First-Order Logic

CS161

Prof. Guy Van den Broeck

Why are we learning about logic?

- Specification α and implementation β
- Show that β entails α using SAT solver

  The implementation is “correct”
- Hardware verification

  CPU computes addition correctly
- Software verification

  Windows driver does not write to unallocated memory
- Cryptography
- Planning

Why are we learning about logic?

engadget

THE NEW REDDIT

comments other discussions (5)

tom's HARDWARE
THE AUTHORITY ON TECH

Mathematics

nature
International weekly journal of science

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Archive > Volume 534 > Issue 7605 > News > Article

Two-hundred-terabyte

19 days ago by CryptoBeer

265 comments share

NATURE | NEWS

Slashdot Stories

Two-hundred-terabyte maths proof is largest ever

Topics: Devices Build Entertainment Technology Open Source Science YRO

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Computer Generates Largest Math Proof Ever At 200TB of Data (phys.org)

Posted by BeauHD on Monday May 30, 2016 @08:10PM from the red-pill-and-blue-pill dept.

143

THE CONVERSATION

Academic rigour, journalistic flair

76 comments

Collqteral May 27, 2016 +2

200 Terabytes. Thats about 400 PS4s.

SPIEGEL ONLINE

(after Marijn J.H. Heule)

Why are we learning about logic?

Probabilistic Reasoning

ELSEVIER

Artificial Intelligence

Volume 172, Issues 6–7, April 2008, Pages 772-799

On probabilistic inference by weighted model counting ☆

Mark Chavira, Adnan Darwiche

Show more

https://doi.org/10.1016/j.artint.2007.11.002

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Abstract

A recent and effective approach to probabilistic inference calls for reducing the problem to one of weighted model counting (WMC) on a propositional knowledge base. Specifically, the approach calls for encoding the probabilistic model, typically a Bayesian network, as a propositional knowledge base in conjunctive normal form (CNF) with weights associated to each model according to the network parameters. Given this CNF, computing the probability of some evidence becomes a matter of summing the weights of all CNF models consistent with the evidence. A number of variations on this approach have appeared in the literature recently, that vary across three orthogonal dimensions. The first dimension concerns the specific encoding used to convert a Bayesian network into a CNF. The second dimensions relates to whether weighted model counting is performed using a search algorithm on the CNF, or by compiling the CNF into a structure that renders WMC a polytime operation in the size of the compiled structure. The third dimension deals with the specific properties of network parameters (local structure) which are captured in the CNF encoding. In this paper, we discuss recent work in this area across the above three dimensions, and demonstrate empirically its practical importance in significantly expanding the reach of exact probabilistic inference. We restrict our discussion to exact inference and model counting, even though other proposals have been extended for approximate inference and approximate model counting.

Deep Learning

A SEMANTIC LOSS FUNCTION FOR DEEP LEARNING WITH SYMBOLIC KNOWLEDGE

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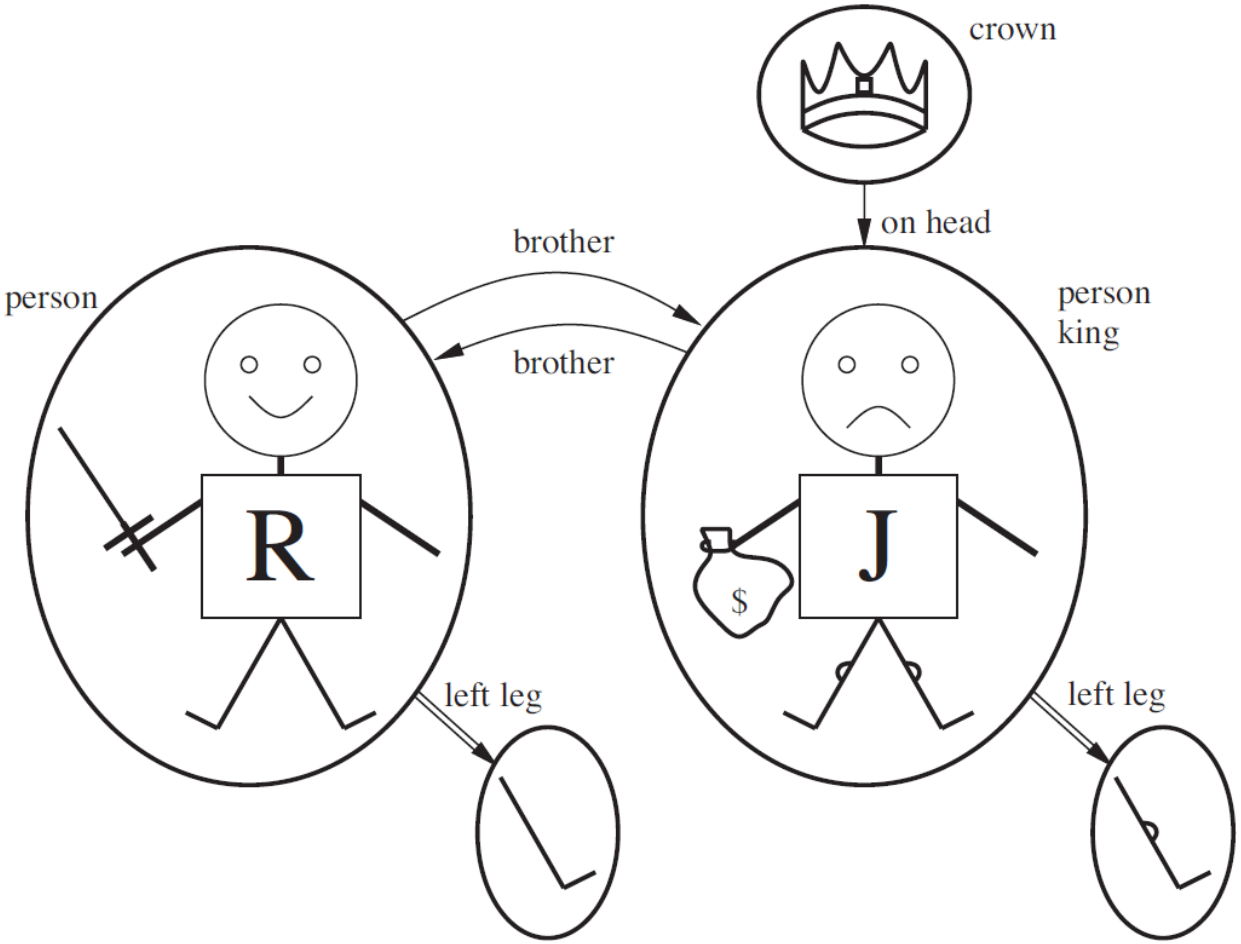
ABSTRACT

This paper develops a novel methodology for using symbolic knowledge in deep learning. From first principles, we derive a semantic loss function that bridges between neural output vectors and logical constraints. This loss function captures how close the neural network is to satisfying the constraints on its output. An experimental evaluation shows that our semantic loss function effectively guides the learner to achieve (near-)state-of-the-art results on semi-supervised multi-class classification. Moreover, it significantly increases the ability of the neural network to predict structured objects, such as rankings and paths. These discrete concepts are tremendously difficult to learn, and benefit from a tight integration of deep learning and symbolic reasoning methods.

First-Order/Predicate Logic

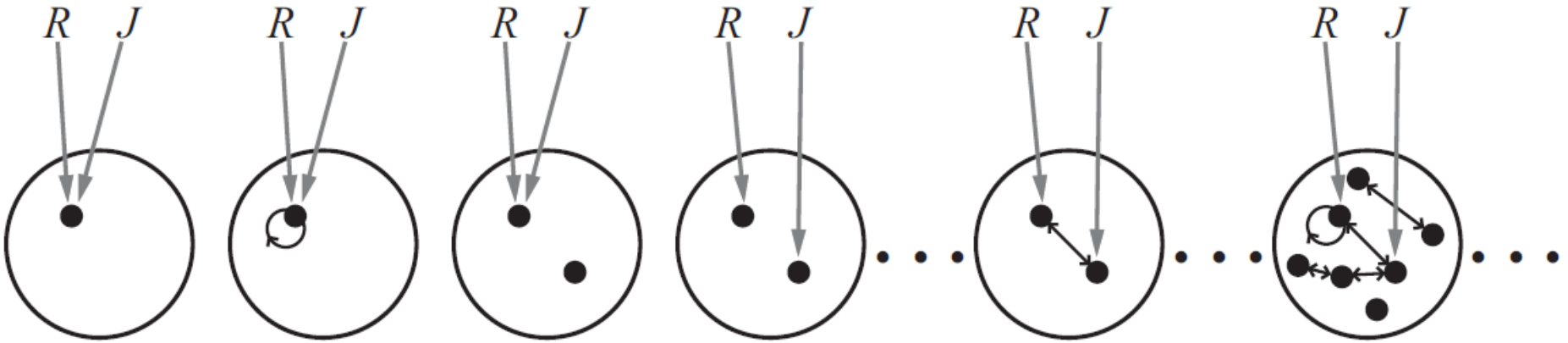
World


crown
on head
brother
brother
person
person
king
R
J
$
left leg
left leg

Syntax and Semantics of First-Order Logic

Models

- Two constants: R, J
- One binary relation



Fun with First-Order Logic

Fun with Quantifiers

Inference Rules: Universal Instantiation

$$\forall x\ \mathrm{King}(x) \wedge \mathrm{Greedy}(x) \Rightarrow \mathrm{Evil}(x)$$

Power of Propositionalization

- Problem: Infinitely many ground terms with function symbols (Father(Father(Father(…))))
- Theorem (Herbrand, 1930):
  If a sentence is entailed by a first-order KB, it is entailed by a finite subset of the propositional KB.
- Idea: Iteratively grow the propositional KB
- Theorem (Church/Turing 1936):
  Entailment in FOL is semi-decidable.

