

# Hair Rendering: Importance Sampling of Dual Scattering Approximation

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July 23, 2020

# Goal of thesis

*"To evaluate whether multiple importance sampling (MIS) the dualscattering method leads to a significant reduction of variance in the rendered results".*

# Background

## Rendering hair

Important for variety of industries

- Animation movie industry: to render realistic hairs in a physically accurate way.
- Game industry: enhance realism and visual effects.
- Clothes manufacturing industry: to render custom fabrics and to compare appearance in different lighting conditions.
- Hair styling: render hair styling products applied to the hair.

# Rendering hair

## Hair fiber representation

Explicit representation vs. Implicit representation

- Explicit representation represents each fiber by geometric primitives (e.g. triangles)
- Implicit representation represents fiber

There are a couple of ways to represent hair fibers:

- Camera-facing triangle strips
- Cylindrical primitives (used in thesis)
- Trigonal prisms
- Ribbons

# Rendering hair

## Rendering challenges

Human hair consists of over hundreds of thousands of hair strands. Leads to rendering challenges:

- Memory consumption: to store all fibers in memory.
- Time: rendering realistic scattering effects requires tracing many samples through the hair volume.
- Aliasing: Hair fibers are very thin, requiring additional samples to be drawn to prevent aliasing.

# Mathematical notation

## Radiometry

- Power (Watts): energy in Joules per second.
- Radiant intensity (steradians): power divided by the solid angle.
- Irradiance (Watts per  $m^2$ ): power per unit area.
- Radiance : Irradiance per solid angle, where solid angle goes to zero (becoming a ray instead of a cone).

# Mathematical notation

## Scattering equations

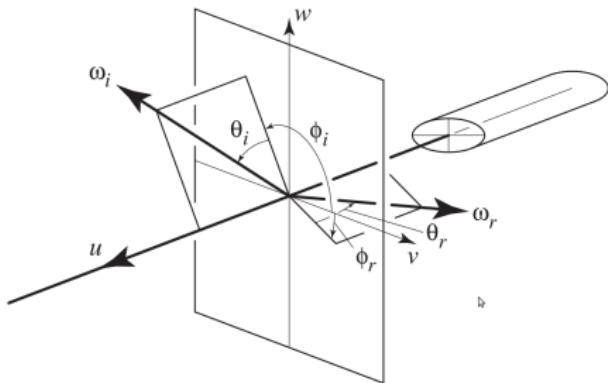
Scattering is represented by a bidirectional reflection distribution function (BRDF):

$$f_r(p, \omega_o, \omega_i) = \frac{dL_o(p, \omega_o)}{dE(p, \omega_i)} = \frac{dL_o(p, \omega_o)}{L_i(p, \omega_i) \cos \theta_i d\omega_i} \quad (1)$$

The BRDF is the fraction of outgoing radiance in direction  $\omega_o$  related to the incident irradiance from direction  $\omega_i$ .

# Mathematical notation

## Coordinates axes for hair fibers



- Coordinate axis are represented using  $uvw$  axes, with  $u$  pointing from base to tip of hair strand and  $vw$  forms the orthogonal plane.
- Longitudinal angles  $\theta$  are with respect to  $u$ , where 0 degrees is perpendicular to the fiber, -90 towards base, and 90 towards tip.
- Azimuthal angles  $\phi$  are formed with respect to the orthogonal plane.
- $\omega_i = (\theta_i, \phi_i)$  incident direction,  $\omega_o = (\theta_o, \phi_o)$  outgoing direction.

