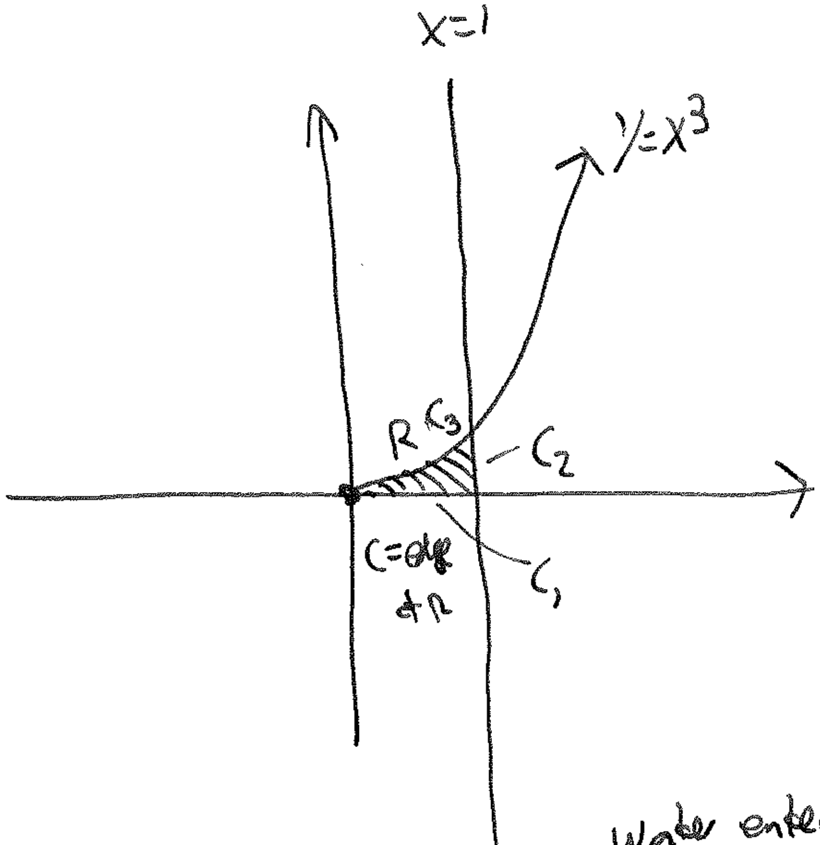


$\vec{F} = M\hat{i} + N\hat{j}$

$\operatorname{div}\vec{F} = M_x + N_y$

$\vec{F} = (1+y^2)\hat{j}$

$$\underbrace{\oint_C \vec{F}\cdot\hat{n}\,ds}_{\text{flow out of } R} = \underbrace{\iint_R \operatorname{div}\vec{F}\,dA}_{\text{water entering through fountain at bottom}}$$

Green's theorem

$$\operatorname{div}\vec{F} = 2y$$

$$\int_{x=0}^{1}\left[\int_{y=0}^{x^3} 2y\,dy\right]dx = \int_{x=0}^{1} x^6\,dx = \frac{1}{7}$$

$$\int_{C_1} \vec{F} \cdot \hat{n}\, ds \qquad \vec{r}(t) = (t, 0)$$

tangent

$$\frac{d\vec{r}}{dt} = (1, 0)$$

rotate 90°

$$\hat{n}\, ds = (0, -1)\, dt$$

$\vec{F}$ but what's y? plug in from $\vec{r}$!

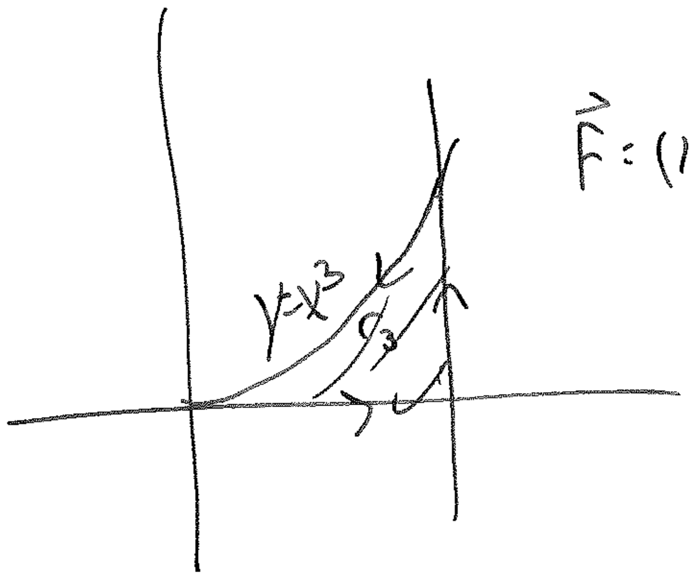
$$= \int_{t=0}^{1} (0, 1+y^2) \cdot (0, -1)\, dt$$