  Idea works if sentence is entailed, loops forever otherwise

Generalized Modus Ponens

Can we do inference immediately?

$$\forall x\ \text{King}(x) \wedge \text{Greedy}(x) \Rightarrow \text{Evil}(x)$$
$$\text{King}(\text{John})$$
$$\text{Greedy}(\text{John})$$

Can we do inference immediately?

$$\forall x\ \text{King}(x) \wedge \text{Greedy}(x) \Rightarrow \text{Evil}(x)$$
$$\text{King}(\text{John})$$
$$\text{Greedy}(\text{John})$$

When do sentences share information?

Substitution and Unification

Most General Unifier (MGU)

Maximum amount of information shared between two sentences

Unification Algorithm

Term-by-term

Depth-First

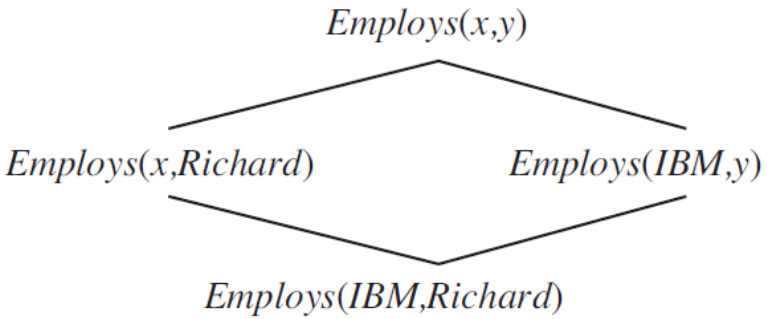
Keep track of substitutions used

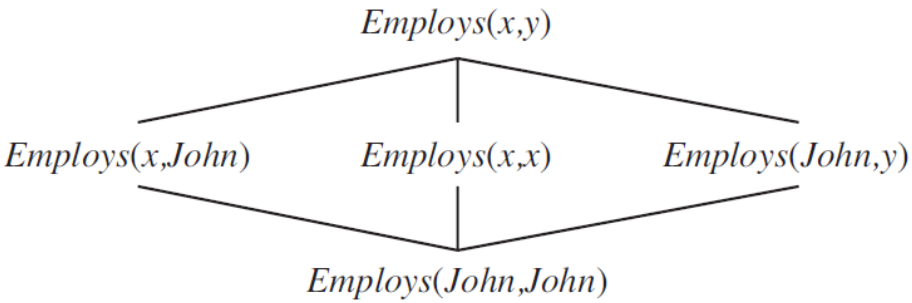
Avoid infinite loops (occurs check)

See book for details.

Subsumption Lattice

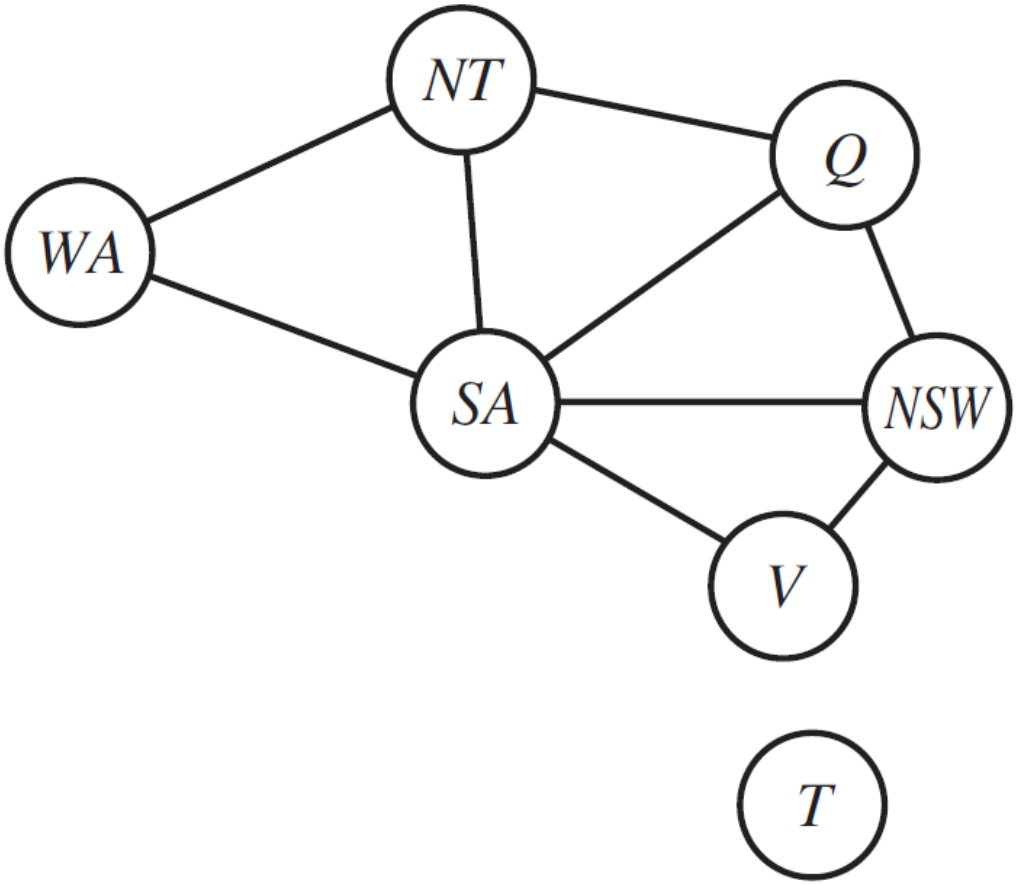
Sentence α subsumes β iff
there exists a substitution θ such that αθ = β





Generalized Modus Ponens

Complexity of Finding Premises



$$Diff(wa, nt) \wedge Diff(wa, sa) \wedge$$
$$Diff(nt, q) \wedge Diff(nt, sa) \wedge$$
$$Diff(q, nsw) \wedge Diff(q, sa) \wedge$$
$$Diff(nsw, v) \wedge Diff(nsw, sa) \wedge$$
$$Diff(v, sa) \Rightarrow Colorable()$$

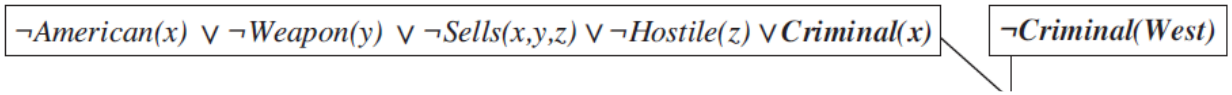
$$Diff(Red, Blue) \quad Diff(Red, Green)$$
$$Diff(Green, Red) \quad Diff(Green, Blue)$$
$$Diff(Blue, Red) \quad Diff(Blue, Green)$$

First-Order Resolution

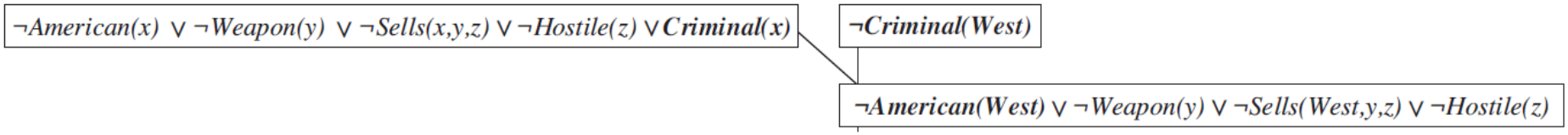
Resolution

$\neg Criminal(West)$

Resolution


¬American(x) ∨ ¬Weapon(y) ∨ ¬Sells(x,y,z) ∨ ¬Hostile(z) ∨ Criminal(x)
¬Criminal(West)

