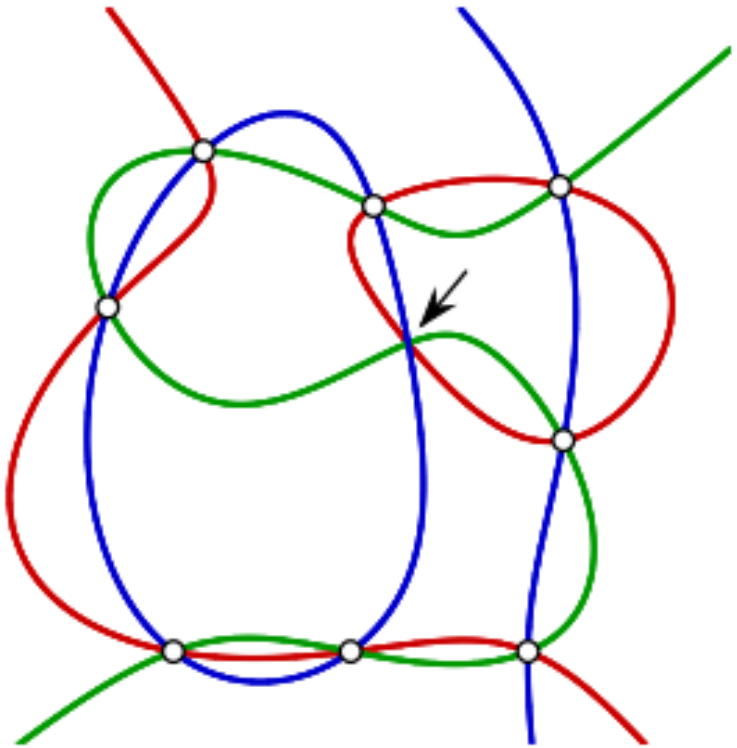
Cremona Transformations

The Cayley–Bacharach Theorem

Theorem 1 (Cayley-Bacharach). *Let $P_1, \ldots, P_8$ be eight distinct points in the plane, no three on a line, and no six on a conic. There exists a unique ninth point P_9 such that every cubic curve through $P_1, \ldots, P_8$ also contains P_9.*



All cubics passing through the eight white points meet in a unique ninth point

(figure lifted from [Ren, Richter-Gebert, Sturmfels])

Formula for the Geiser involution

Cayley-Bacharach Formulas

Qingchun Ren, Jürgen Richter-Gebert and Bernd Sturmfels

Abstract

The Cayley-Bacharach Theorem states that all cubic curves through eight given points in the plane also pass through a unique ninth point. We write that point as an explicit rational function in the other eight.

1 Introduction

This note concerns the following result from classical algebraic geometry.

Theorem 1 (Cayley-Bacharach)**.** *Let $P_1, \ldots, P_8$ be eight distinct points in the plane, no three on a line, and no six on a conic. There exists a unique ninth point P_9 such that every cubic curve through $P_1, \ldots, P_8$ also contains P_9.*

What if you really want the equation?

NOTES ON THE BERTINI INVOLUTION

ETHEL I. MOODY[1]

1. **Introduction.** Given a pencil of plane cubic curves

$$\lambda w(x) + \mu w'(x) = 0 \tag{1}$$

with the vertices of the reference triangle among its base points. Arranged as to (0, 0, 1) the equations may be written

$$w(x) = x_3^2 u_1 + x_3 u_2 + u_3,$$
$$w'(x) = x_3^2 u_1' + x_3 u_2' + u_3',$$

with

$$u_1 = a_1 x_1 + a_2 x_2, \qquad u_1' = a_1' x_1 + a_2' x_2,$$
$$u_2 = b_1 x_1^2 + b_2 x_1 x_2 + b_3 x_2^2, \quad u_2',$$
$$u_3 = c_1 x_1^2 x_2 + c_2 x_1 x_2^2, \quad u_3',$$

and $a_i, a_i', b_i, b_i', c_i, c_i'$ generic constants.

