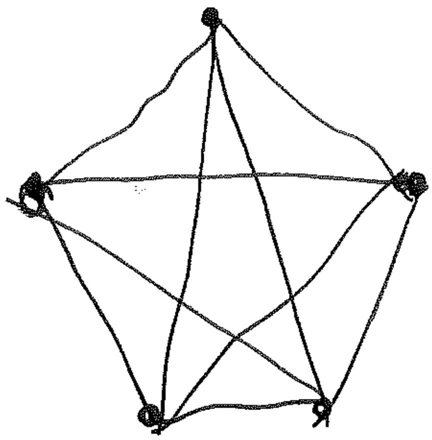
Graph vocab

1. Complete graph on n vertices

$n=5$



how many edges?

$\binom{n}{2}$

2. Regular graph: every vertex has the same degree.

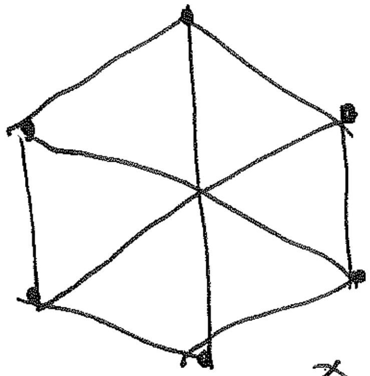
(complete graph is regular)

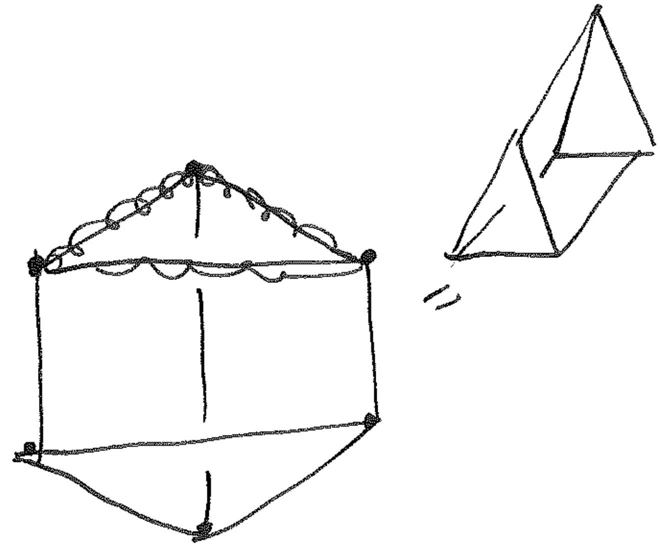
Puzzle: find 3-regular graphs with 6 vertices?

every vertex has 3 edges

one? two? all?

7 vertices?





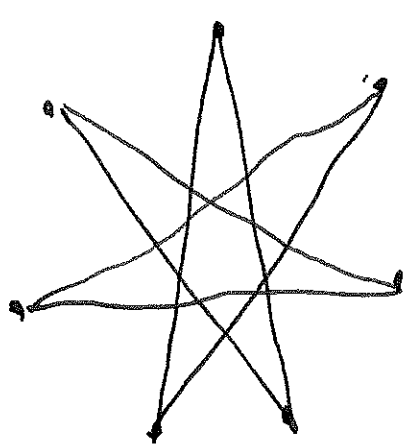
$K_{3,3}$

Why different?

contains loops of length 3!

doesn't.

7 vertices



it's not possible!

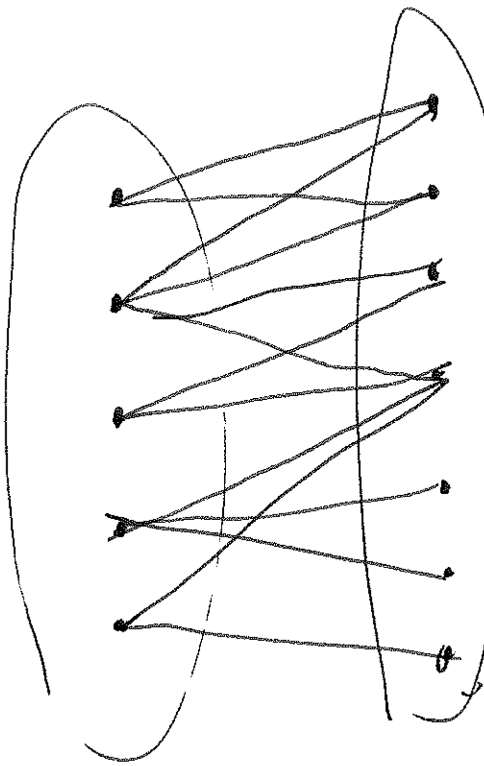
it would need 10.5 edges:

~~sum of edges~~

sum of degrees must be even, but 21 isn't!