$$= \int_{t=0}^{1} (0, 1) \cdot (0, -1)\, dt = \int_0^1 -1\, dt = -1.$$

$$\int_{C_2} \vec{F} \cdot \hat{n}\, ds = 0 \quad \text{since } \vec{F} \text{ is vertical, } \hat{n} \text{ horizontal.}$$

$$\int_{C_3} \vec{F} \cdot \hat{n}\, ds = \iint_R \operatorname{div} \vec{F}\, dA - \int_{C_1} \vec{F} \cdot \hat{n}\, ds - \int_{C_2} \vec{F} \cdot \hat{n}\, ds$$

$$= \frac{1}{7} - (-1) - 0 = \boxed{\frac{8}{7}}$$



$\vec{F} = (1+y^2)\hat{j}$

$\vec{r}(t) = (1-t, (1-t)^3)$

$0 \le t \le 1$

$$\int_{C_3} \vec{F} \cdot \hat{n}\, ds$$

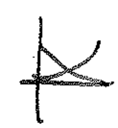
$\dfrac{d\vec{r}}{dt} = (-1, -3(1-t)^2)$ tangent vector

$\downarrow$ rotate

$\hat{n}\, ds = (-3(1-t)^2, 1)$

$\vec{F}$, with substitution for x, y.

$$\int_{t=0}^{1} \left(0, 1+\left((1-t)^3\right)^2\right) \cdot \left(-3(1-t)^2, 1\right)\, dt$$



$u = 1-t$
$du = -dt$

$$= \int_0^1 1+(1-t)^6\, dt = 1 + \int_{t=0}^{t=1} (1-t)^6\, dt$$

$$= 1 + \int_{u=1}^{u=0} u^6(-du) = 1 + \int_0^1 u^6\, du = \frac{8}{7}$$

$$\frac{du\,dv}{dx\,dy} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$$ Jacobian 2×2 determinant

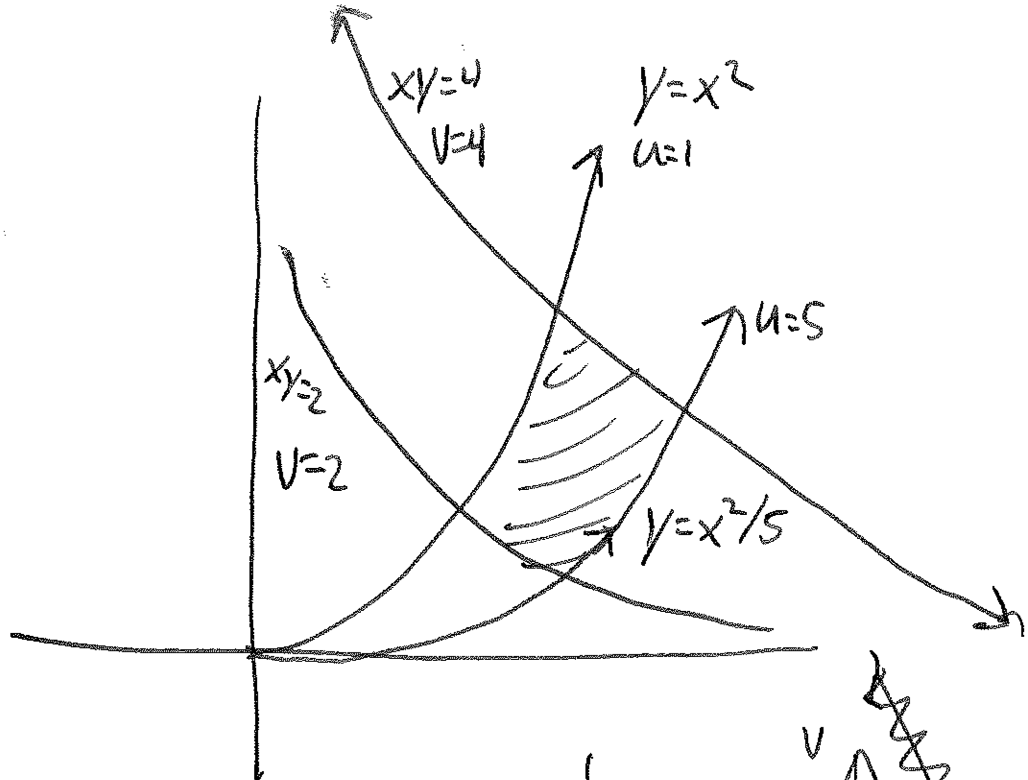
$$= \begin{vmatrix} 2x/y & -x^2/y^2 \\ y & x \end{vmatrix}$$

