Hair Rendering: Importance Sampling of Dual Scattering Approximation

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Important for variety of industries

- Animation movie industry: to render realistic hairs in a physically accurate way.
- Game industry: enhance realism and visual effects.
- Clothes manufacturing industry: to render custom fabrics and to compare appearance in different lighting conditions.
- Hair styling: render hair styling products applied to the hair.

Rendering hair

Hair fiber representation

Explicit representation vs. Implicit representation

- Explicit representation represents each fiber by geometric primitives (e.g. triangles)
- Implicit representation represents fiber

There are a couple of ways to represent hair fibers:

- Connected triangle strips
- Cylindrical primitives
- Trigonal prisms
- Ribbons

Human hair consists of over hundreds of thousands of hair strands. Leads to rendering challenges:

- Memory consumption: to store all fibers in memory.
- Time: rendering realistic scattering effects requires tracing many samples through the hair volume.
- Aliasing: Hair fibers are very thin, requiring additional samples to be drawn to prevent aliasing.

Mathematical notation

Radiometry

- Power (Watts): energy in Joules per second.
- Radiant intensity (steradians): power divided by the solid angle.
- Irradiance (Watts per m^2): power per unit area.
- Radiance : Irradiance per solid angle, where solid angle goes to zero (becoming a ray instead of a cone).

Mathematical notation

Scattering equations

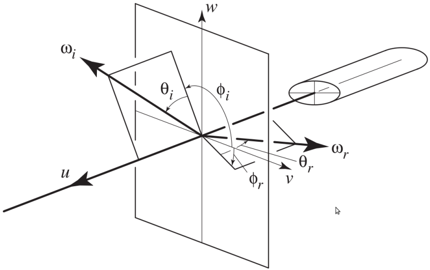
Scattering is represented by a bidirectional reflection distribution function (BRDF):

$$f_r(p,\omega_o,\omega_i) = \frac{dL_o(p,\omega_o)}{dE(p,\omega_i)} = \frac{dL_o(p,\omega_o)}{L_i(p,\omega_i)\cos\theta_i d\omega_i} \quad (1)$$

The BRDF is the fraction of outgoing radiance in direction ω_o related to the incident irradiance from direction ω_i.

Mathematical notation

Coordinates axes for hair fibers



- Coordinate axis are represented using *uvw* axes, with *u* pointing from base to tip of hair strand and *vw* forms the orthogonal plane.
- Longitudinal angles θ are with respect to *u*, where 0 degrees is perpendicular to the fiber, -90 towards base, and 90 towards tip.
- Azimuthal angles ϕ are formed with respect to the orthogonal plane.
- $\omega_i = (\theta_i, \phi_i)$ incident direction, $\omega_o = (\theta_o, \phi_o)$ outgoing direction.

Mathematical notation

Bidirectional curve scattering distribution function (BCSDF)

- Hair fibers are rendered implicitly by 3D curves.
- Curves have no surface area. They have a length, requiring a change to the BRDF formulation.

$$S_r(\omega_o, \omega_i) = \frac{dL_o(\omega_o)}{dE_i(\omega_i)} = \frac{dL_o(\omega_o)}{DL_i(\omega_i)\cos\theta_i d\omega_i} \tag{2}$$

$$L_o(\omega_o) = D \int S(\omega_i, \omega_o) L_i(\omega_i) \cos\theta_i d\omega_i \tag{3}$$

Monte-Carlo integration

Let's say we want to integrate $f(x)$ which is a 1D formula from a to b.

$$\int_a^b f(x)dx \tag{4}$$

This can be evaluated by drawing N uniform samples for $X_i \in [a, b]$.

$$F_N = \frac{b-a}{N}\sum_{i=1}^{N} f(X_i) \tag{5}$$

The Monte-Carlo estimator says that the expected value $E[F_N]$ is equal to the integral.

Possible to integrate any definite higher dimensional integrals.