A point y of the plane fixes the curve of the pencil (1) passing through it, hence

$$w(x)w'(y) - w'(x)w(y) = 0, \tag{2}$$

which may be written in the form

$$W_3(x) = x_3(A_1 x_1 + A_2 x_2) + x_3(B_1 x_1^2 + B_2 x_1 x_2 + B_3 x_2^2) + C_1 x_1^2 x_2 + C_2 x_1 x_2^2 = 0 \tag{3}$$

in which $A_i = a_i w'(y) - a_i' w(y)$, and similarly for B_i and C_i. The tangent to $W_3(x) = 0$ at (0, 0, 1) is

$$A_1 x_1 + A_2 x_2 = 0, \tag{4}$$

which meets the curve again at $R = (r_1, r_2, r_3)$,

Received by the editors August 31, 1942.

[1] Miss Moody, Ph.D. Cornell University, an instructor in mathematics at Pennsylvania State College, was killed in an automobile accident April 11, 1941. I had suggested that she compare my cumbersome method of derivation of the equations of this transformation (Amer. J. Math. vol. 33 (1911) pp. 327–336) with that of employing a pencil of cubic curves. The following notes were found among her posthumous papers sent me recently. The equations of the Bertini involution are simpler than those previously known, and other properties found may be extended by others.

VIRGIL SNYDER

Degtyarev, 2012

THE BERTINI INVOLUTION

ALEX DEGTYAREV

Abstract. We summarize and extend E. Moody's results on the explicit equations related to the Bertini involution.

These notes are the result of my attempt to understand E. Moody's paper [1]. I correct a few misprints in [1] and take the computation a bit further.

I express my admiration to Ethel I. Moody, who managed to perform this tedious computation in the pre-`Maple` era. A `Maple` implementation of most equations is found at `http://www.fen.bilkent.edu.tr/~degt/papers/Bertini.zip`.

Successor to the Free Lance, Established 1887

The Daily Collegian

OF THE PENNSYLVANIA STATE COLLEGE

Weather— Fair And Continued Warm.

VOL. 37—No. 121 FRIDAY MORNING, APRIL 18, 1941, STATE COLLEGE, PA. PRICE THREE CENTS

College May Get $400,000 Airport

Hershey Asks Draft Delay For College Men

See Editorial, Page Two

WASHINGTON, April 17.—Acting Draft Director Lewis B. Hershey announced today that local draft boards will be asked to postpone the induction of college students into the Army "until the end of the scholastic period in which they are called."

A scholastic period was unofficially interpreted as meaning a semester. It was pointed out that under this plan the interruption of classroom study will be prevented.

The announcement emphasized that local boards will not be instructed to postpone student inductions automatically and that full authority still lies with the individual boards.

The 6,500 local boards will receive the recommendation next week. It reads in part:

"The following amendment to Selective Service Regulations has been approved by national headquarters: The time specified for reporting (when called) shall be 10 days, provided, where unusual individual hardship will result, the local board may postpone the time

Bill Now In Legislature Would Allot $100,000 For Training, Research

Federal Money Is Sought To Supplement $300,000 From State

A proposed $400,000 training airport comparable to those at Virginia Polytechnic Institute and Purdue is being sought for the College, officials here announced yesterday.

The first move to secure state and federal funds for its erection was made Wednesday with the introduction in the Legislature of bills appropriating $300,000 by Sen. Joseph Ziesenheim (R., Erie) and Rep. Carleton E. Woodring (D., Northampton).

The bills would provide $200,000 to construct the airport and $100,000 for instruction and research in aeronautical engineering.

A joint Washington committee of the Civil Aeronautics Authority and the Army and Navy is considering the possibility of federal aid to complete the airport.

Fire A Fizzle But Crowd's Big

A thousand students and about 100 student cars raced four pieces of Alpha-Fire Company apparatus to a fraternity section fire call at 7 o'clock last night, only to find a brush pile burning on Fairmount Avenue just west of Lambda Chi Alpha. The fire company allowed the blaze to burn itself out.

Math Prof Killed In Auto Plunge

Miss Ethel I. Moody, popular mathematics instructor, was killed last Friday when her car plunged down a 25-foot embankment near her home in Rushville, N. Y.

No one witnessed the tragedy, but it is believed that in trying to

Women Debate Cornell In Season's Last Meet

In the final intercollegiate debate of the season, Lois E. Notovitz '42 and Sara L. Bailey '43 will uphold the negative side of the proposition, "Resolved, that the nations of the western hemisphere should enter into a permanent union" against Cornell in Room 121 Sparks at 7:30 p.m. Monday.

