

CS161 Homework 5

1.

(a) Neither

Smoke	Fire	$(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
F	F	T
F	T	F
T	F	T
T	T	T

(b) Neither

Smoke	Fire	Heat	$(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire)$
F	F	F	T
F	F	T	F
F	T	F	T
T	F	F	T
F	T	T	T
T	F	T	T
T	T	F	T
T	T	T	T

(c) Valid

Smoke	Fire	Heat	$((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$
F	F	F	T
F	F	T	T
F	T	F	T
T	F	F	T
F	T	T	T
T	F	T	T
T	T	F	T
T	T	T	T

2.

(a)

Knowledge Base

P1: $Mythical \Rightarrow Immortal$

P2: $\neg Mythical \Rightarrow \neg Immortal \wedge Mammal$

P3: $Immortal \vee Mammal \Rightarrow Horned$

P4: $Horned \Rightarrow Magical$

(b)

P1: $\neg \text{Mythical} \vee \text{Immortal}$

P2: $\text{Mythical} \vee (\neg \text{Immortal} \wedge \text{Mammal}) = (\text{Mythical} \vee \neg \text{Immortal}) \wedge (\text{Mythical} \vee \text{Mammal})$

P3: $\neg(\text{Immortal} \vee \text{Mammal}) \vee \text{Horned} = (\neg \text{Immortal} \vee \text{Horned}) \wedge (\neg \text{Mammal} \vee \text{Horned})$

P4: $\neg \text{Horned} \vee \text{Magical}$

(c)

Given the KB, we are not able to prove the unicorn is mythical. However, we are able to prove the unicorn is magical (and also horned).

Proof that unicorn is magical, horned

P5: $(\text{Mythical} \vee \neg \text{Immortal})$	Extracted from P2
P6: $(\text{Mythical} \vee \text{Mammal})$	Extracted from P2
P7: $(\neg \text{Immortal} \vee \text{Horned})$	Extracted from P3
P8: $(\neg \text{Mammal} \vee \text{Horned})$	Extracted from P3
P9: $(\neg \text{Mammal} \vee \text{Magical})$	Resolve P4 and P8
P10: $(\neg \text{Immortal} \vee \text{Magical})$	Resolve P4 and P7
P11: $(\text{Mythical} \vee \text{Magical})$	Resolve P5 and P10
P12: $(\text{Mythical} \vee \text{Horned})$	Resolve P6 and P8
P13: $(\text{Immortal} \vee \text{Mammal})$	Resolve P1 and P6
P14: $\neg \text{Mythical} \vee \text{Horned}$	Resolve P1 and P7
P15: Horned	Resolve P12 and P14
P16: Magical	Resolve P4 and P15

3.

(a)

$\{x/A, y/B, z/B\}$

(b)

No general unifier exists because A cannot be unified with B

(c)

$\{y/\text{John}, x/\text{John}\}$

(d)

No general unifier exists (i.e. we can do $\{x/\text{Father}(y)\}$, but then $\text{Father}(y)$ cannot be unified with y)

4.
(a)

First-Order Logic Formulations

P1: $\forall x, \text{Food}(x) \Rightarrow \text{Likes}(\text{John}, x)$
P2: $\text{Food}(\text{Apples})$
P3: $\text{Food}(\text{Chicken})$
P4: $\forall x, y, \text{Eats}(x, y) \ \& \ \neg \text{Killed}(x, y) \Rightarrow \text{Food}(y)$
P5: $\forall x (\exists y, \text{Killed}(x, y)) \Rightarrow \neg \text{Alive}(x)$
P6: $\text{Eats}(\text{Bill}, \text{Peanuts}) \ \& \ \text{Alive}(\text{Bill})$
P7: $\forall x, \text{Eats}(\text{Bill}, x) \Rightarrow \text{Eats}(\text{Sue}, x)$

(b)

FOL in CNF

P1: $\neg \text{Food}(x) \mid \text{Likes}(\text{John}, x)$
P2: $\text{Food}(\text{Apples})$
P3: $\text{Food}(\text{Chicken})$
P4: $\neg \text{Eats}(x, y) \mid \text{Killed}(x, y) \mid \text{Food}(y)$
P5: $\neg \text{Killed}(x, y) \mid \neg \text{Alive}(x)$
P6: $\text{Eats}(\text{Bill}, \text{Peanuts})$
P7: $\text{Alive}(\text{Bill})$
P8: $\neg \text{Eats}(\text{Bill}, x) \mid \text{Eats}(\text{Sue}, x)$

(c)

Proof that John likes peanuts

P9: $\neg \text{Likes}(\text{John}, \text{Peanuts})$	Proof by contradiction
P10: $\neg \text{Food}(\text{Peanuts})$	Resolve P1 and P9 w/ substitution $\{x/\text{Peanuts}\}$
P11: $\neg \text{Eats}(x, \text{Peanuts}) \mid \text{Killed}(x, \text{Peanuts})$	Resolve P4 and P10 w/ substitution $\{y/\text{Peanuts}\}$
P12: $\neg \text{Eats}(x, \text{Peanuts}) \mid \neg \text{Alive}(x)$	Resolve P5 and P11 w/ substitution $\{y/\text{Peanuts}\}$
P13: $\neg \text{Eats}(\text{Bill}, \text{Peanuts})$	Resolve P7 and P12 w/ substitution $\{x/\text{Bill}\}$
P14: Empty clause (i.e. John likes Peanuts)	Resolve P6 and P13

(d)

What food does Sue eat?