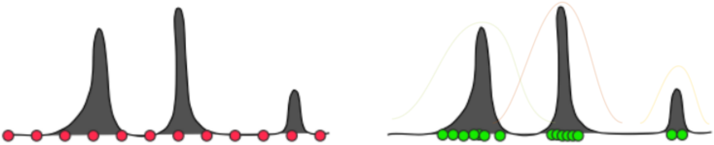
Importance sampling

- Importance sampling prefers to sample high-value contributions.
- The estimator can be rewritten by taking into consideration the probability density function $p(X_i)$
- Samples need to be generated from $p(x)$, by processes such as the inversion method or rejection method.

$$F_N = \frac{1}{N} \sum_{i=1}^{N} \frac{f(X_i)}{p(X_i)} \tag{6}$$

Multiple importance sampling

- Scattering distributions cannot always be represented by closed formulas, or be inverted at all.
- A solution is to match the scattering distribution with multiple functions $f(x)$ and $g(x)$



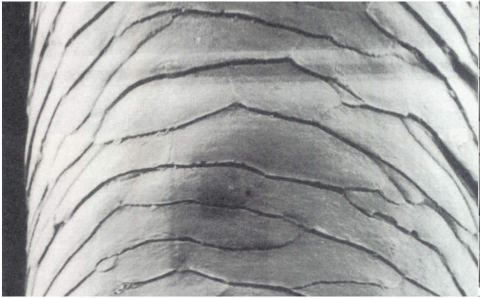
$$\frac{1}{n_f}\sum_{i=1}^{n_f}\frac{f(X_i)g(X_i)w_f(X_i)}{p_f(X_i)}+\frac{1}{n_g}\sum_{j=1}^{n_g}\frac{f(Y_j)g(Y_j)w_g(Y_j)}{p_g(Y_j)} \tag{7}$$

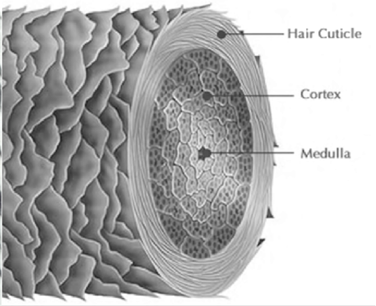
$$w_s(x)=\frac{n_s p_s(x)}{\sum_i n_i p_i(x)} \tag{8}$$

Related work

Structure of hair

- Hair fibers do have cuticle scales tilted by approximately 3 degrees toward the root.
- The core of the fiber (medulla and cortex) consist of pigment which absorbs specific wavelengths in the light.

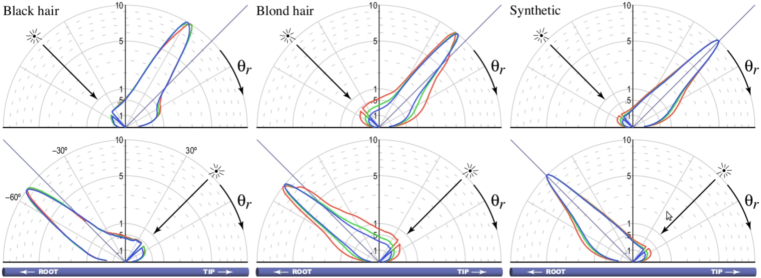




Related work

Measured scattering data

Single fiber scattering model that models three visible scattering components:



Related work

Marschner model

Single fiber scattering model that models three visible scattering components.

R reflection,
TT double transmission,
TRT transmission, reflection, transmission

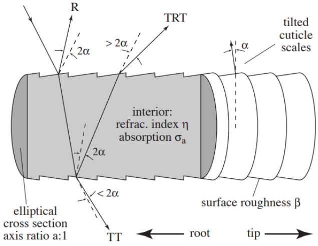


Figure: Longitudinal scattering

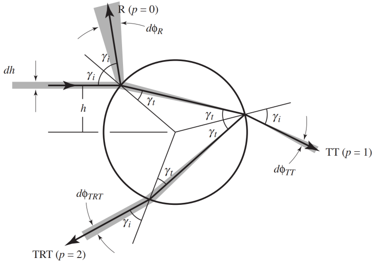


Figure: Azimuthal scattering

The Marschner model scattering equation can be written as:

$$S(\omega_i, \omega_o) = \sum_p M_p(\theta_i, \theta_o) N_p(\eta(\theta_d); \phi_i, \phi_o) / \cos^2 \theta_d \tag{9}$$

For longitudinal scattering

$$M_p(\theta_h) = g(\beta_p; \theta_h - \alpha_p) \tag{10}$$

For azimuthal scattering

$$N_p(\theta_h) = \sum_r A(p, h) |2 \frac{d\phi}{dh}(p, h)| \tag{11}$$

- p is the scattering mode with $p \in R, TT, TRT$
- where θ_h is the half angle between θ_i and θ_r
- where η is the index of refraction.
- r is the number of roots.