Bipartite graph

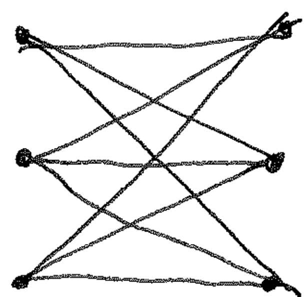
G is bipartite: you can split vertices into two sets (think: color some blue, some red) and every edge connected one vertex from each side.

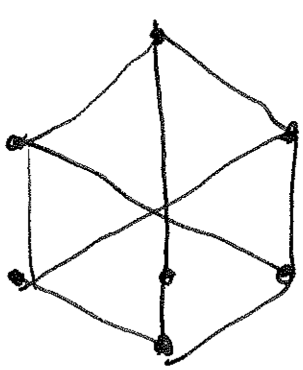


complete bipartite

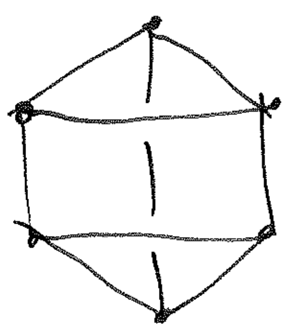
$K_{m,n}$ = m blue, n red

make all possible cross-color connections





bipartite $= K_{3,3}$



not bipartite

contains a triangle

<u>Thm</u> In a bipartite graph, every cycle has even length.

If a graph has no cycles of odd length, it's automatically bipartite.

→ A graph is <u>Eulerian</u> if there's closed trail that touches every edge once.

loop.

