Name: ______________________________

Math 436, Midterm 1
Spring 2021, John Lesieutre

- You have fifty minutes to complete the exam.
- Exam should be submitted on Gradescope once completed. (Scanning and uploading time do not count towards the fifty minutes.)
- You may consult your notes, the course materials on my website, and the textbook.
- Collaboration and all other references are not allowed.
- Although you can write your answers on a copy of the exam, it is not required.
- Justifications or proofs are required for all problems except where indicated otherwise.
- Please either sign below the integrity statement below or copy out this statement at the beginning of your exam.
- I am generous with partial credit! Try not to leave anything blank.
- Good luck!

I affirm that I have complied with all the exam requirements. I have completed the exam within the allotted time and have not consulted any disallowed references.

Signature: ______________________________

Problem	Score	Possible
1		20
2		20
3		20
4		20
5		20
Σ		100

Problem 1. Consider the vector space $\mathcal{P}_3(\mathbb{R})$.

a) (10 points) Give an example of a subspace of $\mathcal{P}_3(\mathbb{R})$ that is neither all of $\mathcal{P}_3(\mathbb{R})$ nor the zero subspace. Prove that your subspace is either closed under addition or closed under scalar multiplication; you do not need to check both.

b) (10 points) Find a basis for your subspace and compute its dimension. You do not need to prove that this is a basis.

Problem 2. a) (10 points) Consider the linear map $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x - y, x + y + z, x)$. What is the nullspace of T? Justify your answer.

b) (10 points) Compute the dimension of the range of T, and prove that your answer is correct. Is this map surjective?

Problem 3. a) (10 points) Suppose that $T : \mathcal{P}_1(\mathbb{R}) \to \mathcal{P}_1(\mathbb{R})$ is a linear map satisfying $T(x+1) = x$ and $T(x-1) = 2x+3$. What is $T(x)$? Justify your answer.

b) (10 points) Prove that the map T is injective.

Problem 4. Consider the map $T : \mathcal{P}_2(\mathbb{R}) \to \mathbb{R}^2$ given by $T(f) = (f(0), f'(3))$. You can assume that this is linear.

a) Prove that T is not injective.

b) Give bases for both vector spaces. For $\mathbb{R}^2$, I will give only half credit if you use $(1, 0)$, $(0, 1)$; try to find another basis. You do not need to prove that your basis is a basis. Compute the matrix for T with respect to your basis.

Problem 5. Consider the two vectors $v_1 = (1, 1, 1)$ and $v_2 = (1, 0, 0)$ in $\mathbb{R}^3$, which are linearly independent.

a) It follows that the list v_1, v_2 can be extended to a basis. Give a vector w such that v_1, v_2, w is a basis, and prove it.

b) Let $U_1 = \text{span}(v_1, v_2)$ and $U_2 = \text{span}(w)$. Prove that $\mathbb{R}^3 = U_1 \oplus U_2$.