MATH 436, MIDTERM 1 SPRING 2021, JOHN LESIEUTRE

- You have fifty minutes to complete the exam.
- Exam should be submitted on Gradescope once completed. (Scanning and uploading time do not count towards the fifty minutes.)
- You may consult your notes, the course materials on my website, and the textbook.
- Collaboration and all other references are not allowed.
- Although you can write your answers on a copy of the exam, it is not required.
- Justifications or proofs are required for all problems except where indicated otherwise.
- Please either sign below the integrity statement below or copy out this statement at the beginning of your exam.
- I am generous with partial credit! Try not to leave anything blank.
- Good luck!

I affirm that I have complied with all the exam requirements. I have completed the exam within the allotted time and have not consulted any disallowed references.

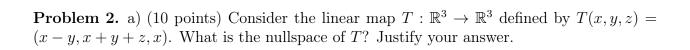
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Problem	Score	Possible
1		20
2		20
3		20
4		20
5		20
Σ		100

Problem	1.	Consider	the	vector	space	\mathcal{P}_{2}	(\mathbb{R})	1
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a) (10 points) Give an example of a subspace of $\mathcal{P}_3(\mathbb{R})$ that is neither all of $\mathcal{P}_3(\mathbb{R})$ nor the zero subspace. Prove that your subspace is either closed under addition or closed under scalar multiplication; you do not need to check both.

b) (10 points) Find a basis for your subspace and compute its dimension. You do not need to prove that this is a basis.



b) (10 points) Compute the dimension of the range of T, and prove that your answer is correct. Is this map surjective?

Problem 3. a) (10 points) Suppose that $T: \mathcal{P}_1(\mathbb{R}) \to \mathcal{P}_1(\mathbb{R})$ is a linear map satisfying T(x+1) = x and T(x-1) = 2x + 3. What is T(x)? Justify your answer.

b) (10 points) Prove that the map T is injective.

Problem 4. Consider the map $T: \mathcal{P}_2(\mathbb{R}) \to \mathbb{R}^2$ given by T(f) = (f(0), f'(3)). You can assume that this is linear.

a) Prove that T is not injective.

b) Give bases for both vector spaces. For \mathbb{R}^2 , I will give only half credit if you use (1,0), (0,1); try to find another basis. You do not need to prove that your basis is a basis. Compute the matrix for T with respect to your basis.

Problem 5. Consider the two vectors $v_1 = (1, 1, 1)$ and $v_2 = (1, 0, 0)$ in \mathbb{R}^3 , which are linearly independent.

a) It follows that the list v_1 , v_2 can be extended to a basis. Give a vector w such that v_1 , v_2 , w is a basis, and prove it.

b) Let $U_1 = \operatorname{span}(v_1, v_2)$ and $U_2 = \operatorname{span}(w)$. Prove that $\mathbb{R}^3 = U_1 \oplus U_2$.