Dynamical systems & chaos

Consider the function

$f(x) = r \cdot x \cdot (1-x)$ where r is a parameter.

What happens when we start iterating?

x_0

$f(x_0)$

$f(f(x_0))$

$f(f(f(x_0)))$

$f(f(f(f(x_0))))$

$\cdots$?

We saw that if you do

$\cos(\cos(\cos(\cos(\cos(\cos(\cos(1)))))))$

it got closer and closer to some specific number. Why?

Try some r-values:

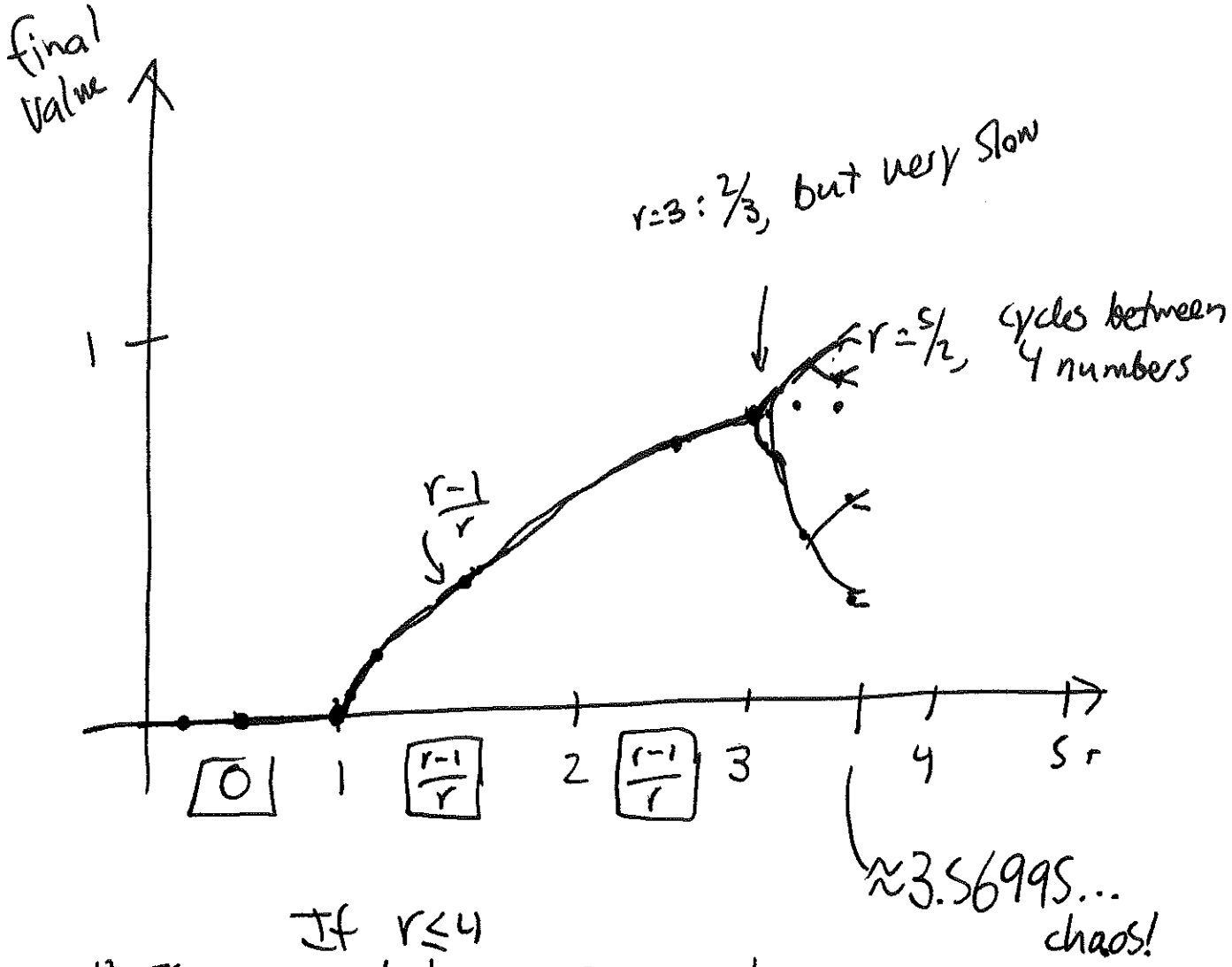
$r = 1/2$ || $r = 3/2$ || $r = 7/2$ || $r = 4$.

Start with $x_0 = 0.1$. $f(x) = rx(1-x)$

Compute $f(f(f(f(f\cdots(x_0)))))$

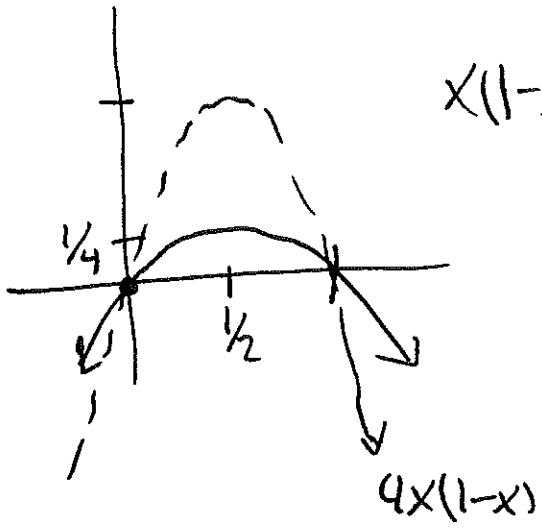
and see what happens.

Can you explain why?



If $r \leq 4$

If x_0 is between 0 and 1, all x's will be:



For $0 < r < 2$, it converges to a point where $f(x) = x$

$$x = r x(1-x)$$

$$1 = r(1-x) = r - rx$$

$$rx = r - 1$$

$$x = \frac{r-1}{r}$$