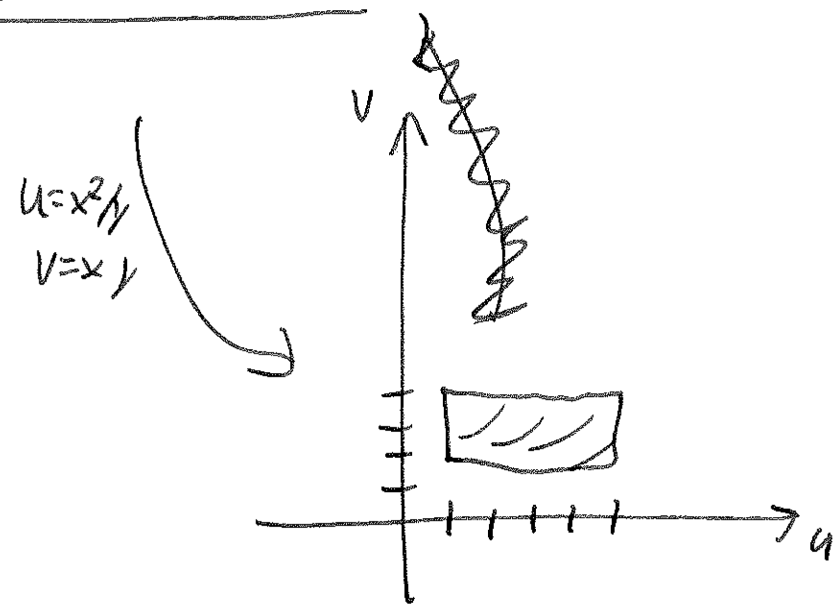
$$= \frac{2x^2}{y} - \left(-\frac{x^2}{y^2}\right)y = \frac{3x^2}{y}$$