Resolution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¬Criminal(West)
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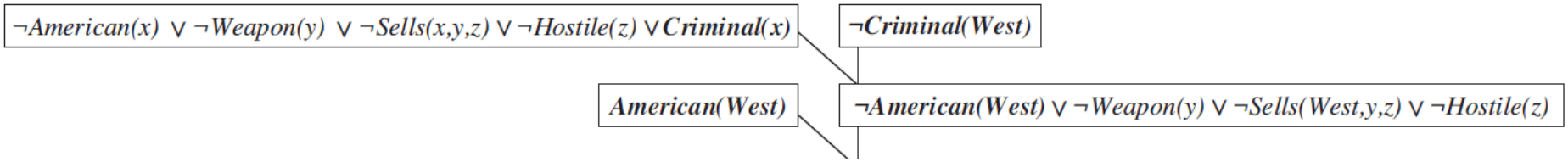
Resolution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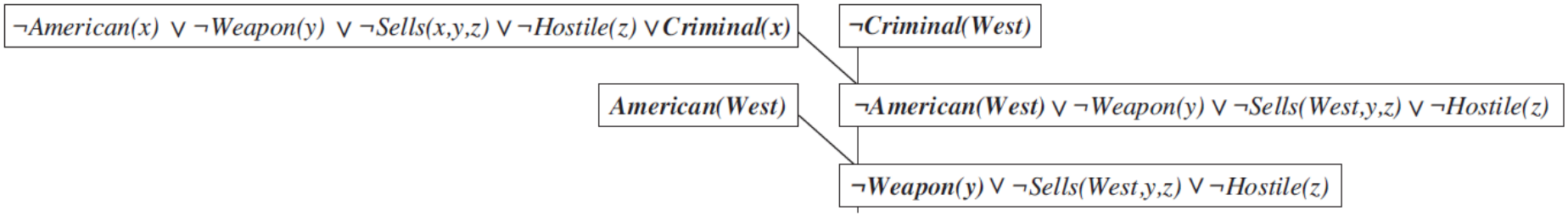
Resolution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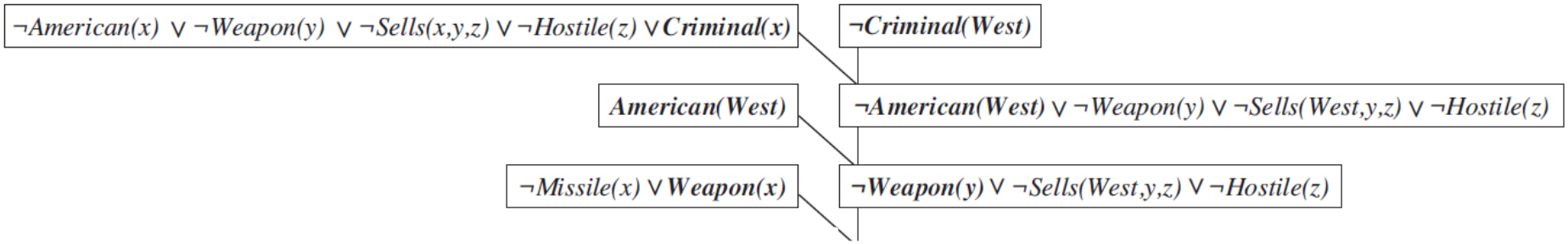
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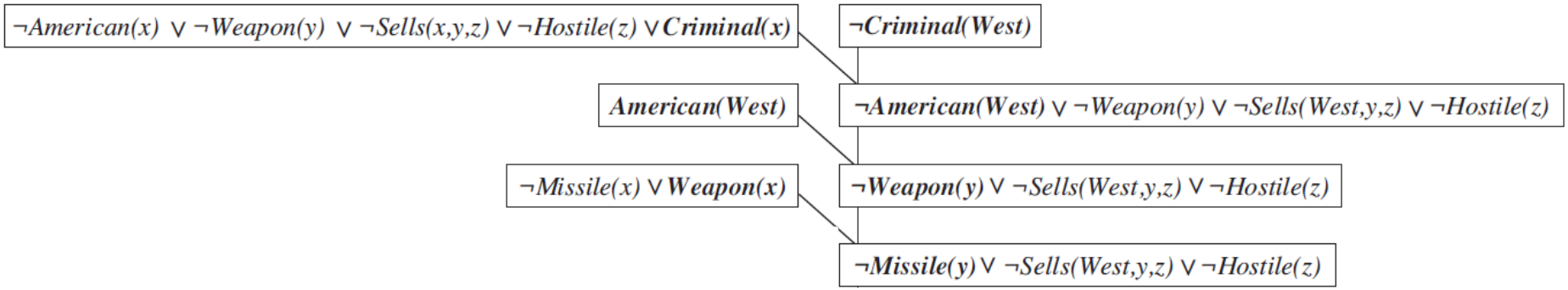
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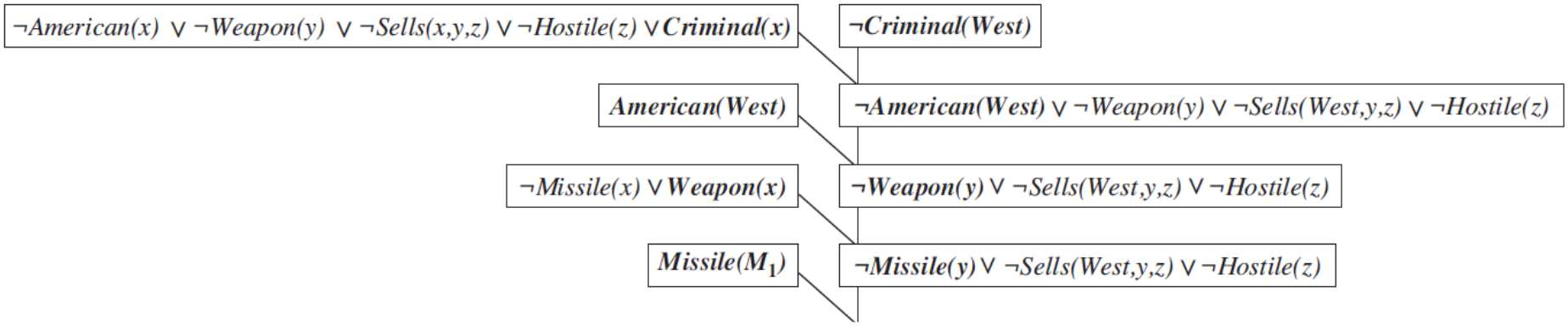
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Missile(M1)