Related work

Dualscattering Approximation

- Marschner model deals with rendering a single fiber.
- Path-tracing have to be performed to render globally illuminated hair volumes.
  - Very costly, takes a lot of time.
  - Requires a lot of samples to reduce noise.

Dualscattering approximation by Zinke et al.(2008) proposes a dual approach to overcome the extensive path tracing.

Dualscattering approximation breaks the scattering computation up into two components

- Global scattering Ψ^G: Computes the irradiance arriving at a point x in the hair volume.
- Local scattering Ψ^L: Accounts for the multiple scattering within the neighborhood of point x

Related work

Dualscattering: global scattering

Global scattering Ψ^G is computed by taking into account:

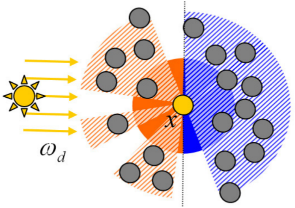
- Transmittance T_f through the volume from direction ω_d
- Spread S_f when scattering through hair strands.

$$\Psi^G(x, \omega_d, \omega_i) \approx T_f(x, \omega_d)\, S_f(x, \omega_d, \omega_i) \tag{12}$$

$$T_f(x, \omega_d) = d_f \cdot \prod_{k=1}^{n} a_f(\theta_d^k) \approx d_f \cdot a_f(\theta_d)^n$$

$$S_f(x, \omega_d, \omega_i) = \frac{g(\theta_d + \theta_i, \sigma_f^2)}{\pi \cos\theta_d}$$

$$\sigma_f^2(x, \omega_d) = \sum_{k=1}^{n} \beta_f^2(\theta_d^k) \approx n \cdot \beta_f^2(\theta_d)$$



Related work

Dualscattering: local scattering

- Local scattering deals with local neighborhood by rendering them using the Marschner model f_s.
- Forward scattering events are dealt with by global scatter component.
- Local scattering needs to deal with at least one backscatter event

$$\Psi^L(x, \omega_d, \omega_i) f_s(\omega_i, \omega_o) \approx d_b \cdot f_{back}(\omega_i, \omega_o) \tag{13}$$

$$f_{back}(\omega_i, \omega_o) = \frac{2}{\cos \theta_d} A_b(\theta_d) \cdot S_b(\omega_i, \omega_o) \tag{14}$$

where A_b is the average backscattering attenuation and S_b the average backscattering spread.

Approach

Goal of thesis:

"To evaluate whether multiple importance sampling (MIS) the dualscattering method leads to a significant reduction of variance in the rendered results".

Theoretical challenges:

- Application of MIS to the dualscattering method.
- Measuring the results for MIS.

Approach

Application of MIS to the dualscattering method.

- Multiple importance sampling has been created for the improved Marschner model.
- Applying the multiple importance sampling strategy to the dualscattering approximation might therefore not be a close fit.

Approach

Measuring the results for MIS.

- Uniform sampling is compared with multiple importance sampling techniques.
- 512 samples per pixel for uniform sampling is assumed as the ground truth to compare against.
- Variance is calculated between the rendered result and the ground truth. Additionally, visual inspection is performed for the results.

Implementation

Multiple importance sampling approach

Multiple importance sampling method is based on the improved Marschner model, for which the equation is simplified as follows:

$$S_p(\omega_i, \omega_o) = M_p(\theta_i, \theta_o) \cdot N_p(\theta_i, \theta_o, \phi) \tag{15}$$

To multiple importance sample this function, the approach is to:

- Sample the longitudinal scattering function M_p
- Sample the azimuthal scattering function N_p

Implementation

Multiple importance sampling approach

- Select a lobe $p \in R, TT, TRT$ dependent on the relative contribution of energy reflected.
- Given a selected lobe p, compute the longitudinal angle θ by sampling M_p.
- An outgoing direction ω_o is given, so we can compute the spherical angles (θ_o, ϕ_o).
- Choose a random offset $h \in [-1, 1]$ along the fiber cross section.