$$du\,dv = \frac{3x^2}{y}\,dx\,dy$$

$$\iint_R 1\,dx\,dy = \int_{u=1}^{5}\int_{v=2}^{4} 1\,\frac{y}{3x^2}\,du\,dv$$

$$= \int_{u=1}^{5}\int_{v=2}^{4} \frac{1}{3u}\,du\,dv = \frac{2}{3}\ln(5)$$





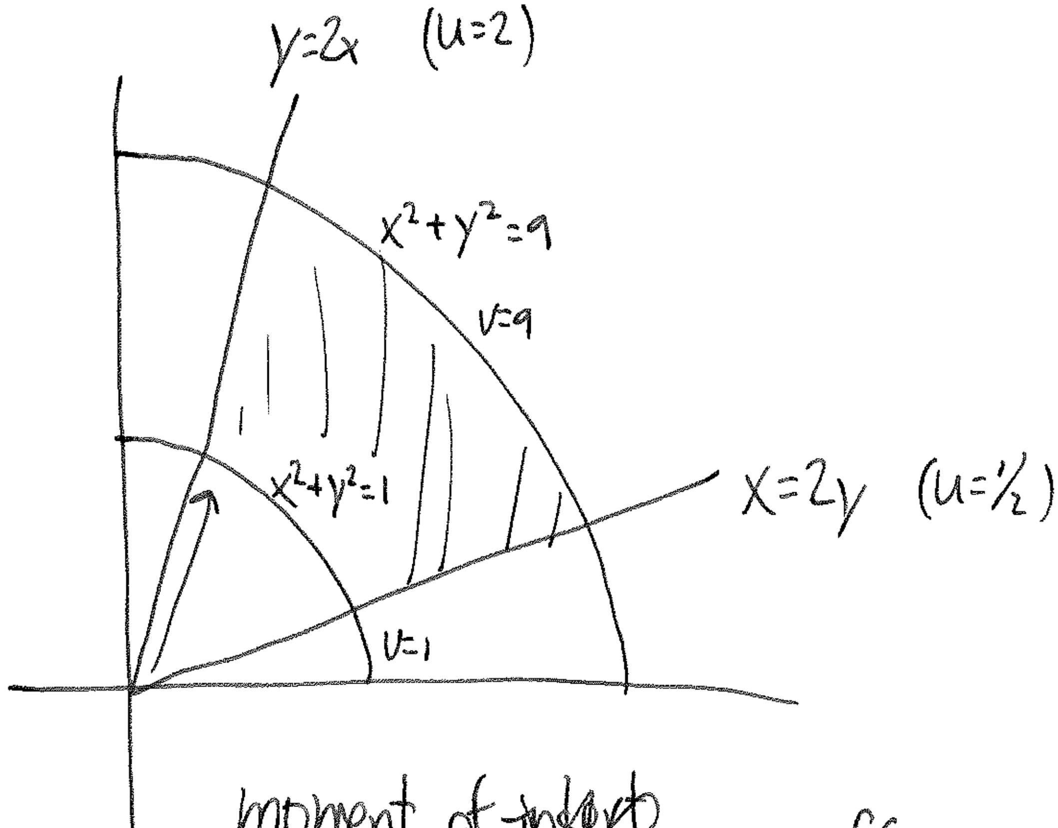
$u = x^2/y$

$v = xy$

$$\iint_R f\,dx\,dy = \int_{u=1}^{5}\int_{v=2}^{4} f\,\overset{\text{JACOBIAN}}{(\quad)}\,du\,dv$$

$$\int_{v=2}^{4}\int_{u=1}^{5} \frac{1}{3u}\,du\,dv$$

$$= \int_{v=2}^{4} \left(\frac{1}{3}\ln(u)\Big|_1^5\right) dv = \int_{v=2}^{4} \frac{1}{3}\ln 5\,dv = \frac{2}{3}\ln 5.$$



$\delta=1$

moment of inertia?
around 0.

$$\iint r^2\delta\,dA$$

$$=\iint x^2+y^2\,dx\,dy$$

$u = y/x$

$v = x^2 + y^2$

$$\frac{du\,dv}{dx\,dy} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = \left\| \begin{matrix} -y/x^2 & 1/x \\ 2x & 2y \end{matrix} \right\| = \left| -\frac{2y^2}{x^2} - 2 \right|$$

$$= \frac{2y^2}{x^2} + 2$$

$$du\,dv = + \frac{2y^2}{x^2} + 2\,dx\,dy$$

$$\iint (x^2+y^2)\,dx\,dy = \int_{u=1/2}^{2} \int_{v=1}^{9} v \left(\frac{2y^2}{x^2} + 2 \right)^{-1} du\,dv$$

need this in u,v!

$=(2u^2+2)^{-1}$

$$= \int_{v=1}^{9} \int_{u=1/2}^{2} \frac{1}{2} \cdot v \cdot \frac{1}{u^2+1}\,du\,dv$$

$$= \frac{1}{2} \int_{v=1}^{9} v\,dv \int_{u=1/2}^{2} \frac{1}{u^2+1}\,du = \left(\frac{1}{2}\right)(40)\left(\tan^{-1} 2 - \tan^{-1} \frac{1}{2}\right)$$

$$= 20\left(\tan^{-1} 2 - \tan^{-1} \frac{1}{2}\right)$$

What's the benefit of conservative field in 3D.

→ work done path independence

$$\int_{C_1} \vec{F}\cdot d\vec{r} = \int_{C_2} \vec{F}\cdot d\vec{r}.$$

$$\rightarrow \quad \int_C \vec{F}\cdot d\vec{r} = f(Q) - f(P)$$

<u>ex</u>

$\vec{G} = \langle y, x, y \rangle$ Show it's <u>not</u> conservative.

M N P

show that one of the conditions on partials fails!

$M_y = N_x, \quad M_z = P_x, \quad N_z = P_y$

✓ ✓ $0 \neq 1$