$$f$$

$$\vec{F} = \langle \underset{M}{\overset{f_x}{e^x yz}},\ \underset{N}{\overset{f_y}{e^x z + 2yz}},\ \underset{P}{\overset{f_z}{e^x y + y^2 + 1}} \rangle$$

Last time: $M_y = N_x$, $M_z = P_x$, $N_z = P_y$ ($\Rightarrow$ conservative)

$$f_x = e^x yz$$

"constant of integration dx"

$$f = e^x yz + g(y,z)$$

$$f_y = e^x z + g_y(y,z) = N = e^x z + 2yz$$

so $g_y = 2yz$ so $\cancel{g(y,z) = y^2 z}$

$$g(y,z) = y^2 z + h(z)$$

so $$f = e^x yz + y^2 z + h(z)$$

$$f_z = e^x y + y^2 + h'(z) = N = e^x y + y^2 + h' = e^x y + y^2 + 1$$

so $h' = 1$. so $h(z) = z$

So $\Rightarrow$ $f = e^x yz + y^2 z + z$

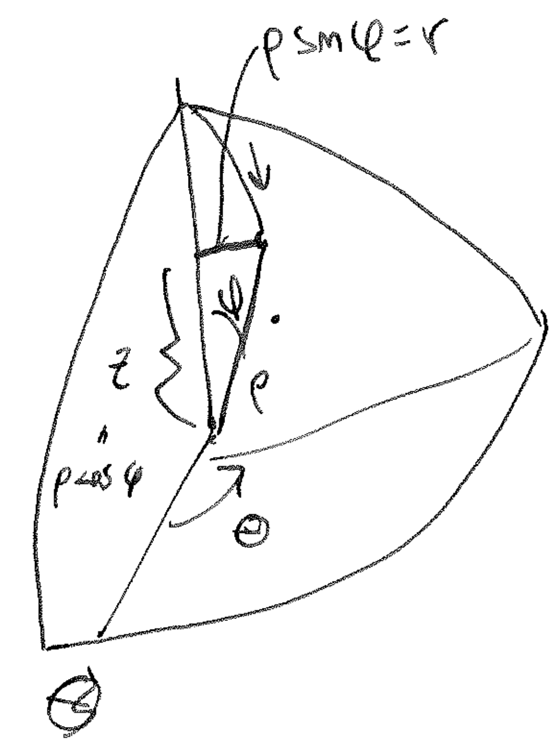
Spherical coords

ρ, θ, φ

ρ = distance to 0

θ = polar angle

φ = angle from north



$$x = r\cos\theta = \rho\cos\theta\sin\varphi$$
$$y = r\sin\theta = \rho\sin\theta\sin\varphi$$
$$z = \rho\cos\varphi$$

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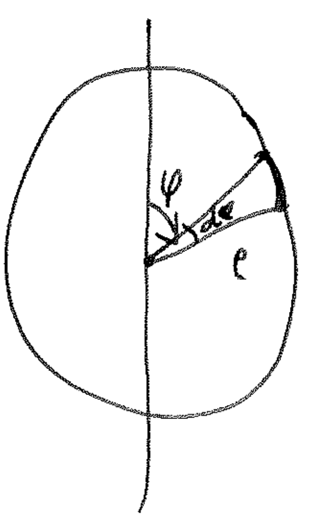
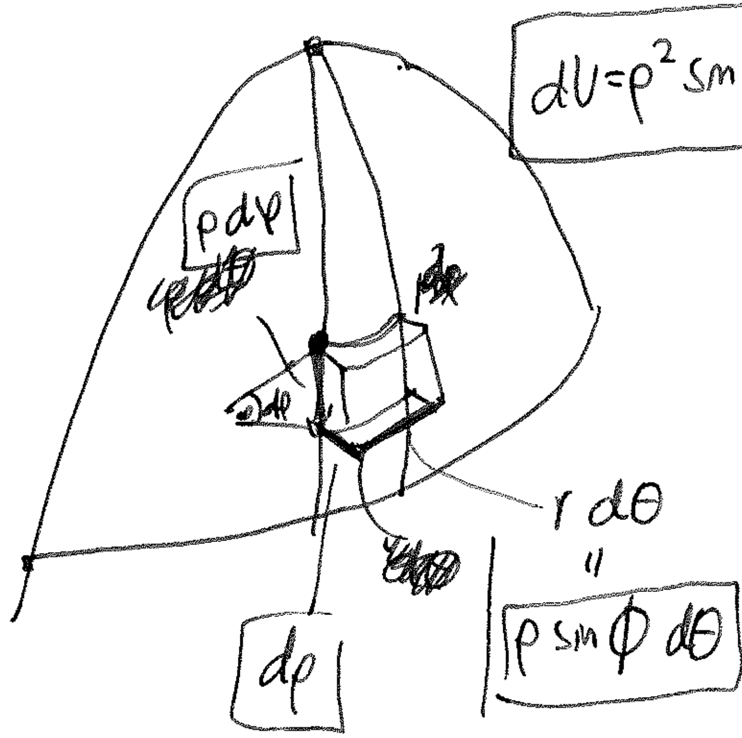
dV in terms of $d\rho$, $d\varphi$, $d\theta$?

