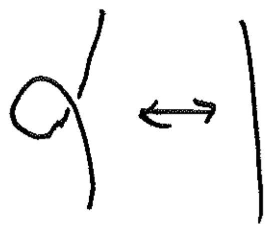
Recap

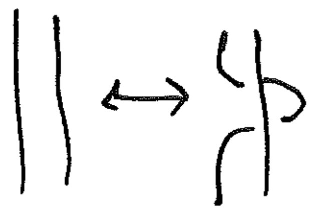
→ We defined a knot and a knot diagram.

→ we defined three Reidemeister moves:



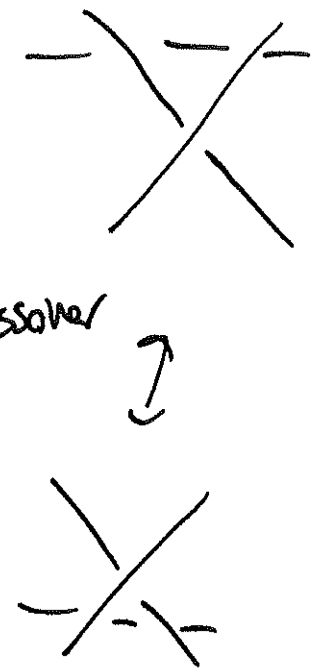
twist

I



crossover

II



passover

III

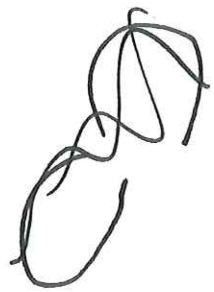
Properties of knots:

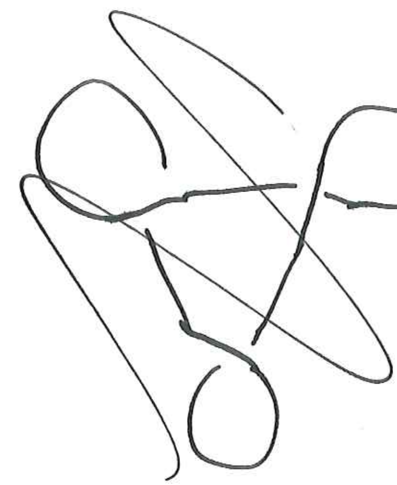
- crossing number (minimal number of crossings in a diagram)
- tricolorability
- chirality (is knot same as reverse?)

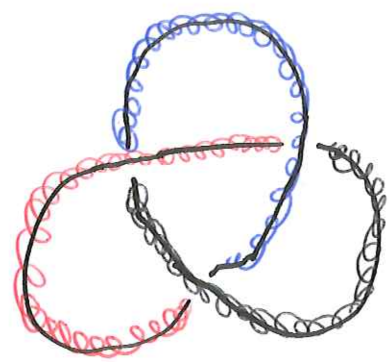
...

Today:
- bridge number
- Alexander polynomial

Trefoil knot







trefoil

Crossing number?

3

tricolorable?

yes

chiral? (different from reverse, or same)

yes (different)

tricolorable:

Figure eight

crossing number?

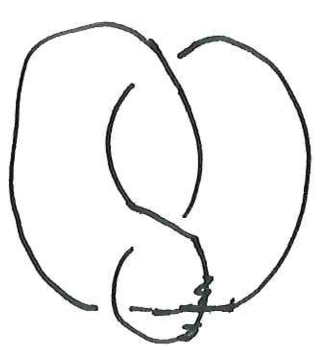
4

tricolorable?

no

chiral?

no (amphichiral/achiral)



Unknot

crossing number?

0

tricolorable?

no

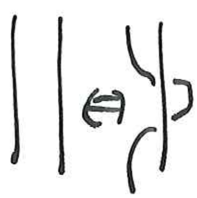
chiral?

amphichiral

- Tricolorability unaffected by Reidemeister moves
- Proofs for crossing numbers.

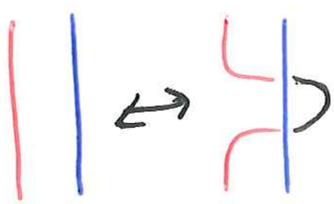
Suppose K is tricolorable. If we perform any Reidemeister moves, it stays that way. (needs checked for all three moves)

ex. <u>Type II</u>. existing tricolor could be either:



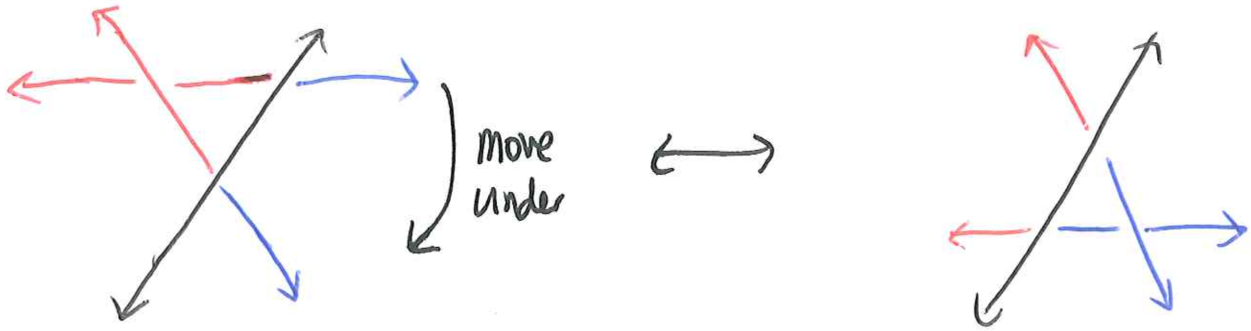
a) 

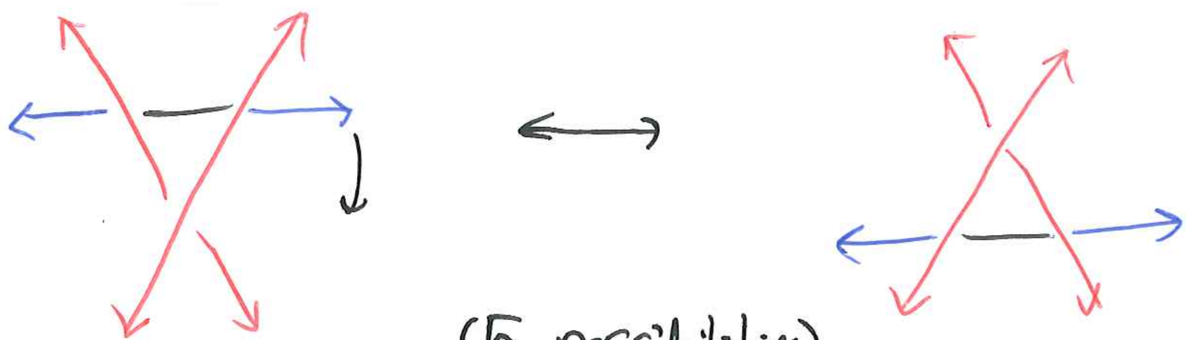
all reversible!

b) 

we need to get a tricoloring on new diagram without changing colors of "outbound" strands.

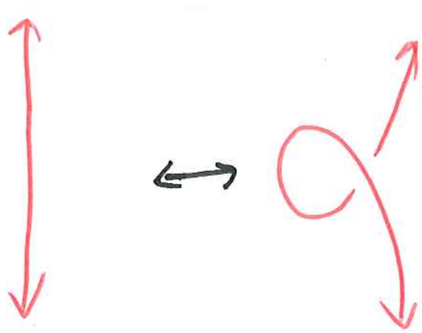
Type III





(5 possibilities)