# Mathematical notation

## Bidirectional curve scattering distribution function (BCSDF)

- Hair fibers are rendered implicitly by 3D curves.
- Curves have no surface area. They have a length, requiring a change to the BRDF formulation.

$$S_r(\omega_o, \omega_i) = \frac{dL_o(\omega_o)}{dE_i(\omega_i)} = \frac{dL_o(\omega_o)}{DL_i(\omega_i) \cos \theta_i d\omega_i}; \quad (2)$$

$$L_o(\omega_o) = D \int S(\omega_i, \omega_o) L_i(\omega_i) \cos \theta_i d\omega_i; \quad (3)$$

# Monte-Carlo integration

Let's say we want to integrate  $f(x)$  which is a 1D formula from  $a$  to  $b$ .

$$\int_a^b f(x) dx \quad (4)$$

This can be evaluated by drawing  $N$  uniform samples for  $X_i \in [a, b]$ .

$$F_N = \frac{b-a}{N} \sum_{i=1}^N f(X_i) \quad (5)$$

The Monte-Carlo estimator says that the expected value  $E[F_N]$  is equal to the integral.

Possible to integrate any definite higher dimensional integrals.

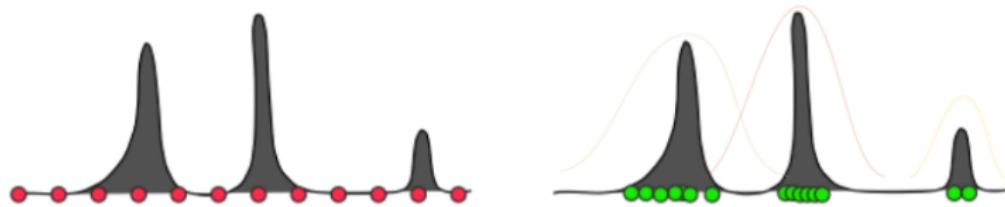
# Importance sampling

- Reduces noise faster by sampling high-value contributions.
- The estimator can be rewritten by taking into consideration the probability density function  $p(X_i)$
- Samples need to be generated from  $p(x)$ , by processes such as the inversion method or rejection method.

$$F_N = \frac{1}{N} \sum_{i=1}^N \frac{f(X_i)}{p(X_i)} \quad (6)$$

# Multiple importance sampling

- Scattering distributions cannot always be represented by closed formulas, or be inverted at all.
- A solution is to match the scattering distribution with multiple simple functions  $f(x)$  and  $g(x)$



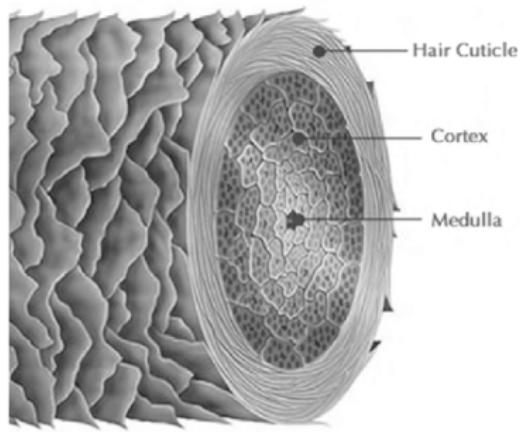
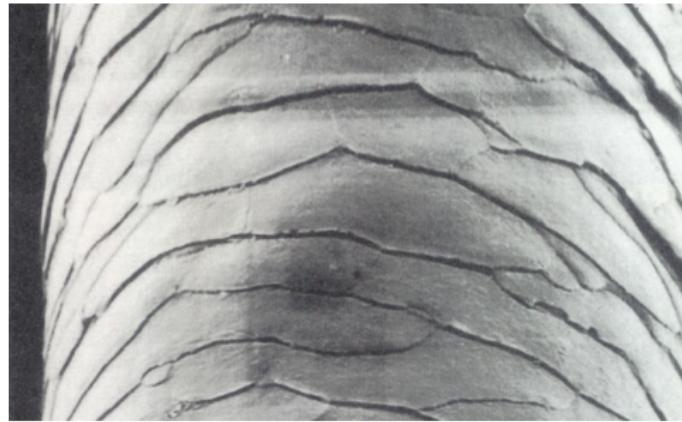
$$\frac{1}{n_f} \sum_{i=1}^{n_f} \frac{f(X_i)g(X_i)w_f(X_i)}{p_f(X_i)} + \frac{1}{n_g} \sum_{j=1}^{n_g} \frac{f(Y_j)g(Y_j)w_g(Y_j)}{p_g(Y_j)} \quad (7)$$

