MATH 436, MIDTERM 2 SPRING 2021, JOHN LESIEUTRE

- You have fifty minutes to complete the exam.
- Exam should be submitted on Gradescope once completed. (Scanning and uploading time do not count towards the fifty minutes.)
- You may consult your notes, the course materials on my website, and the textbook.
- Collaboration and all other references are not allowed.
- Although you can write your answers on a copy of the exam, it is not required.
- Justifications or proofs are required for all problems except where indicated otherwise.
- Please either sign below the integrity statement below or copy out this statement at the beginning of your exam.
- I am generous with partial credit! Try not to leave anything blank.
- Good luck!

I affirm that I have complied with all the exam requirements. I have completed the exam within the allotted time and have not consulted any disallowed references.

Signature:		

Problem	Score	Possible
1		20
2		20
3		20
4		20
5		20
Σ		100

a) (10 points) Give a basis for this space, and compute the matrix for T with respect to your basis.

b) (10 points) Compute $\operatorname{rref}(M)$ for your answer to (a). Is the map invertible? Justify your answer.

Problem 2. Let $V = \mathcal{P}_3(\mathbb{R})$ and $W = \mathcal{P}_1(\mathbb{R})$. Consider the bases $x^3, x^2, x, 1$ for V and x, 1 for W. Suppose that $T : \mathcal{P}_3(\mathbb{R}) \to \mathcal{P}_1(\mathbb{R})$ is a linear map whose matrix with respect to these bases is given by:

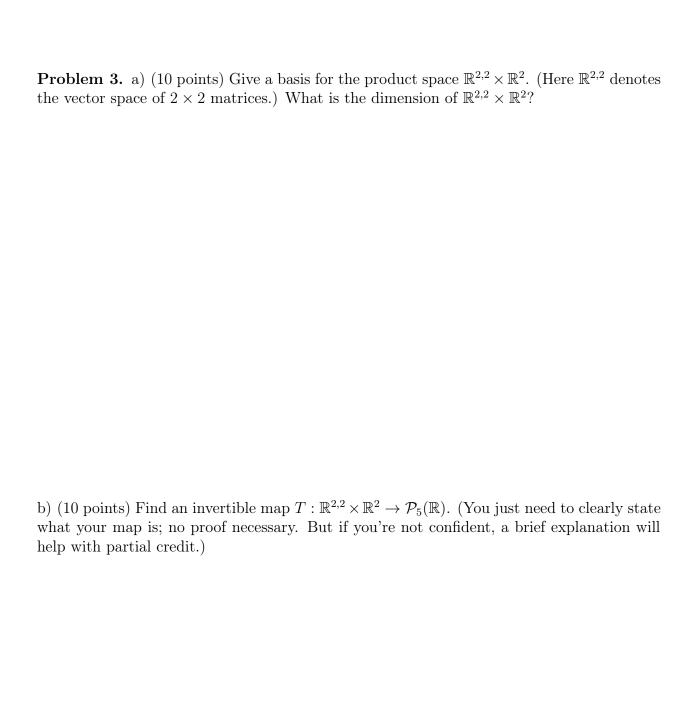
$$M = \left(\begin{array}{ccc} 1 & 2 & 0 & 0 \\ 2 & 6 & 4 & -2 \end{array} \right).$$

This has

$$\operatorname{rref}(M) = \left(\begin{array}{ccc} 1 & 0 & -4 & 2\\ 0 & 1 & 2 & -1 \end{array}\right)$$

a) (10 points) Find bases for the nullspace and the range of T.

b) (10 points) Is the vector 3x - 2 in the range? Justify your answer.



Problem 4. a) (10 points) Suppose that V is a vector space. Define the dual space of V(a definition for any vector space). Give an example of a nonzero element of the dual space $(\mathbb{R}^3)'$.

b) (10 points) Consider the elements λ, μ, ν of $(\mathcal{P}_2(\mathbb{R}))'$ defined by

$$\lambda(f) = \int_{-1}^{1} f \, dx$$
$$\mu(f) = f(0)$$
$$\nu(f) = f''(0)$$

$$\nu(f) = f''(0)$$

Prove that $3\lambda - 6\mu - \nu = 0$ in $(\mathcal{P}_2(\mathbb{R}))'$.

Problem 5. Suppose that $T: V \to V$ is a linear map.

a) (10 points) Suppose that $U \subset V$ is an invariant subspace for T. Prove that U is also an invariant subspace for the composition $T^2 = T \circ T$.

b) (10 points) Let \mathbb{R}^{∞} denote the set of infinite sequences of real numbers. Consider the double-left-shift map $L^2: \mathbb{R}^{\infty} \to \mathbb{R}^{\infty}$ given by

$$L^2(a_0, a_1, a_2, a_3, a_4, \ldots) = (a_2, a_3, a_4, \ldots)$$

Find an eigenvector of this map and the corresponding eigenvalue.