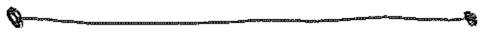
Today: More Heat Equation (+ demos)

L length



Initial temperature $g(x)$ (as a function of x, $0 \le x \le L$)

Hold ends at $0°$, let ~~temp~~ heat flow.

$g(0) = g(L) = 0$.

Let $u(x,t)$ = temp at position x at time t

$\cancel{\frac{\partial}{\partial t}}$

we want to figure out

$$\frac{\partial}{\partial t} u(x,t) = \underset{}{\overset{\text{constant}}{\alpha^2}} \frac{\partial^2}{\partial x^2} u(x,t)$$

We want to find a function $u(x,t)$ satisfying:

- $u(x,0) = g(x)$ (initial temp)
  known
- $u(0,t) = 0$
- $u(L,t) = 0$
- $u_t = \alpha^2 u_{xx}$

Last time: Here are some solutions:

1) $u_1(x,t) = e^{-\frac{\pi^2\alpha^2}{L^2}t}\sin\left(\frac{\pi x}{L}\right)$

2) $u_2(x,t) = e^{-\frac{4\pi^2\alpha^2}{L^2}t}\sin\left(\frac{2\pi x}{L}\right)$

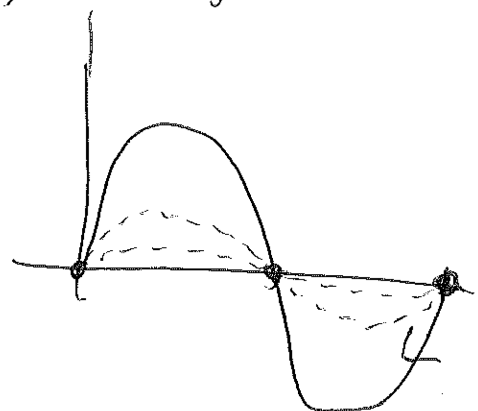
...

$$\boxed{n)\ u_n(x,t) = e^{-\frac{n^2\pi^2\alpha^2}{L^2}t}\sin\left(\frac{n\pi x}{L}\right)}$$

Check:

$$u_n(x,t) = e^{-\frac{n^2\pi^2\alpha^2}{L^2}t}\sin\left(\frac{n\pi x}{L}\right)$$

eg. $n=2$



- $u_n(x,0) = \sin\left(\frac{n\pi x}{L}\right)$

this describes evolution of temp when initial temp is $\sin\left(\frac{n\pi x}{L}\right)$

- $u_n(0,t) = u_n(L,t) = 0$.

✓

IVA.

let $L=\pi$.

If our initial condition was

$$\sin(x) \xrightarrow{\text{sol is}} e^{-\alpha^2 t}\sin(x)$$

$$\sin(2x) \xrightarrow{\text{sol}} e^{-4\alpha^2 t}\sin(2x)$$

$$\sin(3x) \xrightarrow{\text{sol}} e^{-9\alpha^2 t}\sin(3x)$$

If initial condition is

$7\sin(x)-3\sin(2x)$, solution is:

$$u(x,t)=7e^{-\alpha^2 t}\sin(x)-3e^{-4\alpha^2 t}\sin(2x)$$

Given an initial condition $g(x)$,

write Fourier series:

$$g(x) = \sum_{n=1}^{\infty} b_n \sin(nx).$$

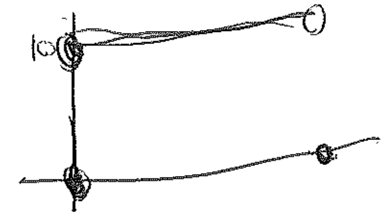
Then solution to heat eqn is

$$\cancel{u(x,t) = \sum_{n=1}^{\infty} b_n e^{-n^2\pi^2\alpha^2 t}}$$

$$\boxed{u(x,t) = \sum_{n=1}^{\infty} b_n e^{-n^2\alpha^2 t} \sin(nx).}$$

Questions

$\boxed{\alpha^2 = 1}$ $\boxed{L = \pi}$



1) A rod is heated to temperature $10°$ C. How long until $u(x,t) < 1°$ C for all x?

2) Can you come up with a initial term function so it warms up in the middle, then cools off?

3) What happens if initial function $g(x)$ doesn't have $g(0) = 0$?

4) ~~How can~~