Implementation

1. Selecting a lobe

- Select a lobe $p \in R, TT, TRT$ dependent on the relative contribution of energy reflected.
- d'Eon et al. (2013) proposes a way to select lobes by taking into account the attenuations A through a smooth, ideally specular fiber.

$$
\begin{aligned}
h &= 2\xi - 1 && \text{where } \xi_h \in [0,1] \\
A(R,h) &= F(\eta, \gamma_i) \\
A(TT,h) &= (1 - F(\eta, \gamma_i))^2 \cdot T(\sigma_a, h) \\
A(TRT,h) &= (1 - F(\eta, \gamma_i))^2 \cdot F(\frac{1}{\eta}, \gamma_t) \cdot T(\sigma_a, h)^2
\end{aligned}
$$

F stands for Fresnel and T for Transmittance

- Original Marschner model longitudinal scattering function M is based on a gaussian function that leaks energy outside the range $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
- d'Eon et al. employs spherical gaussian convolution to conservatively redistribute reflected radiance amongst directions on the sphere.
- Sampling a spherical gaussian is performed using Box-Muller transform, giving us the incident longitudinal angle θ_i.

- ϕ_i^{smooth} is trivial to find if we know the selected lobe p and the offset h from the scattering cross section.
- ϕ_i^{rough} is then found by adding a gaussian offset, sampled again using the Box-Muller transform.

$$\phi_i^{smooth} = \phi_o + \phi(p, h) \tag{16}$$

$$\phi_i^{rough} = \phi_o + \phi(p, h) + g|\beta_p| \tag{17}$$

$$\phi(p, h) = 2p\gamma_t - 2\gamma_i + p\pi \tag{18}$$

Implementation

Voxel grid

Global scattering irradiance is approximated using ray-shooting with voxel grid lookups.

- OpenVDB is used as the voxel grid library.
- Voxel grid is generated in a pre-processing step, storing the densities in the voxel cells
- Voxel grid is simplified by not storing orientation per voxel cell. Instead, the θ_d is used at position x

$$\Psi^G(x, \omega_d, \omega_i) \approx T_f(x, \omega_d)\, S_f(x, \omega_d, \omega_i)$$

$$T_f(x, \omega_d) \approx d_f \cdot a_f(\theta_d)^n$$

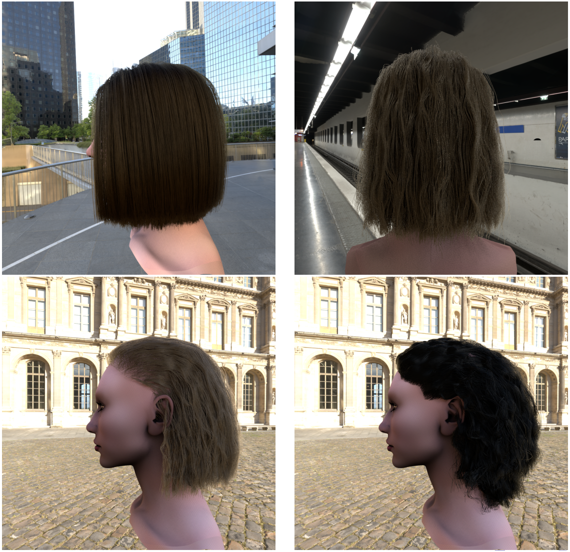
$$S_f(x, \omega_d, \omega_i) = \frac{g(\theta_d + \theta_i, \sigma_f^2)}{\pi \cos \theta_d} \qquad \text{with } \sigma_f^2(x, \omega_d) \approx n \cdot \beta_f^2(\theta_d)$$

Results

Dual scattering approximation IS 512 samples per pixel

Results

Importance sampling results



Results

Real world scenarios (Venice)

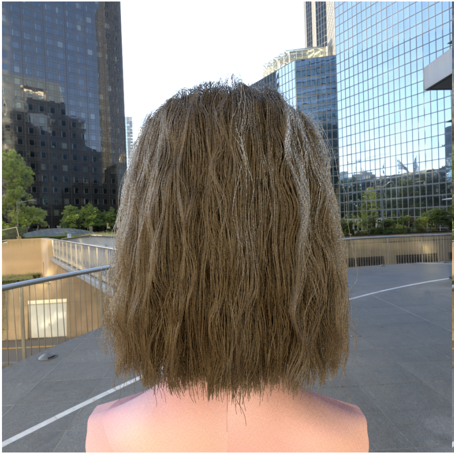
Uniform Sampling (32 spp)

