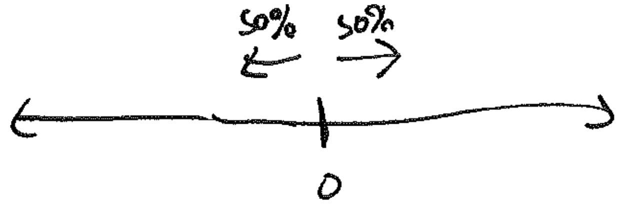
Last time:

Random walk:



Let $u_n =$ probability of coming back to 0 after n steps

Last time: $u_{2n} = \left(\frac{1}{4}\right)^{n}\binom{2n}{n}$.

* If $\sum u_n < \infty$ then the walk may not ever return

If $\sum u_n = \infty$ then guaranteed to return.

Last time: $\sum \left(\frac{1}{4}\right)^{n}\binom{2n}{n}$ diverges (use Stirling's approx)

*

Suppose that some kind of random walk eventually comes back to 0 with probability u. (or disappears forever with prob $1-u$).

What's the probability that it returns to 0 exactly m times?

it's $u^m(1-u)$.

What's the expected number of returns (as a fct of u)

$$\sum_{m=0}^{\infty} \overset{\#\text{times}}{m} \left(\overset{\text{prob of that number of times.}}{u^m(1-u)}\right)$$

$$(1-u)\sum_{m=0}^{\infty} mu^m = u(1-u)\sum_{m=0}^{\infty} mu^{m-1}$$

$$= u(1-u)\frac{d}{du}\left(\sum_{m=0}^{\infty} u^m\right) \leftarrow \text{geometric}$$

$$= u(1-u) \frac{d}{du} \frac{1}{1-u}$$

$$= u(1-u) \frac{1}{(1-u)^2}$$

$$= \frac{u}{1-u}.$$

Expected number of returns is finite if $u<1$

Infinite if $u=1$.

Three things to try:

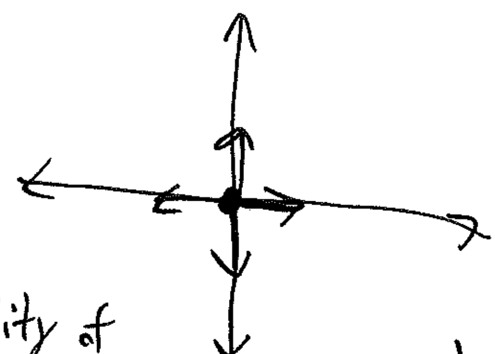
a) What if we use a 1-D random walk,

but go left with probability p &

right with prob. $1-p$.

What is u_n? Does $\sum u_n$ converge?

b) What about 2D?



What is u_n? (Probability of coming back after exactly n steps.)

Does $\sum u_n$ converge?

Hint: $\sum_{k=0}^{n} \binom{n}{k}^2 = ?$

It can be simplified!

c) Program it!

(draw some pictures of 2D walks. how far do you go before coming back?) try 3D and see if it comes.