$$2^{2+i} = ?$$

$$x = 2^{2+i}$$

$$\log x = \log 2^{2+i}$$

$$= (2+i)\log 2$$

$$\cancel{x = e^{2+i \ln \log 2}}$$

$$e^{ix} = \operatorname{cis} x$$
$$= \cos x + i \sin x$$

$$x = e^{(2+i)\log 2}$$

$$= e^{\cancel{2\log 2}}$$

$$e^{2\log 2} \cdot e^{i \log 2} = (e^{\log 2})^2 e^{i\log 2}$$

$$= 4 \cdot \cos(\log 2) + 4i \sin(\log 2)$$

$i^i = x$

$\left((-1)^{1/2}\right)^i = x$

$x = (-1)^{1/2 i}$

$\ln x = \ln (-1)^{1/2 i}$

$= \frac{1}{2} i \ln(-1).$ $\quad e^{i\pi} = -1$ so $\ln(-1) = i\pi$

$= -\frac{1}{2}\pi$

$= e^{-1/2 \pi} = \frac{1}{\sqrt{e^{\pi}}}$

$\boxed{i^i = \frac{1}{\sqrt{e^{\pi}}}.}$

Disclamer: $e^{(2k+1)i\pi} = -1$ so "$\ln(-1)$" isn't really defined. "multi-valued function"

$\ln x = \frac{1}{2} i \left((2k+1) i \pi\right)$

$\ln x = \frac{1}{2}\left(-(2k+1)\pi\right)$

$x = \sqrt{e^{-(2k+1)\pi}}$

Why? TI says

$0.5! = 0.866$

$$\int u\,dv = uv - \int v\,du$$

$$\int_0^\infty e^{-t}\,dt = 1$$

$$\int_0^\infty t e^{-t}\,dt = 1$$

$$\int_0^\infty t^2 e^{-t}\,dt$$

$$\int t e^{-t}\,dt$$

$v = -e^{-t}$

$u = t \qquad \cancel{v = e^{-t}}$

$du = dt \qquad dv = e^{-t}\,dt$

$$\int u\,dv = uv - \int v\,du$$

$$= (-te^{-t}) - \int (-e^{-t})\,dt$$

$$= -te^{-t} - e^{-t}$$

$$\cancel{(2t} \; -(t+1)e^{-t}$$

$$\left(-(t+1)e^{-t}\right)\Big|_0^\infty = (0 - -1) = 1$$

Let

$$C_n = \int_0^\infty t^n e^{-t}\,dt$$

The Γ function:

$$\cancel{\Gamma(z) = \int_0^\infty x^{z-1} e^{-x}\,dx}$$

$$\boxed{\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\,dt} \qquad \Gamma(n) = (n-1)!$$

$C_0 = 1,\ C_1 = 1,\ C_2 = 2,\ C_3 = 6,\ C_4 = 24$

Guess (prove by induction + integration by parts)

$$C_n = n!$$

$$\text{So}\quad \left(\tfrac{1}{2}\right)! = \int_0^\infty t^{1/2} e^{-t}\,dt \overset{\text{I checked}}{=} \frac{\sqrt{\pi}}{2}$$

What you missed:

Plan: Today: Linear programming.

Next: ", Gradient descent

Linear programming:

Chocolate company produces two chocolates:

dark/milk

dark: ~~used~~ uses 1 unit milk + 3 units cocoa

milk: uses 1 unit milk + 2 units cocoa.

Sell dark for \$6, sell milk for \$5

In stock they have 50 milk
120 cocoa

What should they produce to maximize income?

x dark chocolate

y milk chocolate

Linear programming

$x + y \le 50$ (milk constraint)

$3x + 2y \le 120$ (cocoa)

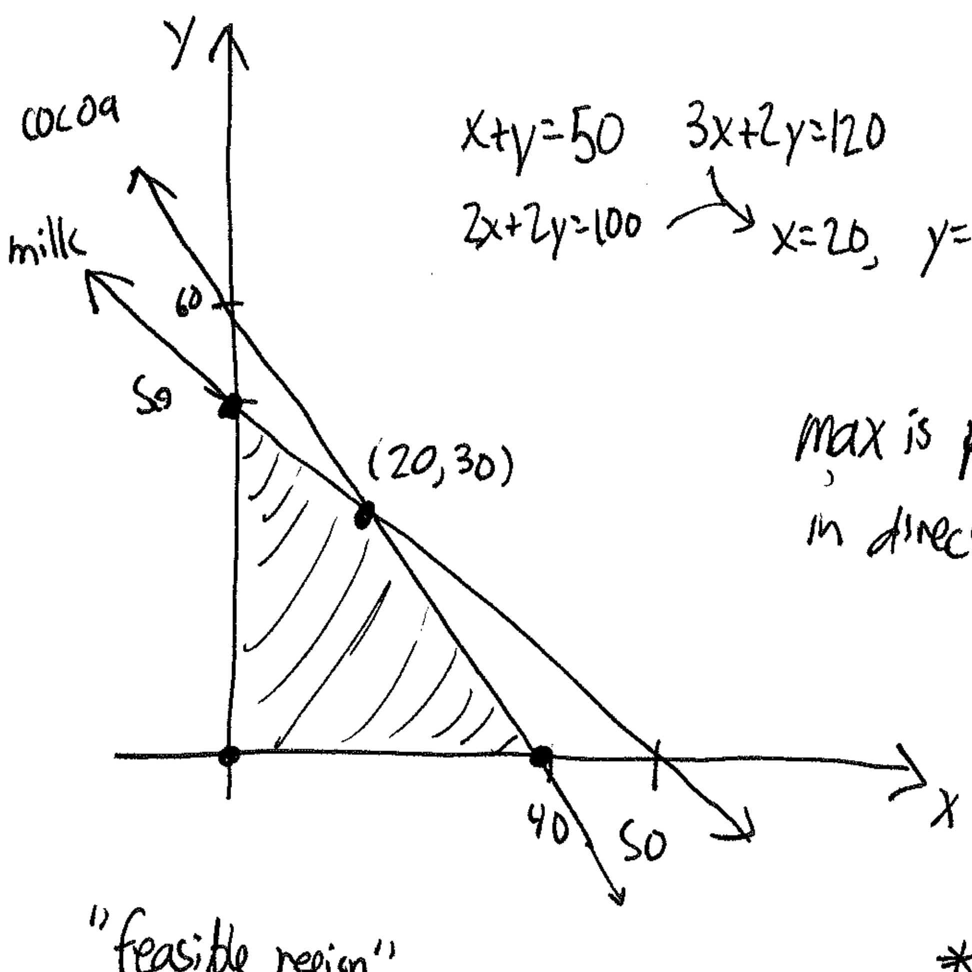
$x \ge 0$

$y \ge 0$

linear constraints

maximize. $6x + 5y$

linear thing to maximize



$x+y=50$ $\quad 3x+2y=120$

$2x+2y=100 \rightarrow x=20, \ y=30$

max is point furthest in direction (6,5).

must be at corner!

$(0,50) \rightarrow 200$

$(40,0) \rightarrow 240$

$*$ $(20,30) \rightarrow 270$

$(0,0) \rightarrow 0$

1000 variables, 2000 inequalities:

How many corners?

$$\sim \binom{2000}{1000}. \quad \text{huge!}$$

$$\frac{200 \times 199 \times 198 \times \cdots}{100 \times 99 \times \cdots}$$