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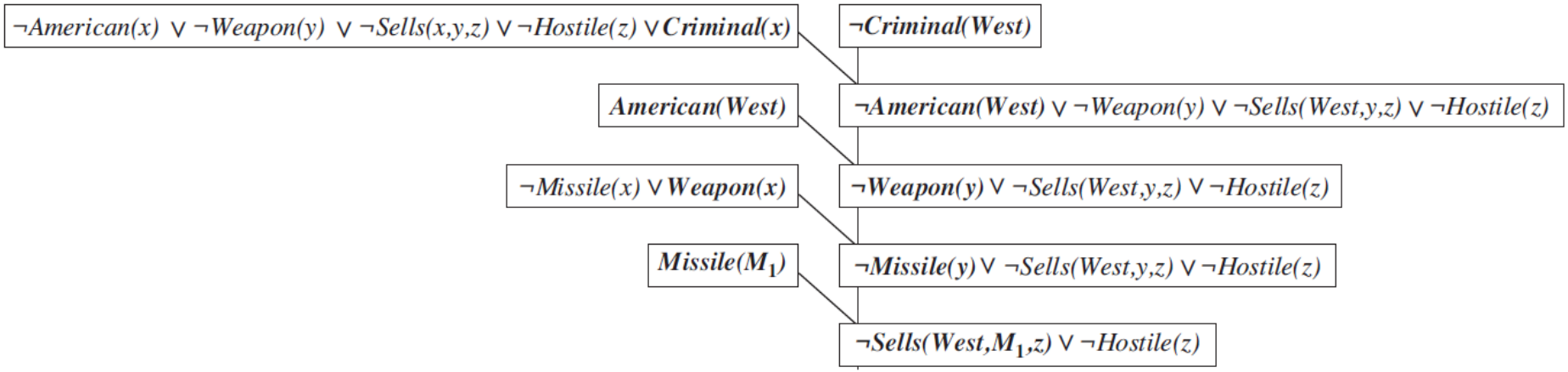
Resolution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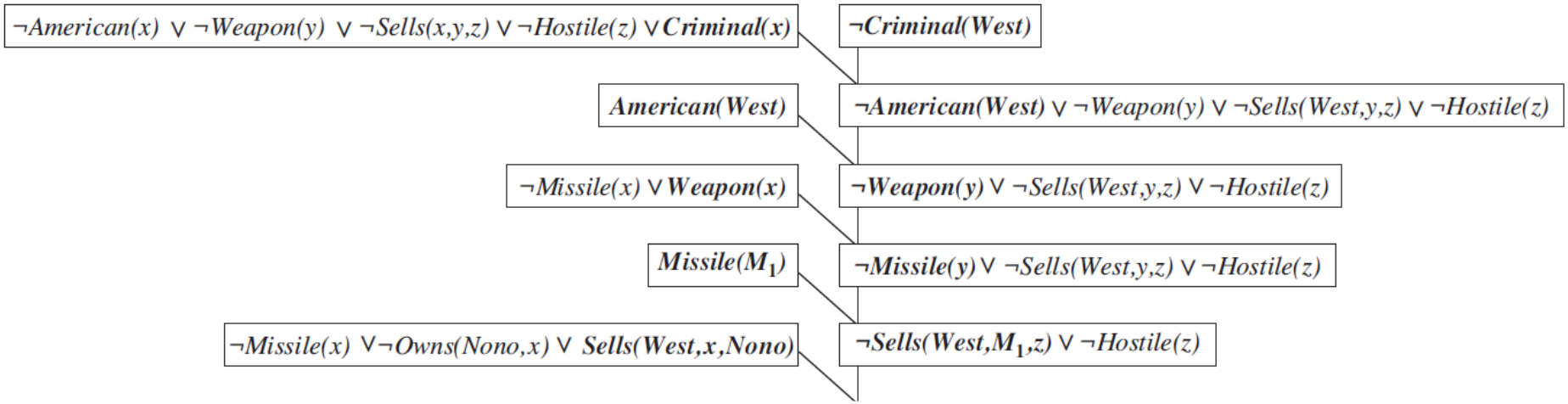
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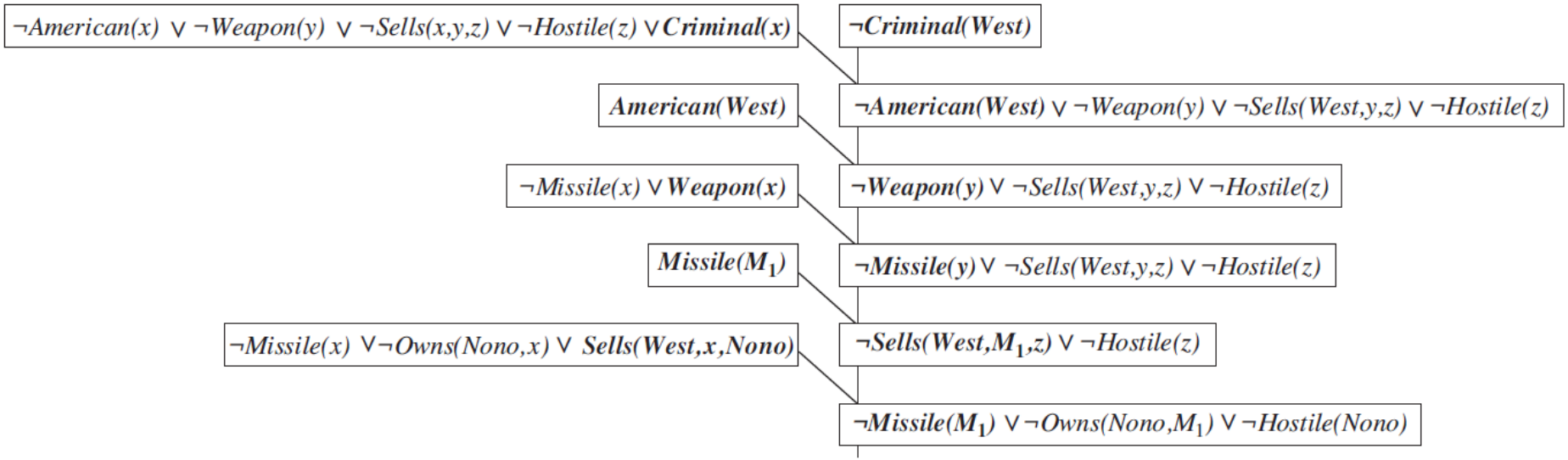
Resolution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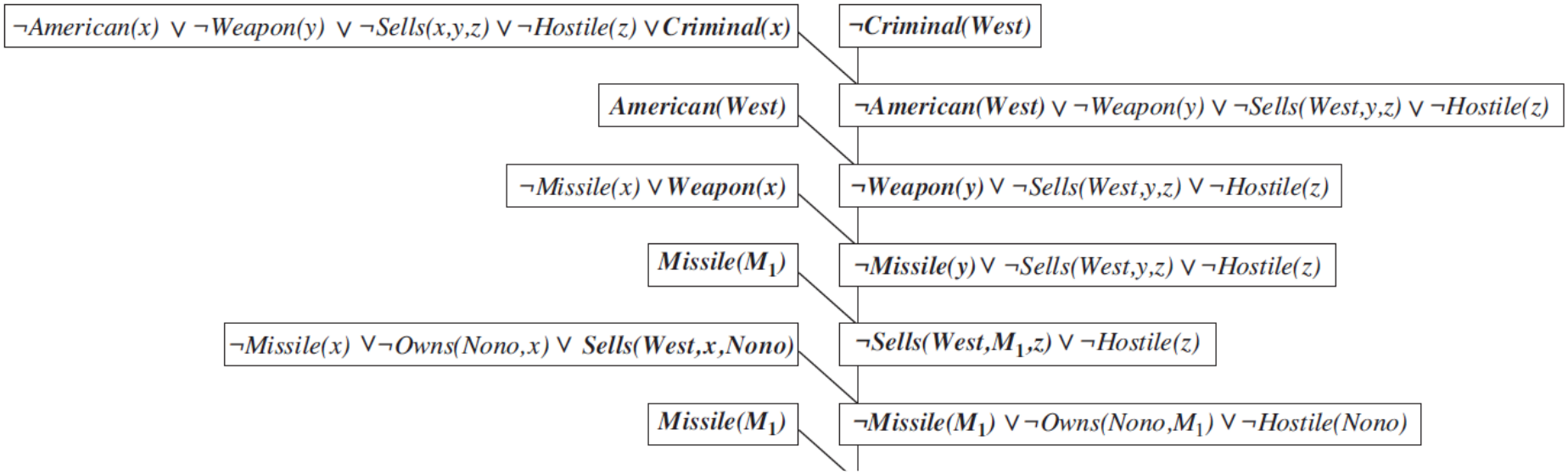
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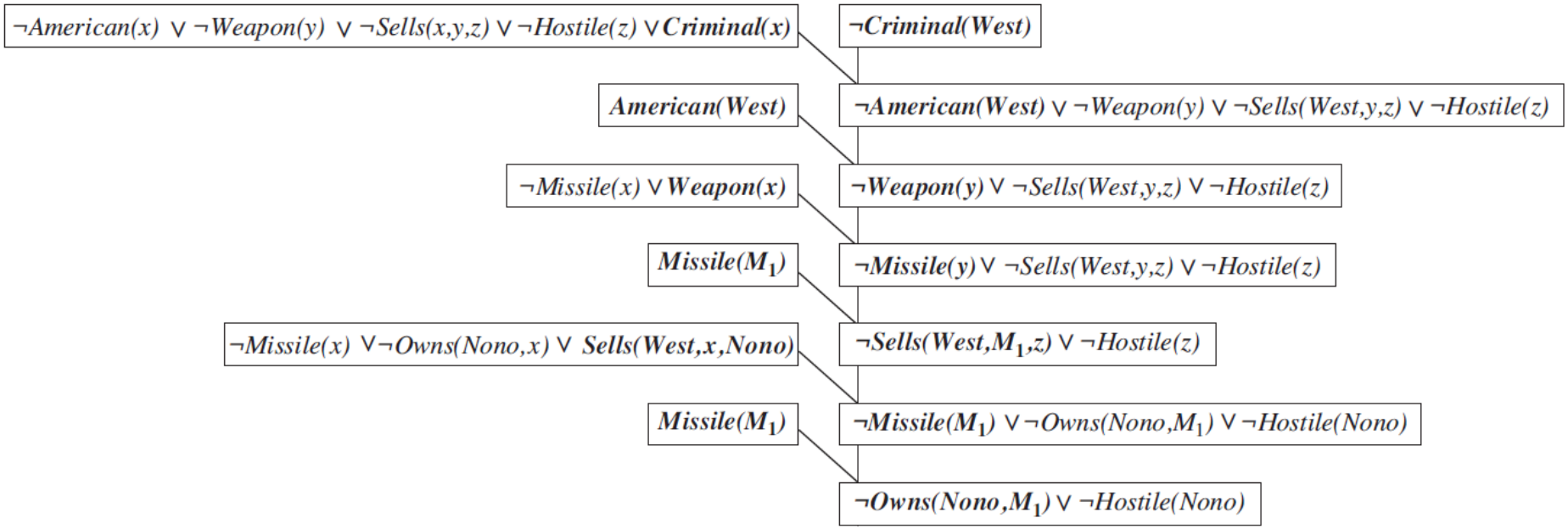
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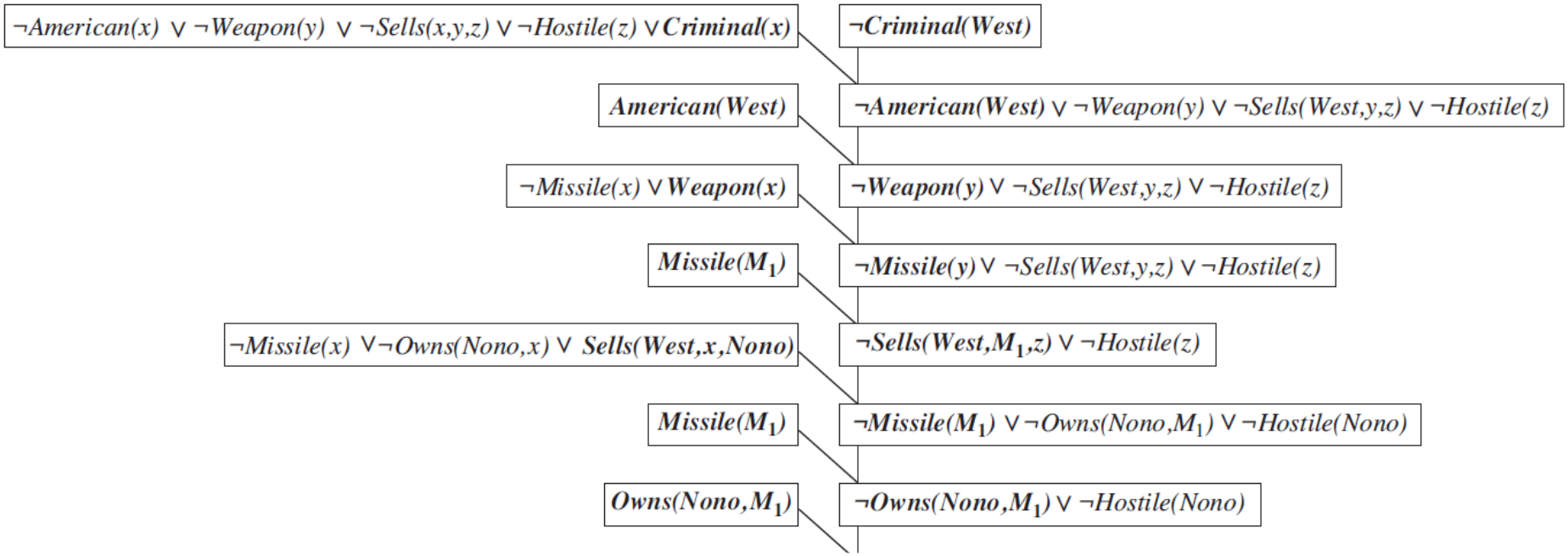