$$w_s(x) = \frac{n_s p_s(x)}{\sum_i n_i p_i(x)} \quad (8)$$

# Related work

## Structure of hair

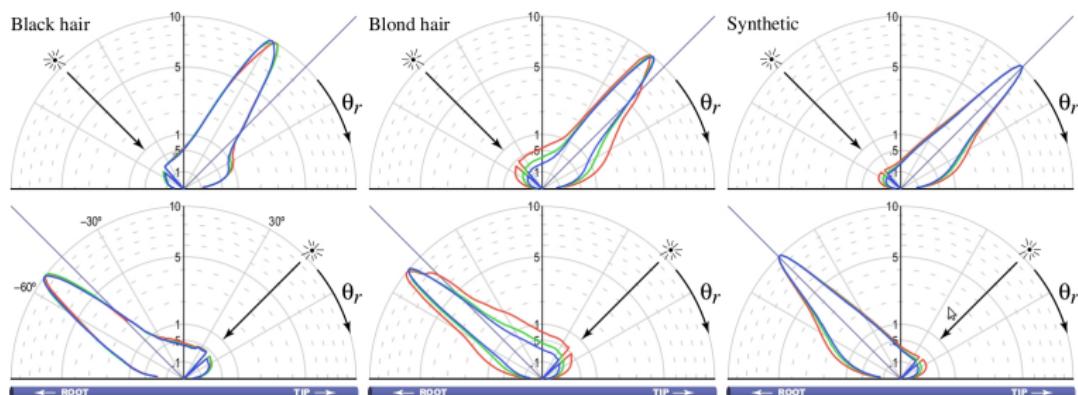
- Hair fibers do have cuticle scales tilted by approximately 3 degrees toward the root.
- The core of the fiber (medulla and cortex) consist of pigment which absorbs specific wavelengths in the light.



# Related work

## Measured scattering data

Single fiber scattering model that models three visible scattering components:



# Related work

## Marschner model

Marschner et al. (2003) proposed a single fiber scattering model for three visible scattering components.

R reflection,

TT double transmission,

TRT transmission, reflection, transmission

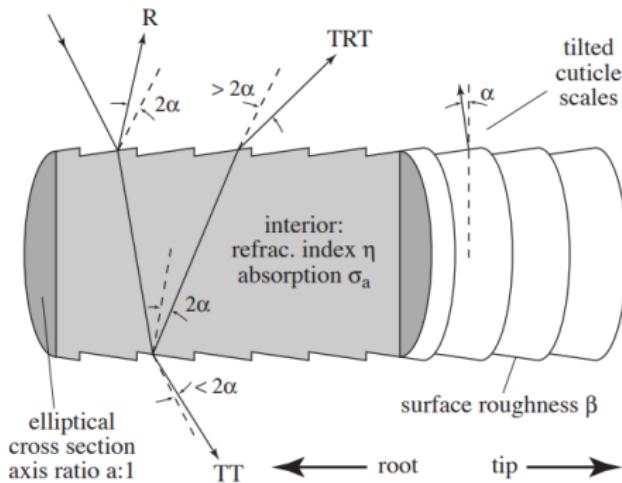


# Related work

## Marschner model: longitudinal scattering M

The Marschner model scattering equation can be written as:

$$S(\omega_i, \omega_o) = \sum_p M_p(\theta_i, \theta_o) N_p(\eta(\theta_d); \phi_i, \phi_o) / \cos^2 \theta_d \quad (9)$$

