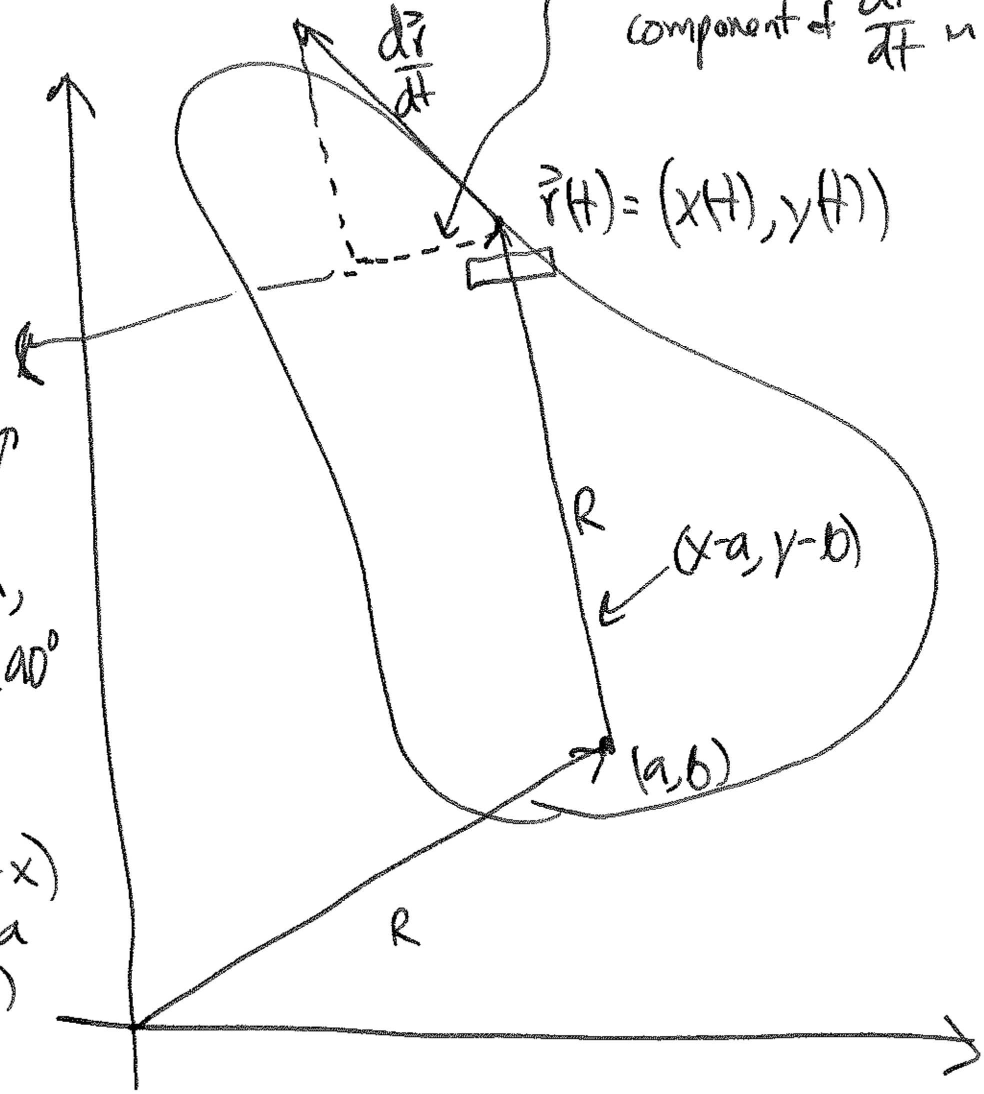
component of $\vec{v}$ in $\vec{u}$ direction is

$$\frac{\vec{v}\cdot\vec{u}}{\|\vec{u}\|}$$

amount the wheel rolls; that's what we integrate

component of $\frac{d\vec{r}}{dt}$ in this direction



2nd arm, rotated 90° is

$(y-b, a-x)$
$b-y, x-a$
(length R)

so we want to integrate

$$\frac{\frac{d\vec{r}}{dt}\cdot(y-b, a-x)}{R}$$

The length the wheel travels is

$$\cancel{\oint_C \frac{1}{R}\, dr}$$

oops sign!

$$\oint_C \left(\frac{b-y}{R}, \frac{x-a}{R}\right) \cdot \frac{d\vec{r}}{dt}\, dt = \oint_C \vec{F} \cdot d\vec{r} \quad \text{where}$$

a, b are nasty fcts of x, y.

$$\vec{F} = \left(\frac{b-y}{R}, \frac{x-a}{R}\right)$$

Why ~~What~~ is

$$\oint \vec{F} \cdot d\vec{r} = \text{area for } \vec{F} = \left(\frac{b-y}{R}, \frac{x-a}{R}\right)?$$

Use Green's theorem!

$$\oint \vec{F}\cdot d\vec{r} = \iint_{S} \operatorname{curl} \vec{F}\, dA$$

S ↑ inside

$$\cancel{\vec{F} = \left(\overbrace{\frac{b-y}{R}}^{M}, \overbrace{\frac{x-a}{R}}^{N}\right)}$$

$$\vec{F} = \left(\overbrace{\frac{b-y}{R}}^{M}, \overbrace{\frac{x-a}{R}}^{N}\right)$$

but a, b are fcts of x, y

$$\operatorname{curl}\vec{F} = N_x - M_y =$$

$$= \frac{1}{R}(1-a_x) - \frac{1}{R}(b_y - 1)$$

$$= \frac{1}{R}(2-a_x-b_y)$$

$$\oint \vec{F}\cdot d\vec{r} = \iint_{S} \frac{1}{R}(2-a_x-b_y)\, dA$$

you could brute force solve for a, b, compute these, and get answer.

Let's use implicit diff instead!

$$a^2+b^2=R^2 \qquad \Big| \qquad (x-a)^2+(y-b)^2=R^2$$

$\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$

$$\frac{\partial}{\partial x}: \quad 2aa_x+2bb_x=0 \qquad 2(x-a)(1-a_x)+2(y-b)(-b_x)=0$$

$$\frac{\partial}{\partial y}: \quad 2aa_y+2bb_y=0 \qquad 2(x-a)(-a_y)+2(y-b)(1-b_y)=0$$

$$\frac{1-a_x}{b_x}=\frac{y-b}{x-a}=\frac{a_y}{1-b_y}$$

$$\frac{1-a_x}{b_x}=\frac{a_y}{1-b_y} \quad \times\frac{a}{b} \longrightarrow \frac{a-aa_x}{bb_x}=\frac{aa_y}{b-bb_y}$$

$$\text{so } \frac{a-aa_x}{-aa_x}=\frac{-bb_y}{b-bb_y} \longrightarrow \frac{1-a_x}{a_x}=\frac{b_y}{1-b_y}$$

$$(1-a_x)(1-b_y)=a_xb_y$$

$$1-a_x-b_y+a_xb_y=a_xb_y \qquad \text{so } 1-a_x-b_y=0.$$

$$\text{Wheel turn} = \oint \vec{F} \cdot d\vec{r} = \iint_S \text{curl}\,\vec{F}\, dA$$

$$= \iint_S \frac{1}{R}(2 - a_x - b_y)\, dA = \iint_S \frac{1}{R}\, dA = \frac{\text{area}(S)}{R} \; !!$$