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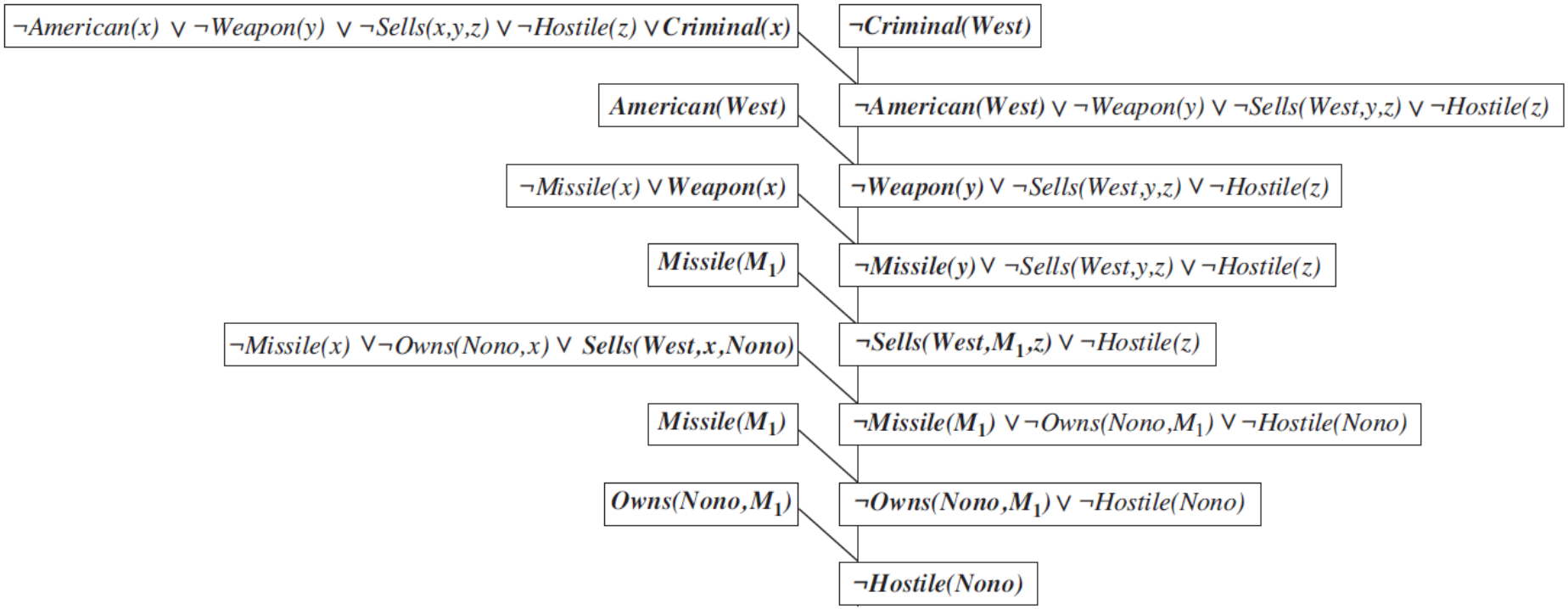
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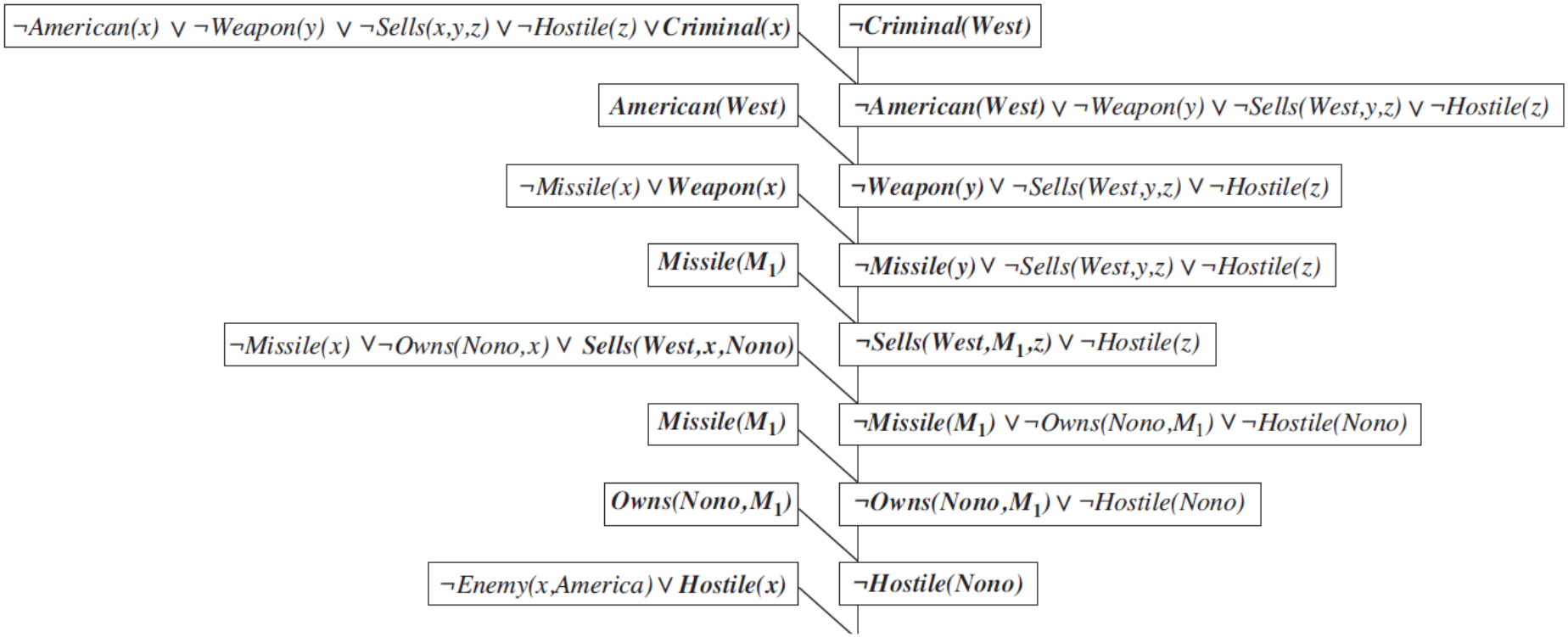
Resolution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Owns(Nono,M1)
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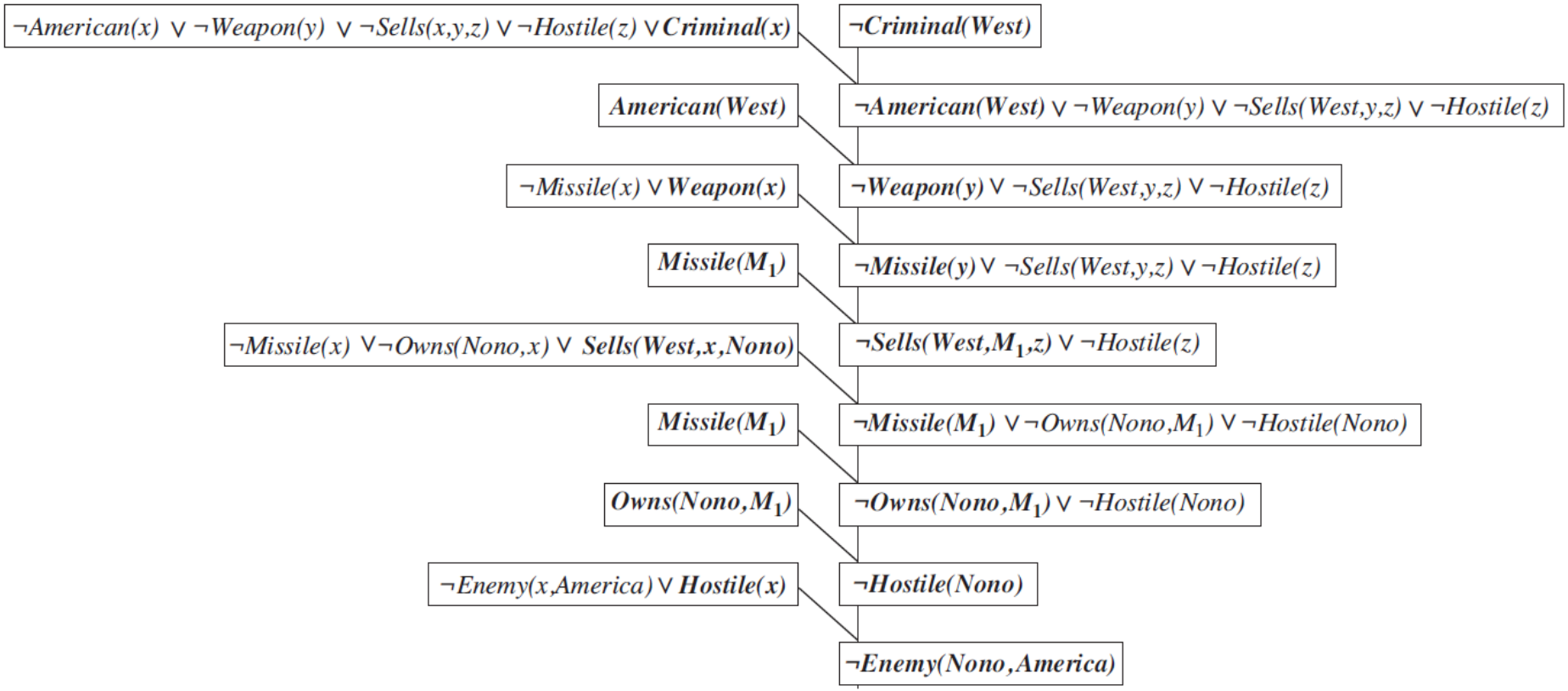
Resolution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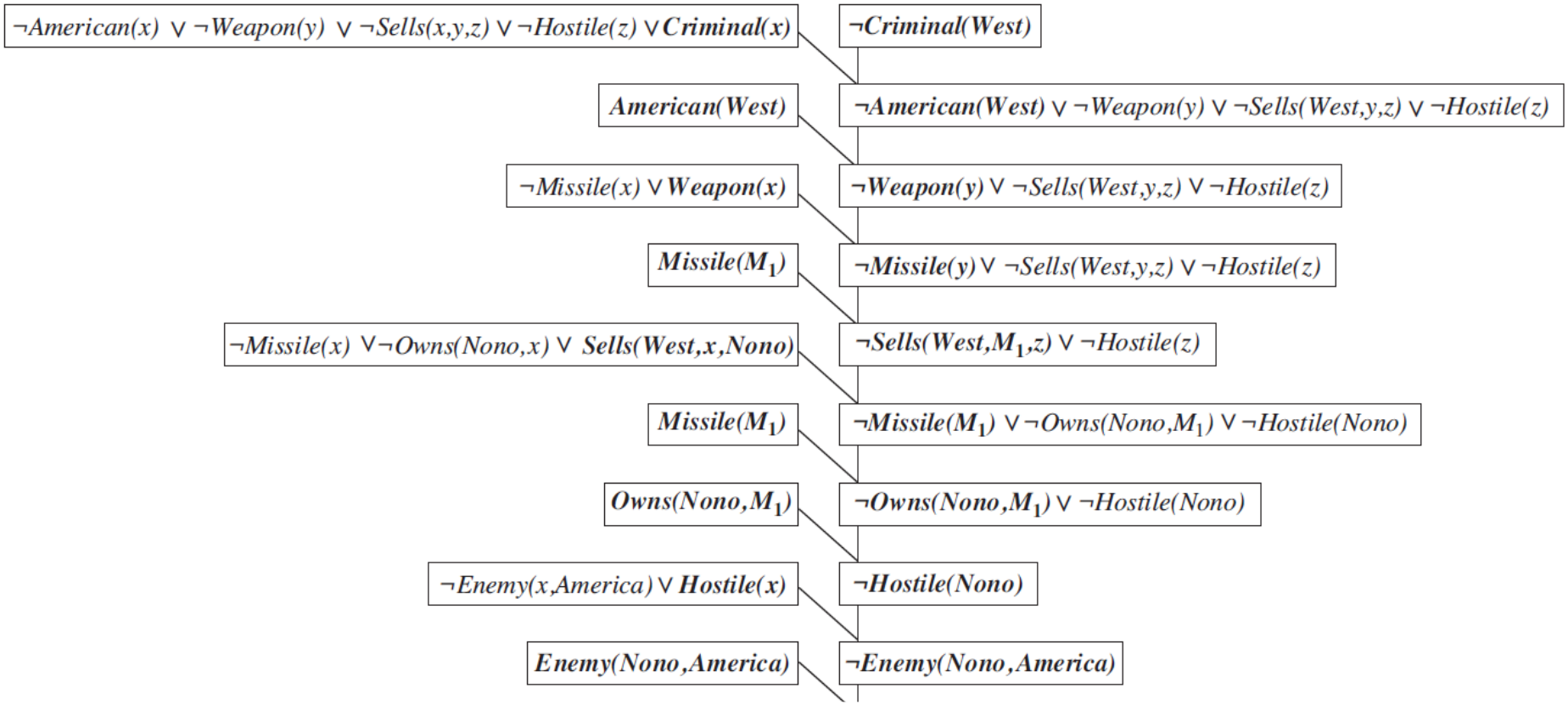
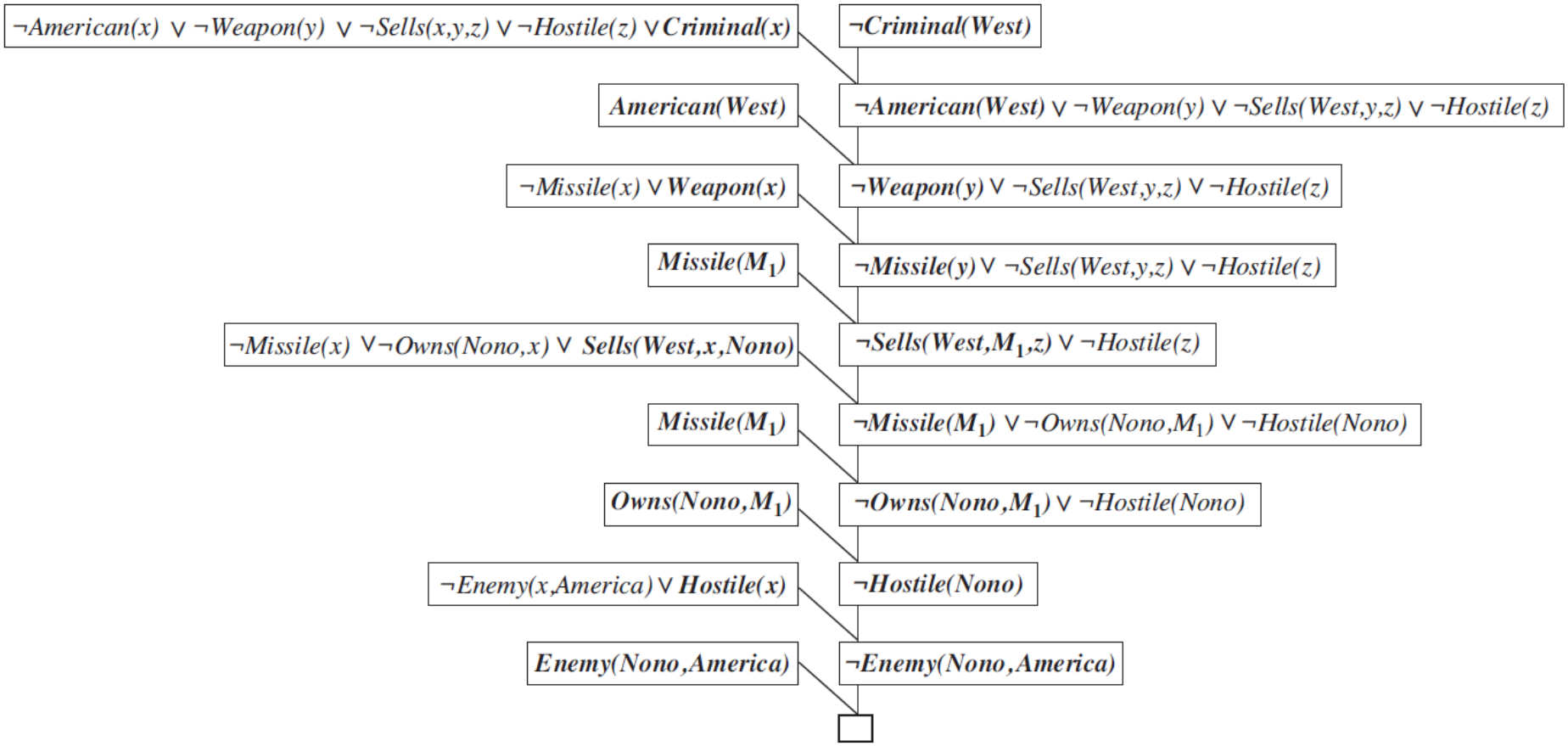
Owns(Nono,M1)
¬Owns(Nono,M1) ∨ ¬Hostile(Nono)
¬Enemy(x,America) ∨ Hostile(x)
¬Hostile(Nono)
¬Enemy(Nono,America)