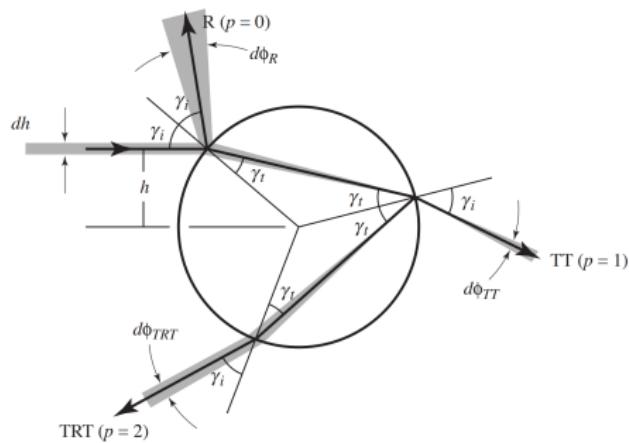


$$M_p(\theta_h) = g(\beta_p; \theta_h - \alpha_p) \quad (10)$$

- $p$  is the scattering mode with  $p \in R, TT, TRT$
- where  $\theta_h$  is the half angle between  $\theta_i$  and  $\theta_o$

# Related work

## Marschner model: azimuthal scattering N



$$N(\phi) = \sum_p N_p(p, \phi)$$

$$N_p(\theta_h) = \sum_r A(p, h) | 2 \frac{d\phi}{dh}(p, h) |$$

- $r$  is the number of roots  $h$ .
- where  $\eta$  is the index of refraction.
- $A$  is the absorption coefficient

# Related work

## Dualscattering Approximation

- Marschner model deals with rendering a single fiber.
- Path-tracing have to be performed to render globally illuminated hair volumes.
  - Very costly, takes a lot of time.
  - Requires a lot of samples to reduce noise.

Dualscattering approximation by Zinke et al.(2008) proposes a dual approach to overcome the extensive path tracing.

# Related work

## Dualscattering approximation

Dualscattering approximation breaks the scattering computation up into two components

- Global scattering  $\Psi^G$ : Computes the irradiance arriving at a point  $x$  in the hair volume.
- Local scattering  $\Psi^L$ : Accounts for the multiple scattering within the neighborhood of point  $x$

# Related work

## Dualscattering: global scattering

Global scattering  $\Psi^G$  is computed by taking into account:

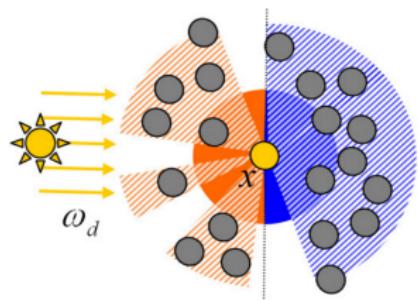
- Transmittance  $T_f$  through the volume from direction  $\omega_d$
- Spread  $S_f$  when scattering through hair strands.

$$\Psi^G(x, \omega_d, \omega_i) \approx T_f(x, \omega_d) S_f(x, \omega_d, \omega_i) \quad (11)$$

$$T_f(x, \omega_d) = d_f \cdot \prod_{k=1}^n a_f(\theta_d^k) \approx d_f \cdot a_f(\theta_d)^n$$

$$S_f(x, \omega_d, \omega_i) = \frac{g(\theta_d + \theta_i, \sigma_f^2)}{\pi \cos \theta_d}$$

$$\sigma_f^2(x, \omega_d) = \sum_{k=1}^n \beta_f^2(\theta_d^k) \approx n \cdot \beta_f^2(\theta_d)$$



# Related work

## Dualscattering: local scattering

- Local scattering  $\Psi^L$  deals with local neighborhood by rendering them using the Marschner model  $f_s$ .
- Forward scattering events are dealt with by global scatter component.
- Local scattering needs to deal with at least one backscatter event

$$\Psi^L(x, \omega_d, \omega_i) f_s(\omega_i, \omega_o) \approx d_b \cdot f_{back}(\omega_i, \omega_o) \quad (12)$$

$$f_{back}(\omega_i, \omega_o) = \frac{2}{\cos \theta_d} A_b(\theta_d) \cdot S_b(\omega_i, \omega_o) \quad (13)$$

where  $A_b$  is the average backscattering attenuation and  $S_b$  the average backscattering spread.

