95
2	71.83
3	72.49
4	72.97
5	73.27
6	73.42
7	73.42
8	73.31
9	73.09
10	72.79
11	72.41



The model line is the "Predicted" column in the residual output.


Three Year Forecast
Homeownership Rate
74
72
70
68
1997
2001
2005
2009
2013
2017
2021
Time Period

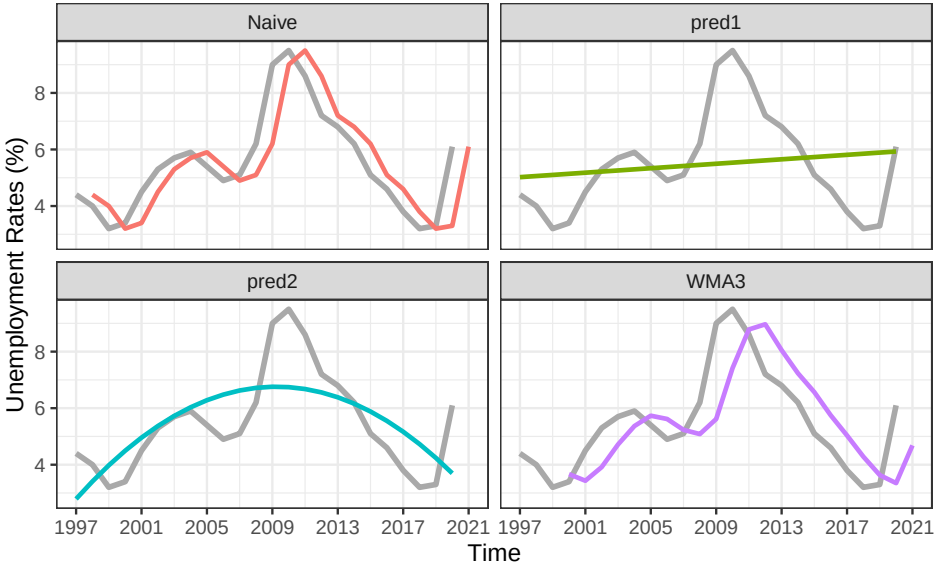

Ten Year Forecast
Homeownership Rate
90
85
80
75
70
1997
2001
2005
2009
2013
2017
2021
2025
2029
Time Period



What is the best forecast model of unemployment rates in MO?

- Naïve
- Weighted MA (3)
- Linear model of time
- Quadratic function of time

Forecast	MSE
OLS	2.95
Weighted MA3	1.98
OLS (Quadratic)	1.67
Naive	1.14


Naive
pred1
pred2
WMA3
Unemployment Rates (%)
4
6
8
1997
2001
2005
2009
2013
2017
2021
Time