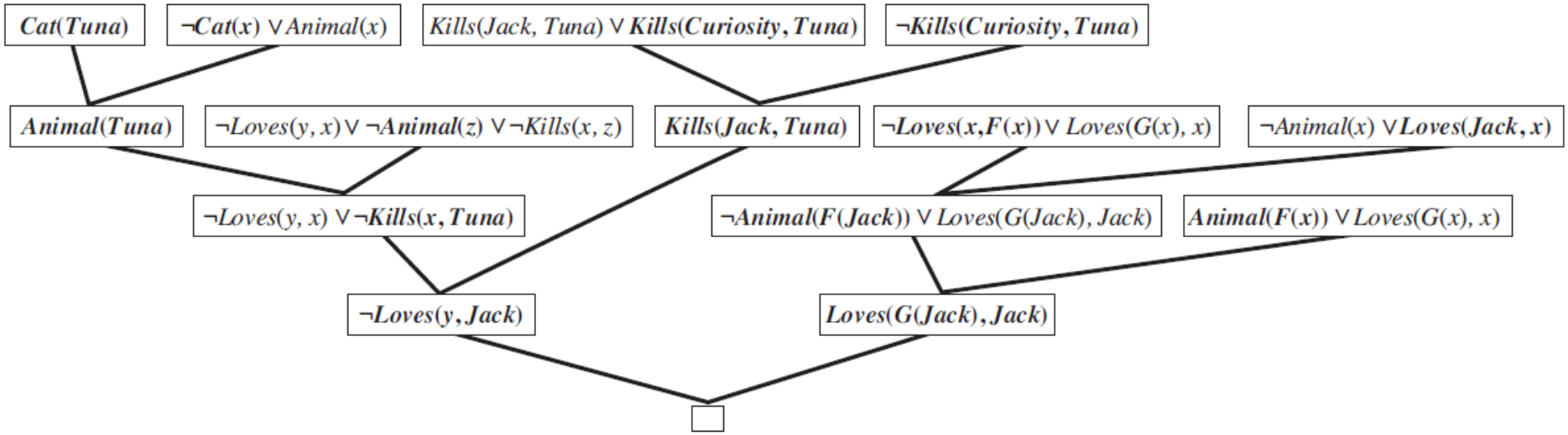
Resolution


¬American(x) ∨ ¬Weapon(y) ∨ ¬Sells(x,y,z) ∨ ¬Hostile(z) ∨ Criminal(x)
¬Criminal(West)
American(West)
¬American(West) ∨ ¬Weapon(y) ∨ ¬Sells(West,y,z) ∨ ¬Hostile(z)
¬Missile(x) ∨ Weapon(x)
¬Weapon(y) ∨ ¬Sells(West,y,z) ∨ ¬Hostile(z)
Missile(M1)
¬Missile(y) ∨ ¬Sells(West,y,z) ∨ ¬Hostile(z)
¬Missile(x) ∨ ¬Owns(Nono,x) ∨ Sells(West,x,Nono)
¬Sells(West,M1,z) ∨ ¬Hostile(z)
Missile(M1)
¬Missile(M1) ∨ ¬Owns(Nono,M1) ∨ ¬Hostile(Nono)
Owns(Nono,M1)
¬Owns(Nono,M1) ∨ ¬Hostile(Nono)
¬Enemy(x,America) ∨ Hostile(x)
¬Hostile(Nono)
Enemy(Nono,America)
¬Enemy(Nono,America)