# Approach

Goal of thesis:

*"To evaluate whether multiple importance sampling (MIS) the dualscattering method leads to a significant reduction of variance in the rendered results".*

Theoretical challenges:

- Application of MIS to the dualscattering method.
- Measuring the results for MIS.

# Approach

Application of MIS to the dualscattering method.

- Marschner model (2003) used in this thesis is not energy conserving
- Multiple importance sampling strategy has been created for the improved model by d'Eon et al (2011).
- Applying the multiple importance sampling strategy to the dualscattering method might therefore not be a close fit.

# Approach

Measuring the results for MIS.

- Uniform sampling is compared with multiple importance sampling techniques.
- 512 samples per pixel for uniform sampling is assumed as the ground truth to compare against.
- Variance is calculated between the rendered result and the ground truth. Additionally, visual inspection is performed for the results.

# Implementation

## Multiple importance sampling approach

Multiple importance sampling method is based on the improved Marschner model, for which the equation is simplified as follows:

$$S_p(\omega_i, \omega_o) = M_p(\theta_i, \theta_o) \cdot N_p(\theta_i, \theta_o, \phi) \quad (14)$$

To multiple importance sample this function, the approach is to:

- Sample the longitudinal scattering function  $M_p$
- Sample the azimuthal scattering function  $N_p$

# Implementation

## Multiple importance sampling approach

- Select a lobe  $p \in R, TT, TRT$  dependent on the relative contribution of energy reflected.
- Given a selected lobe  $p$ , compute the longitudinal angle  $\theta$  by sampling  $M_p$ .
- An outgoing direction  $\omega_o$  is given, so we can compute the spherical angles  $(\theta_o, \phi_o)$ .
- Choose a random offset  $h \in [-1, 1]$  along the fiber cross section.

# Implementation

## 1. Selecting a lobe

- Select a lobe  $p \in R, TT, TRT$  dependent on the relative contribution of energy reflected.
- d'Eon et al. (2013) proposes a way to select lobes by taking into account the attenuations  $A$  through a smooth, ideally specular fiber.

$$h = 2\xi - 1 \quad \text{where } \xi_h \in [0, 1]$$

$$A(R, h) = F(\eta, \gamma_i)$$

$$A(TT, h) = (1 - F(\eta, \gamma_i))^2 \cdot T(\sigma_a, h)$$

$$A(TRT, h) = (1 - F(\eta, \gamma_i))^2 \cdot F\left(\frac{1}{\eta}, \gamma_t\right) \cdot T(\sigma_a, h)^2$$

$F$  stands for Fresnel and  $T$  for Transmittance

# Implementation

## 2. Sampling the longitudinal $M$ function

- Original Marschner model longitudinal scattering function  $M$  is based on a gaussian function that leaks energy outside the range  $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
- d'Eon et al. (2013) employs spherical gaussian convolution to conservatively redistribute reflected radiance amongst directions on the sphere.
- Sampling a spherical gaussian is performed using Box-Muller transform, giving us the incident longitudinal angle  $\theta_i$ .

# Implementation

## 3. Sampling the azimuthal $N$ function

- $\phi_i^{smooth}$  is trivial to find if we know the selected lobe  $p$  and the offset  $h$  from the scattering cross section.
- $\phi_i^{rough}$  is then found by adding a gaussian offset, sampled again using the Box-Muller transform.

$$\phi_i^{smooth} = \phi_o + \phi(p, h) \quad (15)$$

$$\phi_i^{rough} = \phi_o + \phi(p, h) + g|\beta_p| \quad (16)$$

$$\phi(p, h) = 2p\gamma_t - 2\gamma_i + p\pi \quad (17)$$

# Implementation

## Voxel grid

Global scattering irradiance is approximated using ray-shooting with voxel grid lookups.