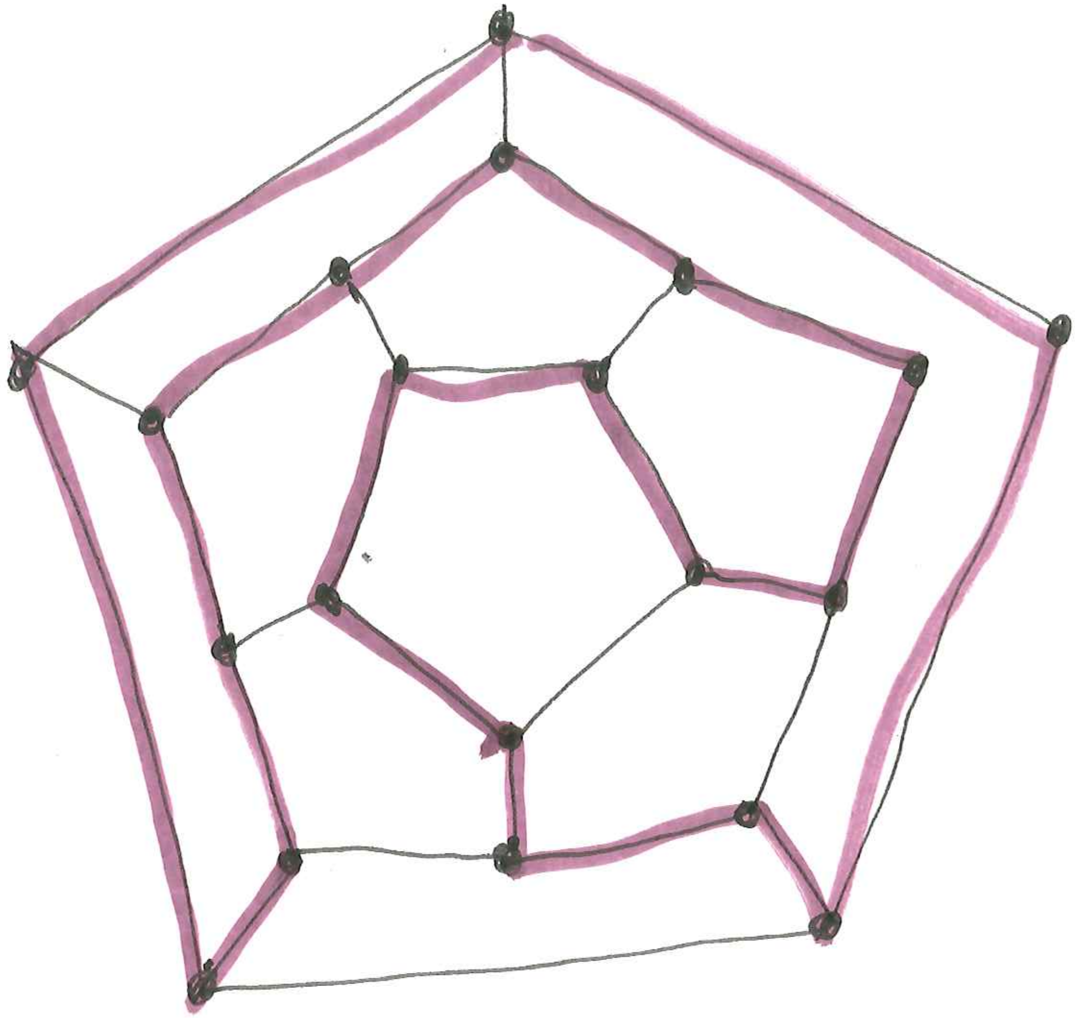
no repeat edges

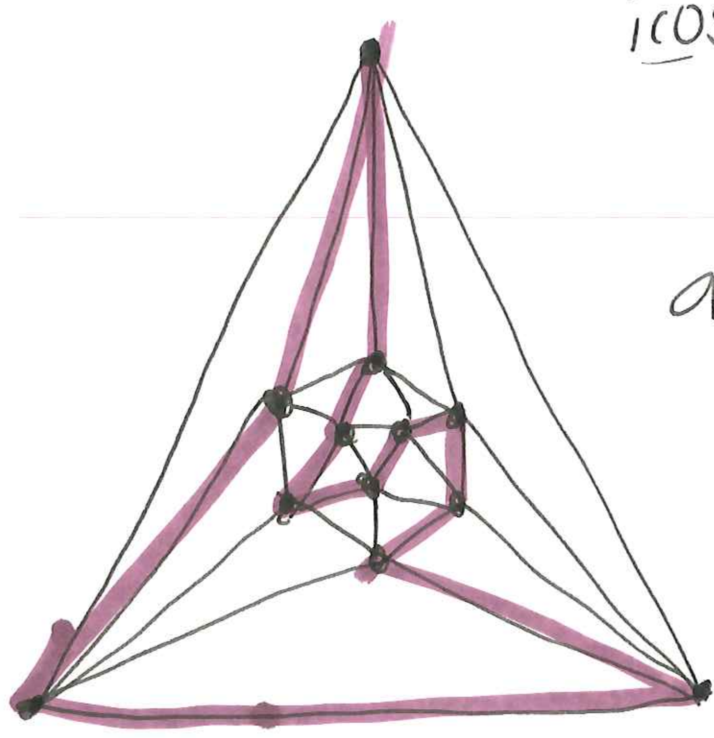
(cf Königsberg)

<u>Thm</u> G is Eulerian if and only if every vertex has even degree.

→ A graph is Hamiltonian if there's a closed trail that touches every <u>vertex</u> once.

How to tell?





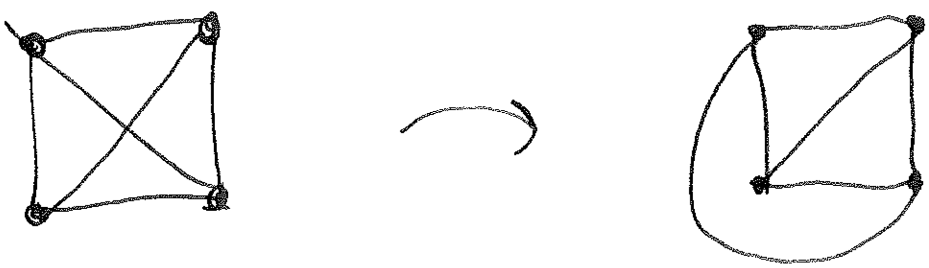
icosahedral graph!

also Hamiltonian

no good algorithm.

Planar graphs:

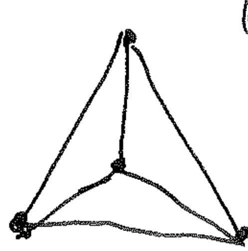
A graph is planar if it can be drawn where edges don't cross:



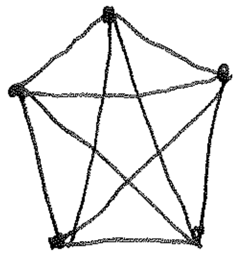
K_4 is planar

Theorem. "Planar" allows ~~bent~~ curved edges.

In fact any planar graph can be drawn with straight non-overlapping edges.

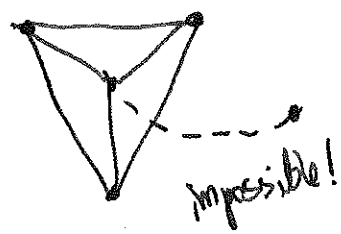


What about K_5?