$$\boxed{dV = \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta}$$

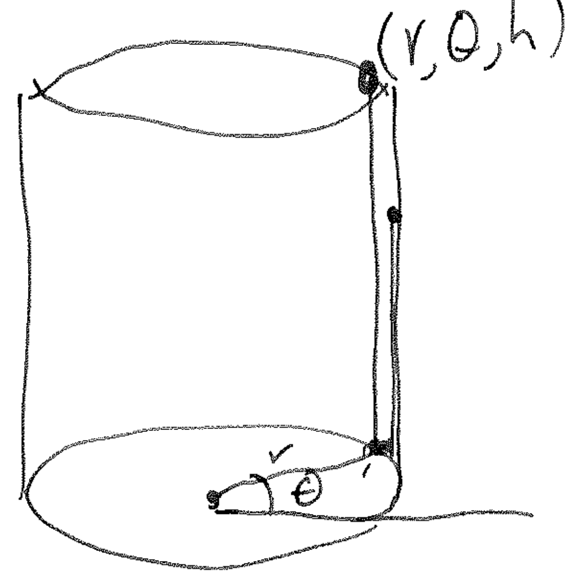




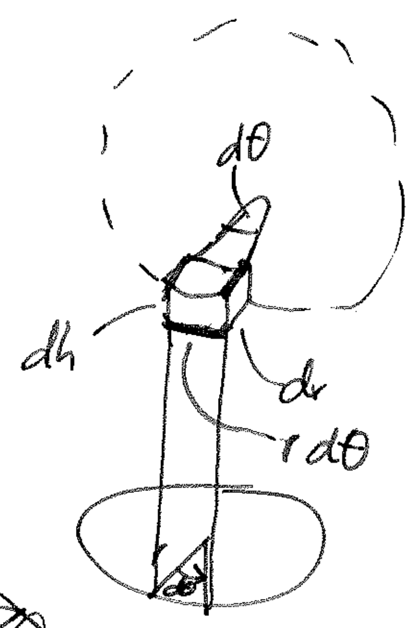
Cylindrical coords

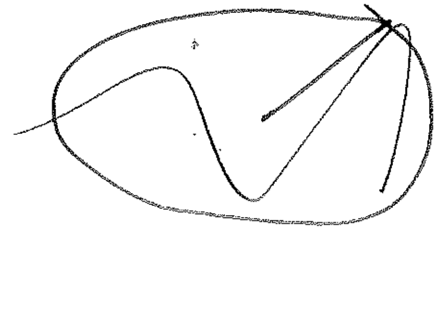
r, θ, h

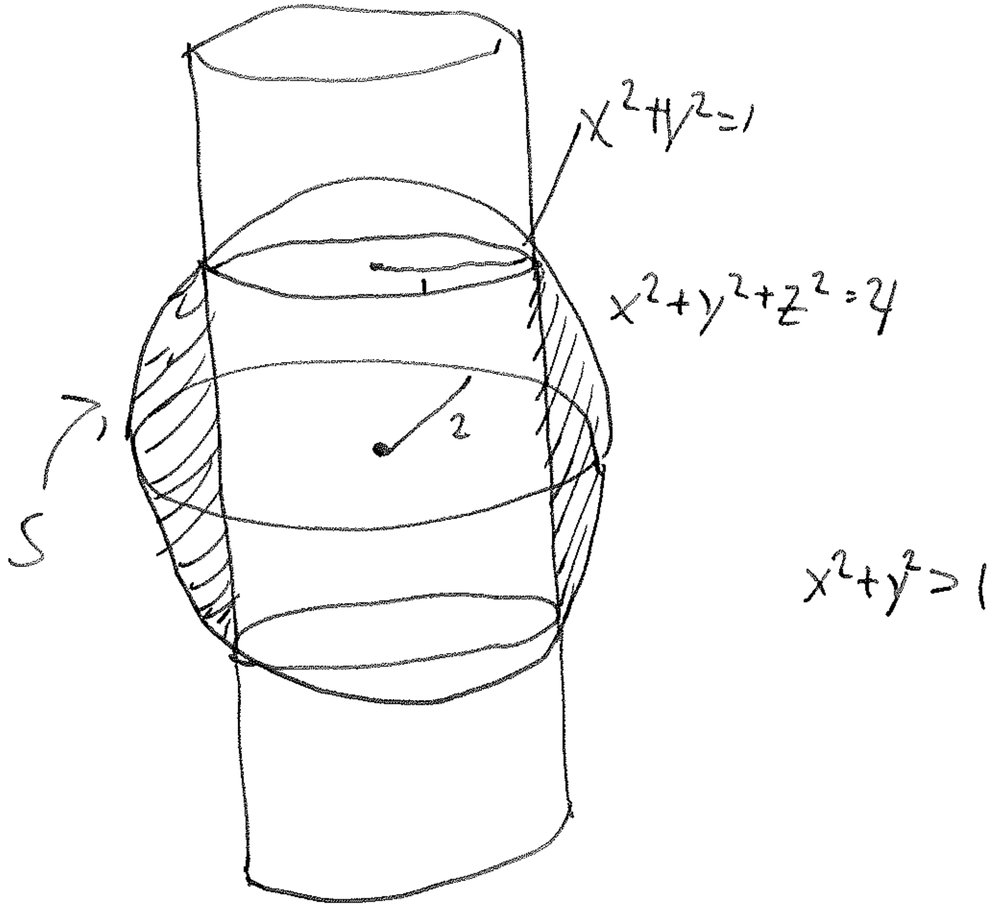
$x = r\cos\theta$
$y = r\sin\theta$
$z = h$



Volume: $dV = r\,dr\,d\theta\,dh$







Flux through S of $y\hat{\imath} - x\hat{\jmath} + z\hat{k}$

$$\iint \vec{F}\cdot\hat{n}\, \underset{}{dS}$$

dS = dA, surface area

$\vec{F}\cdot\hat{n}$: express $\vec{F}\cdot\hat{n}$ in our coordinates

express dS area in our coords

~~$\hat{n} = \frac{1}{2}(x\hat{\imath}+y\hat{\jmath}$~~ $\quad \hat{n} = \frac{1}{2}(x\hat{\imath} + y\hat{\jmath} + z\hat{k})$ ← unit vector

In this problem:

$$\vec{F} = y\hat{\imath} - x\hat{\jmath} + z\hat{k}, \quad \operatorname{div} \vec{F} = 1$$

$$\oiint_S \vec{F}\cdot\hat{n}\,dS = \iint_{\text{spherical}} \vec{F}\cdot\hat{n}\,dS + \iint_{\text{cylindrical}} \vec{F}\cdot\hat{n}\,dS = 4\sqrt{3}\pi + 0 = 4\sqrt{3}\pi.$$