Resolution


¬American(x) ∨ ¬Weapon(y) ∨ ¬Sells(x,y,z) ∨ ¬Hostile(z) ∨ Criminal(x)
¬Criminal(West)
American(West)
¬American(West) ∨ ¬Weapon(y) ∨ ¬Sells(West,y,z) ∨ ¬Hostile(z)
¬Missile(x) ∨ Weapon(x)
¬Weapon(y) ∨ ¬Sells(West,y,z) ∨ ¬Hostile(z)
Missile(M1)
¬Missile(y) ∨ ¬Sells(West,y,z) ∨ ¬Hostile(z)
¬Missile(x) ∨ ¬Owns(Nono,x) ∨ Sells(West,x,Nono)
¬Sells(West,M1,z) ∨ ¬Hostile(z)
Missile(M1)
¬Missile(M1) ∨ ¬Owns(Nono,M1) ∨ ¬Hostile(Nono)
Owns(Nono,M1)
¬Owns(Nono,M1) ∨ ¬Hostile(Nono)
¬Enemy(x,America) ∨ Hostile(x)
¬Hostile(Nono)
Enemy(Nono,America)
¬Enemy(Nono,America)

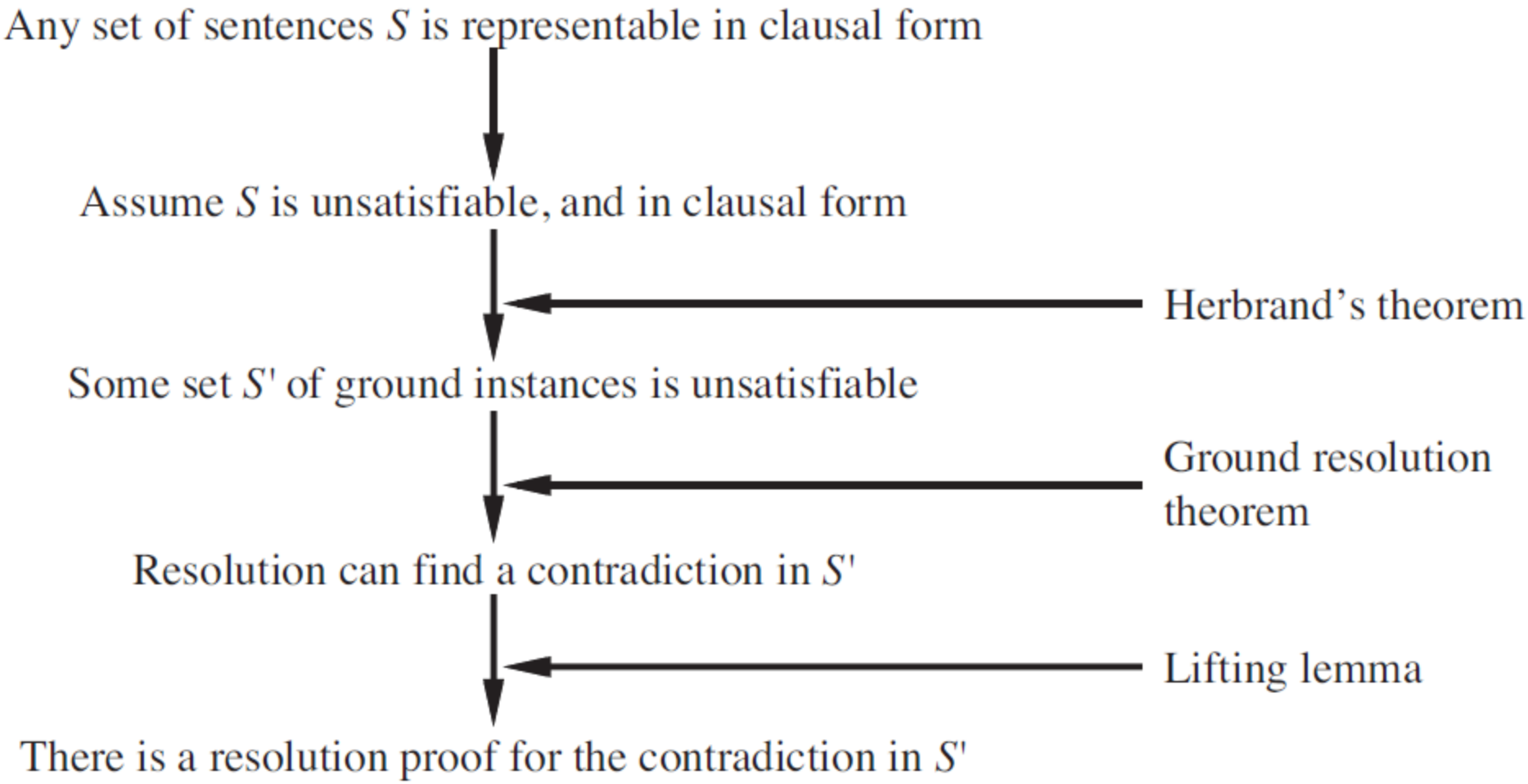
Resolution


Cat(Tuna)
¬Cat(x) ∨ Animal(x)
Kills(Jack, Tuna) ∨ Kills(Curiosity, Tuna)
¬Kills(Curiosity, Tuna)
Animal(Tuna)
¬Loves(y, x) ∨ ¬Animal(z) ∨ ¬Kills(x, z)
Kills(Jack, Tuna)
¬Loves(x, F(x)) ∨ Loves(G(x), x)
¬Animal(x) ∨ Loves(Jack, x)
¬Loves(y, x) ∨ ¬Kills(x, Tuna)
¬Animal(F(Jack)) ∨ Loves(G(Jack), Jack)
Animal(F(x)) ∨ Loves(G(x), x)
¬Loves(y, Jack)
Loves(G(Jack), Jack)
□

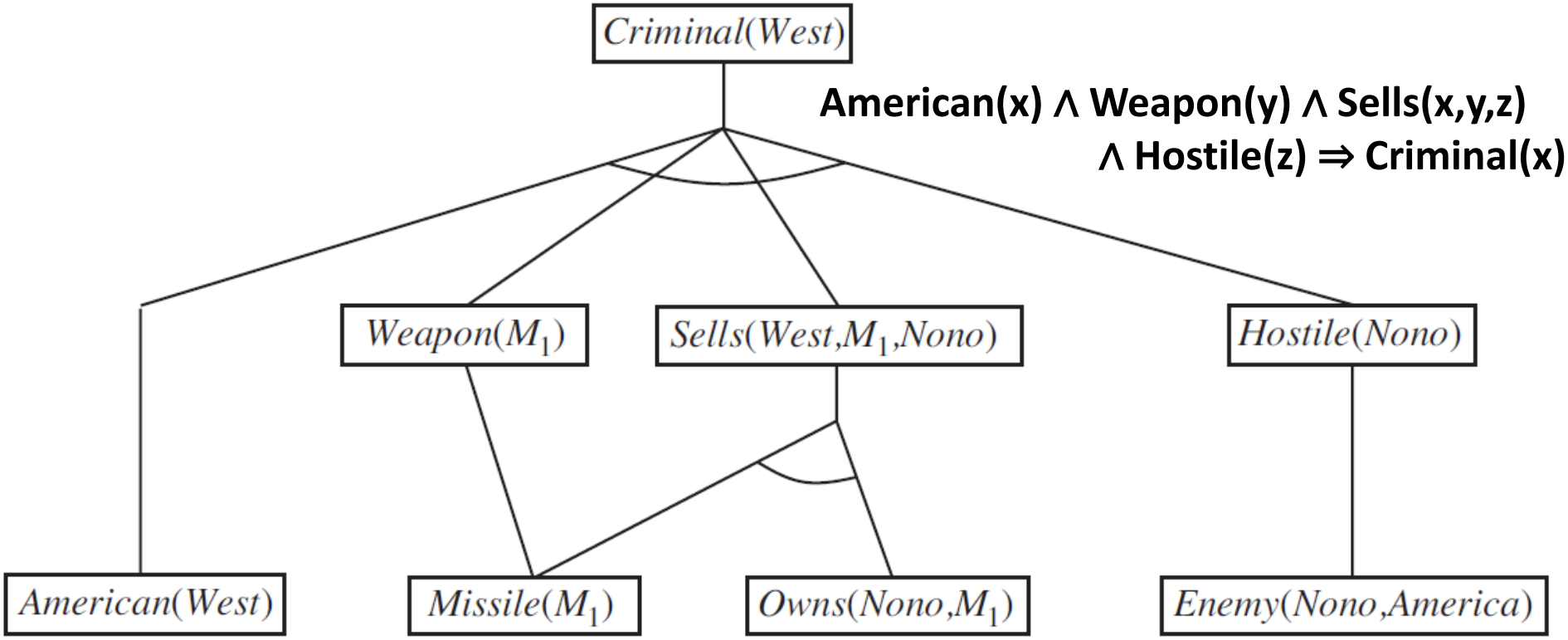
What about ∃?