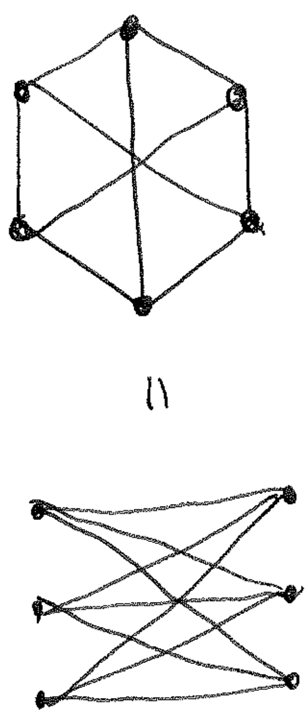
Not planar;

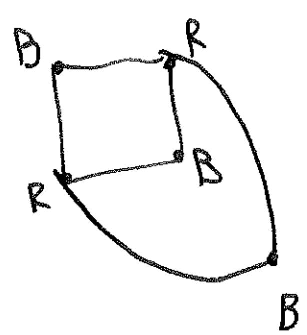
contains K_4, but need to add a point!



What about $K_{3,3}$?

Not planar:



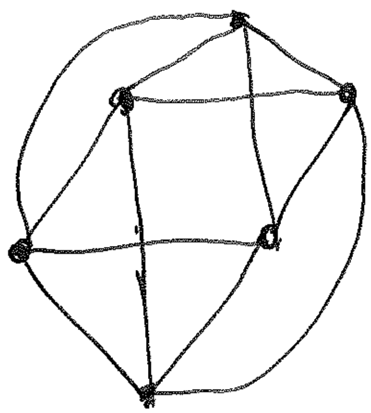


impossible!

I'm convinced.

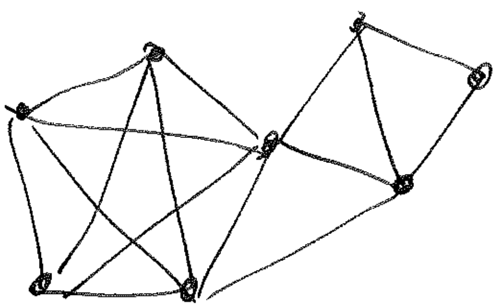
What about O?

is it planar?



vertices + edges of octahedron.

Thm "Kuratowski's theorem" G is planar if and only if it does not contain a copy of K_5 or $K_{3,3}$.



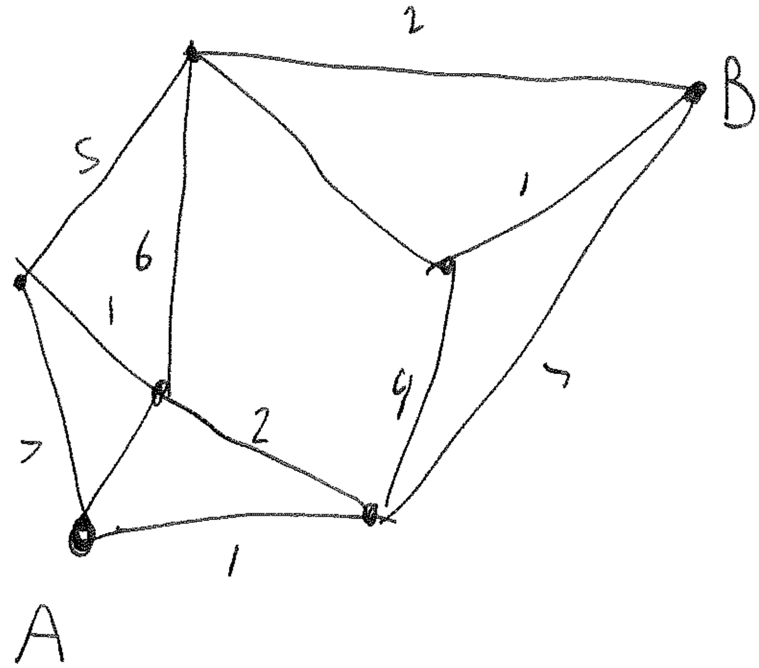
To check planar:

look at every subset of 5. is it K_5?

" of 6. is it $K_{3,3}$?