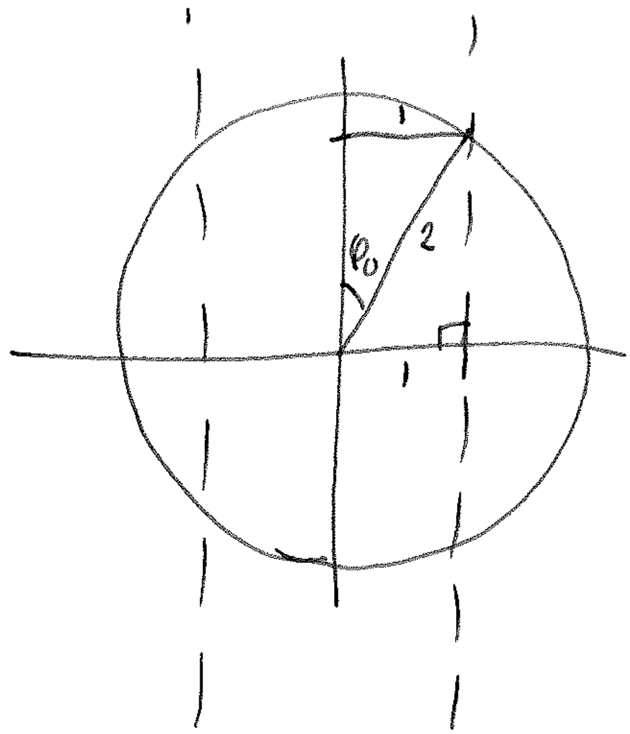
$$\iiint \operatorname{div}\vec{F}\,dV = \iiint dV = \text{volume}(R)$$

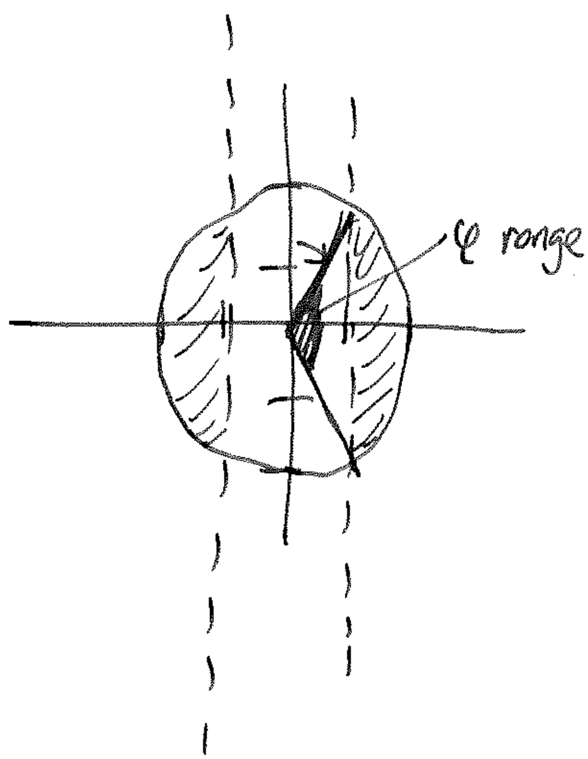
$$\Rightarrow \text{volume} = 4\sqrt{3}\pi$$

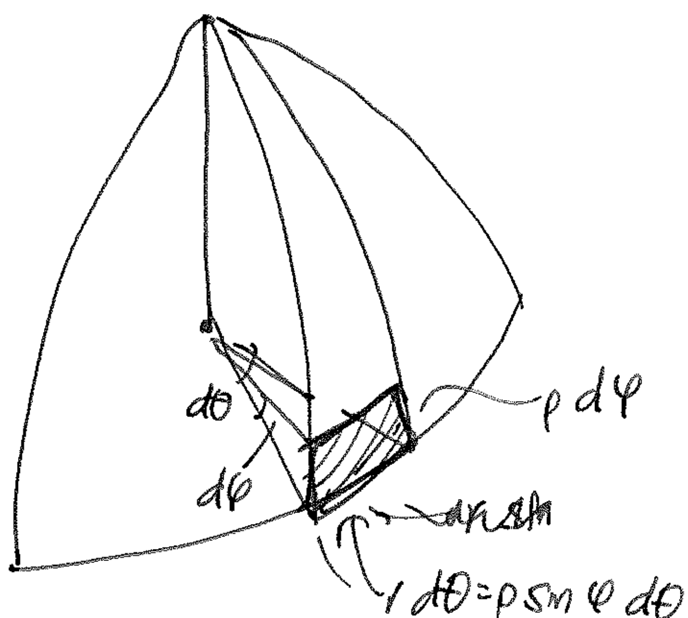
$$\vec{F}\cdot\hat{n} = (y\hat{i} \mp x\hat{j} + z\hat{k})\cdot \tfrac{1}{2}(x\hat{i}+y\hat{j}+z\hat{k})$$

