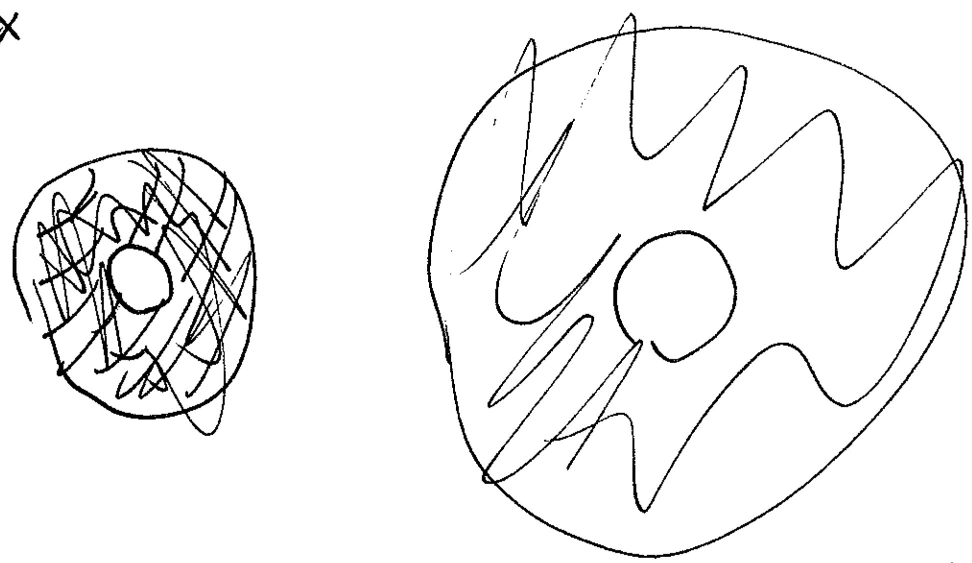
Topology

- basic stuff (today)
- proving some other things using topology
- applications to statistics

{ Brouwer fixed pt
fdtl theorem of algebra?

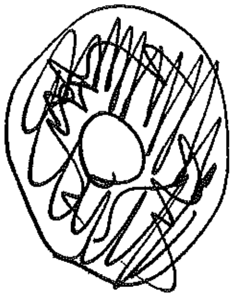
Two topological spaces are "homeomorphic" (the same) if one can be smoothly transformed into the other by stretching, bending, ...
(but no tearing or gluing)

ex

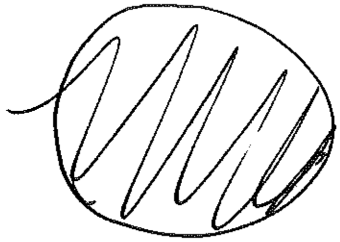


homeomorphic

two annuli of different sizes are homeomorphic



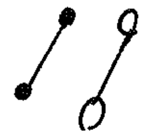
annulus



solid disk

not homeomorphic!

More examples



circle figure 8

X

coffee cup

donut

✓

$\mathbb{R}$ real numbers

(0,1) open interval

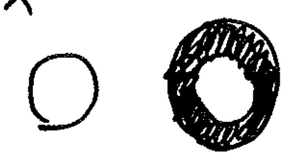
 

annulus biannulus

X

Möbius strip

X



Cylinder  untwisted

X

$\mathbb{R}$ real numbers

vs

$\mathbb{R}^2$

X

~~Möbius strip~~ Cylinder

vs ~~two~~

double-twisted Möbius strip

✓ ?

