Name: ______________________________

Math 436, Midterm 1
Spring 2021, John Lesieutre

- You have fifty minutes to complete the exam.
- Exam should be submitted on Gradescope once completed. (Scanning and uploading time do not count towards the fifty minutes.)
- You may consult your notes, the course materials on my website, and the textbook.
- Collaboration and all other references are not allowed.
- Although you can write your answers on a copy of the exam, it is not required.
- Justifications or proofs are required for all problems except where indicated otherwise.
- Please either sign below the integrity statement below or copy out this statement at the beginning of your exam.
- I am generous with partial credit! Try not to leave anything blank.
- Good luck!

I affirm that I have complied with all the exam requirements. I have completed the exam within the allotted time and have not consulted any disallowed references.

Signature: ______________________________

Problem	Score	Possible
1		20
2		20
3		20
4		20
5		20
Σ		100

Problem 1. Consider the vector space $\mathcal{P}_2(\mathbb{R})$.

a) Consider the subset

$$V = \{f \in \mathcal{P}_2(\mathbb{R}) : f(0) \cdot f(1) = 0\}.$$

Is this a subspace of $\mathcal{P}_2(\mathbb{R})$? Justify your answer.

b) Give an example of a one-dimensional subspace of $\mathcal{P}_2(\mathbb{R})$. You do not need to prove it.

Problem 2. a) Consider the differentiation map $D : \mathcal{P}_3(\mathbb{R}) \to \mathcal{P}_3(\mathbb{R})$ (both subscripts are "3"). Is this map surjective? Is it injective? Justify your answer.

b) Suppose that $T : V \to V$ is a map from a finite-dimensional vector space to itself. Prove that T is injective if and only if it is surjective. (Hint: use the fundamental thorem.)

Problem 3. a) (10 points) Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear map and that $T(1,1) = (1,2)$ while $T(1,-1) = (1,4)$. What is $T(3,-1)$? Justify your answer.

b) Prove that $S : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $S(x, y) = x - y + x^2$ is not a linear map.

Problem 4. Consider the map $T : \mathbb{R}^2 \to M^{2\times 2}$ (the target space is 2×2 real matrices) which is defined by

$$T(x, y) = \begin{pmatrix} x & y \\ x+y & 0 \end{pmatrix}.$$

You may assume this is a linear map.

a) Prove that T is injective.

b) Give bases for both spaces, and compute the matrix for T with respect to those bases.

Problem 5. a) (10 points) Suppose that V is a finite-dimensional vector space and that U_1 and U_2 are two subspaces. Prove that if $\dim U_1 + \dim U_2 \geq \dim V$, then $U_1 + U_2$ is not a direct sum.

b) Give an example of two subspaces of $\mathbb{R}^3$ for which $U_1 + U_2$ is a direct sum. You do not need to prove it, but give a brief justification.