Today:

More diff eq.

But: I have to leave early, sorry!

Warm-up:

$y'' + 6y' + 13y = 0$

$y'' + 2y' + y = 0$

$y'' - 4y = 0$

Find general sol'n.

Hint: try $e^{rt} = y$, see what value of r makes it work.

$r^2 e^{rt} + 6re^{rt} + 13e^{rt} = 0$

$r^2 + 6r + 13 = 0$

...

$r^2 + 2r + 1 = 0$

$r^2 - 4 = 0$

$$y'' - 4y = 0$$

$$r^2 - 4 = 0$$

$$r = \pm 2$$

Two real roots

$$\rightarrow c_1 e^{2t} + c_2 e^{-2t}$$

$$y'' + 2y' + y = 0$$

$$r^2 + 2r + 1 = 0$$

$$(r+1)^2 = 0$$

$$c_1 e^{-t} + c_2 t e^{-t}$$

$\uparrow$

(just remember) this formula

$$y'' + 6y' + 13y = 0$$

$$r^2 + 6r + 13 = 0$$

$$(r+3)^2 + 4 = 0$$

$$(r+3)^2 = -4$$

$$r + 3 = \pm 2i$$

$$r = -3 \pm 2i$$

use real & imag. parts of

$$e^{(-3+2i)t}$$

$$c_1 e^{-3t} \cos 2t + c_2 e^{-3t} \sin 2t.$$

$$y'' - 3y' - 4y = \underbrace{3e^{2t}}$$

not 0!

(this is a mass on a spring, subject to external force.)

$r^2 - 3r - 4 = 0$ $\longrightarrow$ $y'' - 3y - 4y = 0$

$(r-4)(r+1)$ $\qquad$ $c_1 e^{4t} + c_2 e^{-t}$

To find one solution:

guess $y = Ae^{2t}$

from problem

$$y'' - 3y' - 4y = 4Ae^{2t} - 6Ae^{2t} - 4Ae^{2t} = 3e^{2t}$$

$$-6Ae^{2t} = 3e^{2t}$$

$$A = -\frac{1}{2}e^{2t}$$

one solution, with RHS

$$\boxed{y = -\frac{1}{2}e^{2t} + c_1 e^{4t} + c_2 e^{-t}}$$

general sol with RHS=0