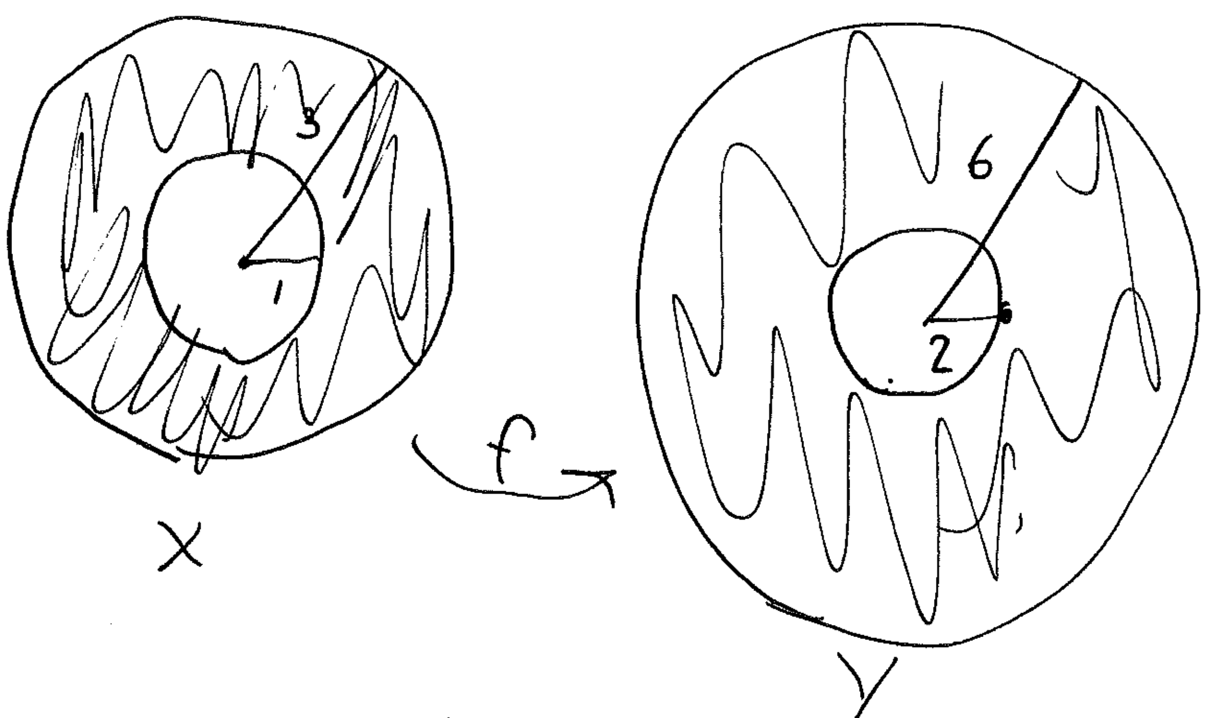
Officially:

Two spaces X and Y are <u>homeomorphic</u>
(sets, basically)

if there's an invertible function

$f: X \to Y$

that's continuous and has a continuous inverse.