Shortest path problem

Suppose G is a graph with a number on each edge ("travel time")



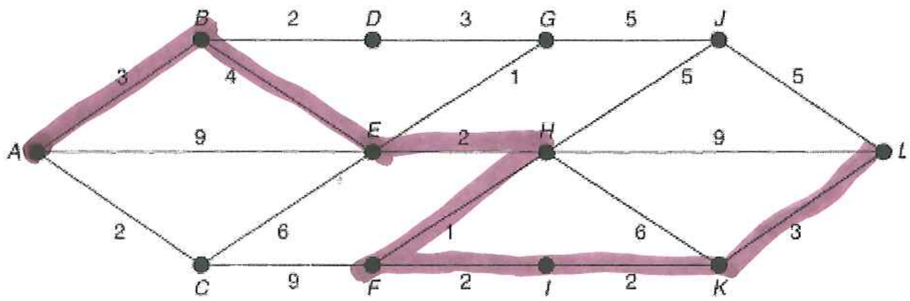
What's the fastest route between A & B?

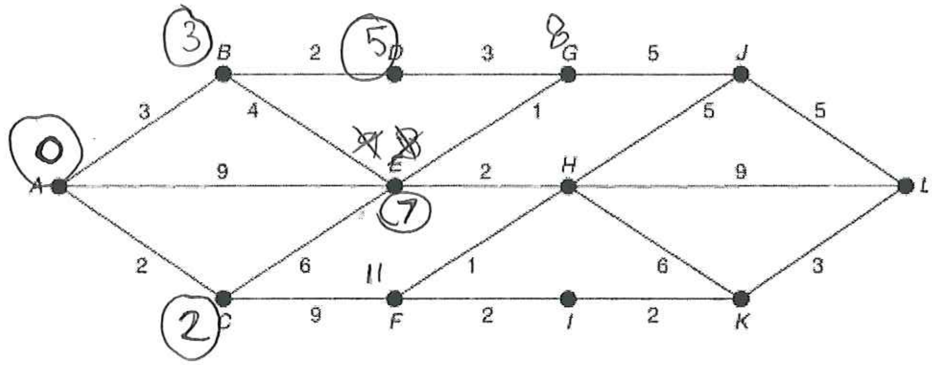
(path on graph minimizing the sum).

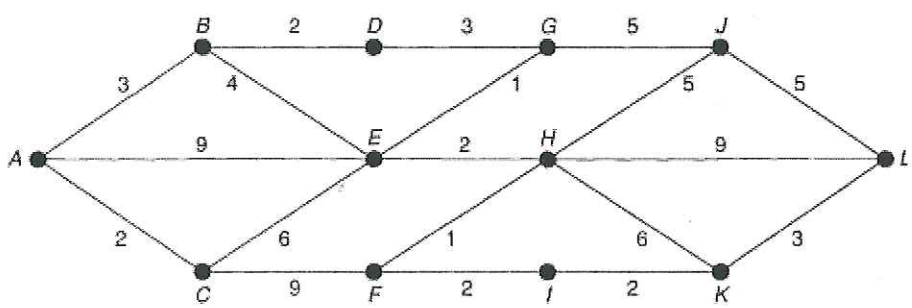
What to do?

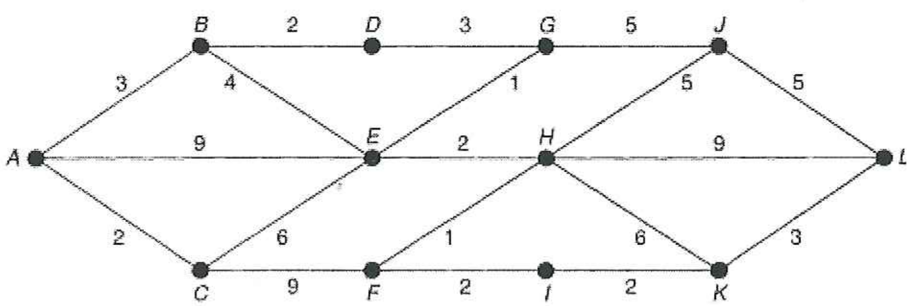
Dijkstra's algorithm.

17


B 2 D 3 G 5 J
3 4 1 5 5
A 9 E 2 H 9 L
2 6 1 6 3
C 9 F 2 I 2 K


3 B 2 5 D 3 8 G 5 J
3 4 1 5 5
0 A 9 E 2 H 9 L
7
2 6 11 1 6 3
2 C 9 F 2 I 2 K


B 2 D 3 G 5 J
3 4 1 5 5
A 9 E 2 H 9 L
2 6 1 6 3
C 9 F 2 I 2 K


B 2 D 3 G 5 J
3 4 1 5 5
A 9 E 2 H 9 L
2 6 1 6 3
C 9 F 2 I 2 K