Conversion to First-Order CNF

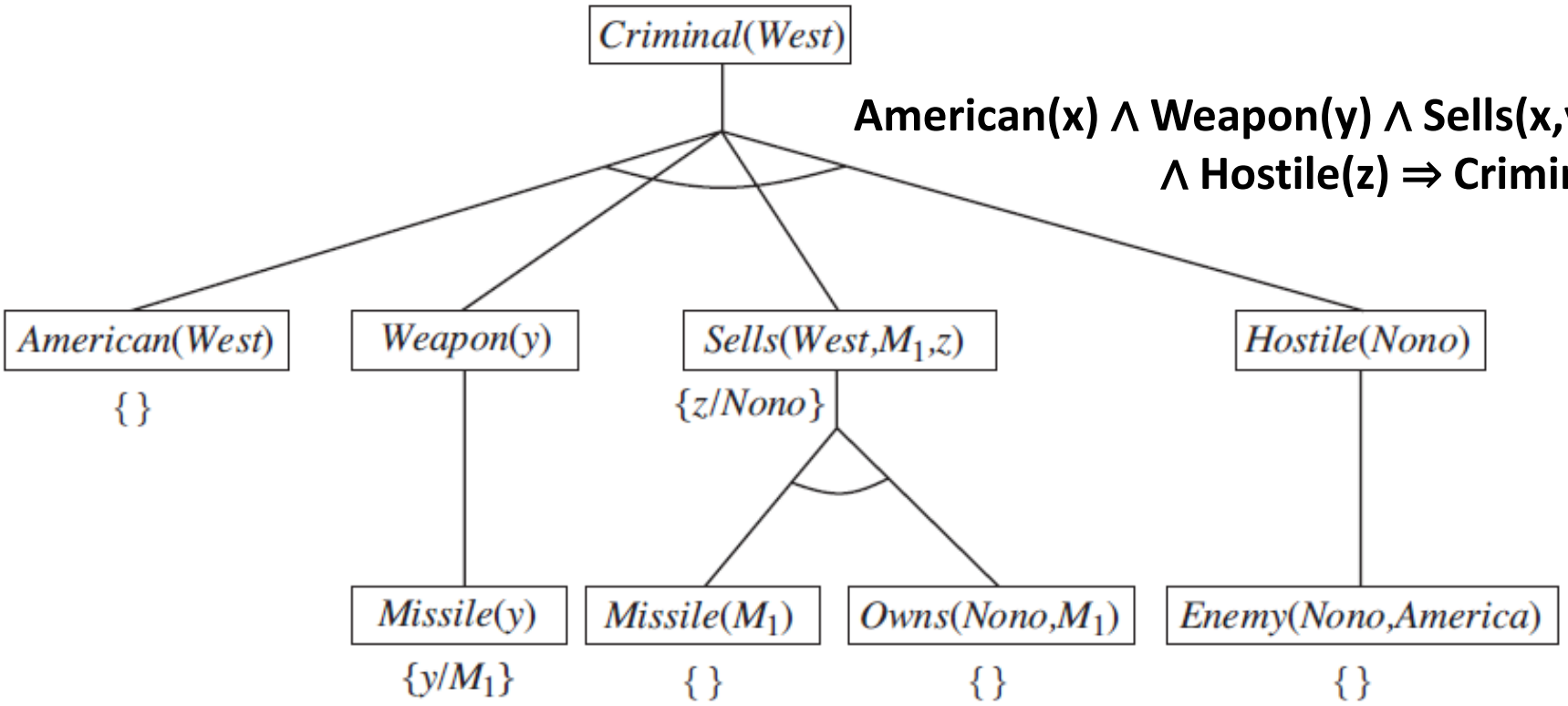
Completeness of Resolution: Intuition



Forward Chaining



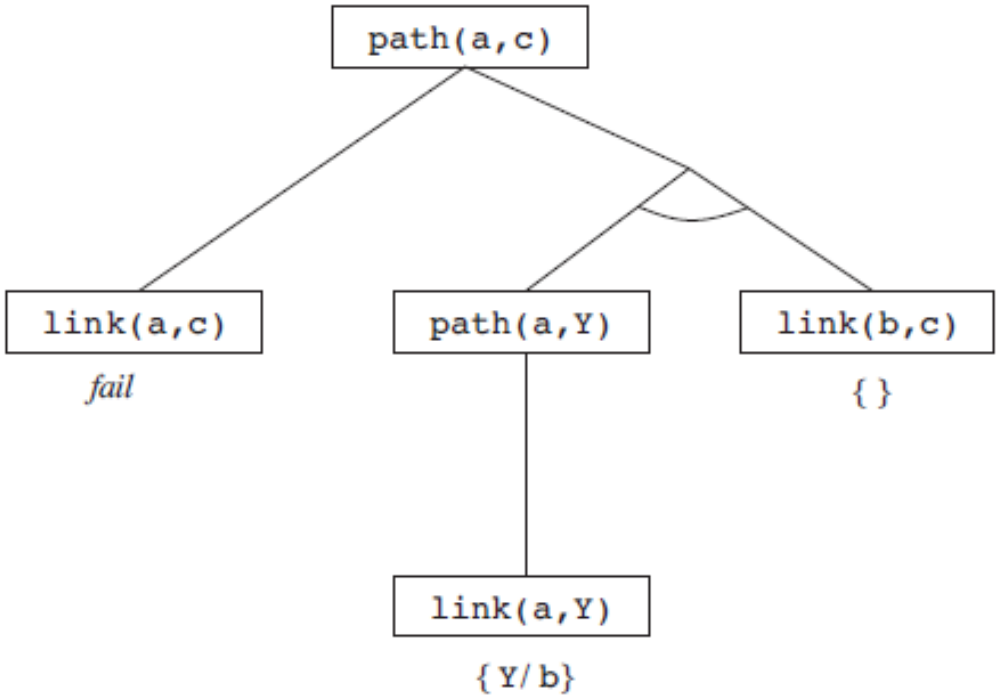
Backward Chaining



Prolog

Link(x,y) ⇒ Path(x,y)
Link(x,z) ∧ Path(z,y) ⇒ Path(x,y)

Link(a,b)
Link(a,d)
Link(b,c)
…

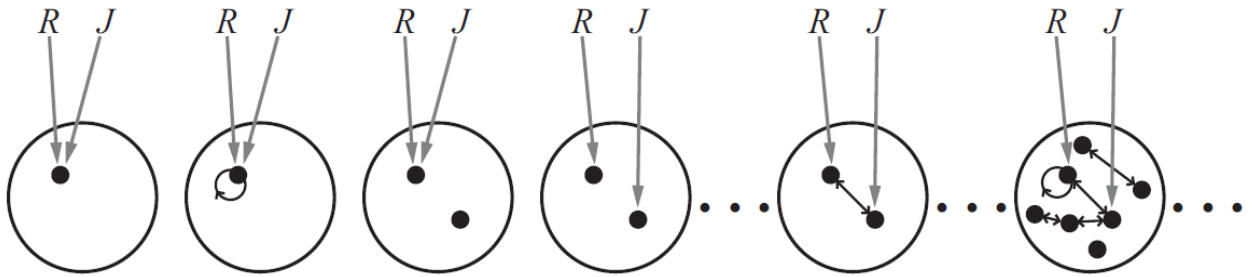


Prolog Programming

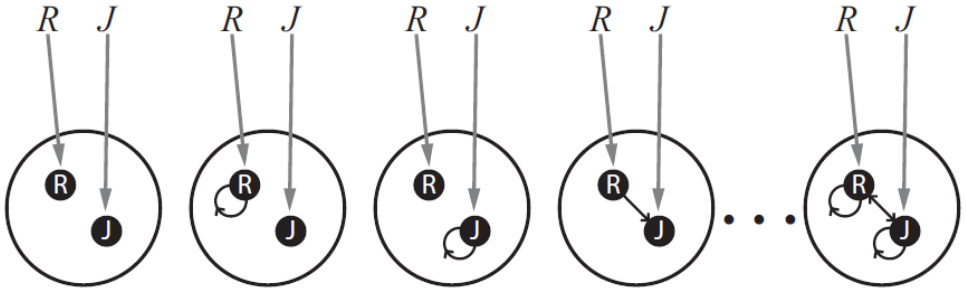
```
quicksort([X|Xs],Ys) :-
  partition(Xs,X,Left,Right),
  quicksort(Left,Ls),
  quicksort(Right,Rs),
  append(Ls,[X|Rs],Ys).
quicksort([],[]).

partition([X|Xs],Y,[X|Ls],Rs) :-
  X <= Y, partition(Xs,Y,Ls,Rs).
partition([X|Xs],Y,Ls,[X|Rs]) :-
  X > Y, partition(Xs,Y,Ls,Rs).
partition([],Y,[],[]).

append([],Ys,Ys).
append([X|Xs],Ys,[X|Zs]) :-
      append(Xs,Ys,Zs).
```

Database Semantics (Prolog+SQL+…)

- Previous semantics:



- Now: unique names assumption



- Also: closed world and domain assumption

Relational Databases

…. are just first-order logic!

SQL or First-order logic

```
SELECT Scientist.X
FROM Scientist, Coauthor
WHERE Scientist.X = Coauthor.X
```

$$Q(x) = \exists y\ Scientist(x) \wedge Coauthor(x,y)$$

Tables are relations

Tuples are positive literals (facts)

Select is exists, join is conjunction, etc.

Why Relational Data?

- Our data is already relational!
  - Companies run relational databases
  - Scientific data is relational:
    - Large Hadron Collider generated 25PB in 2012
    - LSST Telescope will produce 30TB per night

- Big data is big business:
  - Oracle: $7.1BN in sales
  - IBM: $3.2BN in sales
  - Microsoft: $2.6BN in sales

[Gartner'06]