Adv Topics 2 (Lesieutre)
September 13, 2021

Problem 1. Our main goal is to prove that there are some knots that simply can't be untied. How could we define a knot mathematically? What does it mean to untie it?

Problem 2. Consider the trefoil knot.

a) Do you think it is isotopic to the unknot?

b) Is it isotopic to its reverse?

Problem 3. We want to turn the continuous problem of deforming knots into a discrete problem about manipulating knot diagrams. Think about the actions you take when you try to untie a knot. How does these affect a knot diagram for the knot? Can you formalize a list of legal "moves" to perform on a knot diagram in the course of untying it?

Problem 4. Try to convince yourself that none of the Reidemeister moves changes whether or not a knot is tricolorable.

Problem 5. Now we consider our first knot invariant in action: tricolorability.

a) Is the unknot tricolorable?

b) Is the trefoil tricolorable?

c) Is the figure eight knot tricolorable?

Problem 6. Consider the figure eight knot.

a) Do you think it is isotopic to the unknot?

b) Is it isotopic to its reverse?

c) Can you prove that the crossing number is equal to 4?

Problem 7. So far none of our invariants prove that the figure eight knot is nontrivial. We need something fancier: compute the Alexander polynomial of the figure eight knot.