Miss Notovitz is president of Delta Alpha Delta, women's speech professional, and Miss Bailey is manager of women's debate and a member of the Forensic Council. An open forum discussion will follow the debate to which the public is cordially invited.

Hockey, Skiing Managers Named

John D. Clarke '42 has been elected ice hockey manager for 1941-42 and Howard C. Yerger '42 has been named skiing manager, it was announced by the Athletic Association.

Charles R. Sorber, Jr. '43, S. William Kalin '43, and Harry C. Herbert '43 were named hockey first assistants. New skiing first

New Coed Dorm To Open Sunday For Fire Victims

A completely remodelled, newly-furnished rooming house at 238 West College Avenue will be opened Sunday to 26 coeds who were burned out in the $25,000 Anchorage fire Monday night, according to Mrs. Jessie B. Hughes, proprietress of the Anchorage who has leased this new building from Charles Charles Schlow.

Although the structure was undergoing construction into five apartments, a few alterations are being made to fit it for occupancy as quickly as possible. Anchorage residents, all home for Easter vacation, were wired to remain until the house could be completed.

Workmen yesterday removed salvaged furniture, mattresses, kitchen equipment and other articles which escaped damage and cleaned up debris on the upper stories.

Much of the clothing and belongings of the residents was saved with only a few of the occupants losing heavily.

Cause of the fire remains unknown according to Irven M.

Tri-Coble Surfaces

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TRI-COBLE SURFACES AND THEIR AUTOMORPHISMS

JOHN LESIEUTRE
(Communicated by Laura DeMarco)

ABSTRACT. We construct some positive entropy automorphisms of rational surfaces with no periodic curves. The surfaces in question, which we term *tri-Coble surfaces*, are blow-ups of $\mathbb{P}^2$ at 12 points which have contractions down to three different Coble surfaces. The automorphisms arise as compositions of lifts of Bertini involutions from certain degree 1 weak del Pezzo surfaces.

1. INTRODUCTION

Suppose that X is a projective surface over an algebraically closed field K and that $\phi : X \to X$ is an automorphism of X. When $K = \mathbb{C}$, a theorem of Gromov and

(Video break)

- ▶

The Fermat cubic

- X defined by $x^3 + y^3 + z^3 = 1$.
- Parametrized by $f(r, s) =$

$$
\begin{aligned}
&(-r^3 - 2r^2s + rs^2 - s^3 + 2r^2t + s^2t - rt^2 - st^2 \\
&r^3 - 2r^2s + rs^2 + 2r^2t + s^2t - rt^2 - st^2 + t^3 \\
&2r^3 - r^2s + 2rs^2 - 2r^2t - s^2t + rt^2 + st^2 - t^3 \\
&-2r^3 + 2r^2s - rs^2 + s^3 + r^2t - s^2t - 2rt^2 + st^2)
\end{aligned}
$$

Another finite order map

$$\begin{array}{r}
7x^4y - 4x^3y^2 + 6x^2y^3 - xy^4 + y^5 - 7x^4 - 9x^2y^2 + xy^3 - 3y^4 + 4x^3 + 9x^2y + 5y^3 - 6x^2 - xy - 5y^2 + x + 3 \\
\hline
6x^5 - 4x^4y + 10x^3y^2 - 6x^2y^3 + 4xy^4 - y^5 - 11x^4 - 6x^2y^2 - xy^3 + 14x^3 + 3x^2y + 6xy^2 + y^3 - 12x^2 - 2xy - 4y^2 + \\
-3x^5 + 5x^4y - 2x^3y^2 + xy^4 - y^5 - 2x^4 - 3x^3y - 3x^2y^2 - xy^3 + 2x^3 + 6x^2y + 3xy^2 + y^3 - 6x^2 - 2xy - 4y^2 + 2 \\
\hline
6x^5 - 4x^4y + 10x^3y^2 - 6x^2y^3 + 4xy^4 - y^5 - 11x^4 - 6x^2y^2 - xy^3 + 14x^3 + 3x^2y + 6xy^2 + y^3 - 12x^2 - 2xy - 4y^2 +
\end{array}$$

- That's an order 3 map you are unlikely to guess directly.