- OpenVDB is used as the voxel grid library.
- Voxel grid is generated in a pre-processing step, storing the densities in the voxel cells
- Voxel grid is simplified by not storing orientation per voxel cell.  
Instead, the  $\theta_d$  is used at position  $x$

$$\Psi^G(x, \omega_d, \omega_i) \approx T_f(x, \omega_d) S_f(x, \omega_d, \omega_i)$$

$$T_f(x, \omega_d) \approx d_f \cdot a_f(\theta_d)^n$$

$$S_f(x, \omega_d, \omega_i) = \frac{g(\theta_d + \theta_i, \sigma_f^2)}{\pi \cos \theta_d} \quad \text{with } \sigma_f^2(x, \omega_d) \approx n \cdot \beta_f^2(\theta_d)$$

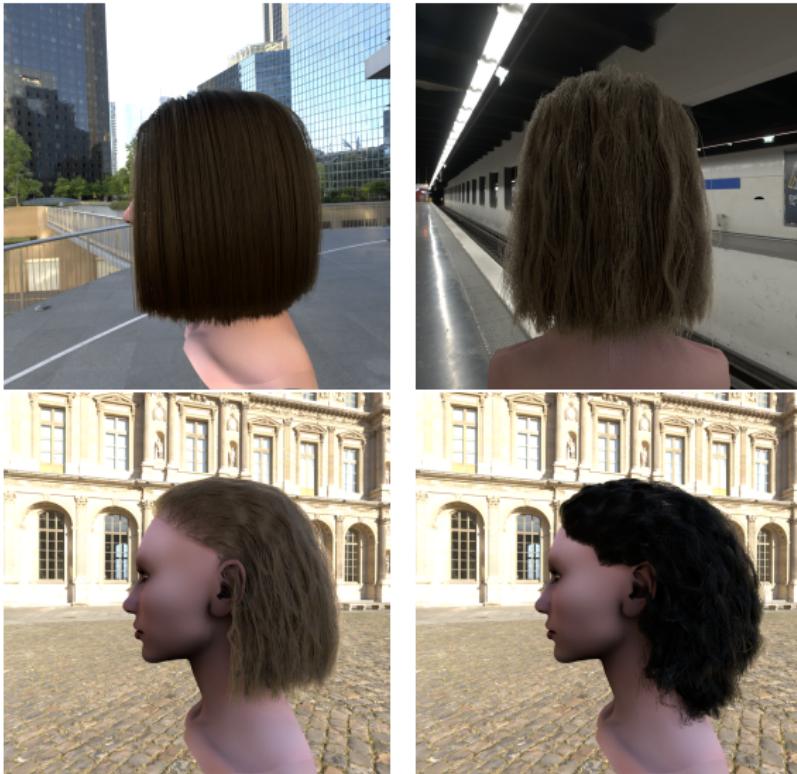
# Results

Dual scattering approximation IS 512 samples per pixel



# Results

## Importance sampling results



# Results

Real world scenarios (Venice)

Uniform Sampling (32 spp)



Importance Sampling (32 spp)



# Results

Real world scenarios (Office Square)

Uniform Sampling (32 spp)