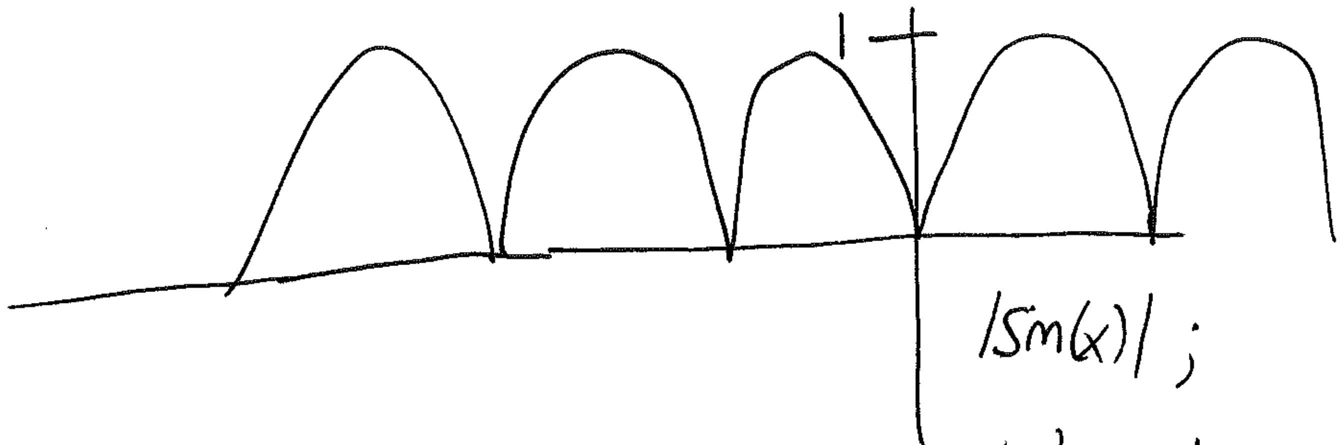
$f(r,\theta) = (2r, \theta)$
in polar

Inverse function is $g(r,\theta) = (\frac{1}{2}, \theta)$

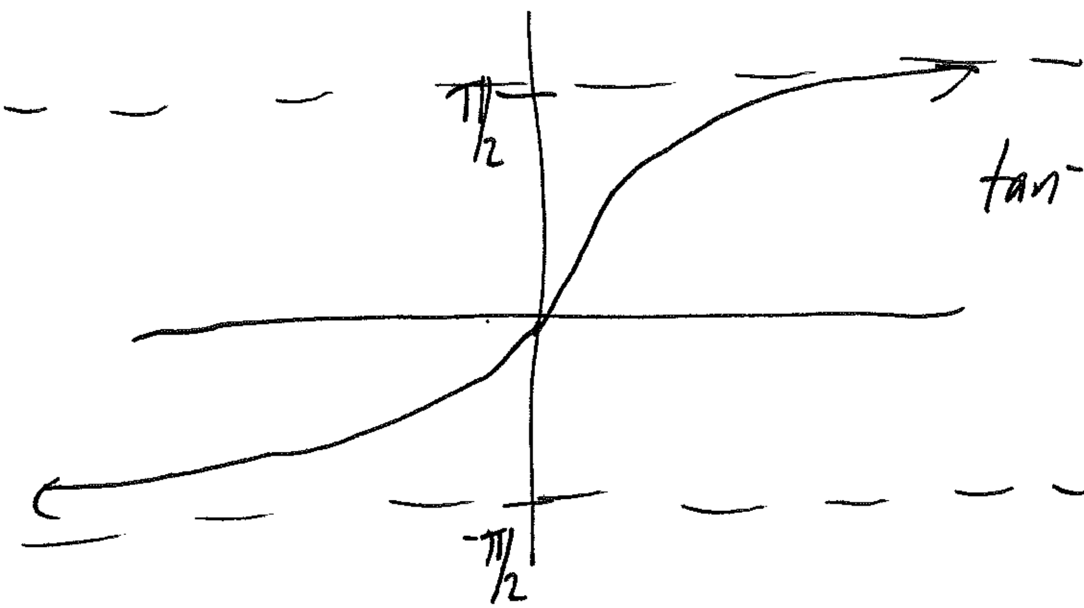
Ex $\mathbb{R}$ and $(0,1)$

Are these homeomorphic?

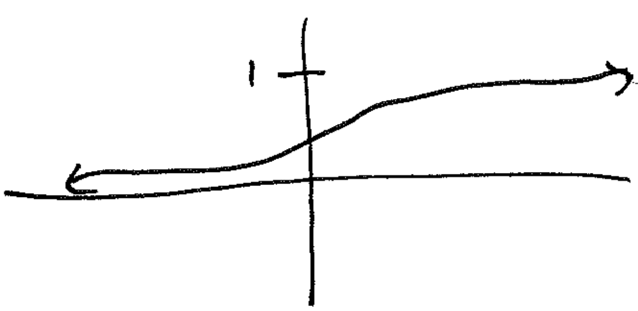
Can you find continuous $f: \mathbb{R} \to (0,1)$ with continuous inverse?