Importance Sampling (32 spp)

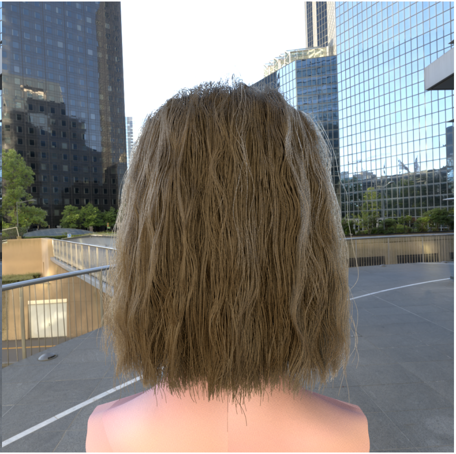
Results

Real world scenarios (Office Square)

Uniform Sampling (32 spp)



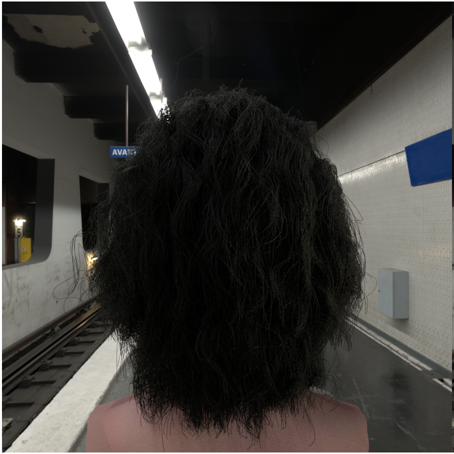
Importance Sampling (32 spp)



Results

Real world scenarios (Subway Station)

Uniform Sampling (32 spp)



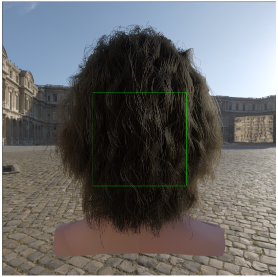
Importance Sampling (32 spp)

Results

Variance computation

- Variance V is computed for a region (w, h) of the image .
- Variance is computed by taking the average distance squared between the image to be analyzed X with the ground truth X'.
- Ground truth is assumed to be uniform sampling at 512 spp.

$$V = \frac{1}{w \cdot h} \sum_{x=0}^{w} \sum_{y=0}^{h} (X_{x,y} - X'_{x,y})^2 \qquad (19)$$



Results

Variance comparison

Results

Variance comparison

Variance plot

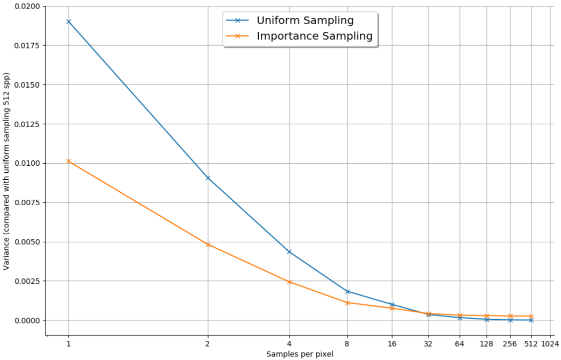


Figure: Logarithmic scale of variance that is reduced as the samples per pixel increase.

- Dual scattering model for this thesis is not fully energy conserving.
- Variance is lower for importance sampling, but not by the extend that it is clearly noticeable.
- Challenge to implement the dualscattering method in PBRT, because dual scattering makes use of an approximated volume (which does not fit a pure ray tracer).
- Involved physics and mathematical knowledge required. That was challenging to understand.

- Choice of rendering framework. Started from scratch, then Pixar Renderman, finally PBRT.
- Voxel grid implementation (OpenVdb) where hair fibers are assumed to be oriented in the same direction.
- Memory and speed requirements. Physics based rendering can take a long time. Feedback cycle is long.
- Non intuitiveness of Marschner model parameters.

Conclusion

"To evaluate whether multiple importance sampling (MIS) the dualscattering method leads to a significant reduction of variance in the rendered results".

Applying importance sampling for the dual scattering method does not significantly reduce the noise compared to rendering using uniform sampling.