$$= \tfrac{1}{2}(xy - xy + z^2) = \frac{z^2}{2} \quad \Big\| \quad dS =$$

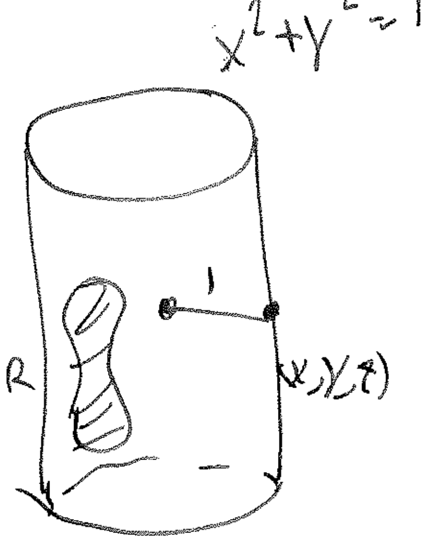
$$\int_{\theta=0}^{2\pi}\int_{\varphi=\pi/6}^{5\pi/6} \frac{z^2}{2}$$







$$dS = \rho^2 \sin\varphi\, d\varphi\, d\theta$$



$$\vec{F} = y\hat{i} - x\hat{j} + z\hat{k}$$

$$\hat{n} = x\hat{i} + y\hat{j}$$

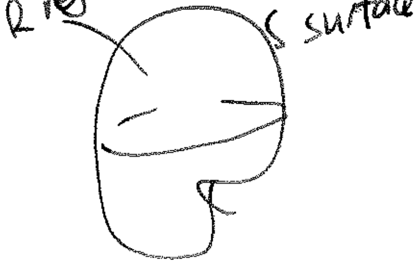
$$\vec{F}\cdot\hat{n} = xy - xy = 0$$

$$\iint_R \vec{F}\cdot\hat{n}\,dS = 0$$

$$\vec{F} = M\hat{i} + N\hat{j} + P\hat{k} \qquad \nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

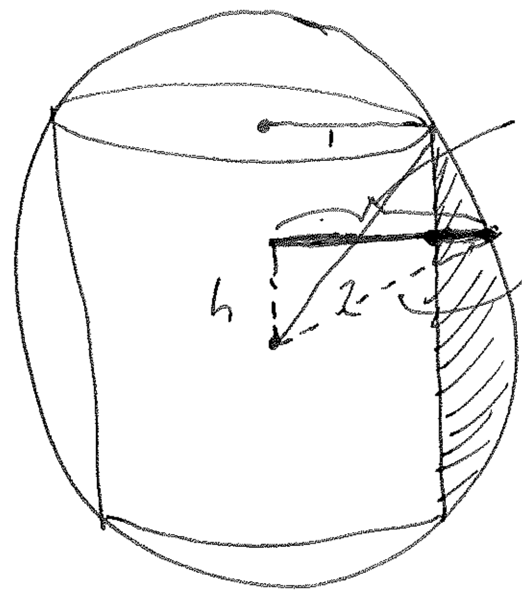
$$\operatorname{div}\vec{F} = \nabla\cdot\vec{F} = M_x + N_y + P_z$$

Divergence theorem



$$\oiint_S \vec{F}\cdot\hat{n}\,dS = \iiint_R \operatorname{div}\vec{F}\,dV$$

Directly:



Cylindrical: $0 \le \theta \le 2\pi$

$-\sqrt{3} \le h \le \sqrt{3}$

$1 \le r \le \sqrt{4-h^2}$

$$\int_{\theta=0}^{2\pi} \int_{h=-\sqrt{3}}^{\sqrt{3}} \int_{r=1}^{\sqrt{4-h^2}} 1 \, r \, dr \, dh \, d\theta$$

$\vec{F} = M\hat{i} + N\hat{j} + P\hat{k}$

curl $\vec{F}$: imagine $\vec{F}$ is water flow

direction of curl $\vec{F}$ = axis a ball would spin on

length of curl $\vec{F}$ = how fast does it spin?

(NB: in 3D, curl $\vec{F}$ is another vector field!

in 2D it's just a function.)