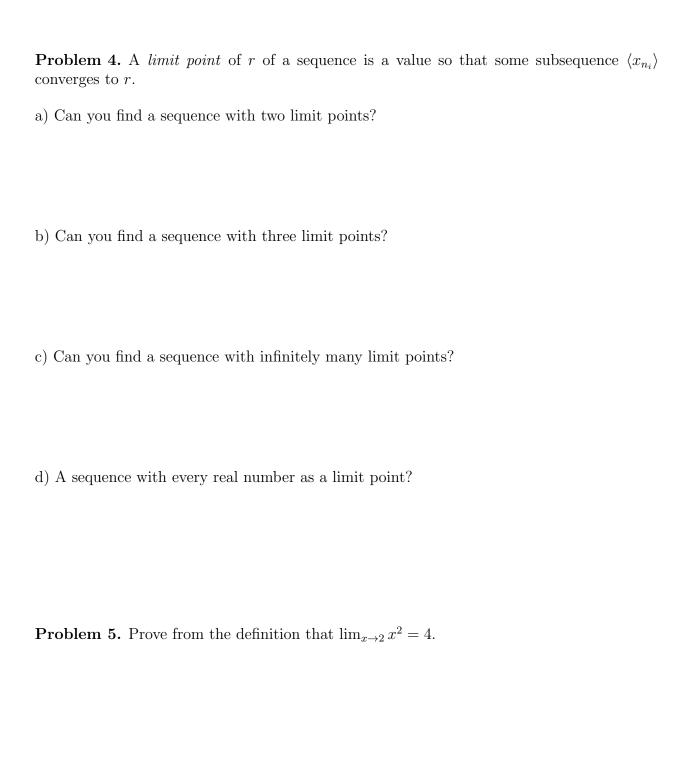
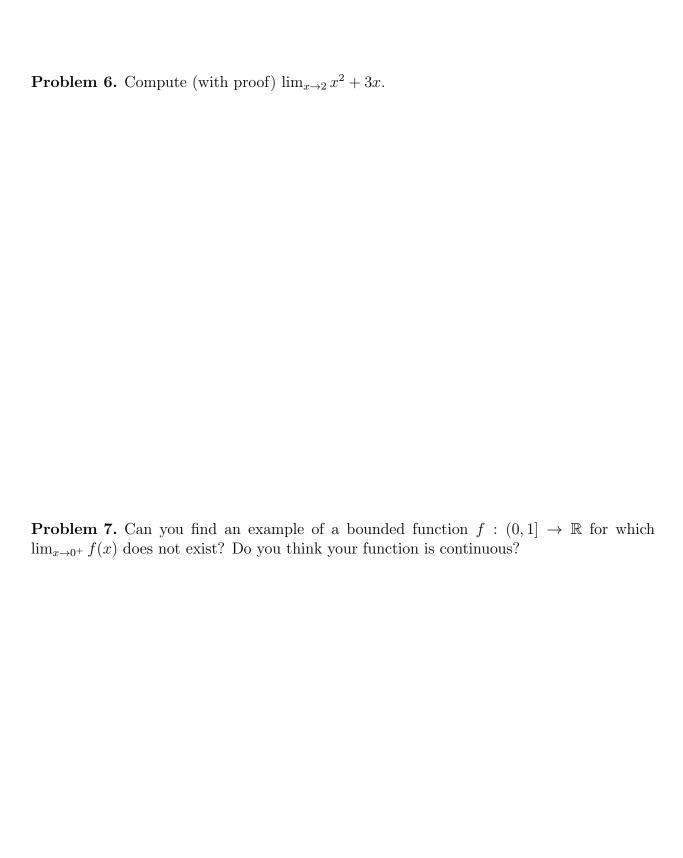


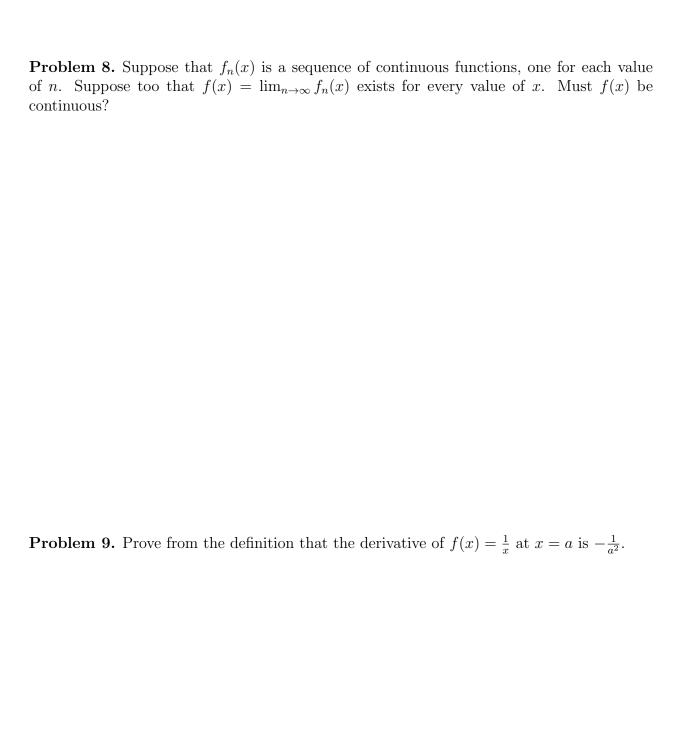
**Problem 1.** Consider the sequence  $b_n = \pi + \frac{1}{\sqrt{n}}$ . Does it have a limit? Prove it.

**Problem 2.** Find, with proof, the limit of the sequence  $a_n = \frac{n+1}{n-1}$ .

**Problem 3.** Suppose that the  $x_n$  converges to L and  $y_n$  converges to M. Prove that  $x_ny_n$  converges to LM.







<b>Problem 10.</b> This problem is about a function called the "Devil's staircase". Define $f(x)$ by the following procedure:	
1.	Write $x$ in base 3.
2.	If there is a 1 in the expansion, turn every digit after the 1 into a 0.
3.	Turn all the 2s into 1s.
4.	Interpret the result as a binary number.
a) Compute a few values and try to plot the function.	
b) For what values of $x$ is the function continuous? Try to convince yourself, even if you don't write out a careful proof.	
c) For	what values of $x$ is $f(x)$ differentiable? What is the derivative?
d) Is t	here anything you find concerning about this function?

**Problem 11.** Define a function by 
$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x > 0 \\ 0 & \text{if } x \le 0 \end{cases}$$

Prove that f is infinitely differentiable for all values of x, and check that  $f^{(n)}(0) = 0$  for all values of n.

(For this problem, you don't need to give a proof for every limit you use, as long as you make clear what limit you are taking.)

(Hint: prove by induction that for x>0,  $f^{(n)}(x)=P_n\left(\frac{1}{x}\right)e^{-1/x^2}$  where  $P_n$  is some polynomial. You can assume that  $\lim_{x\to 0^+}\frac{1}{x^d}e^{-1/x^2}=0$ , which is not so hard to prove by L'Hôpital's rule.)

(The upshot is that there are nonconstant functions for which all the derivatives are 0. The Taylor series for this function at  $\xi = 0$  is the zero function, even though the function itself is not 0; it just grows very very slowly near 0.)