$|\sin(x)|$; not one-to-one



$\tan^{-1}(x)$



$f(x) = \frac{1}{\pi}\tan^{-1} x + \frac{1}{2}$

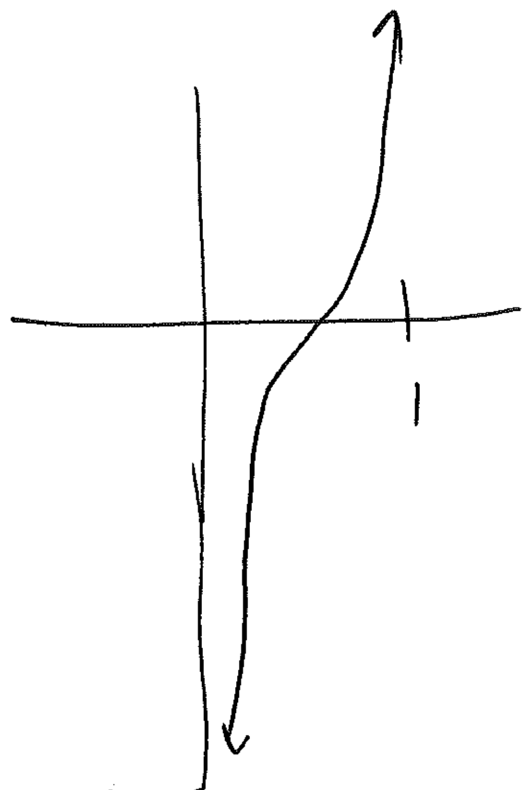
inverse?

$$x = \frac{1}{\pi}\tan^{-1} y + \frac{1}{2}$$

$$\pi\left(x - \frac{1}{2}\right) = \tan^{-1} y$$

$$y = \tan\left(\pi\left(x - \frac{1}{2}\right)\right).$$

$$g(y) = \tan\left(\pi\left(x - \frac{1}{2}\right)\right)$$



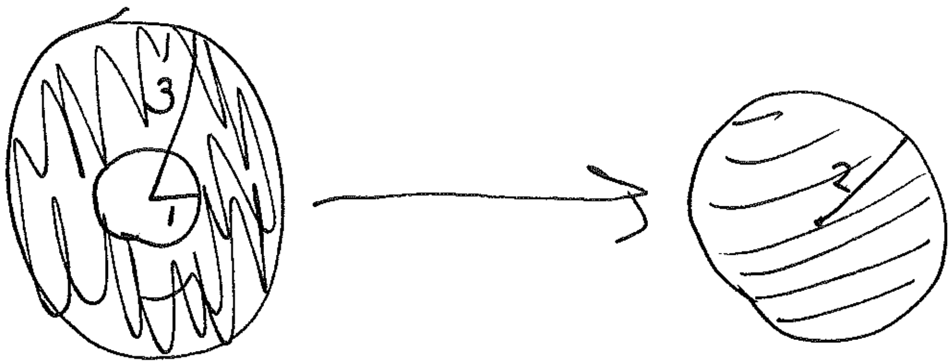
$\mathbb{R}$ and $(0,1)$ are homeomorphic!

A strategy to show spaces are homeomorphic (or not).

To show X and Y are not homeomorphic:
find a "characteristic" of spaces that's unaffected by homeomorphisms.

("topological invariants")

Not example



$f(r,\theta) = (r-1, \theta)$

Doesn't count!
This f is not invertible.

For g to be inverse of f means that

$g(f(p)) = p$ for any p

and $f(g(p)) = p$ for any p.

If
$g(r,\theta) = (r+1, \theta)$
then
$g(0, \pi/4) = (1, \pi/4)$
$g(0, \pi/2) = (1, \pi/2)$

same pt in polar!
g doesn't make sense when $r=0$.