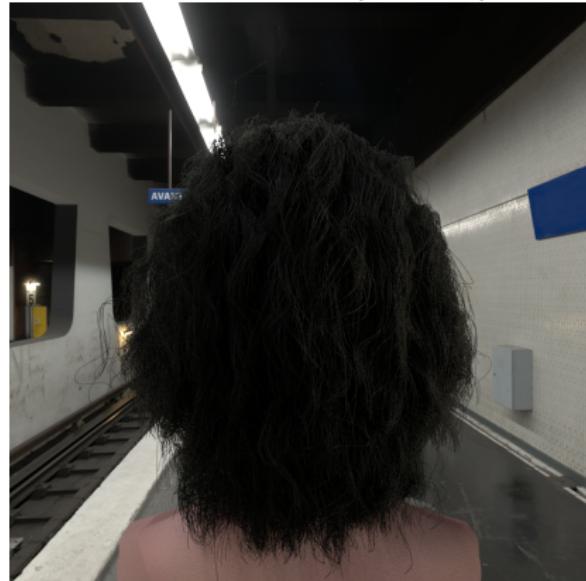
Importance Sampling (32 spp)



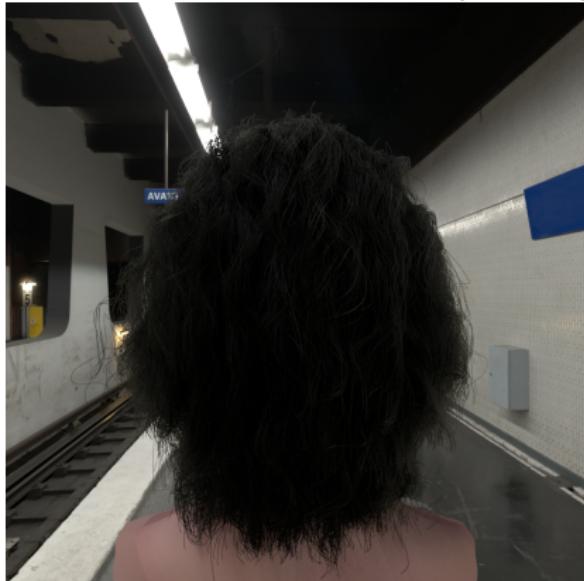
# Results

## Real world scenarios (Subway Station)

Uniform Sampling (32 spp)



Importance Sampling (32 spp)

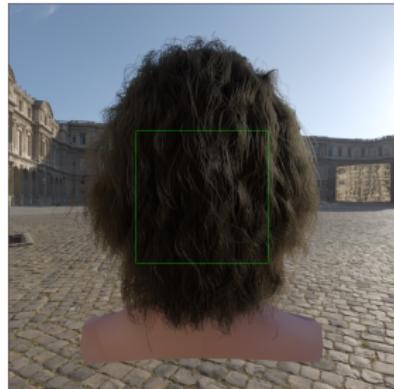


# Results

## Variance computation

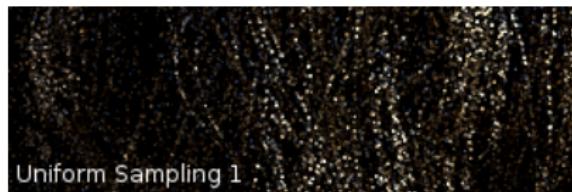
- Variance  $V$  is computed for a region  $(w, h)$  of the image .
- Variance is computed by taking the average distance squared between the image to be analyzed  $X$  with the ground truth  $X'$ .
- Ground truth is assumed to be uniform sampling at 512 spp.

$$V = \frac{1}{w \cdot h} \sum_{x=0}^w \sum_{y=0}^h (X_{x,y} - X'_{x,y})^2 \quad (18)$$