First-Order Logic Formulation: $\exists x, \text{Food}(x) \ \& \ \text{Eats}(\text{Sue}, x)$

P9: $\neg \text{Food}(x) \mid \neg \text{Eats}(\text{Sue}, x)$	Formulation in CNF
P10: $\neg \text{Eats}(\text{Bill}, x) \mid \neg \text{Food}(x)$	Resolve S8 and S9
P11: $\neg \text{Food}(\text{Peanuts})$	Resolve S6 and S10 w/ substitution $\{x/\text{Peanuts}\}$

Then the same proof follows from (c) starting w/ P10 in (c) (i.e. Sue eats Peanuts)

(e)

First-Order Logic Formulations

P1: $\forall x, \text{Food}(x) \Rightarrow \text{Likes}(\text{John}, x)$
P2: $\text{Food}(\text{Apples})$
P3: $\text{Food}(\text{Chicken})$
P4: $\forall x, y, \text{Eats}(x, y) \ \& \ \neg \text{Killed}(x, y) \Rightarrow \text{Food}(y)$
P5: $\forall x (\exists y, \text{Killed}(x, y)) \Rightarrow \neg \text{Alive}(x)$
P6: $\forall x, y, \neg \text{Eats}(x, y) \Rightarrow \text{Die}(x)$
P7: $\forall x, \text{Die}(x) \Rightarrow \neg \text{Alive}(x)$
P8: $\text{Alive}(\text{Bill})$
P9: $\forall x, \text{Eats}(\text{Bill}, x) \Rightarrow \text{Eats}(\text{Sue}, x)$

FOL in CNF

P1: $\neg \text{Food}(x) \mid \text{Likes}(\text{John}, x)$
P2: $\text{Food}(\text{Apples})$
P3: $\text{Food}(\text{Chicken})$
P4: $\neg \text{Eats}(x, y) \mid \text{Killed}(x, y) \mid \text{Food}(y)$
P5: $\neg \text{Killed}(x, y) \mid \neg \text{Alive}(x)$
P6: $\text{Eats}(x, y) \mid \text{Die}(x)$
P7: $\neg \text{Die}(x) \mid \neg \text{Alive}(x)$
P8: $\text{Alive}(\text{Bill})$
P9: $\neg \text{Eats}(\text{Bill}, x) \mid \text{Eats}(\text{Sue}, x)$

What food does Sue eat?

First-Order Logic Formulation: $\exists z, \text{Food}(z) \ \& \ \text{Eats}(\text{Sue}, z)$

P10: $\neg \text{Food}(z) \mid \neg \text{Eats}(\text{Sue}, z)$	Formulation in CNF
P11: $\neg \text{Eats}(\text{Bill}, z) \mid \neg \text{Food}(z)$	Resolve P9 and P10 w/ substitution $\{x/z\}$
P12: $\text{Die}(\text{Bill}) \mid \neg \text{Food}(z)$	Resolve P6 and P11 w/ substitution $\{x/\text{Bill}, y/z\}$
P13: $\neg \text{Alive}(\text{Bill}) \mid \neg \text{Food}(z)$	Resolve P7 and P12 w/ substitution $\{x/\text{Bill}\}$
P14: $\neg \text{Food}(z)$	Resolve P8 and P13
P15: $\neg \text{Eats}(x, y) \mid \text{Killed}(x, y)$	Resolve P4 and P14 w/ substitution $\{z/y\}$
P16: $\neg \text{Eats}(x, y) \mid \neg \text{Alive}(x)$	Resolve P5 and P15
P17: $\neg \text{Eats}(\text{Bill}, y)$	Resolve P8 and P16 w/ substitution $\{x/\text{Bill}\}$
P18: $\text{Die}(\text{Bill})$	Resolve P6 and P17 w/ substitution $\{x/\text{Bill}\}$
P19: $\neg \text{Alive}(\text{Bill})$	Resolve P7 and P18 w/ substitution $\{x/\text{Bill}\}$
P20: Empty clause (i.e. Sue eats everything Bill eats)	Resolve P8 and P19

Sue eats everything Bill eats