Type I

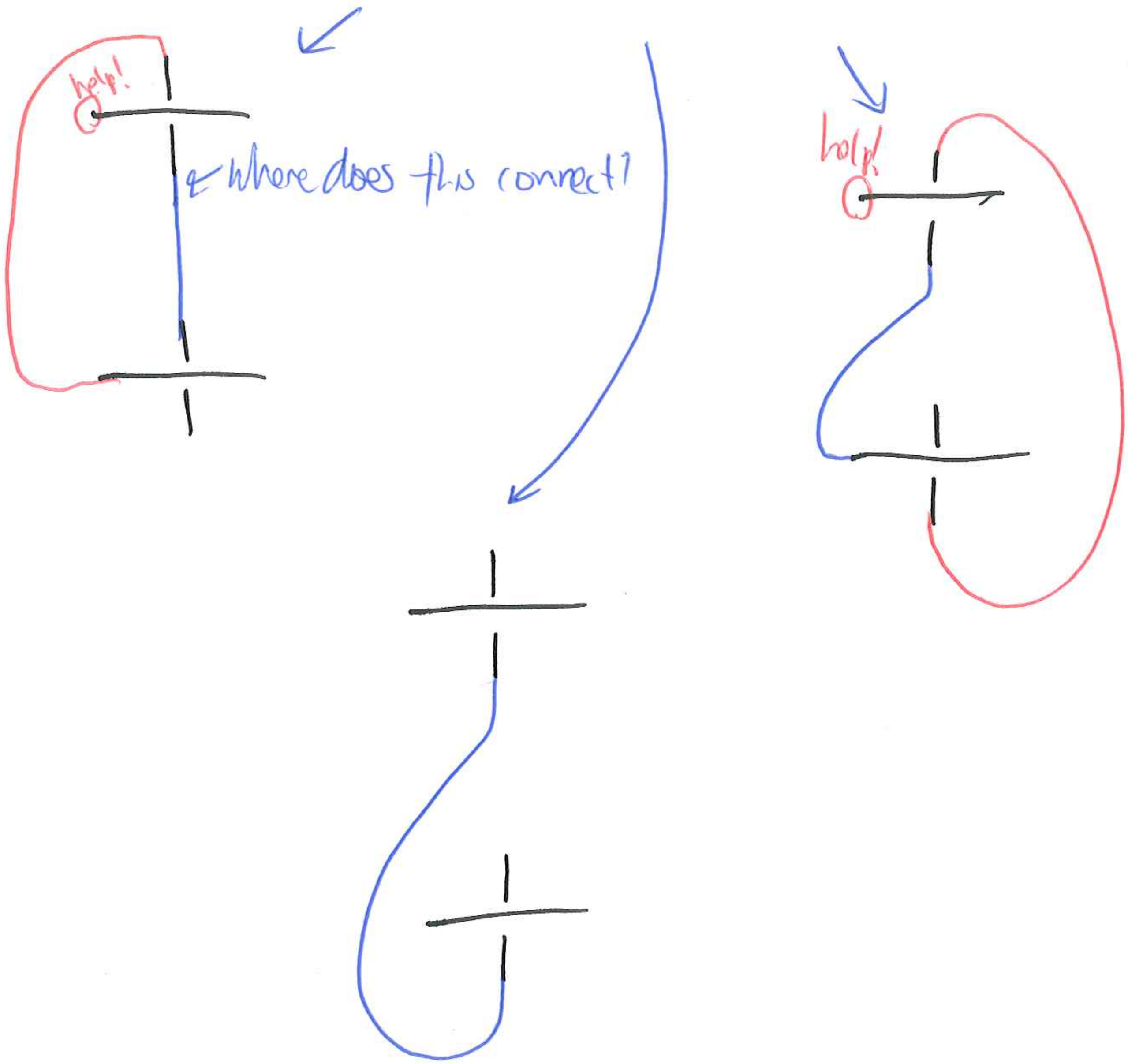


Two-crossing knots

first step

2nd

Is there a knot with crossing number two?



No! All are unknots.

→ Crossing number

Theorem Trefoil is ~~actually~~ not isotopic to the unknot, and $cr(\text{trefoil})=3$.

Pf. It's tricolorable, and the unknot isn't.

(and tricolorability is isotopy invariant)

$cr(K) \leq 3$ because we can draw it with 3 crossings.

$cr(K) \neq 2$ because any knot with crossing number 2 is unknot (but trefoil is not unknot)

$cr(K) \neq 1, 0$ for same reason.

How to prove crossing number of fig 8 is 4?

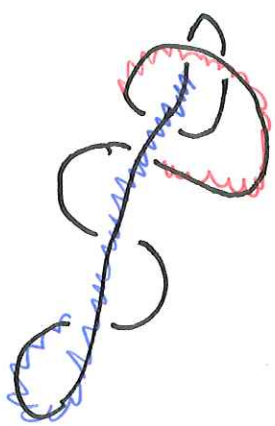
(Prove anything with 3 is trefoil or unknot
and prove figure eight is not unknot)

Crossing number too hard to compute!! Useless invariant. Tricolorability not precise enough.

We need more invariants!

(these have same defect as crossing #; hard to compute)

Bridge number

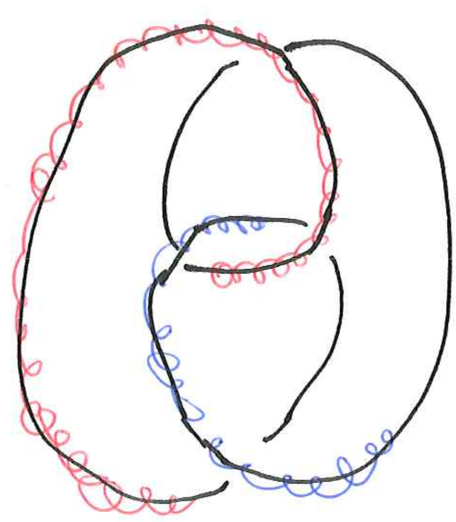


a *bridge* in a knot diagram is an arc that makes 1 or more overcrossings

the bridge number of a knot is the minimum number of bridges in a diagram for the knot.

Trefoil?

at most 3...



it's actually just 2!

Next time:

Come up with more invariants that aren't changed by

Reidemeister moves, since much easier to calculate

Mostly associate a polynomial to a knot: Alexander poly

Jones poly

("Skein relations") HOMFLY poly

...