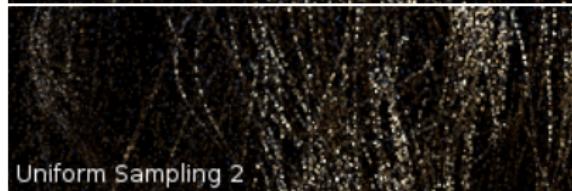


# Results

## Variance comparison



Uniform Sampling 1



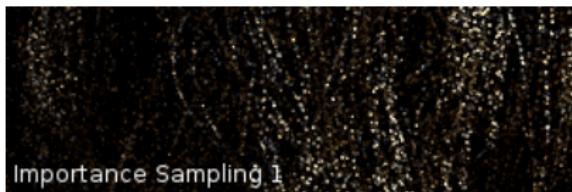
Uniform Sampling 2



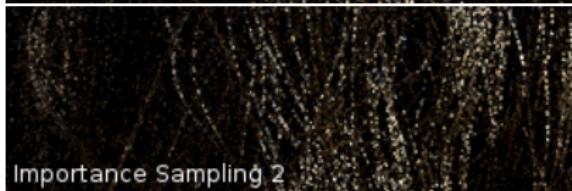
Uniform Sampling 4



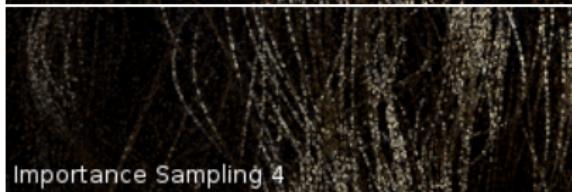
Uniform Sampling 8



Importance Sampling 1



Importance Sampling 2



Importance Sampling 4



Importance Sampling 8

# Results

## Variance comparison



Uniform Sampling 16



Importance Sampling 16



Uniform Sampling 32



Importance Sampling 32



Uniform Sampling 64



Importance Sampling 64



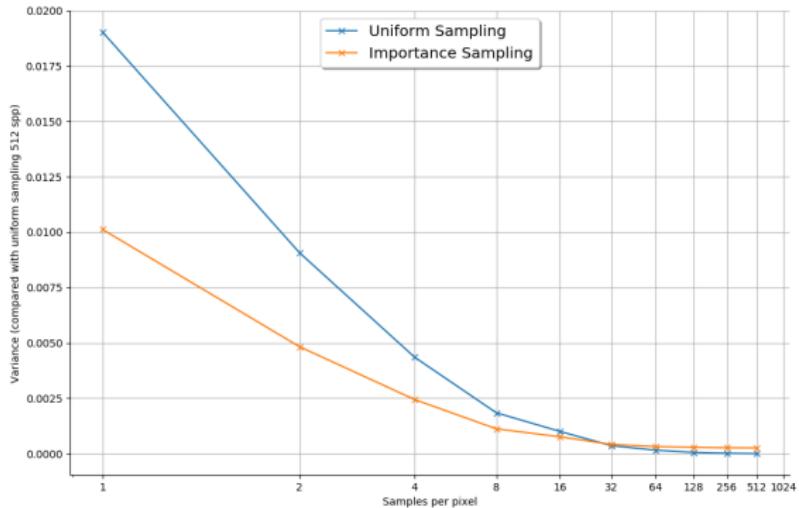
Uniform Sampling 128



Importance Sampling 128

# Results

## Variance plot



**Figure:** Logarithmic scale of variance that is reduced as the samples per pixel increase.

# Conclusion

## Theoretical Challenges

- Dual scattering model for this thesis is not fully energy conserving.
- Variance is lower for importance sampling, but not by the extend that it is clearly noticeable.
- Challenge to implement the dualscattering method in PBRT, because dual scattering makes use of an approximated volume (which does not fit a pure ray tracer).
- Involved physics and mathematical knowledge required. That was challenging to understand.

# Conclusion

## Practical Challenges

- Choice of rendering framework. Started from scratch, then Pixar Renderman, finally PBRT.
- Voxel grid implementation (OpenVdb) where hair fibers are assumed to be oriented in the same direction.
- Memory and speed requirements. Physics based rendering can take a long time. Feedback cycle is long.
- Non intuitiveness of Marschner model parameters.

# Conclusion

*"To evaluate whether multiple importance sampling (MIS) the dual scattering method leads to a significant reduction of variance in the rendered results".*

Applying importance sampling for the dual scattering method does not significantly reduce the noise compared to rendering using uniform sampling.